Oscillations in the solar atmosphere: the 5-min peak as a consequence of wave reflection at the photosphere

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SUMMARY
The vertical propagation of compression waves from beneath the solar atmosphere is considered. It is shown that at the step-up in critical frequency which occurs at the photosphere, the phase difference between reflected and incident waves is strongly dependent on frequency, and is zero at about 3 mHz. This can explain why velocity power spectra deduced from photospheric spectral lines, have maxima at that frequency, period 5 min, if all modes are excited to equal velocity amplitudes, that is, to equal energy density, in the region just beneath the photosphere.

Higher in the atmosphere, there is a step-down in critical frequency into the corona, the whole constituting a quasi-rectangular barrier in critical frequency. Standing-wave components can be set up, by partial reflection, at frequencies higher than the top of the barrier which is at about 4.5 mHz. The first of these has a period of about 200 s, and may be responsible for the peak observed at that period in power spectra from chromospheric spectral lines. For waves of frequency higher than 4.5 mHz, the relatively cool layer between the photosphere and corona acts as an acoustic interference filter which is most transparent at those frequencies at which standing waves are set up within it.

1 INTRODUCTION
High-resolution velocity power spectra deduced from Doppler observations of solar spectral lines show many very sharp peaks corresponding with standing wave modes in the interior of the Sun. Each of the observed peaks occurs at a frequency which can be matched very closely with that calculated for a particular standing wave mode in a standard solar model. For reviews see Deubner & Gough (1984), Leibacher et al. (1985) and Christensen-Dalsgard (1988).

There is, however, no very convincing explanation of the relative amplitudes of the different modes, in particular, of why the power spectra are dominated by modes of frequency about 3 mHz, period 5 min. It is generally held that this is a consequence of the way the waves are excited. Two classes of excitation mechanism have been proposed. In one class it is claimed that 5-min modes are overstable. In the other it is proposed that they are driven by motions in the convection zone. Libbrecht (1988), has given a clear account of the shortcomings of both classes of mechanism in providing an explanation of the 5-min peak. Briefly, if these modes are overstable, what limits their amplitudes to the very small values that they have? If driven by convective motions why the rather sharp peak at 5-min period?

Since the oscillations are detected in the atmosphere it is important to take very carefully into account the effects of the structure of the atmosphere on the observed amplitudes. This requires numerical computation but the results can then be difficult to interpret. In Section 2 it is argued that as far as compression waves are concerned the solar atmosphere can be approximated by a rectangular barrier in critical frequency. This allows solutions of the wave equation to be written in closed form which helps us to understand the effects found. To the same end only vertical propagation is considered. This is not as restrictive as it may appear at first sight since waves in all modes of moderate degree, \( \ell \), propagate almost vertically at the solar surface as a consequence of the very rapid decrease in the sound speed with height.

2 WAVE PROPAGATION IN THE SOLAR ATMOSPHERE
The wave equation for the perturbation velocity, \( u \), in acoustic waves travelling vertically in an atmosphere is

\[
\frac{\partial^2 u}{\partial z^2} + \frac{\gamma g}{c^2} \frac{\partial u}{\partial z} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2},
\]

where \( z \) is a coordinate measured positive upwards, \( g \) is the gravitational acceleration, \( c \) is the speed of sound and \( \gamma \) is the adiabatic exponent.
The equation has solutions which are oscillatory in time, with frequency \( \omega \), of the form
\[
u = Z(z) e^{i \omega t},
\]
where \( Z \) satisfies
\[
\frac{d^2 Z}{dz^2} - \frac{yg}{c^2} \frac{dZ}{dz} + \frac{\omega^2}{c^2} Z = 0.
\]
(1)

In this equation the coefficients of the second and third terms are functions of \( z \) through the variation of the sound speed \( c \) with height.

One consequence of the stratification of the atmosphere is the existence of a critical frequency, such that wave propagation is impossible at lower frequencies. This can be shown by a straightforward transformation of equation (1). If we put
\[
Z(z) = \xi(z) \exp \left( \frac{1}{2} \int \frac{yg}{c^2} \frac{dz}{dz} \right)
\]
(Whittaker & Watson 1927), then equation (1) becomes
\[
\frac{d^2 \xi}{dz^2} = - \left( \frac{\omega^2 - \frac{y^2 g^2}{4 c^4} + \frac{yg d(1/c^2)}{2}}{c^2} \right) \xi,
\]
or
\[
\frac{d^2 \xi}{dz^2} = - \left( \frac{\omega^2 - \omega^2_i}{c^2} \right) \xi,
\]
(1')

where
\[
\omega^2_i = \frac{y^2 g^2}{4 c^4} + \frac{c^2 yg d(1/c^2)}{2}.
\]

The solution \( \xi \) (or \( Z \)) is oscillatory in \( z \), corresponding to travelling waves, only if \( \omega > \omega_i \). This critical, or transition, frequency is given by
\[
\omega^2 = \frac{y^2 g^2}{4 c^4} + \frac{c^2 yg d(1/c^2)}{2} - \omega^2_i + \frac{\alpha}{c} \frac{d(c^2)}{dz},
\]
where \( \omega = yg/2c \) is the acoustic cut-off frequency for an isothermal atmosphere.

