Intensity correlation of ionizing background at high redshifts

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SUMMARY
We derive the joint probability distribution of intensities of ionizing field at two different places. The intensity correlation function $\xi_J$ is calculated for randomly distributed QSOs as the main ionizing sources, taking into account the absorption produced by QSO absorption-line systems. We show that the Ly$\alpha$ forest line clouds can be used as intensity indicators to reveal $\xi_J$ at high $z$. We measure the equivalent width correlation $\xi_{1/W}$ for eight selected QSOs and find, in some cases, significant $1/W$ correlations produced mainly by the lines near the QSO emission redshifts. Such correlation may be generated by the non-uniform spectral signal-to-noise ratio, or it may be caused by the enhanced ionizing field near the QSOs. If the latter explanation is correct, then from the affected wavelength range in Q0000 – 26.3 we conclude that $J_{-21} \leq 0.1$ at $z = 4$. We point out some difficulties involved in comparing the observed $\xi_{1/W}$ with the calculated $\xi_J$. We need superb observational data and simulated spectra to examine the non-uniform signal-to-noise ratio effect. When better data eventually become available, the method we describe here can be used to set important constraints on the dominating ionizing sources at early epochs.

Key words: intergalactic medium – quasars: absorption lines – quasars: general – diffuse radiation.

1 INTRODUCTION
Studies of the metagalactic ionizing background and its evolution with redshift provide important clues to the formation and evolution of quasars and galaxies. This ionizing field may have been responsible for the re-ionization of the intergalactic medium, the ionization of the Ly$\alpha$ clouds, and even the quasi-stellar object (QSO) metal absorption-line systems. At present we do not know what kind of sources dominated the ionizing field at early epochs. The suggested candidates include quasars and young, star-forming galaxies, among many others (see Bechtold et al. 1987; Miralda-Escudé & Ostriker 1990; and references therein). Since the space densities of different ionizing sources are expected to be very different, we expect that the intensity fluctuations they produce in the ionizing field also differ significantly. In the first of this series of papers (Zuo 1992) we have shown that absorptions produced by QSO absorption-line systems reduce the total number of ionizing sources involved in generating the ionizing background and therefore enhance the $J_n$ fluctuations significantly, if QSOs are the main ionizing sources at early epochs. In this paper we discuss a related subject, the correlation of the ionizing field at high redshifts. We examine the relation between the intensity correlation and the equivalent width correlation of QSO Ly$\alpha$ forest lines and explore the possibility that the observable equivalent width correlation may be used to constrain the source space density.

2 DERIVATION OF INTENSITY CORRELATION FUNCTION $\xi_J$

2.1 Markov’s method and the joint probability distribution $W(J_n, J_J)$

Suppose there are $N$ ionizing sources which are randomly distributed in a region of volume $V$. Here we only consider Euclidean space. At high redshifts the absorption produced by QSO absorption-line systems can effectively block ionizing flux from the distant sources and the ionizing field at the Lyman limit frequency $\nu_L$ is a local phenomenon (see Section 3.2). So it is good enough to use results derived for a Euclidean space at high redshift. We assume that the occurrence of ionizing sources with a Lyman limit frequency luminosity $L$ at the position $r$ is governed by the probability

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distribution function $f(r, L)$, here $\int f(r, L) \, d^3r \, dL = 1$. Given two positions $r = 0$ and $r_1$, we want to know the probability $w_n(J_0, J_1) \, d\sigma \, dL$ that the mean Lyman limit frequency intensity at $r = 0$ is between $J_0$ and $J_0 + dJ_0$, and at the same time the mean Lyman limit frequency intensity at $r_1$ is between $J_1$ and $J_1 + dJ_1$. Using Markoff’s method (see Chandrasekhar 1943) we can easily write down the probability distribution:

$$w_n(J_0, J_1) = \frac{1}{(2\pi)^n} \int d\rho \int d\sigma \quad \times \exp\left(-i\rho J_0\right) \exp\left(-i\sigma J_1\right) A_n(\rho, \sigma)$$

(2.1) where $A_n(\rho, \sigma) = I^n$ and

$$I = \int dL \int \exp(i\rho J_0) \exp(i\sigma J_1) f(r, L) \, d^3r.$$

(2.2)

