Three-dimensional simulation of polytropic accretion discs

D. Molteni,1 G. Belvedere2 and G. Lanzafame2

1Dipartimento di Fisica dell’Università, via Archirafi 36, I-90139 Palermo, Italy
2Istituto di Astronomia dell’Università, viale A. Doria 6, I-95125 Catania, Italy

Accepted 1990 December 3. Received 1990 November 13; in original form 1990 July 17

SUMMARY
We carried out three-dimensional simulations of the formation and evolution of accretion discs in close binary systems, using the Smoothed Particle Hydrodynamics method to solve the fluid dynamic equations. Although the runs presented here refer to an ideal gas with different polytropic indexes, and constitute the first stage of more physically complex forthcoming simulations, they nevertheless give some interesting results: the disc structure and dynamics are in agreement with standard models only for small $\gamma$-values; as a consequence of the $z$-resolution it is found that disc formation is inhibited for $\gamma \geq 1.2$, which means that some 2D simulations of polytropic discs are meaningless; the disc is slightly asymmetric in the $x$–$y$ plane and its profile is irregular and pulsating at the outer boundary.

1 INTRODUCTION
Although the mass transfer processes from a normal star to a compact object in a close binary system are fairly well understood as are the main physical phenomena which may occur in accretion discs, the dynamic structure of the disc itself, especially when the magnetic field is taken into account, is far from being clear. From the observational point of view, direct observational evidence for discs is well established only in the case of cataclysmic variables (e.g. Pringle & Wade 1985). For strong, compact, galactic X-ray sources, indirect arguments are generally available, such as the high efficiency of the accretion mechanism in converting gravitational energy to account for their powerful X-ray emission ($L_x \geq 10^{36}$ erg s$^{-1}$), and the large angular momentum transport needed to speed up the pulsation rate exhibited by X-ray pulsars.

The structure and energetics of accretion discs of collapsed objects have been studied extensively in the last 20 yr starting from the classical works by Lynden-Bell (1969), Pringle & Rees (1972), and Shakura & Sunyaev (1973), who first described the angular momentum transport inside the disc in terms of viscous stress in Keplerian rotating discs. Since then, the interest has mainly been focused on suggesting various parameterizations of the viscous stress and cooling processes, assuming substantial equilibrium along $z$.

Generally all accretion disc models assume Keplerian rotation. Perhaps this is not so obvious and attempts to follow different approaches have been made by some authors (Paczynski & Wiita 1980; Jaroszynski, Abramowicz & Paczynski 1980; Belvedere & Molteni 1984; Belvedere, Molteni & Pidatella 1984). Most models of accretion discs have been elaborated in the thin disc approximation, which is perhaps supported by some observational evidence from close binary systems (e.g. King 1989), while no observational evidence exists in practice for discs around supermassive black holes which are currently considered as the central powerhouse of active galactic nuclei. In the latter case it is a just matter of speculation, although arguments in favour of the formation of thick discs (accretion tori) have been suggested (e.g. Begelman 1989). However, thin disc models are basically limited in so far as they assume axial symmetry and no structure along the $z$-axis, i.e. no $z$-dependence of the physical quantities. Of course, the actual astrophysical situation may be quite different, suggesting deeper investigation and a more careful description of the structure, dynamics and energetics of accretion discs without assuming any preferred symmetry. This can be accomplished by means of 3D simulations. Unfortunately, these are very computationally expensive, and therefore 2D simulations are usually carried out.

Here we want to present the results of 3D simulations of accretion discs, in the framework of the Smoothed Particle Hydrodynamics (SPH) method, that is particularly convenient for its Lagrangian formulation and reduced computer memory needs.

This work is to be considered as a first step in a long-term research programme on 3D disc simulations which is outlined in the following items.

(i) Structure and dynamics of a polytropic gas disc without radiative transport and losses.
(ii) Structure and dynamics of a polytropic gas disc with radiative transport and losses.
(iii) Introduction of self-induction effects in a plasma disc, leading to magnetic field generation and interaction of the self-induction generated disc magnetic field with an external...
field (the magnetic field of the primary star), in order to highlight the often neglected role of the magnetic field in the structure, dynamics and energetics of accretion discs (Belvedere & Moleteni 1984).

2 THE MODEL SIMULATION IN THE FRAMEWORK OF THE SPH METHOD

The SPH method (Monaghan 1985) is essentially a Lagrangian scheme in which points represent parcels of fluid moving according to pressure and body forces. This method makes use of an interpolation kernel, instead of a mesh of points as usual in current techniques, and has the advantage of allowing a substantial computer time saving, assuring comparable high-quality results. The interpolation value of a certain physical quantity $A(r)$ related to a plasma particle at position $r$ is defined in this formalism as

$$A(r) = \sum_{j}^{N} \int_{D} A(r') W(r, r', h) d^{3}r',$$

where $W(r, r', h)$ is the interpolation kernel, a continuous function of range $h$, whose limit for $h \to 0$ is the delta distribution function.