The wave propagation characteristics of the outer solar envelope and atmosphere are determined by the variation of \( \omega \) with height and this is shown for a particular model in Fig. 1. Here \( h = 0 \) corresponds to \( \tau_{5000} = 1 \) in the Harvard-Smithsonian Reference Atmosphere (Gingerich et al., 1971), and that model provides the data for \( 0 \leq h \leq 1830 \) km. For the region beneath the photosphere, \( h < 0 \), data are taken from the model of Ulrich (1970), and for the upper chromosphere and transition region, \( h > 1830 \) km, from Athay (1974). At the solar surface \( g = 274 \) m s\(^{-2}\) and for this figure \( g \) has been taken to be 1.3. The effective value of \( g \) is less than 5/3, appropriate for a cool monatomic gas, because at the temperature of the solar atmosphere atoms can absorb energy in electronic excitation. A change of \( \pm 0.1 \) in \( g \) changes \( \omega_i \) at all heights by about \( \pm 4 \) per cent.

Fig. 1 has two peculiar features. The 'crevasse' at \( h = -140 \) km is due to the large, negative temperature gradient just below the photosphere. The 'spikes' near \( h = 2000 \) km arise because of the very large, positive temperature gradients into the narrow transition region and thence into the corona. Of course it is doubtful that the chromosphere and lower corona can be represented by a homogeneous model, and the height and width of these 'spikes' must be expected to vary with time and position. Also magnetic effects are likely to be significant at this height and above.

There is considerable uncertainty also in the temperature structure beneath the photosphere. However, the most important feature of Fig. 1, the sudden increase in \( \omega_i \), at the photosphere to a 'plateau' at \( \omega_i = 0.03 \) s\(^{-1}\), is common to all models. There is direct evidence for it in the results of Lites & Chipman (1979) and Lites, Chipman & White (1982). They made observations simultaneously in pairs of spectral lines which sample the oscillatory velocity at different heights, and measured the phase difference between the oscillations at these heights as a function of frequency. Their results show a sudden transition from evanescent waves (phase difference zero or very small) to travelling waves at \( \omega = 0.03 \) s\(^{-1}\). We can also conclude from these results that the 'spikes' in \( \omega_i \), at \( h = 2000 \) km are effectively transparent to waves of frequencies well below their peaks.

Thus, the low-temperature region between the photosphere and corona constitutes a barrier to acoustic waves travelling up from below the photosphere. This barrier can be approximated roughly by an asymmetric, rectangular barrier as drawn on Fig. 1, where the critical frequency jumps from \( \omega_i \) below the photosphere to \( \omega_i \) at the photosphere and drops back to \( \omega_i \) in the corona. The thickness of the barrier is about 2000 km. Note that photospheric spectral lines, with which most investigations of solar oscillations have been carried out, respond to the wave velocity at, or only just inside, the lower edge of the barrier. Also the lower edge of the approximating rectangular barrier is not at \( h = 0 \) but at \( h = 120 \) km.

To this approximation the solar atmosphere is being represented by regions in each of which the critical frequency is constant, so that the solution of equation (1) in each region can be approximated by that for an isothermal atmosphere in which the effective sound speed is given by \( yg/2\omega_i \). That is, in equation (1'), the quantity \( (\omega^2 - \omega^2_i)/c^2 \) is being taken as constant, for a given frequency, within each region. Clearly this is a reasonable approximation in the region of the temperature minimum between the photosphere and corona. It is not so obviously so in the other two regions. But consider the region below the photosphere; here \( \omega^2 - \omega^2_i \) slowly decreases with height but \( c^2 \) also decreases with height keeping the ratio approximately constant. In the corona \( \omega^2 - \omega^2_i \) slowly increases with height, but so does \( c^2 \), again keeping the ratio approximately constant.

3 SOLUTIONS OF THE WAVE EQUATION

In the case of a constant sound speed, equation (1) has a wavelike solution, \( Z = e^{iKz} \), if the wavenumber \( K \) satisfies the dispersion equation:
\[
c^2 K^2 + iygK - \omega^2 = 0;
\]
that is, if
\[
K = -i \frac{yg}{2c} \pm \frac{1}{c} \left( \omega^2 - \frac{y^2 g^2}{4c^4} \right)^{1/2}.
\]
For $\omega > \gamma g/2c_1 = \omega_1$, (the cut-off frequency) $K$ has a real part and can be written

$$K = -ix \pm k$$

where $x = \gamma g/2c^2 = \omega_1/c$ and $k = (\omega^2 - \omega_1^2)^{1/2}/c$. The perturbation velocity, $u$, is then given by

$$u = e^{ikz} e^{i\omega t} = e^{ix} e^{i(\omega t \pm kz)},$$

and represents travelling waves with amplitude varying as $e^{ix}$, which maintains constant kinetic energy density.

For $\omega < \gamma g/2c_1$, $K$ has no real part, but

$$K = -ix_+ \quad \text{or} \quad -ix_-,$$

where

$$x_\pm = x \pm (\omega_2^2 - \omega^2)^{1/2}/c,$$

and the perturbation velocity is given by

$$u = e^{ix_-} = e^{ix} e^{-(\omega_2^2 - \omega^2)^{1/2}/c},$$

or

$$u = e^{ix_+} = e^{ix} e^{(\omega_2^2 - \omega^2)^{1/2}/c}.$$

These are standing waves of exponential profile. In the former the energy density in the wave decreases with increasing $z$ and in the latter it increases with $z$, but both $x_+$ and $x_-$ are positive at all frequencies, and the amplitude always increases with height.

