The hypothesis that magnetic fields buoyantly convect in neutron stars

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SUMMARY
The hypothesis that a magnetic field convects out of the superconducting interior of a neutron star on a time-scale $10^6$–$10^7$ yr because of the buoyancy of its individual quantized magnetic flux tubes is critically examined. The hypothesis in its current form equates the buoyancy force of a magnetic flux tube to the viscous force produced by electrons scattering elastically off the flux tube. Examination of the relevant processes with emphasis on the gravity-induced electric field shows that: (i) the picture of flux tubes as independent buoyant entities is invalid in a neutron star because the electron mean free path greatly exceeds the lattice spacing of the flux tubes and the magnetic field is frozen in the electron gas; (ii) the gravity-induced electric field ties the electron and proton gases together and, therefore, the magnetic field is frozen in a quasi-neutral plasma; (iii) the buoyancy of the magnetic flux tubes is already included in the hydrostatic equation and does not require special treatment; (iv) the Lorentz retarding force acting on the lattice owing to its drift motion through the plasma greatly exceeds the viscous force due to electron scatter; and (v) the diffusion time-scale of the magnetic field is of order $10^{18}$ yr, in approximate agreement with original estimates.

1 INTRODUCTION
Typically, isolated pulsars have surface dipole magnetic fields of order $10^{12}$ G, and their observed spin-down rates indicate a radio emission lifetime of a few million years (reviewed by Manchester & Taylor 1977). Several authors have explored the possibility that the decay of the magnetic field limits the radio-emission lifetime of pulsars (Ostriker & Gunn 1969; Gunn & Ostriker 1970; Lyne, Ritchings & Smith 1975; Lyne, Anderson & Salter 1982; Lyne, Manchester & Taylor 1985; Stollman 1987). Theoretical studies show, however, that the decay of the magnetic field in a neutron star occurs on a time-scale exceeding the age of the Universe (Baym, Pethick & Pines 1969a,b; Gentile 1970; Channugam & Gabriel 1971; Ewart, Guyer & Greenstein 1975; Flowers & Itoh 1976; Sang & Channugam 1987).

In an attempt to resolve the conflict between observation and theory, Muslimov & Tsygan (1985) reconsidered the idea of magnetic buoyancy, originally introduced by Parker (1955) to explain sunspots. These authors proposed that the tubes of quantized magnetic flux convect out of the superconducting interior of a neutron star because of their buoyancy as individual entities. The magnetic pressure inside a flux tube reduces the proton and electron pressures (and hence densities) and accounts for the buoyant state of the flux tube. Muslimov & Tsygan equated the buoyancy and viscosity forces acting on individual magnetic flux tubes and found a diffusion time of the order $10^6$ yr. They calculated a viscous drag force from the rate that momentum transfers to electrons elastically scattering off the moving magnetic flux tubes. Harvey, Ruderman & Shaham (1986) showed that the viscous force due to elastic electron scattering is independent of temperature and obtained essentially similar results (their diffusion time $10^6$ yr is an arithmetical error). Jones (1987) has studied the buoyancy convection mechanism in detail and found that the magnetic field is frozen in the electron gas, and the magnetic field and electron gas jointly diffuse out of the superconducting interior on a time-scale of order $10^6$ yr. Wendell (1988) has invoked the buoyancy hypothesis in order to explain the comparatively weak magnetic fields in millisecond pulsars.

According to the general picture of a neutron star (Baym & Pethick 1979; Pines & Alpar 1985), the core consists of superfluid neutrons, superconducting protons, and degenerate relativistic electrons. This n–e quantum fluid is bounded on the outside by an inner crust of neutron-rich ions (density $\rho < 2 \times 10^{14}$ g cm$^{-3}$), and on the inside by an inner core of more exotic composition (density $\rho > 5 \times 10^{14}$ g cm$^{-3}$). The core comprises two distinct fluid components: one fluid consists of superfluid neutrons; the other consists of superconducting protons and relativistic electrons forming a quasi-neutral plasma having a density an order of magnitude
less than the density of the neutrons. Both fluids are in chemical equilibrium with each other, and each is in independent hydrostatic equilibrium. The two fluids interact through proton–neutron nuclear interactions and through the electron interactions with excitations (Feibelman 1971) and spontaneous magnetizations (Sauls, Stein & Serene 1982) inside the cores of quantized neutron vortices. No radial momentum exchange can occur between the two fluids, despite these interactions, and each fluid exists in independent hydrostatic equilibrium in a gravitational field determined primarily by the neutrons.

