Models of radio galaxies with tangled magnetic fields – II. Numerical simulations and their interpretation

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SUMMARY
This paper describes and discusses numerical simulations of the radio emission from radio galaxies or quasars. In most simulations made hitherto, the emphasis has been on the fluid dynamics of the jet interacting with the ambient circum or intergalactic gas and, where radio images have been shown, they have generally represented the radio emissivity simply as some function of total pressure. In this paper, the fluid dynamics is conventional; the emphasis is on the development of the magnetic field and its consequences for the brightness distribution and polarization of observable radio images. In the models, a randomly tangled magnetic field and an energy spectrum of relativistic electrons are introduced with the jet. The fields and electrons are convected passively with the fluid, and synchrotron losses in the electron energy distribution are followed in detail. We find that the magnetic field strength (and therefore also the radio emissivity) varies widely through the source, in ways not directly related to the pressure distribution. The models predict images with high polarization around the periphery of the lobes, and $E$-vectors normal to the periphery, in agreement with observation, but the polarization over most of the rest of the model source is much higher than is generally observed. Hotspots appear naturally in the model sources, although no diffusive particle acceleration at a ‘working surface’ is assumed.

1 INTRODUCTION

The methods we have used in producing simulations of magnetic field development in model radio galaxies, and for deriving radio images from them, have been described in Matthews & Scheuer (1990, hereafter Paper I). In the present paper we report the results of running simulations with a variety of input parameters, and compare the salient features of the radio images with those observed in real radio galaxies and quasars. We discuss to what extent the general features found in our models may be expected to extend to less idealized models lacking axial symmetry, and what additional physical processes must be important in real radio sources.

First, we recapitulate the physical assumptions of the model. The models computed are axisymmetric; a jet of plasma of density $\rho_p$, Mach number $M$ and radius $r_j$ grid units enters the left-hand side of the computing grid of fluid cells, initially filled with uniform plasma of density $\rho_o$. Reflecting boundary conditions are applied along the left edge of the grid, which corresponds to the natural assumption that the active nucleus also ejects a similar jet in the opposite direction. Relativistic electrons with a power-law energy distribution, and a magnetic field, are both injected with the jet of ‘thermal’ plasma, and in our computations we assume that neither contributes substantially to the total pressure, so that the fluid dynamics are simply those of a non-relativistic gas of adiabatic index 5/3. As we know nothing about the origin of magnetic field in radio galaxies, we keep our assumptions about the injected field as non-commital as possible: we assume that it is a tangled field with no preferred direction. (As described in Paper I, we also assume a Gaussian distribution of injected magnetic field strengths within each volume element when computing synchrotron emissivity after significant synchrotron radiation losses.) We assume that the magnetic field remains frozen into the plasma as the jet interacts with the ambient gas, and is thus distorted by the fluid flow into anisotropic configurations in which the volumes with fields $B$ and $-B$ are equal, but there is a preferred axis. Furthermore, strong distortion can increase the field strength by a large factor.

In real sources, the field distortion and amplification will be limited by magnetic forces resisting the fluid flow, and also by field line reconnection; these are discussed later in this paper. Excessively large shears occurring at a few points in the simulations are prevented from dominating the radio images by introducing synchrotron loss into the models; as the frequency of the synchrotron loss bend varies as $|B|^{-3}$, a very little synchrotron loss suppresses radio emission from
unphysically large fields. At the same time, the large range in the amount of shear in the material that came from the jet is hard to avoid, and forces us to view radio source structure in a new light.

2 CHOICE OF JET SIMULATION PARAMETERS

The morphology of the flow pattern is determined by a number of dimensionless parameters, some of which are much more important than others in the region of parameter space of interest to us.

Mach number $M = v_j/c_j$

We aim to model only highly supersonic jets, and previous work [in particular that of Williams (1985) using the same fluid code] indicates that the form of the flow is not very sensitive to $M$ so long as $M \gg 3$. Accordingly we set $M = 10$ in all the simulations, as in the ‘basic model’ of Williams (1985) and Leahy & Williams (1984).

Pressure contrast $p_j/p_a$

The pressure $p_a$ of the ambient gas is unimportant provided that it is low enough for the bow shock to be a strong shock, for its initial internal energy then contributes a negligible amount to the internal energy of the shocked gas. We therefore set $p_a = p_0$ throughout, but the results will be valid for any low value of ambient pressure.