### 3.1 Step barrier

The step-up in critical frequency at the photosphere turns out to have a crucial effect on velocity power spectra. So consider first an atmosphere of just two regions:

- **region I:** $z < 0$, $\omega_i = \omega_1 = \gamma g/2c_1$,
- **region II:** $z > 0$, $\omega_i = \omega_2 = \gamma g/2c_2 > \omega_1$.

#### 3.1.1 Frequencies lower than top of barrier

For frequencies $\omega$ such that $\omega_1 < \omega < \omega_2$ there are travelling waves in region I but standing waves of exponential profile (evanescent waves) in region II. The solutions of equation (1) can be written:

region I: $Z_1 = A e^{(\omega_1 + ik_1)z} + Be^{(\omega_1 - ik_1)z}$

where $k_1 = (\omega_2^2 - \omega^2)^{1/2}/c_1$ and $\omega_1 = \gamma g/2c_1^2$.

region II: $Z_2 = Ce^{-\omega z}$

where $\omega_2 = \omega_1 - (\omega_2^2 - \omega^2)^{1/2}/c_2$ with $\omega_2 = \gamma g/2c_2^2$. There is no term in $e^{\omega z}$ in the solution for region II because the energy density must go to zero as $z \to \infty$.

At the boundary between the two regions $Z$ must be continuous, and, since neither $Z$ nor $dZ/dz$ can become infinite, we can see from the wave equation (1) that $d^2Z/dz^2$ must remain finite and hence $dZ/dz$ must be continuous also.

Applying these boundary conditions at $z = 0$ gives

$$C = A + B,$$

$$\kappa_- C = (\omega_1 - ik_1) A + (\omega_1 + ik_1) B.$$ Solving for $B$ and $C$ in terms of $A$ we find

$$B = e^{i\phi} A,$$

where $\tan \phi = \frac{2(\omega_1 - \kappa_-) k_1}{k_1^2 - (\omega_1 - \kappa_-)^2}$, (2)

$$C = \frac{2k_1 e^{i\phi} A}{[(\omega_1 - \kappa_-)^2 + k_1^2]^{1/2}},$$

(3)

where $\tan \theta = (\omega_1 - \kappa_-)/k_1$.

Thus $|B| = |A|$, indicating total reflection at the step, as must be the case, but the phase difference, $\phi$, between the reflected and incident waves varies very rapidly with frequency. Fig. 2 shows the relevant wavenumbers as a function of $\omega$, for $\omega_1 = 0.0145$ s$^{-1}$ ($c_1 = 12.3$ km s$^{-1}$) and $\omega_2 = 0.028$ s$^{-1}$ ($c_2 = 6.36$ km s$^{-1}$), which corresponds approximately to the solar case.
As \( \omega \to \omega_r, \) \( k_1 \to 0 \) and \( (\kappa_z - k_z) > 0 \), so that from equation (2), \( \phi = \pi \); thus at frequencies just higher than \( \omega_r \) the reflected and incident waves are almost in antiphase at the step so that the amplitude of the oscillations at the step is small, tending to zero as \( \omega \to \omega_r \).

As \( \omega \) increases from \( \omega_r \), the phase difference \( \phi \) decreases, and becomes zero at the frequency \( \omega_i \), for which \( \kappa_z = k_1 \). This frequency is given by

\[
\omega_i = \omega_i \left( \frac{2 - \omega_r^2}{\omega_i^2} \right)^{1/2}.
\]

At this frequency there is an antinode at the step with the oscillations having an amplitude \( 2A_r \), as is confirmed by equation (3), from which \( C \to 2A_r \) as \( \kappa_z \to k_1 \).

As the frequency increases from \( \omega_i \), the reflected and incident waves move progressively out of phase again and so the amplitude of the oscillations at the step decreases. Thus, even if the velocity amplitude, \( A \), of upgoing waves below the step is the same at all frequencies we would expect a maximum in the amplitude of oscillations at the step at the in-phase frequency \( \omega_i \). With \( \omega_1 = 0.0145 \) s\(^{-1}\) and \( \omega_2 = 0.028 \) s\(^{-1}\), \( \omega_i = 0.0191 \) s\(^{-1}\), corresponding to \( \nu = 3.04 \) mHz or period 330 s.

Since total reflection occurs at the step, there are pure standing waves below it as well as above, and it is interesting to examine their profiles. These are shown, for waves of three different frequencies, in Fig. 3; in each case the upgoing wave has unit amplitude at \( z = 0 \) (i.e. \( A = 1 \) at each frequency). We can see, from the shape of these profiles, that in the region of the step the velocity oscillations have maximum amplitude at the in-phase frequency \( \omega_i \). This maximum disappears a few hundred kilometres above the step because the higher the frequency the greater the rate at which the amplitude increases with \( z \), according to the factor \( e^{\kappa_z z}; \kappa_z \) increases rapidly with frequency (see Fig. 2).