All physical quantities are described as extensive properties smoothly distributed in space and computed by interpolation at $r$. In SPH terms we have

$$A_i = \sum_{j=1}^{N} (A_i/n_j) W(r, r', h),$$

where the sum is extended to all particles included in the domain $D$, $n_j = \rho_j/m_j$ is the number density relative to the $j$th particle and $W$ has the meaning of a statistical weight.

The basic equations we solved here, in the framework of the Lagrangian SPH method, are

$$\frac{D\rho}{Dt} + \rho \nabla \cdot v = 0,$$

continuity equation, \hspace{2cm} (1)

$$\frac{Dv}{Dt} = -\nabla p \rho + \frac{1}{\rho} \left[ -2\omega \times v + \omega \times (\omega \times r) - \nabla \Phi_{grav} \right],$$

momentum equation, \hspace{2cm} (2)

$$\frac{D\rho}{Dt} = -\rho \nabla \cdot v,$$

energy equation, \hspace{2cm} (3)

$$p = \frac{\rho}{m} k_{B} T,$$

perfect gas equation. \hspace{2cm} (4)

Here, most symbols have the usual meaning, $D/Dt$ stands for the Lagrangian derivative, $\epsilon$ is the thermal energy per unit mass, and $\omega$ is the angular velocity of the rotating reference frame, corresponding to the rotation period of the binary system.

Note that the pressure term includes the artificial viscosity contribution and that in the present model, which is the first step of our 3D SPH investigation programme, we did not consider the radiative transfer in the energy equation, as stated in Section 1. In the SPH formalism, equations (1)-(3) take the following form:

$$\frac{Dv_{i}}{Dt} = -m \sum_{j=1}^{N} \left[ \frac{p_{i} + p_{j}}{\rho_{i} \rho_{j}} \right] \nabla_{i} W_{ij} + g_{i},$$

$$\frac{Dv}{Dt} = m \sum_{j=1}^{N} \left[ \frac{p_{i} + p_{j}}{\rho_{i} \rho_{j}} \right] v_{i} \cdot \nabla_{i} W_{ij},$$

where $g_{i} = -2\omega \times v_{i} + \omega \times (\omega \times r) - \nabla \Phi_{grav}$, $v_{i} = v_{i} - v_{c}$, and $m$ is the mass of a particle. In this scheme the continuity equation is identically satisfied. An artificial viscosity is, however, needed to handle and smooth discontinuities. We adopted the artificial viscosity of Lattanzio et al. (1985), with an artificial thermal diffusion term that reduces shock fluctuations. It takes the form

$$\eta_{i} = a \mu_{i} + b \mu_{i}^{2},$$

where

$$\mu_{i} = \frac{2hv_{i}^{2} r_{i}}{(c_{i} + c_{f})(r_{i}^{2} + \eta^{2})},$$

with $c_{i}$ the sound speed at the $i$th particle, $r_{i} = |r_{i} - r|$, \hspace{2cm} (5)

$\eta^{2} \ll h^{2}$, and $a$ and $b$ of the order of the unity.

The characteristics of the binary systems, in which the primary star (1) is a compact object and the secondary star (2) a main-sequence dwarf, are defined by the stellar masses $M_{1} = M_{2} = M_{s}$, the stellar radii $R_{1} = 0.02\ h$ and $R_{2} = 0.5\ h$, where $d = 2.4 \times 10^{11}$ cm is the distance between the stellar centres. The description of the plasma motion is treated in the rotating reference frame centred at the primary star and having a 12.6-h rotation period, that is the orbital period of the binary system. The temperature of the secondary star’s atmosphere is taken $10^{4}$ K. The following normalization factors have been adopted: $M = M_{1} + M_{2}$ for masses; $d = 2 \times 10^{11}$ cm for lengths; $v_{0} = [G(M_{1} + M_{2})/d]^{1/2} = 3.3 \times 10^{7}$ cm s$^{-1}$ for the velocity, so that the orbital period is normalized to $2\pi v_{0} = 10^{-9}$ g cm$^{-3}$ for the density; $\rho_{0} = \rho_{0} v_{0}^{2}$ for the pressure; $T_{0} = (\gamma - 1) v_{0}^{2} m / k_{B}$ for the temperature; $v_{0}^{2}$ for the thermal energy per unit mass. The adopted kernel width (or resolution length) is $h = 0.0075\ d$ and the integration (or interpolation) domain is a sphere of radius $0.55\ d$ centred at the primary star.