Density contrast $\eta = \rho_j/\rho_a$

The ratio $\eta$ of jet density $\rho_j$ to ambient density $\rho_a$ is important, since it is the main factor determining the ratio of the gas volume injected by the jet to the cube of the distance advanced by the bow shock, at any given time. Thus, in an axisymmetric model, $\eta$ determines how fat or thin the bow-shock of shocked jet material will be; to obtain reasonably broad lobes such as are observed in most radio galaxies one needs a ‘light’ jet with $\eta \leq 1$.

Resolution $r_j$

Since the jet radius $r_j \Delta x$ is the only physical input parameter with the dimensions of length, it would seem that $r_j$ is not a physical dimensionless parameter but rather one that affects the resolution of the numerical code, through the number of grid cells of size $\Delta x$ across the jet. However, this resolution determines the rate of numerical diffusion of mass and momentum in the simulation, and is therefore perhaps better regarded as another physical parameter which introduces some guess at the amounts of small-scale mixing and dissipation, through fluid turbulence and various brands of plasma turbulence which are not taken account of in the simulation. In any event, we should try to find out how much changes in resolution affect the results, particularly as they do not correspond to well-defined physics. Clearly, increasing the resolution will diminish diffusion, and thus make shear layers thinner, but will also make the shear within those layers stronger and hence tend to produce thinner regions of stronger, more distorted magnetic field. Unfortunately the amount of computing time required rises steeply with resolution, and only one run was made at twice the standard resolution.

Epoch $t$

The stage of development of the flow may be measured in several ways, such as the number of jet radii in $v_j t$. Here we use the dimensionless parameter $t' = (p_j/\rho_j)^{1/2} t/\Delta x$, which is the measure of time used in the computer program, and has the advantage that the distance by which the bow shock penetrates into the ambient medium, which is roughly $v_j \eta^{1/2} t' = My^{1/2} \Delta x t'${\textsuperscript{†}} is almost independent of the density contrast $\eta$ for given $M$ and $t'$.

A jet was introduced by imposing boundary conditions along the left-hand edge of the computing grid ($z=0$) of $v_z = c_j = v_j$ within $r_j$ grid cells of the origin. Filling the grid at $t=0$ was a uniform ambient fluid. Marker particles were inserted into the jet (from $t=0$), at $z=0$, one per grid cell, every eight iterations. Time intervals between injections were recorded since these varied a little over time. Associated with each marker particle were two poloidal vectors $a$ and $b$, initially orthogonal unit vectors, the parameter $1/r_0$ (initial radial position) for keeping track of the toroidal vector $c$, and the synchrotron loss parameter $b$ (defined in equation 3.2 of Paper I). At a specified model time, these data and the fluid parameters of velocity, internal energy and density were stored in files which served as input to the map simulation program.

Radio map simulations of total and polarized flux and of polarization angles were computed; they are displayed as split images, with intensity contours and polarization vectors in the upper half and a grey-scale representation of intensity in the lower half. Normalized flux units are used such that a cube of size $\Delta x = \text{grid cell size emitted 2/3 of a flux unit per hertz per steradian}$ when it was injected. Occasionally trouble arises because the mapping algorithm (see Paper I) deems all the flux from the volume element associated with a marker particle to be concentrated at the marker particle; when the volume element is large and/or contains a particularly strong magnetic field due to shear, a prominent ring appears in the image. We shall mention these cases as they arise.

Fluid data are displayed in a split diagram with pressure contours in the upper half and density contours in the lower half, with marker particles superposed. The marker particles are arranged in ‘tracks’, each track consisting of marker particles which started at the same place. Tracks are numbered with increasing initial radial position, i.e. track 1 starts at $r = 0.5$, track 2 at $r = 1.5$ etc.

Table I shows the parameters of the fluid simulations which were made; as already stated, $M = 10$ and $p_a = p_0$ throughout. From each fluid flow simulation, one can make radio maps for sources suffering various amounts of synchrotron loss by scaling the loss parameter $b$ (which is equivalent to changing the injected rms field strength or the observing frequency), as well as the same source seen from directions making various angles $\theta$ with the jet axis.

The results of these runs will be described in the following
Radio galaxies with tangled magnetic fields – II 625

Table 1. The run numbers are those used by Matthews (1988); not all of the runs are described here. The time units in the last column are used in computing synchrotron losses, as described in the discussion of Fig. 6 in Section 3.1, and at the beginning of Section 3.2.