We can understand the reason for the peak in amplitude at the step as follows. Above the step in critical frequency the standing wave has an exponential profile, \( Z_{II} = Ce^{\kappa_z z} \), but \( \kappa_z \) increases rapidly with frequency. At the step the slope of this profile is \( \kappa_z C \). Below the step there is a standing wave of sinusoidal profile with an amplitude increasing with \( z \) according to the factor \( e^{\kappa_z z} \), and \( \kappa_z \) is independent of frequency. At the frequency \( \omega_r \), at which \( \kappa_z = k_1 \), in order that the slopes of the profiles are continuous across the boundary, the sinusoidal part of the standing wave below the step must make no contribution to the slope; that is, there must be an antinode of the sinusoidal standing wave at the step. For \( \omega < \omega_r, \kappa_z > \kappa_z \) so the sinusoidal profile must make a negative contribution to the slope at the step; hence the amplitude is smaller than at \( \omega_r \). For \( \omega > \omega_r, \kappa_z < \kappa_z \) and the sinusoidal profile must make a positive contribution to the slope, and again the amplitude is smaller than at \( \omega_r \).

### 3.1.2 Frequencies higher than top of barrier

For frequencies \( \omega > \omega_z \) there are travelling waves in both regions, though only upgoing waves in region II. So equation (II) in Section 3.1.1 above should be replaced by

\[ Z_{II} = Ce^{(\kappa_z - i \kappa_1) z}, \]  

(II)

![Figure 2](https://example.com/figure2.png)

**Figure 2.** The variation of wavenumbers with frequency for \( \omega_1 = 0.0145 \) s\(^{-1}\) and \( \omega_2 = 0.028 \) s\(^{-1}\).

![Figure 3](https://example.com/figure3.png)

**Figure 3.** Velocity profiles of standing waves of three different frequencies across a step-up in critical frequency from \( \omega_1 = 0.0145 \) s\(^{-1}\) to \( \omega_2 = 0.028 \) s\(^{-1}\) at \( z = 0 \); \( \cdots \cdots \omega = 0.0146 \) s\(^{-1}\) (just above minimum frequency for propagation below the step); \( \omega = 0.0191 \) s\(^{-1}\) (the in-phase frequency); \( \cdots \cdots \omega = 0.0275 \) s\(^{-1}\) (just below frequency at top of the step).
where \( k_2 = (\omega^2 - \omega_2^2)^{1/2}/c_2 \) and \( \kappa_2 = \gamma g/2c_2^2 \).

The boundary conditions give
\[
C = A + B,
\]
\[
(k_2 - ik_2) C = (\kappa_1 - ik_1) A + (\kappa_1 + ik_1) B.
\]

The solutions are
\[
B \frac{[(k_1 + k_2)^2 - (\kappa_2 - \kappa_1)^2]^{1/2} + 4k_1[k_2 - \kappa_1]}{(k_1 + k_2)^2 + (\kappa_2 - \kappa_1)^2} e^{i\phi},
\]
where
\[
\tan \phi = \frac{-2k_1(\kappa_2 - \kappa_1)}{k_1^2 - k_2^2 - (\kappa_2 - \kappa_1)^2},
\]
\[
\tan \theta = \frac{-2k_1}{k_1 + k_2}.
\]

Fig. 4 shows calculated power spectra at \( z = 0 \) and \( z = 50 \) km; that is, at, and just above the step in critical frequency (corresponding to observations with photospheric spectral lines). These are calculated using equation (3) for \( \omega_1 < \omega < \omega_2 \) and equation (4) for \( \omega > \omega_2 \), with \( \omega_1 = 0.0145 \) s\(^{-1}\) and \( \omega_2 = 0.028 \) s\(^{-1}\), taking \( A = 1 \).

### 3.2 Rectangular barrier

Now consider the effect of including the step-down in critical frequency which occurs at the temperature rise into the corona. That is to take an atmosphere of three regions:

region I: \( z < 0 \), \( \omega_0 = \omega_1 = \gamma g/2c_1 \), \( \kappa_1 = \gamma g/2c_1^2 \),

region II: \( 0 < z < L \), \( \omega_0 = \omega_2 = \gamma g/2c_2 > \omega_1 \), \( \kappa_2 = \gamma g/2c_2^2 \),

region III: \( z > L \), \( \omega_0 = \omega_3 = \gamma g/2c_3 < \omega_1 \), \( \kappa_3 = \gamma g/2c_3 \).

#### 3.2.1 Frequencies lower than top of barrier

For frequencies \( \omega \) such that \( \omega_1 < \omega < \omega_2 \) there are up- and downgoing travelling waves in region I, evanescent waves in region II and upgoing travelling waves in region III. The solutions of the wave equation can be written:

region I: \( Z_1 = A e^{(\kappa_1 - i\omega_1)z} + B e^{(\kappa_1 + i\omega_1)z} = A e^{\kappa_1 z} + B e^{\kappa_1 z} \),

region II: \( Z_{II} = Ce^{\kappa_2 z} + De^{\kappa_2 z} \),

region III: \( Z_{III} = E e^{(\kappa_3 - i\omega_3)z} = E e^{\kappa_3 z} \),

where
\[
k_1 = (\omega^2 - \omega_1^2)^{1/2}/c_1,
\]
\[
k_2 = (\omega^2 - \omega_2^2)^{1/2}/c_2,
\]
\[
k_3 = (\omega^2 - \omega_3^2)^{1/2}/c_3.
\]

Both real, exponential solutions are needed in region II because \( z \) does not extend to infinity for this solution.