In the above expression,

$$j_0 = \frac{1}{4\pi} \frac{L}{4\pi r^2} e^{-\hat{r}}$$

(2.3)

and

$$j_1 = \frac{1}{4\pi} \frac{L}{4\pi |r-r_1|^3} \exp\left[-\tau(|r-r_1|)\right]$$

(2.4)

are the mean Lyman limit frequency intensities at $r = 0$ and $r_1$, respectively, produced by a source of luminosity $L$ at the position $r$. Here we have assumed that the optical depth $\tau$ at $n_1$ between $r_1$ and $r$ depends only on the distance between the two positions. Actually the absorption along various lines of sight can be different because of the fluctuations in the number of absorbing clouds intersected. In this paper we neglect the fluctuations in the ionizing field produced by such an effect. We will discuss briefly the validity of this assumption in Section 3.2.

Now consider that the total number of sources in the volume $V$ is not a fixed number but obeys a Poisson distribution with a mean total number $\langle N \rangle$, that is,

$$P(N, \langle N \rangle) = \frac{(\langle N \rangle)^N}{N!} e^{-\langle N \rangle}.$$  

(2.5)

In this case the probability distribution becomes

$$W(J_0, J_1) = \sum_{N=0}^{\infty} P(N, \langle N \rangle) w_n(J_0, J_1)$$

$$= \frac{1}{(2\pi)^n} \int d\rho \int d\sigma \exp(-i\rho J_0) \exp(-i\sigma J_1) A(\rho, \sigma)$$

(2.6)

where $A(\rho, \sigma) = \exp(-C_1(\rho) - C_2(\rho, \sigma))$ with

$$C_1(\rho) = \int dL \int \varphi(L, r) \, d^3r, \quad (2.7)$$

and

$$C_2(\rho, \sigma) = \int dL \int e^{i\rho(L, r)} \Phi(L, r) \, d^3r.$$

(2.8)

In the above expressions $\Phi(L, r) = \langle N \rangle f(r, L)$ is the source luminosity function. Since

$$\delta(s) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dx e^{-i\sigma x}$$

(2.9)

and $A(\rho, \sigma) = e^{-C(\rho)}$, one can easily get the probability density of $J_0$ at $r = 0$:

$$P(J_0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\rho \exp[-i\rho J_0 - C_1(\rho)].$$

(2.10)

A detailed discussion of $P(J_0)$ produced by randomly distributed sources at high $z$ can be found in Zuo (1992). Using the relation

$$\delta'(s) = -\frac{i}{2\pi} \int_{-\infty}^{\infty} xe^{-isx} \, dx$$

(2.11)

and integrating by parts we get the mean intensity at $r = 0$:

$$\langle J_0 \rangle = \int_0^\infty J_0 P(J_0) \, dJ_0 = \int dL \int \varphi(L, r) \, d^3r.$$

(2.12)

It is also not difficult to calculate two more lower moments of $W(J_0, J_1)$. We have

$$\int_0^\infty J_1 W(J_0, J_1) \, dJ_1 = \frac{1}{2\pi} \int d\rho \exp[-i\rho J_0 - C_1(\rho)]$$

$$\times \int dL \int \exp(i\rho J_1) \Phi(L, r) \, d^3r$$

(2.13)

and

$$\langle J_0 J_1 \rangle = \int_0^\infty J_0 J_1 W(J_0, J_1) \, dJ_1$$

$$= \int dL \int \varphi(L, r) \, d^3r \langle J_0 \rangle \langle J_1 \rangle.$$

(2.14)