<table>
<thead>
<tr>
<th>Run name</th>
<th>Density contrast $\eta = \rho_j/\rho_a$</th>
<th>Jet radius $r_j$</th>
<th>Epoch $(\rho_a/\rho_j)^{\Delta x^2/\gamma}$</th>
<th>Unit of $t^{1/2}$</th>
<th>$t/t^{1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A14–t10</td>
<td>0.1</td>
<td>10</td>
<td>10</td>
<td>$10^{1.2}$ s</td>
<td></td>
</tr>
<tr>
<td>A14–t12.5</td>
<td>0.1</td>
<td>10</td>
<td>12.5</td>
<td>$10^{1.2}$ s</td>
<td></td>
</tr>
<tr>
<td>A14–t15</td>
<td>0.1</td>
<td>10</td>
<td>15</td>
<td>$10^{1.3}$ s</td>
<td></td>
</tr>
<tr>
<td>A16–t20</td>
<td>0.1</td>
<td>20</td>
<td>20</td>
<td>$5 \times 10^{1.2}$ s</td>
<td></td>
</tr>
<tr>
<td>A18–t10</td>
<td>0.01</td>
<td>10</td>
<td>10</td>
<td>$10^{1.3}$ s</td>
<td></td>
</tr>
<tr>
<td>A18–t12.5</td>
<td>0.01</td>
<td>10</td>
<td>12.5</td>
<td>$10^{1.3}$ s</td>
<td></td>
</tr>
</tbody>
</table>

section, and more general conclusions will be drawn in Section 4.

3 MODELS AND THEIR RADIO IMAGES

3.1 The basic model: $\eta = 0.1$, $r_j = 10$

The distributions of density and pressure, and of the marker particles, at $t = 10$, are shown in Fig. 1. The jet enters the grid on the left, and terminates in a conical jet-shock at about $z = 100$. Jet and ex-jet material is marked by dots representing marker particles, which flow up the jet, through the jet-shock and round a bend into the backflow. When they reach the left-hand reflective boundary, most of them spread outwards and accumulate in a ‘reflection disc’ around the base of the jet, but a few turn inwards and flow into the cavity between the jet and the backflow.

A roughly parabolic bow shock is travelling into the ambient gas. The highest pressure and density is just downstream of the jet shock in a small region which underlies the radio hotspot. Within the bow shock is shocked ambient gas surrounding a ‘cocoon’ of shocked ex-jet material. The boundary between the two, the ‘contact layer’, is clearly seen in the density contours where the density falls off rapidly as one moves into the cocoon speckled with marker particles. This is not evident in the pressure contours; evidently there is approximate pressure balance between the cocoon and shocked ambient fluid.

When the jet enters the grid, it is overexpanded since, although it is pressure-matched with the undisturbed ambient medium, it is surrounded by higher pressure material within the bow shock. It responds by being squeezed inwards slightly and forming an oblique internal shock which in 3-D is a conical shock. There is another internal shock which is not so clear in Fig. 1. A series of internal shocks is a consistent feature of jets of this type (e.g. Falle & Wilson 1985). The jet shock is not a simple Mach disc, but has a ‘triple-shock’ configuration, pointed out by Norman et al. (1982), of a small Mach disc and two conical shocks, the strongest of which is the boundary of the high-pressure ‘hotspot’ region.

After being decelerated through this shock, ex-jet material passes through its point of highest pressure at the furthest point of the bend it goes through, and is then accelerated down a pressure gradient into the backflow, which does not flow uniformly into the cocoon, but forms a narrow sheath of laminar flow compressed against the contact layer. The backflow thus forms a cylindrical shell rather than a uniformly filled volume.

As mentioned above, some fluid flows into the interior of the cocoon between the jet and the backflow. This region is in the plateau of roughly constant pressure within the bow shock; its density is lower than that of the backflow and its temperature is higher.

Beyond the jet-shock (i.e. at larger $z$) the ambient medium forms a conical intrusion into the jet flow, along the jet axis. This intrusion fluctuates with time, as was seen at later epochs of this model source (e.g. Fig. 7).

Most of the marker particles remain in the jet and cocoon, but a few cross the contact layer and become mixed into ambient material. In Fig. 1, two tracks (1 and 2) have done this, and may be seen as accumulations of particles in the conical intrusion of ambient medium ahead of the jet shock. Looking at the density contours it is clear that these particles have moved from light jet material into more dense ambient medium. There is a possibility that this is due to errors in marker particle trajectories, although the algorithm computes a fairly accurate trajectory given a bilinearly interpolated velocity field. It is not clear whether this is attributable more to small errors in the scheme for moving marker particles or to numerical mass diffusion; the latter possibility reminds us that there could indeed be a corresponding real physical effect due to mixing at the contact layer. Whatever the case, the effect is due to details of the flow model rather than its gross structure, and we shall need
to watch its influence on the magnetic field configuration and simulated radio emission carefully.