In this discussion I stress the importance of the gravity-induced electric field that binds the proton and electron gases together as a quasi-neutral fluid in hydrostatic equilibrium (Rosseland 1924; Eddington 1926; Bally & Harrison 1978). The effect of this electric field on the physics of neutron stars has hitherto been neglected. In the e–p plasma, the relativistic electrons contribute most of the pressure, the protons contribute most of the mass, and the gravity-induced electric field that couples together the protons and electrons as a single fluid exists even when the protons are superconducting.

Jones (1987) showed that the magnetic field is tightly coupled to the electrons because the electron mean free path is large compared with the separating distance between the quantized magnetic flux tubes. I follow Jones and show in addition that, because the electrons and protons are coupled together by an electric field, the magnetic field is frozen in the e–p plasma. This plasma, with its electric and magnetic fields, in chemical equilibrium with the neutron fluid, floats hydrostatically in the gravitational field of the star, and the magnetic stress gradients contribute to the support of the plasma. The diffusion velocity of the magnetic field is governed by dissipative electron–proton encounters in the cores of magnetic flux tubes, yielding a decay time of order $10^{18}$ yr. Hence, if magnetic-field decay correctly accounts for the cut-off in the radio-emission lifetime of pulsars, then a mechanism other than that proposed by Muslimov & Tsygan must be the cause.

Magnetic inhomogeneities on scales much larger than an electron mean free path remain free to convect buoyantly as in turbulent convection and Parker’s sunspot mechanism. These large-scale convective processes lie outside the scope of this discussion.

## 2 Characteristic Quantities

### 2.1 Decay of the magnetic field

The proton London penetration depth

$$\lambda = (m_p c^2/4\pi n_p e^2) = 1 \times 10^{-11} \text{ cm}$$

(1)

exceeds the proton coherence length

$$\xi = 2E_f/p \pi k_f(p) \Delta = 1 \times 10^{-12} \text{ cm},$$

(2)

(more precisely $\lambda > 2\xi$) and the protons form a type II superconductor. In these relations, $m_p$ denotes the proton mass, $E_f(p) = \hbar^2 k_f(p)/2m_p$, the proton Fermi energy (less the rest-mass energy $m_p c^2$), $k_f(p) \propto n_p^{1/3}$ the proton Fermi wave-number, $n_p$ the proton number density, and $\Delta$ the proton superconducting energy gap. Typically, $n_p \sim 10^{37} \text{ cm}^{-3}$, and $\Delta = 0.7 \text{ MeV}$ for a $15_6$ energy gap. The critical magnetic fields $H_{c_1}$ and $H_{c_2}$ have values

$$H_{c_1} = (\phi_0/4\pi \lambda^2) \ln(\lambda/\xi) = 3 \times 10^{14} \text{ G},$$

(3)

$$H_{c_2} = (\phi_0/2\pi \xi^2) = 3 \times 10^{16} \text{ G}.$$  

(4)

When the magnetic induction, $B$, is less than $H_{c_1}$, expulsion of the field is energetically advantageous, as in the Meissner effect; when $B$ lies between the critical values $H_{c_1}$ and $H_{c_2}$, the fluid nucleates into flux tubes or filaments of quantized flux

$$\Phi_0 = h c/2 e = 2 \times 10^{-7} \text{ G cm}^2;$$

(5)

and when $B$ exceeds $H_{c_2}$, superconductivity is destroyed and protons exist in a normal degenerate state.