### 2.2 The intensity correlation function $\xi_{ij}$

In the following discussions we make some further approximations. We assume that the ionizing sources are uniformly (randomly) distributed in the volume $V$ and the luminosity of an ionizing source is independent of its position, so that $\Phi(L, r) = n\delta(L)$, where $n = \langle N \rangle / V$ is the mean number density of sources and $\delta(L) \, dL = 1$. Since the absorption produced by QSO absorption-line systems is significant, especially at high redshifts (see Section 3.2), we can extend
the integration to the whole space. We thus get
\[ \langle J \rangle = \langle J_0 \rangle = \langle J \rangle = \frac{n(L)}{4\pi} \int_0^\infty e^{-r} \, dr, \tag{2.15} \]
where \( \langle L \rangle = \int L \phi(L) \, dL \) is the mean luminosity of the ionizing sources. We define the Lyman limit frequency intensity correlation function as
\[ \xi_J(r_i) = \frac{\langle J_0 J \rangle}{\langle J \rangle^2} - 1. \tag{2.16} \]
Using the above approximations and introducing the new variable \( l = |r - r_i| \), we get
\[ \xi_J(r_i) = \frac{1}{8\pi n} \left[ \int_0^\infty e^{-r} \, dr \right] \frac{\langle L^2 \rangle}{\langle L \rangle^2} \frac{1}{r_i} I_J(r_i), \tag{2.17} \]
where
\[ I_J(r_i) = \int_0^\infty e^{-r} \frac{dr}{r} \int_0^{r+r_i} e^{-l} \, dl. \tag{2.18} \]
The integration area in \( I_J \) is the shaded region in Fig. 1. We can choose a new coordinate system \((x, y)\) to simplify the integration (see Fig. 1). With
\[ x = \frac{1}{\sqrt{2}} (r - l + r_i), \tag{2.19a} \]
\[ y = \frac{1}{\sqrt{2}} (r + l - r_i), \tag{2.19b} \]
we get
\[ I_J(r_i) = \int_0^\infty dx \int_0^y dy \frac{\exp[-\tau(r) - \tau(l)]}{[(1/\sqrt{2}) y + (r_i/2)]^2 - [(1/\sqrt{2}) x - (r_i/2)]^2}. \tag{2.20} \]
To proceed further a particular form of \( \tau(r) \) must be known. Here we assume that the optical depth takes the form \( \tau(l) = r/r_0 \), where \( r_0 \) is the distance at which the effective optical depth at \( v_1 \) is unity (see Section 3.2). With this expression for the absorption we have \( \tau(r) + \tau(l) = (r + r_i)/r_0 \). This leads to
\[ I_J(u_i) = 2 \int_0^1 \ln \left( \frac{u + u_i}{u - u_i} \right) e^{-u} \, du, \tag{2.21} \]
where \( u_i = r_i/r_0 \). By introducing another new variable:
\[ v = \ln \left( \frac{u + u_i}{u - u_i} \right), \tag{2.22} \]
the \( I_J \) integration becomes
\[ I_J(u_i) = 2 \int_0^\infty v \exp \left[ -u_i \frac{1 + e^{-v}}{1 - e^{-v}} \right] dv. \tag{2.23} \]
For the mean intensity we have
\[ \langle J_0 \rangle = \frac{n(L)}{4\pi} r_0, \tag{2.24} \]
and the intensity correlation function is given by
\[ \xi_J(r_i) = \frac{1}{8\pi n^3} \int_0^\infty \frac{\langle L^2 \rangle}{\langle L \rangle^2} \frac{1}{u_i} I_J(u_i). \tag{2.25} \]
We show in Fig. 2 the numerical calculations of \( I_J(u_i)/u_i \) as a function of \( u_i \). Since \( I_J(u_i = 0) = \pi^2/2, \xi_J(r_i) \propto 1/r_i \) when \( r_i \) approaches zero.

Figure 1. The coordinate systems \((r, l)\) and \((x, y)\) and the integration area (shaded area) for calculating \( I_J(r_i) \).

Figure 2. \( I_J(u_i)/u_i \) as a function of \( u_i \).
3 SOURCE LUMINOSITY FUNCTION, ABSORPTION AND $\xi_P(r_1)$

3.1 Ionizing source luminosity functions and $\xi_P(r_1)$

To understand the effects produced by source luminosity functions we first consider a simple situation. Suppose that there are only two kinds of sources, one with a higher luminosity $L_h$ and a space density $n_h$, and another with a lower luminosity $L_l$ and a space density $n_l$. In this case the total number density of sources is $n = n_h + n_l$ and the luminosity distribution is

$$\phi(L) = \frac{n_h \delta(L - L_h) + n_l \delta(L - L_l)}{n}. \tag{3.1}$$

We have

$$\langle L \rangle = \frac{n_h L_h + n_l L_l}{n}, \tag{3.2}$$

and

$$\frac{1}{n} \langle L^2 \rangle = \frac{n_h L^2_h + n_l L^2_l}{n_h L_h + n_l L_l}. \tag{3.3}$$

We see that if the high-luminosity objects dominate $\langle L \rangle$, thus also dominating the mean intensity $\langle J \rangle$, then

$$\langle L^2 \rangle / (n \langle L \rangle^2) \approx 1/n_h.$$  

On the other hand, if the low-luminosity sources dominate $\langle L \rangle$ then

$$\langle L^2 \rangle / (n \langle L \rangle^2) \approx (1/n_l)[1 + n_h L^2_h / (n_l L^2_l)].$$

Roughly speaking, the intensity correlation function $\xi_P$ is inversely proportional to the mean number of dominating ionizing sources within a spherical region of radius $r_0$ (see equation 2.25).