Applying the boundary conditions at \( z = 0 \) and \( z = L \) gives
\[
C + D = A + B,
\]
\[
E e^{\kappa_1 L} = C e^{\kappa_2 L} + D e^{\kappa_2 L},
\]
\[
\kappa_1 C + \kappa_2 D = K_1 A + K_2 B,
\]
\[
K_1 E e^{\kappa_1 L} = \kappa_2 C e^{\kappa_2 L} + \kappa_2 D e^{\kappa_2 L}.
\]

These equations can be solved for \( B, C, D \) and \( E \) in terms of \( A \), the amplitude of the upgoing wave in region I. The solutions for \( C \) and \( D \) are given in the Appendix.

#### 3.2.2 Frequencies higher than top of barrier, \( \omega > \omega_2 \)

For \( \omega > \omega_2 \) there are travelling waves in all three regions: thus equation (II) in Section 3.2.1 above should be replaced by
\[
Z_{II} = C e^{(\kappa_2 - i\omega_2)z} + D e^{(\kappa_2 + i\omega_2)z} = C e^{\kappa_2 z} + D e^{\kappa_2 z},
\]
where
\[
k_2 = (\omega^2 - \omega_2^2)^{1/2}/c_2.
\]

Applying the boundary conditions at \( z = 0 \) and \( z = L \) gives
\[
C + D = A + B,
\]
\[
E e^{\kappa_1 L} = C e^{\kappa_2 L} + D e^{\kappa_2 L},
\]
\[
K_1 C + K_2 D = K_1 A + K_2 B,
\]
\[
K_1 E e^{\kappa_1 L} = K_2 C e^{\kappa_2 L} + K_2 D e^{\kappa_2 L}.
\]

The solutions for \( C \) and \( D \) are in the Appendix.
Fig. 5 shows velocity power spectra at \( z = 0 \) and \( z = 50 \) km calculated from equations (A1) and (A2) of the Appendix with \( \omega_1 = 0.0145 \text{ s}^{-1} \), \( \omega_2 = 0.028 \text{ s}^{-1} \) and \( \omega_3 = 0.009 \text{ s}^{-1} \). For frequencies less than \( \omega_3 \) these spectra differ from those given in Fig. 4, for the step barrier, only very close to \( \omega_2 \), that is they are dominated by a broad peak with maximum at \( \omega = 0.019 \text{ s}^{-1} \) \( (\nu = 3 \text{ mHz}) \). At frequencies greater than \( \omega_2 \), higher than the top of the barrier, the spectra are strongly influenced by the effect of the resonances which occur when the thickness, \( L \), of the barrier is a whole number of half-wavelengths (analogous to the Ramanauer effect in quantum mechanics for a potential well).

Fig. 6 shows the calculated power spectrum at \( z = 500 \) km, corresponding to the low chromosphere. Here the resonant modes across the top of the barrier dominate. The resonant frequencies are indicated.

At \( z = 1000 \) km, at the middle of the barrier, for which the calculated power spectrum is shown in Fig. 7, the fundamental resonant mode of wavelength \( 2L \) at \( \omega = 0.02973 \text{ s}^{-1} \) \( (\nu = 4.73 \text{ mHz}) \), period 211 s dominates very strongly. (In plotting Figs 6 and 7 the amplitudes have been 'normalized' by dividing by the frequency-independent factor \( e^{\omega z} \).)

4 COMPARISON WITH OBSERVATIONS

As is well known, velocity power spectra deduced from observations of photospheric spectral lines have maxima at a frequency close to 3 mHz. High-resolution spectra, from long time series of Doppler observations, split into individual modes at those frequencies for which the waves reflected downwards at the photosphere superpose with the upgoing waves to form standing-wave modes in the solar interior. The broad peak in the calculated power spectra of Figs 4 and 5 is to be understood as the envelope to these modes. It was pointed out in Section 3.1.1 how this peak arises as a consequence of the boundary condition on the slope of the wave profiles at the photospheric step in critical frequency. Thus, the conclusion here is that photospheric velocity power spectra are dominated by modes at about 3 mHz because that is the frequency at which the reflected and incident waves are in phase at the photosphere.

Velocity power spectra deduced from chromospheric spectral lines have a maximum at a higher frequency corresponding to a period of about 200 s (Evans & Michard 1962; Orrall 1966; Mein & Mein 1976). This might be due to the fundamental resonant mode across the top of the barrier.