Previous simulated radio maps which assumed isotropic radiation had the structure of the pressure distribution, commonly a bright hotspot after the jet shock and a cocoon around the jet though the pressure distribution itself can take on a variety of forms (Smith et al. 1985).

Fig. 2 shows a part of the fluid half-plane with poloidal distortion vectors $a$ and $b$ attached to their associated marker particles. These indicate the magnetic field configuration. The long lines across the diagram are vectors within the cavity which have been greatly stretched. Tracks 1–9 in the jet have vectors which are largely undistorted from their initial configuration, except for some compression in the radial direction.

In the jet, most of the shear occurs at the surface, mainly

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**Figure 3.** Graph of energy densities (magnetic, fluid internal, fluid kinetic) at marker particles, plotted against distance (in grid cell widths) along a track of marker particles (track no. 4) in jet simulation A14–t10 (cf. Fig. 1). Notation: $E_{\text{fluid},i}$ = energy density (magnetic, internal, kinetic) at marker particle. $E_i^0$ = initial energy density at injection position in base of jet. Ratios plotted are – solid line: $(E_{\text{mag}}/E_0^0)$, dashed line: $(E_{\text{int}}/E_0^0)$ and dotted line: $(E_{\text{kin}}/E_0^0)$.
Radio galaxies with tangled magnetic fields – II

1990 MNRAS. 242. 623 M

Affecting track 10. Compression occurs downstream of the jet shock, especially in a direction normal to the flow lines. In the backflow, the poloidal vectors are aligned with the flow direction, stretched in this direction and compressed radially.

Toroidal field is amplified as the jet diverges when it passes through the jet shock into the backflow, since a loop with radius \( r_0 \) centred on the axis is amplified by a factor of \( r/r_0 \) by the time its radius is \( r \) in the backflow. This effect is discussed in more detail in Section 4.1; it is strongest for loops initially close to the jet axis, so that the backflow has embedded in it a toroidal field which is strongest at the surface.

Powerful shear occurs in mixing layers, notably in the contact layer between cocoon and ambient fluid, and in the cocoon interior, where regions of very different velocities are adjacent to each other. The result is very strong amplification of poloidal field in these mixing regions, especially where marker particles have crossed the contact layer, and where they have travelled from the end of the backflow at the base of the jet into the large eddy interior to the cocoon. However, as the latter region has a very low density of fast particles, it has less influence on the radio emission. The most important feature of the magnetic field configuration is a cylindrical sheath, compressed perpendicular to flow lines and amplified toroidally and along flow lines. Although the toroidal vector \( \mathbf{E} \) is not displayed in Fig. 2, it is easy to visualize: it is normal to \( \mathbf{a} \) and \( \mathbf{b} \), with a length \( r/r_0 \).

Fig. 3 is a graph of fluid internal energy density, kinetic energy density and magnetic energy density following a track of marker particles (track 4). This is almost, but not quite, a flow line. A track is really a line ‘frozen-in’ to the fluid, but serves as a tracer through the flow. Thus Fig. 3 shows how the energies vary, more or less along a flow line. Compression occurs in a ‘knot’, where the first internal shock is focused, and in the jet shock. The pressure continues to rise as the track goes through the bend and then falls off as it enters the backflow. Magnetic field gains proportionally more energy than the thermal fluid. Magnetic energy density is amplified by a factor of \( \approx 150 \) in the high-pressure region, and retains an amplification of \( \approx 20 \) in the backflow, about five times that of the thermal fluid internal energy density. (Note that Fig. 3 shows the variation of these energy densities along a track, not absolute values; in particular, as the magnetic field is assumed to be dynamically unimportant throughout, its energy density is simply proportional to the magnetic energy density injected into the jet, and the latter must be kept low enough to maintain the validity of passive correction throughout the source.)