At high redshifts, quasars are the only known ionizing sources which are observed in large quantities. We now examine how their luminosity functions would affect the intensity correlation function $\xi_P$. For definiteness we will adopt an Einstein-de Sitter universe ($q_0 = 0.5$) and use $H_0 = 50$ km s$^{-1}$ Mpc$^{-1}$ in the following discussions and throughout the paper. According to Boyle, Shanks & Peterson (1988) the QSO luminosity function at $z < 2.2$ is most simply parametrized by two power laws, with $\Phi(L) \propto L^{-3.7 + 0.1}$ at high luminosities and $\Phi(L) \propto L^{-14.0 + 0.2}$ at low luminosities. The characteristic luminosity $L^* \propto z^2$ with redshift $z$. At higher redshifts the results for the QSO luminosity function are still controversial. For simplicity we assume here that at $z \geq 2.2$ quasars have a constant comoving space density and there was no luminosity evolution for individual sources. Thus the proper QSO luminosity function at $z \geq 2.2$ can be expressed as

$$\Phi(L, z) = \begin{cases} \Phi(L^*)(L/L^*)^{-1.4}(1 + z)^3 & \text{if } L \leq L^*, \\ \Phi(L^*)(L/L^*)^{-3.7}(1 + z)^3 & \text{if } L \geq L^*. \end{cases} \tag{3.4}$$

where $L$ is the monochromatic luminosity at $v_\lambda$. It is clear that those sources with $L \sim L^*$ dominate the mean intensity $\langle J \rangle$. Using the above expression we get

$$\frac{1}{n} \langle L^2 \rangle = \frac{1}{2.5} \frac{1}{\Phi(L^*)} \frac{L^*}{(1 + z)^3}. \tag{3.5}$$

For the quasar spectrum we adopt $L_{\odot} \propto v^{-0.5}$ for $\lambda > 1216$ Å and $L_{\odot} \propto v^{-1}$ for $\lambda < 1216$ Å. The Lyman limit frequency luminosity $L$ and the absolute $B$ magnitude $M_B$ are then related by $L = 10^{50}h^2 M_B + 20.33$ erg s$^{-1}$ Hz$^{-1}$. From Boyle et al. (1988) we know that at $z = 2.2$ the characteristic absolute $B$ magnitude is $M_B = -26.5$ and the corresponding comoving space density is about $10^{-6}$ Mpc$^{-3}$ mag$^{-1}$. Using these figures we get

$$\langle L^2 \rangle / (n \langle L \rangle^2) = 0.4 \times 10^6 (1 + z)^{-3} \text{ Mpc}^{-3} \text{ for } z \geq 2.2.$$  

A better estimate may be obtained by using Boyle et al.’s (1988) model B QSO luminosity function. By adopting a lower luminosity cut-off, $M_B = -20$, and a higher luminosity limit, $M_B = -30$, we get, for $z \geq 2.2$,

$$\langle L^2 \rangle / (n \langle L \rangle^2) = 0.3 \times 10^6 (1 + z)^{-3} \text{ Mpc}^{-3},$$

which makes only a slight modification. This result is not very sensitive to the assumed lower and higher cut-offs.

3.2 The absorption length-scale $r_0$

We now discuss the effects produced by QSO absorption-line systems. It is known that the Lyman continuum absorption from quasar absorption-line systems significantly attenuates the ionizing radiation background, especially at high redshifts (see Zuo 1991 and references therein). A useful description of absorption produced by discrete clouds between $z_{\text{abs}}$ and $z_c$ at frequency $\nu_{\text{obs}}$ is the effective optical depth, $\tau_{\text{eff}}(\nu_{\text{obs}}, z_{\text{abs}}, z_c)$, which is defined by $e^{-\tau_{\text{eff}}} = (e^{-\tau})$, where the average is along various lines of sight. Let $\eta/(N) dN dV$ be the mean number of absorbing clouds along various lines of sight in the redshift interval $z - z + dz$ and with a neutral hydrogen column density in the range $N \rightarrow N + dN$. We have, for $\lambda_{\text{abs}} \leq 912$ Å:

$$\tau_{\text{eff}}(\nu_{\text{obs}}, z_{\text{abs}}, z_c) = \int_{z_{\text{obs}}}^{z_c} \int_{N_{\text{min}}}^{N_{\text{max}}} \eta(N)[1 - \exp[-\tau_{\text{eff}}(\nu_{\text{obs}}, N)]] dN dz, \tag{3.6}$$

where

$$\tau_{\text{eff}}(\nu_{\text{obs}}, N) = 6.3 \times 10^{-18} N[(1 + z_{\text{abs}})/(1 + z)]^3(\lambda_{\text{abs}}/912)^3$$

is the optical depth of a single cloud, and $N_{\text{min}}$ and $N_{\text{max}}$ are the lower and upper column density limits, respectively. Observationally the distribution function $\eta/(N)$ is not well determined. For our purpose here we use the distribution function adopted by Miralda-Escudé & Ostriker (1990) in their Model A2:

$$\eta/(N) = 5.6 \times 10^8 N^{-1.5} \left(\frac{1 + z}{4}\right)^{3.4},$$  

for $10^{14} < N < 1.59 \times 10^{17}$ cm$^{-2}$;  

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\[ \eta_s(N) = 4.6 \times 10^8 N^{-1.5} \left( \frac{1+z}{4} \right)^{0.5}, \]

for \( 1.59 \times 10^{17} < N < 10^{22} \) cm\(^{-2} \).

(3.7b)

Since \( \eta_s(N) \propto N^{-1.5} \) and \( \tau_s(N) \propto N \), those clouds with \( N \sim 10^{17}-10^{18} \) cm\(^{-2} \) contribute most to \( \tau_{\text{eff}}(v_L, z_{\text{obs}}, z) \). We show in Fig. 3 our calculated \( \tau_{\text{eff}}(v_L, z_{\text{obs}}, z) \) curves (the solid lines). We see that \( \tau_{\text{eff}}(v_L, z_{\text{obs}}, z) \) increases very rapidly with increasing \( z \).

If we use equation (3.7b) and extrapolate it to both small and large column densities we can get an analytical approximation for \( \tau_{\text{eff}} \).

\[ \tau_{\text{eff}}(v_{\text{obs}}, z_{\text{obs}}, z) \approx 2 \ln \left( \frac{1+z}{1+z_{\text{obs}}} \right) \left( \frac{(1+z_{\text{obs}})\lambda_{\text{obs}}}{912} \right)^{3/2}. \]

(3.8)

We also plot this approximation for \( v_L \) in our Fig. 3 (the dashed lines). We see that this approximation is quite good for \( z_{\text{obs}} \approx 2.5 \) but underestimates the absorption at high redshifts. This approximate absorption can be obtained by approximating, in equation (3.6), \( e^{-\tau} \approx 1 - \tau \) for \( \tau < 1 \) and \( e^{-\tau} = 0 \) for \( \tau > 1 \). This approximation is shown by dotted lines in Fig. 3 for \( v_L \). We see that this approximation always gives a higher \( \tau_{\text{eff}} \) value.

In order to estimate the absorption length-scale \( r_0 \) we introduce \( \xi_{\text{obs}}(z_{\text{obs}}) \) by demanding that \( \tau_{\text{eff}}(v_L, z_{\text{obs}}, z_{\text{obs}}) = 1 \), i.e., at frequency \( v_L \) the effective optical depth between \( z_{\text{obs}} \) and \( z_{\text{obs}}(z_{\text{obs}}) \) is unity. Since at high redshifts the absorption is significant and \( z_{\text{obs}}(z_{\text{obs}}) - z_{\text{obs}} \) is small, we get

\[ r_{0}(z_{\text{obs}}) = 6 \times 10^3 (1+z_{\text{obs}})^{-2.5}(z_{\text{obs}} - z_{\text{obs}}) \text{Mpc}. \]

(3.9)