5 CHARACTER OF WAVES AT FREQUENCIES HIGHER THAN TOP OF BARRIER

In Section A2 of the Appendix we notice the occurrence of several terms in \( \sin 2k_2 L \) and \( \cos 2k_2 L \) which lead us to expect some kind of periodic behaviour corresponding to \( 2k_2 L = 2\pi n \) where \( n \) is an integer. Since \( k_2 = 2\pi/\lambda \), this means \( L = n\lambda/2 \). The frequencies \( \omega_2 \), at which this condition is satisfied can be found by putting \( k_2 = (\omega^2 - \omega_2^2)^{1/2}/c_2 \), and are given by

\[
\omega_2^2 = \omega_1^2 + n^2 \left( \frac{\pi c_2}{L} \right)^2.
\]

For \( \omega_2 = 0.028 \text{ s}^{-1} \) and \( L = 2000 \) km the first three of these frequencies are 0.02973 s\(^{-1}\), 0.03440 s\(^{-1}\) and 0.04102 s\(^{-1}\) and are indicated in Figs 6 and 7.

![Figure 5](https://academic.oup.com/mnras/article-abstract/251/3/427/996619/1)

**Figure 5.** Calculated velocity power spectra at \( z = 0 \) and \( z = 50 \) km for an asymmetric rectangular barrier in critical frequency, as in Fig. 1. \( \omega_1 \) and \( \omega_2 \), as for Fig. 4; \( \omega_3 = 0.009 \text{ s}^{-1} \); width of barrier 2000 km.

![Figure 6](https://academic.oup.com/mnras/article-abstract/251/3/427/996619/2)

**Figure 6.** Calculated velocity power spectrum at \( z = 500 \) km for asymmetric, rectangular barrier; \( \omega_1 \), \( \omega_2 \) and \( \omega_3 \) as for Fig. 5. Frequencies of resonant modes across top of barrier are indicated by (1), (2), (3). Amplitude has been 'normalized' by dividing by \( e^{\omega z} \).
Fig. 8 shows the ‘normalized’ amplitude of the oscillation at 0.02973 s⁻¹ as a function of height. This is not the profile of the wave; the relative phase varies with height because the standing wave has a node within this height range and because of the travelling wave component. The two components can be separated as follows. The perturbation velocity can be written

\[ u_{\text{II}} = e^{\pm \omega t} [ |C| \cos(\omega t - k_2 z + \alpha_c) + |D| \cos(\omega t + k_2 z + \alpha_D)], \]

where |C| and |D| are given in Section A2 of the Appendix, \( \alpha_c = \phi_c \) and \( \alpha_D = \phi_D - 2k_2 L \). (The real parts of the complex exponential solutions have been taken to represent the waves.) Writing \(|C| = |D| + (|C| - |D|)

\[ u_{\text{II}} = e^{\pm \omega t} \left[ |D| \cos(\omega t - k_2 z + \alpha_c) + |D| \cos(\omega t + k_2 z + \alpha_D) \right] + e^{\pm \omega t} (|C| - |D|) \cos(\omega t - k_2 z + \alpha_c). \]

The first two terms represent a standing wave of ‘normalized’ amplitude \( 2|D| \), and the third a travelling wave of amplitude \(|C| - |D|\).

This equation can be rearranged to make it easy to locate the nodes of the standing wave:

\[ u_{\text{II}} = e^{\pm \omega t} 2|D| \cos \left( \frac{k_2 z - (\alpha_c - \alpha_D)}{2} \right) \cos \left( \frac{\omega t + \alpha_c + \alpha_D}{2} \right) \]

\[ + e^{\pm \omega t} (|C| - |D|) \cos(\omega t - k_2 z + \alpha_c). \]

Fig. 9 shows the ‘normalized’ profile of the standing wave and the amplitude of the travelling wave. The wavelength of the standing wave is \( 2L_s = 4000 \) km, but the nodes do not occur at the edges of the barrier. In this case, the lowest frequency mode, the single node occurs at \( z = 218 \) km. (The higher frequency modes similarly have nodes displaced relative to the edges, the lowest ones being close to \( z = 200 \) km.) As a result there are oscillations of relatively large amplitude at the upper edge of the barrier at these resonant frequencies and these modes correspond to transmission resonances as we can see from Fig. 10, which shows the reflection and transmission coefficients for the barrier. The values shown are calculated from the ‘normalized’ amplitudes, but the sum of the ‘coefficients’ is not unity, as it is for a symmetric barrier, because the wave velocities are different above and below the barrier.

It has been mentioned before that the behaviour that has been found at frequencies higher than the top of the barrier is somewhat analogous to the Ramsauer effect in quantum mechanics, but a closer analogy, at these higher frequencies, is the optical interference filter, in which a low-velocity layer (sheet of dielectric) is placed in the path of the waves.

6 DISCUSSION

Several authors (for example, Ulrich & Rhodes 1977; Provost & Mein 1979; Leibacher, Gouttebroze & Stein 1982) have suggested that the oscillation at about 200 s⁻¹ period in the chromosphere is due to wave trapping in a chromospheric ‘cavity’ between photosphere and corona, arguing that wave reflection is total at the rapid temperature rise into the corona. But while there is certainly a tempera-
Figure 10. Reflection and transmission at the asymmetric, rectangular barrier of Fig. 1. Curve labelled ‘reflection’ shows $|B|^2/|A|^2$; that labelled ‘transmission’ shows $A_q(2000)/|A|^2$ where $A_q(2000)$ is the normalized amplitude of oscillation at the upper edge of the barrier.