The radio image for this model, with the jet axis 15° out of the plane of the sky, is shown in Fig. 4. This looks very reasonable at first sight, with a dominant hotspot and a low-brightness lobe, but an image made for \( \theta = 0 \) (i.e. looking down the jet axis) showed a ring-shaped hotspot. Compression at the jet-shock is the prerequisite for producing a hotspot, but the greatest compression occurs not at the jet shock itself, but a little downstream, where the flow begins to turn into the backflow. In Fig. 4, however, it is clear that the dominant contribution comes not from the region of greatest compression, but from the region (ring-shaped in 3D) of large computed field where tracer particles have strayed into the ambient gas. That impression is confirmed on making a radio image omitting tracks 1, 2 and 10, shown in Fig. 5. This has the same contour levels as Fig. 4, relative to maximum brightness, but shows much more extended emission, in which the hotspot is associated with the ring-shaped region of highest compression.

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**Figure 4.** Simulated radio map (without synchrotron loss) of part of the source A14-10 shown in Fig. 1 (the region shown in Fig. 2), viewed at 75° to the jet axis. Coordinates are in units of grid cell width; the origin does not coincide with that in Fig. 1. Total intensity (maximum = 1.21 \( \times \) 10^{21}) is represented as a grey-scale below the projection of the jet axis (black = maximum, white = 0) and as contours above: levels are at (0.01, 0.1, 0.2, …, 0.9) \( \times \) (maximum). Polarization E-vectors have length proportional to percentage polarization (maximum within the lowest contour = 70.09 per cent); 100 per cent is represented by the bar in the top left-hand corner.

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**Figure 5.** As in Fig. 4, except that contributions to the emission associated with marker particle tracks 1, 2 and 10 have been omitted; maximum total intensity = 5.24 \( \times \) 10^{21}, maximum percentage polarization within lowest contour = 73.93 per cent.

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Radio images incorporating various degrees of synchrotron loss are shown in Fig. 6. For the purposes of computing synchrotron losses, the unit of time, $\Delta t \equiv (p_j/\rho_j)^{-1/2}$, in which $\Delta t$ is measured, was set equal to a particular time, in most cases $10^{13}$ s; this allows the injected rms field strength $B_j$ and the observing frequency $\nu$ to be specified in conventional units rather than combining them into a less illuminating dimensionless parameter. As expected, Fig. 6(a) shows little evidence of synchrotron loss, and is similar to Fig. 4. Fig. 6(b) and (c), corresponding to an injected field of 1 nT, bear a striking resemblance to Fig. 5. Synchrotron loss is much less sensitive to frequency than to magnetic field strength, so it is not surprising that, while the model shows some spectral steepening in the backflow, there is little change in the appearance of the hotspot between 1 and 10 GHz. On the other hand, increasing the field injected into the jet by a further factor of 10 (or, equivalently, observing at 1000 GHz), as in Fig. 6(d), destroys emission from plasma that has passed through the high-field regions in the hotspot, and allows the very much weaker emission from the jet itself to stand out clearly.

As expected from the stretching illustrated in Fig. 2, the polarization ‘vectors’ (E-field) are normal to the periphery of the image, as is usually observed (e.g. Leahy, Pooley & Riley 1986). Even in the interior of the image of the lobe, where the toroidal component of the field contributes, it appears that the poloidal shear exceeds the toroidal stretching, but the fractional polarization is not as great; however, this feature is clearly dependent on the details of the model. The robust features of the polarization are

(i) the $E$-vectors are approximately normal to the periphery, as observed, and
(ii) the percentage polarization over most of the source is much greater than percentages typical of real radio galaxies.

These features will be found in all the simulations in this paper, and will be discussed in Section 4.

Next we consider the time dependence of the model, but
Radio galaxies with tangled magnetic fields – II 629

This is not new; other authors (e.g. Smith et al. 1985; Norman 1989) have found that the flow does not approach a steady state. Instead, the flow pattern fluctuates in a manner depending on the position of the terminal jet shock relative to the pattern of conical shocks within the jet and on the shedding of vortices behind that shock.

The radio image with no synchrotron loss, shown in Fig. 9(a), is now dominated by a single narrow ring within the lobe, where the magnetic field has been sheared particularly strongly, and the flux has been concentrated by the mapping algorithm as noted near the end of Section 2. As Fig. 9 shows, synchrotron loss removes this ring even when the rms injected field is only 0.1 nT, but the image is then changed little by a tenfold increase in field from Fig. 9(b)–(c).