Some calculated \( z_{\text{obs}}(z_{\text{obs}}) \) and \( r_{0}(z_{\text{obs}}) \) values are listed in Table 1. In this table we also list \( J_{-22}(z_{\text{obs}}) \), the mean Lyman-limit intensity of ionizing field at redshift \( z_{\text{obs}} \), expressed in units of \( 10^{-22} \) erg cm\(^{-2} \) s\(^{-1} \) Hz\(^{-1} \) sr\(^{-1} \). In this calculation we have used the Boyle et al. (1988) model B QSO luminosity function at \( z = 2.2 \) and have assumed that for higher redshifts the comoving space density is fixed and there was no luminosity evolution for individual quasars. These assumptions are also used to calculate \( \langle L^* \rangle/(8\pi r_{\text{obs}}^2 n(L^*) \) for various \( z_{\text{obs}} \), which are listed in column 5 of Table 1. In the sixth column we give \( r_{\text{corr}} \), the intensity correlation length-scale at which the intensity correlation function \( \xi_{\text{corr}} \) is unity. The last column of Table 1 lists

\[ \Delta \lambda_{\text{corr}} \approx \lambda_a H_0(1+z)^{2.5} r_{\text{corr}}/c, \]

where \( \lambda_a = 1215.67 \) Å. This is the separation of two Ly\( \alpha \) lines at \( z = 0 \) produced by two clouds separated by a distance \( r_{\text{corr}}(z_{\text{obs}}) \) at \( z_{\text{obs}} \) along a line of sight. Using the Table 1 values we have calculated \( 1 + \xi_{\text{corr}}(r) \) as a function of separation \( r \) at various redshifts. Our results are shown in Fig. 4. Note that from the assumed QSO luminosity function we see that at \( z = 4 \) there are only about three QSOs, on average, with \( M_B \) brighter than \( -26.5 \) mag in a spherical region of radius \( r_0 \). More accurately we have \( 4\pi r_{\text{obs}}^2 n(L^*)/(3L^2) = 10 \) at \( z = 4 \). This is the reason why the fluctuations in the ionizing field are significant at high \( z \).

We now give a brief discussion of the absorption law we have used in our calculations. From equations (3.7a,b) it can be shown that, at redshift \( z_{\text{obs}} = 3 \), the absorptions produced by the optically thin clouds and the optically thick Lyman-limit systems (LLS) are comparable. For \( z_{\text{obs}} > 3 \) the optically thin clouds dominate the absorption, because of the more rapid increase in the line number density with redshift. In this case we expect the absorption variations along different lines of sight to be less significant. Monte Carlo simulations may be used in future work to improve upon our current absorption treatment.

Finally we should point out that the recent work by Lanzetta (1991) has shown that the LLS may evolve much faster than we have assumed \( \langle dN/dz \rangle \propto (1+z)^{0.5} \) at high \( z \). Also, Schmidt, Schneider & Gunn (1991) have recently shown that bright QSOs are endangered species at early epochs and their comoving space density declines steeply for redshifts larger than 3. Hence we may have considerably underestimated the fluctuations and intensity correlations in the ionizing field at high \( z \), if bright QSOs are the dominating sources.

![Figure 3. The effective optical depth \( \tau_{\text{eff}}(v_L, z_{\text{obs}}, z) \) as a function of \( z \) for various \( z_{\text{obs}} \). The dashed and dotted lines are the two approximations (see the text).](image-url)

Table 1. Some calculated results.

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<th>( J_{-22}(z_{\text{obs}}) )</th>
<th>( \langle L^* \rangle/(8\pi r_{\text{obs}}^2 n(L^*) )</th>
<th>( r_{\text{corr}} ) (kpc)</th>
<th>( \Delta \lambda_{\text{corr}} ) (Å)</th>
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4 QSO ABSORPTION LINES AND $\xi_J(r)$

4.1 Ly$\alpha$ forest line clouds as intensity indicators

In the previous section we have shown that if the ‘observed’ quasars dominate the ionizing field then the small-scale correlation of the background intensity is significant, especially at high redshifts. According to standard models the Ly$\alpha$ forest lines observed in quasar absorption spectra are produced by intervening clouds which are highly ionized by the ionizing background. Therefore these clouds may serve as intensity indicators at high $z$ and help us to determine the intensity correlation function observationally. We now discuss how this could be done in practice. We assume that the majority of Ly$\alpha$ forest lines are on the linear part of the curve of growth and the rest equivalent width $W$ of an absorption line is proportional to the Lyman limit frequency intensity $J$ of the ionizing field. We may write $W = A/J$, where $A$ depends only on the intrinsic properties of an absorbing cloud, i.e., $A \propto f_{HI} n_{HI}^2 d l$ for ionized clouds with $f_{HI}$ denoting the hydrogen recombination coefficient and $n_{HI}$ the total hydrogen number density. We point out that when $J$ is small and $A$ is large, our assumption that the line is on the linear part of the curve of growth is no longer valid. The saturation effect must be considered. But since the large-$W$ lines contribute little to the rest equivalent width correlation function $\tilde{\xi}_{1/w}$, which we will discuss below, our approximation should be a good one. Let the probability distribution of $A$ be $f(A)$. We then get