A magnetic field of $B = 10^{12}$ G in a neutron star should exhibit the Meissner effect and be expelled from the superconducting core into the inner crust. The high conductivity of the electron gas, however, greatly delays the expulsions (Baym et al. 1969a) and the magnetic field nucleates into quantized filaments of flux and has the properties of a type II superconductor.

The characteristic decay time of a magnetic field in a spherical conductor of radius $R$ and conductivity $\sigma$ is (Cowling 1945)

$$t_0 = 4 \sigma R^2/\pi c^2.$$  

(6)

The electrical conductivity of an electron gas is

$$\sigma = n_e e^2 c^2 \tau/E_f(e),$$

(7)

in which $\tau$ denotes the transport relaxation time and $E_f(e)$ the Fermi energy of the electrons. A minimum value for $t_0$ can be obtained by using the conductivity of electrons interacting with normal degenerate protons at temperature $T$:

$$\sigma_n = n_e e^2 c^2 \tau_n/E_f(e) \sim 1 \times 10^{23} \text{ s}^{-1},$$

(8)

where the relaxation time is given by $\tau_n = 2 \times 10^2 T^{-2}$ s (Baym et al. 1969b). The magnetic field decays in a minimum time $t_0 \sim 3 \times 10^{13}$ yr for $T = 10^8$ K and $R = 10^6$ cm. This result neglects the steady decrease in temperature because of cooling and hence underestimates the normal decay time. Moreover, in a proton superconductor, the electrons are free to interact with normal protons only in the cores of magnetic flux tubes and the decay time is further increased by several orders of magnitude (Section 2.4).

We may interpret $t_0$ as a magnetic diffusion time. In a normal plasma, the induced electric field $E \sim V_0/B/c$, expressed in terms of a diffusion velocity $V_0$ transverse to $B$, drives an electric current $j = \sigma E/c$; from $4\pi j = \nabla \times B$, or $j = -B/4\pi R$, we obtain

$$V_0 = c^2/4\pi \sigma R,$$

(9)

giving a diffusion time $t_D = R/V_0$ in approximate agreement with equation (6). The equivalence of the diffusion and decay time-scales also applies when the protons are superconducting.

### 2.2 Electron chemical potential

The electron chemical potential $\mu_e$ (the energy required to add an additional electron) in a neutron star is

$$\mu_e = \left(E_f(e) - e \varphi \right) / g_0,$$

(10)

where the electron Fermi energy $E_f(e)$ includes $m_e c^2$, $\varphi$ is the gravity-induced electric potential, and $g_0 = \exp(2\varphi/c^2)$ is the
time component of the metric tensor in which $\psi$ becomes the
Newtonian gravity potential when $|\psi| \ll c^2$. Equation (10)
 omitted the comparatively small thermal energy. The chemical
 potentials are constant in hydrostatic equilibrium and equal
to their surface values. The electron chemical potential therefore has the constant value

$$\mu_e = m_e c^2 (g_{90}/8)_{1/2}$$

(11)

where $(g_{90}/8)_{1/2} = 1 - 2GM/Rc^2$ at the surface of a spherical star of mass $M$ and it is assumed that $\psi$ vanishes at the surface
(only electric potential differences are important). Thus the electron chemical potential $\mu_e$ has a value close to 0.5
MeV and is comparatively small in the interior of a neutron star. This conclusion is contrary to the usual assumption that
$\mu_e$ equals the electron Fermi energy.

In the deep interior, where $E_\psi(e)$ is of order 100 MeV, the electron chemical potential is negligible and the radial
gravity-induced electric field, of order $\psi/R$, or $E_\psi(e)/eR$,
equals roughly $10^5$ V cm$^{-1}$ for $R \approx 10^6$ cm. This electric field, essential to the structure of the star, binds together the electrons (which contribute most of the plasma pressure) and the protons (which contribute most of the plasma mass).