3.2 Increasing the resolution

Next we double the resolution, by increasing $r_j$ the number of grid cells in a jet radius, from 10 to 20. In order to arrive at the stage of evolution depicted at $r_j = 10$ for the basic model, we now have to run the simulation to $r_j = 20$, and therefore, for the same amount of synchrotron loss at given field strength, we must make unit $r_j$ correspond to only $5 \times 10^{12}$ s.

The state of the fluid flow at $r_j = 20$ is shown in Fig. 10 and the corresponding poloidal distortion vectors are shown in Fig. 11. Fig. 10 should be directly comparable with Fig. 1, and indeed the only obvious changes are that the shocks within the jet have become better defined and the large-scale wave on the surface of the cocoon has become more distinct. Close examination of the computer output shows that in this case no marker particles have crossed from the jet material into the ambient gas, but (see inset in Fig. 10) the surface layers of the backflow show a folding pattern, and indeed, near this surface, there are clear velocity oscillations which are the immediate cause of the folding behaviour. Thus we get very strong amplification of the field at intervals along the contact layer, and corresponding to these there are sharp rings of radio emission in radio images made without any synchrotron loss – one of which is shown in Fig. 12. The

Figure 7. Fluid data and marker particles in jet simulation A14–t15 (a later stage of the jet shown in Fig. 1); Mach number $M = v/c_s = 10$, density contrast $\rho_0/\rho_\infty = 0.1$, jet radius $r_j = 10$, at age $r_j = 15$ (ambient medium has density $\rho_\infty$, pressure $p_\infty$). The jet enters the grid on the left; marker particles are in jet and ex-jet material only. Contours: pressure above axis (maximum $= 85.4 p_\infty$), density below (maximum $= 4.12 \rho_\infty$). Levels are at $1/2, 1/4, \ldots, (1/2)^{10}$ (maximum). Coordinates are in units of grid cell width.

briefly, as it shows little that is essentially new; details may be found in Matthews (1988). The state of the fluid flow at $r_j = 15$ is shown in Fig. 7; the corresponding distortion of fluid elements is shown in Fig. 8, for a region near the hotspot. The most striking change is that the conical intrusion of ambient gas ahead of the jet shock has become much smaller.

Figure 8. Poloidal distortion vectors from a region within the flow plane in jet simulation A14–t15 (cf. Fig. 7, $M = 10$, $\rho_0/\rho_\infty = 0.1$, $r_j = 10$, $r_j = 15$). The vectors indicate the distortion by the flow of unit cubes (and hence the 'frozen-in' magnetic field) starting at the base of the jet. Coordinates are in units of grid cell width $\Delta x$. Unit vectors have length $\Delta x/2$. 

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outer ring in this image is the result of this process, whereas the fainter inner rings are there largely because of the artificial concentration of flux mentioned in Section 2. Again, a very little synchrotron loss removes the rings, as shown in Fig. 13, and generates images similar to those of Fig. 6(b) and (c). Nevertheless, it is noticeable that changes which appear quite minor in the flow pattern give rise to much more obvious changes in the radio images, a point to which we shall return in the next section.

3.3 Increasing the density contrast: $\eta = 0.01$

A run with a jet density only 0.01 of the ambient density shows most of the features of the basic model plus several of the changes which are to be expected or are at least reasonable. The most striking change is the increase in the width of the cocoon. At $\eta = 10$ (not illustrated here) there is a deep conical intrusion of shocked ambient gas beyond the jet shock, and in the absence of synchrotron loss the radio emission is concentrated into a small hotspot. At $\eta = 12.5$ the conical intrusion has almost disappeared; Fig. 14 shows the fluid flow at that stage. The cocoon is wider than that of the $\eta = 0.1$ run; also, there are more marker particles, and a greater accumulation of them in the bulge near $z = 0$, reflecting the greater volume of low-density gas that has to be injected to excavate a cocoon of the same length. The instabilities in the contact layer now show clearly in the density distribution, not only in the clumping of marker particles.

An interesting consequence of the broader lobe appears in the radio images, such as Fig. 15. The $E$-vectors around the periphery of the image are still perpendicular to the edge. In the interior of the lobe, however, the polarization vanishes and then the $E$-vectors become parallel to the $z$-axis, except near the axis, where the contribution of the jet itself is important. This behaviour reflects the fact that the poloidal...
shear is less strong in the interior of the cocoon, and the
toroidal field component predominates there; this
contribution to the emission was smaller in the narrower lobes of the
$\eta = 0.1$ models, so that there the polarization did not change