$$\left\langle \frac{1}{W} \right\rangle = \int \frac{1}{A} f(A) P(J) dA dJ = \left\langle \frac{1}{A} \right\rangle \langle J \rangle,$$

and

$$\frac{1}{W} \left\langle \frac{1}{W_1 W_2} \right\rangle = \int \int \int \frac{J_1}{A_1} \frac{J_2}{A_2} f(A_1) f(A_2) \times W[J_1, J_2] dA_1 dA_2 dJ_1 dJ_2 = \left( \frac{1}{A} \right)^2 \langle J_1 J_2 \rangle,$$

where the two subscripts represent two different positions 1 and 2, separated by a distance $r$. We see that

$$\frac{\left\langle \frac{1}{W_1} \frac{1}{W_2} \right\rangle}{\left\langle \frac{1}{W} \right\rangle^2} = 1 + \xi_{1/w}(r) = \left\langle \frac{J_1 J_2}{J} \right\rangle = 1 + \xi_J(r).$$

This indicates that we may get the intensity correlation function $\xi_J(r)$ by simply measuring the $1/W$ correlation of QSO Ly$\alpha$ forest lines. Unfortunately this is not the whole story. In the above equations we have assumed that we are able to detect all lines, including very weak lines, and the integration is over the whole range of $J$ and $A$. Observationally only those absorption lines with $W$ larger than a threshold or cut-off value $W_c$ can be positively detected. In this case the integration region for calculating $\langle 1/W \rangle$ is the shaded area shown in Fig. 5. The above equations also need to be modified. Letting $A_{\min}$ and $A_{\max}$ be the lower and upper limits of $A$ and denoting $J_A = A_{\min}/W_c$ and $J_{\max} = A_{\max}/W_c$, we then get

$$\left\langle \frac{1}{W} \right\rangle = \frac{C}{C_b},$$

where

$$C = \int_0^{J_A} J P(J) dJ \left\langle \frac{1}{A} \right\rangle + \int_{J_A}^{J_{\max}} J P(J) dJ \int_{J_{\min}}^{J_{\max}} \frac{f(A)}{A} dA,$$

and

$$C_b = \int_0^{J_A} J P(J) dJ + \int_{J_A}^{J_{\max}} J P(J) dJ \int_{J_{\min}}^{J_{\max}} f(A) dA.$$

We also have

$$\left\langle \frac{1}{W_1 W_2} \right\rangle = \frac{D}{D_b},$$

where

Figure 5. The integration region (shaded) for calculating equations (4.1) and (4.2) when a cut-off $W_c$ is imposed.
where

\[
D = \int_0^J dJ_1 \int_0^{J_2} W(J_1, J_2) J_1 J_2 dJ_1 dJ_2 \left( \frac{1}{A} \right)^2
\]
\[
+ \int_{J_1}^{J_{\text{max}}} dJ_1 \int_{J_2}^{J_{\text{max}}} W(J_1, J_2) J_1 J_2 dJ_1 dJ_2
\]
\[
\times \left[ \frac{1}{A_1} \int_{J_1 W_1}^{J_1} A_2 \frac{f(A_1)}{dA_1} \int_{J_2 W_2}^{J_2} A_2 \frac{f(A_2)}{dA_2} \right]
\]
\[
+ 2 \int_0^J dJ_1 \int_{J_2}^{J_{\text{max}}} W(J_1, J_2) J_1 J_2 dJ_1 dJ_2
\]
\[
\times \left[ \frac{1}{A_2} \int_{J_1 W_1}^{J_1} A_2 \frac{f(A_2)}{dA_2} \int_{J_2 W_2}^{J_2} \left( \frac{1}{A} \right) \right],
\]
\]
\[
(4.8)
\]
and