For the same reason the reflectivity of the barrier decreases as $\omega$ approaches $\omega_z$, making a frequency-dependent contribution to the damping of the waves, so that the lifetimes of the interior $p$-modes would be expected to decrease with increasing frequency; or, equivalently, the half-widths of the sharp peaks corresponding to individual $p$-modes in the power spectra should increase. This effect is very clear in the results of Libbrecht & Zirin (1986).

Power spectra having a strong similarity to those found here were deduced from computations by Provost (1976). In that work the mean statistical energy of perturbations was prescribed at an arbitrary base to the atmosphere, 1000 km beneath the photosphere for the spectra shown in the figure in the paper (the spectrum of perturbations at this base is not ‘white’) and reflection was assumed to occur at a free surface at the transition to the corona. The variations in power spectra with height which were found in these computations were attributed to ‘filtering’ by the atmosphere. This phenomenon has been discussed by Soufiani (1966). He pointed out that the atmosphere acts as a high-pass filter on waves generated beneath, so that higher in the atmosphere the high-frequency components should be more prominent.

The point of view of this paper is very different. The oscillations in the atmosphere are due mainly to standing waves, and the power spectrum at a particular height depends crucially on where the velocity nodes and antinodes are relative to that height. For frequencies lower than about 4.5 mHz the standing waves are the atmospheric tails of solar global modes; at higher frequencies there are standing waves set up by partial reflection at the photosphere and at the transition to the corona. The atmosphere is acting as a filter at these higher frequencies but it is an interference filter. These higher frequency standing-wave components may also be atmospheric tails to global modes, but the atmospheric interference filter is most transparent at just those frequencies at which these standing waves are set up, so perhaps global modes are most likely at intermediate frequencies at which the filter is most opaque.

The paper by Provost, referred to above, appeared shortly after that by Deubner (1975), in which observations were reported which, for the first time, demonstrated the existence of ‘ridges’ in the $\omega$--$k$ diagram. Thereafter attention shifted to properties of the global modes and away from atmospheric effects.

7 CONCLUSION

The simple rectangular barrier model of the variation of critical frequency with height in the solar atmosphere gives a natural explanation of why velocity power spectra deduced from photospheric spectral lines peak at around 5-min period. For the calculated power spectra the velocity amplitude in the upgoing waves beneath the photosphere has been taken to be the same at all frequencies; this corresponds to equal energy at all frequencies in the region just beneath the photosphere. The implication of the result obtained here is that the dominant 5-min solar $p$-modes are not overstable, but dominate the observed power spectra because standing waves of this period have velocity antinodes in the region of observation. The amplitudes in the convection zone are small, and it is possible that modes of all frequencies in the range ~2 to ~5 mHz are excited, presumably by convec-
tion, to more or less equal energy density in the upper convection zone. Taking a broader view, such waves should exist in all stars with a convective zone, whether in the envelope or core.

The amplitude of the oscillations increases with height at all frequencies, but the rate of increase is greater the higher the frequency, up to the frequency corresponding to the top of the barrier. As a result the 5-min peak can be expected to be wiped out a few hundred kilometres above the photosphere, and at greater heights the power spectra are dominated by those resonant modes for which the thickness of the barrier is an integral number of half-wavelengths. These modes are set up at frequencies that are higher than the top of the barrier by partial reflection at its edges, rather than by total reflection in a chromospheric cavity. The first of these has a period of about 200 s and might be responsible for the peak at this period found in power spectra deduced from chromospheric spectral lines.

Another effect of the barrier is to channel the wave energy that propagates into the corona, at frequencies above the top of the barrier, into those frequencies corresponding to the resonant modes, the relatively low-temperature region between the photosphere and the corona acting as an acoustic interference filter.

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APPENDIX

A1 Solutions for asymmetric barrier $\omega_1 < \omega < \omega_2$

The solutions for $C$ and $D$ of the equations of Section 3.2.1 are

$$C = \frac{2k_1 \left[ \left( \kappa_+ - \kappa_3 \right) F - k_3 G \right]^2 + \left[ k_3 F + \left( \kappa_+ - \kappa_3 \right) G \right]^2 \right]^{1/2}}{G^2 + F^2} e^{i\theta_C} = \frac{C}{A} e^{i\theta_C},$$

where

$$\tan \theta_C = \frac{k_3 F + \left( \kappa_+ - \kappa_3 \right) G}{\left( \kappa_+ - \kappa_3 \right) F - k_3 G},$$

$$D = \frac{2k_1 e^{-i\left( \kappa_+ - \kappa_5 \right) L} \left[ \left( \kappa_5 + \left( \kappa_+ - \kappa_- \right) F \right]^2 + \left[ \kappa_5 + \left( \kappa_+ - \kappa_- \right) G - k_3 F \right]^2 \right]^{1/2}}{G^2 + F^2} e^{i\theta_D} = \frac{D}{A} e^{i\theta_D},$$

where

$$\tan \theta_D = \frac{\left( \kappa_5 - \kappa_- \right) G - k_3 F}{k_5 G + \left( \kappa_5 - \kappa_- \right) F},$$