### 2.3 Chemical equilibrium

The chemical potential of a proton (neglecting thermal energy) is

$$\mu_p = [E_\psi(p) + e\psi + W_p + BH_{21}/4\pi n_p]_{(g_{90})_{1/2}}$$

(12)

where $E_\psi(p)$ represents the Fermi energy of the protons
(including $m_p c^2$) and $W_p$ the interaction energy of a proton with neighbouring nucleons. The magnetic term follows from the work by Josephson (1966), Jones (1975), and Easson & Pethick (1977) and its derivation assumes that $B \ll H_{21}$ (this assumption justifies the neglect of the magnetic contribution to $\mu_p$).

Also, the chemical potential of a neutron is

$$\mu_n = [E_\psi(n) + W_n]_{(g_{90})_{1/2}}$$

(13)

where $E_\psi(n)$ denote the Fermi energy of the neutrons (including $m_n c^2$) and $W_n$ the interaction energy of a neutron with neighbouring nucleons. For the modified Urca reactions

$$n + n = p + e^- + \tau_e$$

(14a)

$$n + p + e^- = n + n + \tau_e$$

(14b)

(Chiu & Salpeter 1964; Bahcall & Wolf 1965), the chemical potentials have the relation

$$\mu_n = \mu_p + \mu_e$$

(15)

If we neglect the small electron rest-mass energy, equation
(15) reduces to $\mu_n = \mu_p$. Omission of $\mu_e$ does not affect the
previous estimates of the chemical composition in neutron stars because of the $e\psi$ term in $\mu_p$.

The chemical potentials have identical values inside and outside the magnetic flux tubes. The magnetic pressure in the filaments, of order $H_{21}/4\pi$, reduces the proton and electron number densities (which are roughly equal) and creates a Fermi energy difference

$$\Delta [E_\psi(n) - E_\psi(p) - E_\psi(e)] = BH_{21}/4\pi n_p$$

of order $10^4$ eV for $n_p = 10^{37}$ cm$^{-3}$. Despite the Fermi energy difference, neutrons inside the filaments cannot beta decay

because of the additional magnetic energy required to create
a proton and an electron.

#### 2.4 Scalelengths

The significant scalelengths, in addition to the proton coherence length $\xi \approx 10^{-12}$ cm (equation 2) and the penetration depth $\lambda \approx 10^{-11}$ cm (equation 1), are:

$$L = (\phi_0/B)_{1/2} \approx n_p^{-1/2} \sim 10^{-10}$$

(16)

$$\Gamma = \phi_0/2B\lambda = (2n_p\lambda)^{-1} \sim 10^{-8}$$

(17)

$$\xi = (2\pi n_p\xi)^{-1} \sim 10^{-7}$$

(18)

$$\tau = c\tau_N H_{21}/B \sim 10$$

(19)

The quoted numerical values are for $B = 10^{12}$ G, proton density $n_p = 10^{37}$ cm$^{-3}$, and temperature $T = 10^8$ K. In a plane perpendicular to the magnetic field, the flux filaments form a triangular lattice of density $n_p = B/\phi_0$ per unit area and lattice constant $L = (2\pi n_p)^{1/2}$, which is large compared with the penetration depth.

An electron moving perpendicular to the magnetic lattice travels a mean distance $\Gamma = 10^{-8}$ cm between encounters with magnetic filaments of radius $\lambda$. Also, an electron travels a mean distance $\Gamma = 10^{-7}$ cm between encounters with the cores of filaments of radius $\xi$ that contain normal protons. Finally, electrons at Fermi energy and temperature $10^8$ K travel a distance $\Gamma \sim 10$ cm in a relaxation time $\tau$ determined by interactions with normal protons in the cores of magnetic flux tubes.

Because dissipative $e$-$p$ interactions occur only in the flux tube cores of radius $\xi$, the relaxation time $\tau_N$ of equation (8), which assumes that all protons are normal, must be increased by a factor of $\Gamma/\xi$, roughly equal to $H_{21}/B \sim 3 \times 10^4$. Thus

$$\tau = (H_{21}/B)\tau_N$$

(20)

and the decay (or diffusion) time increases from $3 \times 10^{13}$ to $10^{18}$ yr.