4 DISCUSSION AND CONCLUSIONS

4.1 Generalization

We regard it as our first task in the whole of Section 4 to
discuss which of the features of our numerical models are
crucially dependent on our various assumptions and on the
code for fluid flow simulation, and which are special cases of
very general physical processes which are likely to occur in

Figure 11. Poloidal distortion vectors from a region within the flow plane in jet simulation A16–120 (cf. Fig. 10, $M=10$, $\rho/\rho_0 = 0.1$, $r_0 = 20$, $r_1=20$). The vectors indicate the distortion by the flow of unit cubes (and hence the 'frozen-in' magnetic field) starting at the base of the jet. Coordinates are in units of grid cell width $\Delta x$. Unit vectors have length $\Delta x$. 
real sources. This discussion is necessary before we attempt any comparison with observation, for it is difficult (perhaps impossible) to find examples of radio galaxies (or quasars) with hotspots that are axisymmetric even in first approximation, while our simulations have shown that very similar flows can generate a wide variety of shapes, sizes and intensities of hotspots.

We begin by discussing the magnetic field configuration. An example which lends itself to a particularly clear discussion is the toroidal stretching and amplification of magnetic flux, already explained in Section 3.1. Fluid elements initially very close to the jet axis become widely separated in a ring on the surface of the cocoon, and the stretching factor (final radius/initial radius) becomes arbitrarily large for fluid elements initially arbitrarily close to the axis. Clearly this phenomenon is not a result of axial symmetry; it will occur wherever there is a place (which in a stationary flow would be a stagnation point) at which a fluid flow separates. It is a topological feature, which should occur in any source unless the whole of the jet material is deflected into one more or less curved trajectory through a succession of oblique shocks. There is also a good reason why the resulting growth of magnetic field has not been noted in other simulations: when the field inserted into an axisymmetric simulation has a toroidal or helical pattern (Norman 1989; Kössl 1989), the toroidal field component (which is the component that is amplified by the stretching) goes through a zero on the jet axis, and grows linearly (if it is non-zero) with radius near the jet axis. Thus, ordered initial field configurations concentric with the axis of the flow may be expected to lead to a more or less uniform toroidal field strength when the jet is turned inside-out in the backflow.

In addition to the toroidal field stretching, our models all exhibit spectacular poloidal shear, for several reasons. Soon after passing through the shock terminating the jet, the jet material follows a strongly curved trajectory leading into the backflow. Even with uniform speed, the material initially on the outside of the jet will overtake the material starting nearer the axis, simply because it runs on an ‘inside track’ round the bend. Complications arise because the obliquity of the shock (and hence the speed of the shocked plasma) varies with radius (and time). This shear is in the same sense as that imposed by the adhesion between jet material in the backflow and the shocked ambient gas. The latter is modified according to the effective viscosity of the plasma, which is controlled by the numerical diffusion (and hence by the resolution) of the numerical code, although as we have seen,
increasing the resolution does not reduce the shear, but merely causes stronger shear in a thinner shear layer.

A further candidate region for large field amplification is the neighbourhood of the slip line (or rather, in the axisymmetric configuration beyond a Mach disc, a slip cone) that is formed downstream from the junction of two shock fronts (e.g. Landau & Lifshitz 1959, section 102), but we cannot claim to have distinguished this in our simulations.

In view of the preceding discussion, we expect the main features of the magnetic field configuration to be insensitive to the details of the fluid model, the numerical code and even the form of the injected magnetic field; except for highly idealized cases, a fairly well-ordered initial field should still lead to a field configuration in the lobes that is strongest near the surface of the lobes, and predominantly tangential there.

Unlike the toroidal stretching, the poloidal shear near the contact discontinuity (or mixing layer?) depends on our assumption of continuity of tangential velocity rather than slip between the two fluids. If we were able to increase the numerical resolution indefinitely, we should expect the shear layer to become thinner, with correspondingly greater shear and amplification of the field, until a stage is reached where magnetic forces become important, and begin to involve a thicker layer of plasma in the shear again. Is there any way in which the shear layer could remain very thin, without amplifying the field so much as to make it dynamically important? One possibility is the fast reconnection of field lines, which may be expected to limit the strength of small-scale field structures. As we do not know the size spectrum of the magnetic fields (and probably too little of the physics of reconnection) we have to leave this possibility an open question.
4.2 Polarization

Qualitatively the models agree very well with the observation that the electric vectors are normal to the edge of the image, and that the polarization is strongest around the edge. Conversely, the clearest disagreement between model and observation comes in the fractional polarization. As illustrated by Fig. 16, all the simulated images show extremely high fractional polarization over much of the image (indeed, where synchrotron losses are significant, it can exceed the standard 70–75 per cent maximum for the usual range of power-law spectra). Observations of radio sources often show strong polarization, 50 per cent or even more, at the edges of the lobes, but over most of the source the polarization is usually less than 25 per cent (e.g. Leahy et al. 1986). Here something significant must be missing from our physical model. It could be small-scale fluid turbulence, but that would itself further amplify a frozen-in field and be inhibited by the magnetic forces. We consider that field-line reconnection is more likely to play the crucial role in reducing the anisotropy of the field.