We simulate the physical conditions at the inner (i) and outer (ii) disc edges as follows.

(i) The free inflow condition is realized by elimination of particles once they enter the sphere of radius $R_{1} + 4h$, centred at the primary star. This alters the disc structure a few $h$ values near the inner edge.

The emission or injection of ‘new’ particles from the inner Lagrangian point towards the inside of the primary star’s Roche lobe is simulated by the generation of them in fixed points, called ‘injectors’, placed within a conic volume having the Lagrangian point as vertex, an aperture of $\approx 57^{\circ}$, and a size of $0.01\ d$. This conic volume represents a portion of the secondary star’s atmosphere, whose temperature ($10^{4}$ K) determines the initial velocity of the injected particles. ‘New’ particles are generated in the injectors whenever ‘old’ particles leave an injector free.

3 RESULTS AND DISCUSSION

We considered a polytropic gas ($p \propto \rho^{\gamma}$) and examined three cases in detail, taking $\gamma = 1.01, 1.1$ and 1.2. For all simula-
tions a stationary state was reached after three orbits or less, the total number of particles being about 10 000, 2500 and 1500 respectively, which is a computationally low number of points compared to that of mesh points needed for finite difference methods with similar spatial resolution. The figures shown hereafter are kept at about the same evolutionary stage corresponding to four orbital periods.

For the $\gamma = 1.01$ and 1.1 cases, the simulated disc structure is consistent with the thin disc approximation. Indeed the disc thickness and temperature are in agreement with the analytical solutions that can be derived for a polytropic thin disc model, i.e. $z = r(3(\gamma - 1))^{1/3}$ and $T = (3/2)(m/k_B) \times (\gamma - 1)(GM/r)$, independent of any assumption on the stress tensor $\tau_{\phi\phi}$ provided that the dissipated mechanical energy goes entirely into thermal energy.

Figs 1, 2 and 3 show the disc $x$-$y$ plane for $\gamma = 1.01$, 1.1 and 1.2, respectively. Figs 4, 5 and 6 show the corresponding $y$-$z$ view, within the strip defined by $x_1 = -0.02$ and $x_2 = +0.02$.

From the latter figures it is clearly apparent that a linear relationship exists between $z$ and $r$ for $\gamma = 1.01$ and 1.1, whereas for $\gamma = 1.2$ a large dispersion occurs, due to the increasing particle vector velocity as a result of the temperature increase with $\gamma$. The dispersion was extreme in a further simulation performed for $\gamma = 1.67$.

It is evident from these results that large $z$-motions, inhibiting disc formation, occur for large $\gamma$ values, so that some previous 2D simulations with $\gamma > 1.1$ (Sawada et al. 1987) are not strictly meaningful.

All this may be seen even better from some 'artificial' plots shown in Figs 7, 8 and 9, where we first wound up all particles round the $z$-axis, then 'squeezing them flat' against the $x$-$z$ plane.

Figs 10, 11 and 12 show an enlarged $x$-$y$ view of the stream particles colliding with the disc. For $\gamma = 1.01$ the stream seems to deviate and disappear as soon as it collides with the disc, whereas for $\gamma = 1.1$ and 1.2 it seems to conserve its identity deep in the disc. In our opinion the latter effect can be a consequence of the dispersion of particles for higher $\gamma$ that inhibits dense disc formation, rather than something resembling the spiral shocks claimed by Sawada et al.

---

**Figure 2.** $x$-$y$ view of the $\gamma = 1.1$ disc at time 25.17 (= 4 orbital periods). Some 2057 of the total of 2372 particles are shown.

**Figure 3.** $x$-$y$ view of the $\gamma = 1.2$ disc at time 26.88 (= 4.3 orbital periods). Some 1286 of the total of 1579 particles are shown.
al. (1987). This interpretation is clearly supported by the advantage of having z-resolution, lacking in the models by Sawada et al.

Our simulations also revealed some asymmetries. For instance Fig. 1 shows that the $\gamma = 1.01$ disc is slightly asymmetric in the $x$-$y$ plane, i.e. it extends more in the positive $y$ direction than in the $x$ direction, the scale ratio being 1.25. Moreover, a curious feature is shown in Fig. 13, where only particles with $z > 0.03$ in the $x$-$y$ plane are present: if the disc were symmetric the particles ought to be distributed circularly, but the figure shows a larger number of particles in the region where the impact of the stream with the disc occurs.
Figure 8. Artificial representation in the x–z plane of all disc particles (γ = 1.1), after winding up round the z-axis. Parameters are the same as in Fig. 5. For some particles the vector velocity is visible. The small segment, coming out from a particle, points toward the direction of the motion.