In a plane perpendicular to the magnetic lattice, an electron travels a mean distance $\Gamma$ between successive encounters with magnetic filaments of radius $\lambda$, and at each encounter at energy $E_\psi(e)$ the electron deflects through a small angle $\Delta \theta = \lambda/a_1 \sim 10^{-7}$, where $a_1 = E_\psi(e)/eH_{21}$ is the gyration radius in a uniform field $H_{21}$. Repeated small-angle deflections $\Delta \theta$ separated by distances $\Gamma$, describe a polychiral orbit of radius $\Gamma/\Delta \theta$, equal to the gyration radius $a_1 = (H_{21}/B) a_1 \sim 3 \times 10^{-7}$ cm in a uniform field $B$. On substituting $E_\psi(e) = e\psi$ from equation (10), using the approximation $\mu_e = 0$, we find that the polychiral orbit has a radius

$$a_1 = \phi/B$$

(21)

The mean radius of curvature of the polychiral orbit is constant when $B$ varies as $\phi$, or as $n_p^{1/3}$. When, however, $B$ varies spatially in some other manner, there exists an azimuthal electron drift current that affects the distribution of the magnetic field. It is interesting to speculate on the possibility that this drift current initially tends to adjust the distribution of $B$ such that it conforms to $n_p^{1/3}$.

In a proton type II superconductor, Fermi-energy electrons intersect $\Gamma/\Gamma \sim 10^8$ magnetic filaments of radius $\lambda$ in a relaxation time $\tau$, and hence the magnetic lattice is

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frozen in the electron gas (Jones 1987). The gravity-induced electric field binds together the electron and proton gases and the magnetic field is therefore frozen in the e–p plasma.

3 MAGNETIC CONVECTION

3.1 Buoyancy and magnetic pressure

To the hydrostatic equation of an e–p plasma,

$$\nabla P_{\text{ep}} = -n_{\text{ep}} \psi$$

($P_{\text{ep}} = \rho_{\text{ep}} c^2$), let us add the magnetic stress gradients, while keeping the plasma density $n_{\text{ep}}$, pressure $P_{\text{ep}}$, and $\psi = \frac{e}{c^2} \ln(\phi_0)$ constant. Hence

$$\nabla (P_{\text{ep}} + P_\text{M}) = -n_{\text{ep}} \nabla \psi - F_\text{b},$$

where a hydrostatic pressure term $P_\text{M}$ represents the magnetic stresses, and a frictional force per unit volume $-F_\text{b}$ now couples the plasma to a neutral medium. These last two equations give the buoyancy force

$$F_\text{b} = -\nabla P_\text{M}$$

acting on the plasma. For $B \ll H_{\text{ci}}$, the magnetic pressure has the value $B H_{\text{ci}}/4 \pi$ (Jones 1975; Easson & Pethick 1977). In ambipolar diffusion (Spitzer 1962), magnetic stress gradients buoyantly expel the plasma and a magnetic field from an impeding neutral medium. Such diffusion cannot occur in a neutron star because the plasma density is determined by n–p–e chemical equilibrium and because of the negligible nature of the frictional coupling between the plasma and neutron fluid.

Instead, we must suppose that the addition of the magnetic field in the hydrostatic equation perturbs the density and pressure of the plasma, while the gravitational field of the star remains fixed and determined primarily by the neutron fluid. With no frictional coupling ($F_\text{b} = 0$), we have

$$\nabla (P_{\text{ep}} + \Delta P_{\text{ep}} + P_\text{M}) = -\nabla (n_{\text{ep}} \Delta \rho_{\text{ep}}) \nabla \psi.$$  

Equations (22) and (25) yield

$$\nabla (\Delta P_{\text{ep}} + P_\text{M}) = -\nabla \Delta \rho_{\text{ep}} \nabla \psi,$$

and therefore, because of the addition of the magnetic field, the plasma inflates ($\Delta P_{\text{ep}} < 0$, $\Delta \rho_{\text{ep}} < 0$) to a new equilibrium configuration.