We should like to be able to use the polarization of the lobes as a diagnostic for distinguishing between models such as ours which display strong poloidal shear in the contact layer, and those which assume ideal slip between ex-jet material and shocked ambient gas, in which the toroidal stretching of the field should predominate. Unfortunately both predict electric vectors perpendicular to the edge of the radio lobes. The distinction should come in the polarization of the interior of the lobes, where the line-of-sight strikes the backflow at right angles (for sources with jet axes in the plane of the sky). Here, models with ideal slip would predict electric vectors aligned with the jet axis, whereas our models can (as we saw in Section 3) lead to a predominance of either poloidal or toroidal field. Observations should show the polarization of the interior of the radio lobes more cleanly than simulations made so far, for the observed jets are generally much thinner relative to the lobes than the jets in our models, and therefore do not contaminate the radio emission near the centre line of the lobes nearly as much. However, observations that we are aware of do not offer clear-cut evidence either way.

We must also note that, among the few examples of hotspots which have been mapped in detail, it is common for the jet to appear to curl round in one piece, rather than dividing at anything resembling a stagnation point. To shed light on these cases (perhaps the majority), three-dimensional simulations will be needed.

4.3 Brightness distribution

One striking feature of the models is that the hotspots stand out very strongly, while the radio emission from the jets is faint unless synchrotron loss is very severe, and this despite the fact that we have incorporated no mechanism of particle acceleration at the ‘working surface’. A little thought shows that this behaviour is quite reasonable. At a strong shock in ordinary gas with adiabatic index 5/3, four-fold compression raises the two transverse components of magnetic field by a factor of 4, and the $K$ of the particle spectrum by a factor of $4^{1+2\nu/3}$ (cf. equation 2.7 of Paper 1), so the volume emissivity can easily rise by two orders of magnitude, and more if the magnetic field is sheared significantly. We face the possibility that adding diffusive shock acceleration would increase the brightness contrast between jet and hotspot beyond that observed! The fact that many sources contain large-scale radio jets, though generally faint ones, shows that these jets already contain a significant population of relativistic electrons. The standard models of diffusive shock acceleration will only accelerate particles with Larmor radii larger than the shock thickness, which, if there are protons present and the jet speed is even mildly relativistic, means that only electrons which already have high Lorentz factors $\gamma$ can be accelerated further. Thus it need not surprise us unduly if diffusive shock acceleration at the jet shock is not always as important as used to be thought.

4.4 The importance of the magnetic field configuration

Conditions for the stretching and hence the growth of the magnetic field are strongly localized, and may depend on details of instabilities, or inhomogeneities in the initial ambient gas, which can cause the magnetic field energy density to become a substantial part of the total in some sheets and filaments occupying a small part of the total volume of the source. (Since force-free field configurations exist, the magnetic energy density could locally even exceed the total energy density in its surroundings.) In general terms, such regions will be near the contact discontinuities of the corresponding idealized flows, and in any regions of fluid turbulence there may be. In such regions, a randomly tangled field, such as that we have used as input, will be subject to fast field-line reconnection, which will limit the growth of field strength, but will also provide a further possibility, that of re-accelerating relativistic electrons (e.g. Biskamp 1989).

Probably the most important result of all this work is to draw attention to the crucial role of the magnetic field structure in determining the observed radio emission. The traditional recipes, according to which the radio emission is just some power of the energy density (e.g. pressure$^2$, or the 7/4 power as implied by the equipartition rule), were inadequate and misleading. While our simulations tend to exaggerate the amplification of magnetic field by distortion, because they ignore field-line reconnection (and indeed we have not been able to guess at a plausible algorithm for incorporating it), that amplification is likely to occur, and the observed prevalence of filamentary structures of high volume emissivity in radio sources suggests that it is important.

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Radio galaxies with tangled magnetic fields – II