Figure 9. Artificial representation in the x–z lane of all disc particles (γ = 1.2), after winding up round the z-axis. Parameters are the same as in Fig. 6. For some particles the vector velocity is visible.

Figure 10. Enlarged x–y view of the stream particles (γ = 1.01). The reference velocity is 0.5 \( v_0 \). Some 3417 of the total of 9899 particles are shown.

Figure 11. Enlarged x–y view of the stream particles (γ = 1.1). The reference velocity is 0.5 \( v_0 \). Some 630 of the total of 2372 particles are shown.

and on the opposite side. This density peak moves along the outer border as evidenced by looking at frames both preceding and following in time, thus suggesting an oscillation in thickness of the disc edge. For discs with higher \( \gamma \)s the particles are too disperse in \( z \) for such feature to appear.

It is not easy to explain these regime asymmetries, as they persist well after the transient phase. In fact, during the latter phase, they should clearly be explained in terms of single-particle orbital motion in the context of a three-body problem in a rotating frame. However, when time goes on and the number of particles increases, the effect of collisions and the
presence of the artificial viscosity should result in circularizing orbits, and in the smoothing out asymmetries. Therefore, we are led to guess that the source of asymmetry is to be found in the violent impact of the gas stream, coming from a preferred direction on to the disc (Fig. 1), this having as a consequence the propagation of a disturbance affecting the disc body and producing the oscillation in thickness (Fig. 13).

Another interesting feature shown by our simulations for $\gamma = 1.01$ and 1.1 concerns the kinematic field inside the disc. If we look at the distribution of particle vector velocities in the disc central region ($x$--$y$ plane) and then subtract the Keplerian field, we get a non-zero distribution. Therefore the kinematic field which comes from solving our model equations is not strictly Keplerian. This is partially due to the rotating reference frame, but, to some extent, should be imputed to collisions between particles. This was confirmed by plots of the disc in the $y$--$z$ plane, where a number of particles were pushed back from the central region due to particle impacts.

Moreover, we want to remark that our simulations allow one to determine an expression for the artificial viscosity. In our models the only quantity affected by the viscous stress is the density. Of course the simulated density profile is different from that characteristic of $ap$ models, in so far as it reflects the (intrinsically) artificial viscosity, which is the only viscosity present in our runs. Indeed, fitting the simulation data, we found that our density profile is consistent with a stress tensor $T_{\rho \theta} = \eta_{\text{num}} \frac{\partial v_\rho}{\partial r},$ where $c_s$ is the sound speed, $\omega_k$ is the Keplerian angular velocity, and $h$ is of the order of the particle size, that is $\eta_{\text{num}} = c_s \omega_k h.$ Such an expression is the simulated counterpart of the thin disc model and has the same order of magnitude.

Finally, the injection rate and the accretion rate have been evaluated by counting the number of particles injected at the Lagrangian point and the particles captured at the inner radius. Since the conversion factor from particles per time to $g s^{-1}$ is equal to $\rho_0 h^2 \langle d/v_0 \rangle = \rho_0 (0.0075) ^2 v_0 = 7.7 \times 10^{14}$, typical values of the injection rate and the accretion rate ($\gamma = 1.1$) are $\approx 10^{20} g s^{-1}$ and $\approx 5 \times 10^{19} g s^{-1}$, respectively.
In conclusion, the results of our simulations, although limited to the polytropic gas model, clearly show the following.

(i) The thin disc approximation is confirmed only for low $y$ values.

(ii) For higher $y$ the disc formation is inhibited. This indicates that some 2D simulations are meaningless as they lack $z$-resolution. These results were confirmed by an additional run with $h = 0.00375 \, d$, half of the previous one, giving a higher resolution (see Figs 14, 15 and 16).

(iii) Disc asymmetries are present even in regime situation.

(iv) Due to particle interaction the kinematic field is not strictly Keplerian.

(v) A numerical expression for the artificial viscosity, resembling the one characteristic of thin disc models, is provided.

Further simulations are in progress, now taking into account the radiative contribution to pressure and energy.

ACKNOWLEDGMENTS

We are gratefully indebted to Professor J. J. Monaghan for supplying the core of its SPH code and to Professor L. Scarsi, director of the Palermo CNR IFCAI Institute whose computer facilities we used.

REFERENCES


© Royal Astronomical Society • Provided by the NASA Astrophysics Data System