When a plasma is frictionally coupled to a second fluid, instead of equation (25), we have

$$\nabla (P_{\text{ep}} + \Delta P_{\text{ep}} + P_\text{M}) = -\nabla (\rho_{\text{ep}} + \Delta \rho_{\text{ep}}) \nabla \psi - F_\text{b},$$

and therefore, from equation (22),

$$\nabla \Delta P_{\text{ep}} + P_\text{M} = -\nabla \Delta \rho_{\text{ep}} \nabla \psi - F_\text{b}.$$  

This result defines what we usually mean by a buoyancy force. Thus, when magnetic inhomogeneities buoyantly convect as isolated entities (because their dimensions greatly exceed the relevant particle mean-free paths), we have $\Delta P_{\text{ep}} = -P_\text{M}$, and the buoyancy force from equation (28) is

$$F_\text{b} = -\nabla \Delta \rho_{\text{ep}} \nabla \psi.$$  

Muslimov & Tsygan (1985) observed that the magnetic pressure in the quantized flux tubes reduces the internal density of the plasma and creates a microscopic buoyancy force $F_{\text{filament/b}}$ per unit volume given by equation (29). With

$$\delta P_{\text{ep}} = (\partial \rho_{\text{ep}} / \partial P_{\text{ep}}) \Delta P_{\text{ep}}$$

and $\Delta P_{\text{ep}} = -P_\text{M}$, they showed that this equation becomes

$$F_{\text{filament/b}} = (\frac{H_{\text{ci}}}{4 \pi} \frac{\partial P_{\text{ep}}}{\partial P_{\text{ep}}} \nabla \psi,$$

and because $\nabla (\partial P_{\text{ep}} / \partial P_{\text{ep}}) \nabla \psi = \nabla |\nabla \psi|^{-1} / \rho$, we obtain a force per unit length

$$f_\text{b} \sim \frac{1}{\Delta \rho_{\text{ep}}} \frac{C}{\nabla |\nabla \psi|} \frac{\Delta \rho_{\text{ep}}}{\nabla |\nabla \psi|}$$

similar to that previously obtained by Muslimov & Tsygan (1985) and Harvey et al. (1986). The weak buoyancy force $F_\text{b} \sim n_{\text{b}} f_\text{b}$, per unit volume is given by

$$F_\text{b} \sim \frac{1}{\Delta \rho_{\text{ep}}} \frac{C}{\nabla |\nabla \psi|} \frac{\Delta \rho_{\text{ep}}}{\nabla |\nabla \psi|}$$

from equation (32). With $|\nabla \psi| = B H_{\text{ci}}/4 \pi \rho_\text{b}$ substituted in equation (33), the approximately derived result corresponds to the plasma buoyancy force defined by equation (24).

We have seen (Section 2.3) that the tubes of magnetic flux in a neutron star have a separating distance $L$ very much smaller than the electron mean free path $\lambda$. If $L$ were less than $\lambda$, and the magnetic interaction between the filamentary vortices were negligible, the flux tubes or filaments would act freely as isolated entities, as assumed in the buoyancy hypothesis. However, when $L$ greatly exceeds $\lambda$, as in a neutron star, the electron orbits lapse the flux tubes together into a buoyant lattice, and the buoyancy force with the tension in the flux tubes becomes the gradient of the macroscopic magnetic stresses that contribute to the hydrostatic support of the plasma, as in equation (25). No radial dynamic friction couples the plasma and the neutron fluid, and the buoyancy effect is therefore zero. The magnetic field causes the plasma to inflate slightly ($\Delta \rho_{\text{ep}} < 0$, $\Delta P_{\text{ep}} < 0$), and hence, according to equations (22) and (26),

$$\nabla (\rho_{\text{ep}} \Delta \rho_{\text{ep}}) \nabla P_{\text{ep}} - \nabla \Delta P_{\text{ep}} = \nabla P_\text{M},$$

and throughout the plasma the beta decay equilibrium of equation (16) is preserved because of the magnetic energy term in equation (12).

3.2 Diffusion time

The question remains: at what speed does the filamentary lattice diffuse through the plasma?