Here

$$G = \left( \kappa_+ - \kappa_3 \right) \left( \kappa_5 - \kappa_- \right) e^{-i\left( \kappa_+ - \kappa_5 \right) L} - k_3 k_5 (1 - e^{-i\left( \kappa_+ - \kappa_- \right) L})$$

and

$$F = \left( \kappa_+ - \kappa_5 \right) k_5 e^{-i\left( \kappa_+ - \kappa_- \right) L} + \left( \kappa_5 + \left( \kappa_5 - \kappa_- \right) k_5 + \left( \kappa_5 - \kappa_- \right) k_1 e^{-i\left( \kappa_+ - \kappa_- \right) L}. $$

REFERENCES


Taking the amplitude of the upcoming wave to be unity at \( z = 0 \), that is; \( A = 1 \) for all \( \omega \), the solution for \( Z \) in region II can be written

\[
Z_{\text{II}} = |C| e^{i \phi_c} + |D| e^{i \phi_d} = |C| e^{i \phi_c} \exp \left[ - \left( \frac{\omega_z^2 - \omega^2}{c_z^2} \right)^{1/2} z \right] + |D| e^{i \phi_d} \exp \left[ \left( \frac{\omega_z^2 - \omega^2}{c_z^2} \right)^{1/2} z \right]
\]

\[
= A_c(z) e^{i \phi_c} + A_d(z) e^{i \phi_d}
\]

where

\[
A_c(z) = |C| e^{i \phi_c} \exp \left[ - \left( \frac{\omega_z^2 - \omega^2}{c_z^2} \right)^{1/2} z \right]
\]

and

\[
A_d(z) = |D| e^{i \phi_d} \exp \left[ \left( \frac{\omega_z^2 - \omega^2}{c_z^2} \right)^{1/2} z \right].
\]

At a particular height the resultant amplitude, \( A_R(z) \), is given by

\[
A_R(z) = A_c(z) + A_d(z) - 2 A_c(z) A_d(z) \cos(\pi - (\theta_d - \theta_c))
\]

and the phase of the resultant oscillation relative to that of the upcoming wave at \( z = 0 \) is \( \theta_R \), where

\[
\tan \theta_R = \frac{A_c(z) \sin \theta_c + A_d(z) \sin \theta_d}{A_c(z) \cos \theta_c + A_d(z) \cos \theta_d}
\]

\[A2 \quad \omega > \omega_z\]

The solutions for \( C \) and \( D \) of the equations of Section 3.2.2 are

\[
C = \frac{2 k_1 [[(\kappa_2 - \kappa_3) Y - (k_2 + k_3) X]^2 + [(\kappa_2 - \kappa_3) X + (k_2 + k_3) Y]^2]^{1/2}}{X^2 + Y^2},
\]

where

\[
\tan \phi_c = \frac{(\kappa_2 - \kappa_3) X + (k_2 + k_3) Y}{(\kappa_2 - \kappa_3) Y - (k_2 + k_3) X},
\]

\[
D = \frac{2 k_1 [-(\kappa_2 - \kappa_3) Y - (k_2 + k_3) X]^2 + [(\kappa_2 - \kappa_3) X - (k_2 + k_3) Y]^2]^{1/2}}{X^2 + Y^2},
\]

where

\[
\tan \phi_d = \frac{-(\kappa_2 - \kappa_3) Y + (k_2 + k_3) X}{(\kappa_2 - \kappa_3) X - (k_2 + k_3) Y},
\]

Here

\[
\theta = 2 k_2 L, \quad X = -\alpha + \delta \cos \theta + \epsilon \sin \theta, \quad Y = -\beta - \delta \sin \theta + \epsilon \cos \theta,
\]

with

\[
\alpha = (\kappa_2 - \kappa_1)((\kappa_2 - \kappa_3) + (k_2 + k_3)(k_2 + k_3)), \quad \beta = (\kappa_2 - \kappa_1)(k_2 + k_3) - (\kappa_2 - \kappa_3)(k_1 + k_2),
\]

\[
\delta = (\kappa_2 - \kappa_1)((\kappa_2 - \kappa_3) + (k_1 - k_3)(k_2 - k_3)), \quad \epsilon = (\kappa_2 - \kappa_1)(k_3 - k_2) - (\kappa_2 - \kappa_3)(k_1 - k_2).
\]

Taking the amplitude of the upcoming wave to be unity at \( z = 0 \); that is; \( A = 1 \) for all \( \omega \), the solution for \( Z \) in region II it can be written

\[
Z_{\text{II}} = C e^{i \phi_c} + D e^{i \phi_d} = |C| e^{i \theta_c} + |D| e^{i \theta_d} = A_c(z) e^{i \theta_c} + A_d(z) e^{i \phi_d},
\]

where

\[
A_c(z) = |C| e^{i \theta_c}, \quad A_d(z) = |D| e^{i \theta_d}, \quad \theta_c(z) = \phi_c - k_2 z, \quad \theta_d(z) = \phi_d - \theta + k_2 z.
\]
At a particular height the resultant amplitude, $A_R(z)$, is given by

$$A_R^2(z) = A_C^2(z) + A_D^2(z) - 2A_C(z) A_D(z) \cos[\pi - (\theta_D - \theta_C)] \quad (A2)$$

and the phase $\theta_R$, relative to that of the upgoing wave at $z = 0$ is given by

$$\tan \theta_R = \frac{|C| \sin \theta_C(z) + |D| \sin \theta_D(z)}{|C| \cos \theta_C(z) + |D| \cos \theta_D(z)}.$$