In a laboratory type II superconductor, the electrons flowing across the cores of magnetic flux tubes become normal and interact dissipatively with the ions of the solid-state lattice (Kim, Hempstead & Strnad 1965; Bardeen & Stephen 1965). In a type II proton superconductor of a neutron star, normal electrons flowing across the cores of magnetic flux tubes interact dissipatively with normal protons. The retarding Lorentz force acting on the drifting magnetic flux tubes is basically the same in both cases, and the principal difference, as Jones (1987) points out, is the absence of a rigid solid-state lattice. In both cases, the diffusion of the magnetic flux tubes through an electron gas creates in the cores of the flux tubes an electric field that drives a normal electron current. This current in a neutron star interacts with the normal protons in the filamentary cores on a dissipative time-scale $\tau$.  

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given by equation (22), and the lattice diffuses in the super-conductor at velocity $V_D$ determined by equation (9). This magnetic diffusion time, $t_{D}$, is of order $10^{18}$ yr.

The same result can be obtained in the microscopic drifting flux tube picture (Kim et al. 1965). We assume that the drift velocity $V_D$ is perpendicular to a flux tube. An electric field $E = V_D H_{B_1}/c$ drives an electron current $j = σE/c$ in the core of the flux tube. The retarding Lorentz force acting on a unit length of the flux tube is $f_{L} = |j × \phi_h| = σ V_D H_{B_1}/c^2$.

Hence, the macroscopic force per unit volume of the lattice is

$$F_1 = n_{s} f_{L} = σ V_D H_{B_1}/c^2,$$

and equals the gradient of the magnetic stresses $|\nabla P_M| ∼ BH_{B_1}/4πR$ in the present approximation. The drift velocity is therefore

$$V_D = c^2/4πσ R,$$

in agreement with equation (9).

### 3.3 Comparison of Lorentz and viscous forces

Electrons elastically scattered off a drifting magnetic filament exert a retarding or viscous force per unit length

$$f_{vm} = n s e^2 \Phi_0 V_D/c E_{||}(Φ)$$

(Harvey et al. 1986). A comparison of equations (35) and (37) shows

$$f_{L} / f_{vm} = (Γ/4πλ) ln(λ/ξ) ∼ 10^{12},$$

and the Lorentz retarding force greatly exceeds the viscous force, thus accounting for the wide difference between the diffusion time-scales of $10^8$ and $10^{18}$ yr. When the electron mean free path $Γ$ is greater than the lattice spacing $L$, the lattice is tied to the conducting fluid (the e−p plasma) and the magnetic flux tubes cannot behave freely as isolated buoyant entities. In this case the Lorentz force dominates. The viscous force dominates when the electron mean free path is less than the radius of a flux tube; in this case the flux tubes behave as buoyant entities.

## 4 CONCLUSION

The buoyancy convection hypothesis with its alternative viewpoint of magnetic diffusion in a type II proton superconductor does not constitute a new theory of rapid magnetic diffusion in neutron stars. The quantized magnetic field diffuses through the electron–proton plasma in a time of order $10^{18}$ yr. This time is greatly reduced in the buoyancy convection hypothesis by ignoring the Lorentz retarding force acting on the magnetic filaments drifting relative to the plasma.

Possibly the cut-off in the radio lifetime of isolated pulsars is a consequence not of magnetic diffusion and decay but of the spin and magnetic dipole axes evolving into alignment on a time-scale of $10^8 – 10^{17}$ yr (Flowers & Ruderman 1977; Kundt 1981; Lyne & Manchester 1988). Possibly other mechanisms of magnetic transport exist unrelated to the present analysis. For example, Jones (1987) has proposed an interesting unipolar diffusion mechanism, in which magnetic stress gradients expel the magnetic field with the electron gas, but not the proton gas. This proposal, at face value, is ruled out by the gravity-induced electric field that binds together the electrons and protons. Conceivably, this constraint can be evaded in other and more subtle diffusion processes (Harrison, in preparation).

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### REFERENCES