\[
D_b = \int_0^J dJ_1 \int_0^{J_2} W(J_1, J_2) dJ_2 + \int_{J_1}^{J_{\text{max}}} dJ_1 \int_{J_2}^{J_{\text{max}}} W(J_1, J_2) dJ_1 dJ_2
\]
\[
\times \left[ \frac{1}{A_1} \int_{J_1 W_1}^{J_1} A_2 \frac{f(A_1)}{dA_1} \int_{J_2 W_2}^{J_2} A_2 \frac{f(A_2)}{dA_2} \right]
\]
\[
+ 2 \int_0^J dJ_1 \int_{J_2}^{J_{\text{max}}} W(J_1, J_1) dJ_1 dJ_2 \int_{J_2 W_2}^{J_2} A_2 \frac{f(A_2)}{dA_2}.
\]
\]
\[
(4.9)
\]
Now we see that the measured

\[1 + \xi_{1/W} = \langle 1/(W_1 W_2) \rangle / \langle 1/W \rangle^2
\]
also depends on \( W_1 \) and \( f(A) \).

4.2 Equivalent width cut-off effect – a top-hat toy model

To understand better the equivalent width cut-off we consider a simple one-dimensional toy model here. Suppose some identical ionising sources are randomly distributed in a straight line. The mean source number per unit length is \( n \). Assume the ‘intensity’ produced by such sources is of the top-hat shape with a range of effect \( 2I \), that is, the intensity at \( x \) produced by a source located at \( x_i \) is given by

\[
j = \begin{cases} 
J & \text{for } x_i - I < x < x_i + I, \\
0 & \text{otherwise},
\end{cases}
\]
\]
\[
(4.10)
\]
The mean intensity is \( \langle J \rangle = \langle N_i \rangle J \), where \( \langle N_i \rangle = 2I n \) is the mean total source number within a length \( 2I \). Since the probability of finding \( N \) sources in a length \( 2I \) is \( \langle N_i \rangle^N e^{-\langle N_i \rangle} / N! \), the intensity probability distribution \( P(J) \) is

\[
P(J) = e^{-\langle N_i \rangle} \sum_{N=0}^{\infty} \frac{\langle N_i \rangle^N}{N!} \delta(J - J_{\text{th}}).
\]
\]
\[
(4.11)
\]
The intensities at two points 1 and 2, separated by a distance \( r > 2I \), are not correlated and we have \( W(J_1, J_2) = P(J_1) P(J_2) \) in this case. But for separations \( r < 2I \), the intensities are indeed correlated. The correlation is generated because the sources falling within the length \( 2I - r \) near the positions 1 and 2 can affect the intensities at both points. With this physical picture in mind the joint probability distribution

\[
W(J_1, J_2) = \exp\left[ - (nr + \langle N_i \rangle) \right] \sum_{M=0}^{\infty} \sum_{N=0}^{\infty} \sum_{K=0}^{\infty} \frac{(nr)^N (N_i)^K}{N! M! K!} \]
\[
\times \delta(J_1 - (N + M) j_{\text{th}}) \delta(J_2 - (K + M) j_{\text{th}}).
\]
\]
\[
(4.12)
\]
Using the above equation the intensity correlation function \( \xi(r) \) can be shown to be

\[
\xi_{1/W}(r) = \begin{cases} 
1 - \frac{nr}{\langle N \rangle} & \text{for } 0 \leq r \leq 2I, \\
0 & \text{for } r \geq 2I.
\end{cases}
\]
\]
\[
(4.13)
\]
We can also show that Markoff’s method applied to the one-dimensional problem leads to the same results.

In our Fig 6 the calculated intensity correlation function \( \xi_{1/W}(r) \) is shown by dashed lines as a function of \( r \). In the calculations we have used \( J = 1, 2I = 1 \) and \( n = 10 \). We have adopted different \( f(A) \) distributions to investigate the cut-off effect. For clouds having an identical \( A = A_0 \) value, the condition \( W > W_c \) means that we can detect an absorption line only at places where \( J < J_{\text{th}} = A_0 / W_c \). Fig 6(a) shows the calculated \( \xi_{1/W}(r) \) curves for various \( J_{\text{th}} \) values. Our results for uniform and power-law \( f(A) \) distributions are plotted in Figs 6(b), (c) and (d). In Fig 6(d) the small open circles are results of our Monte Carlo simulations for \( W_c = 0, 0.08 \) and 0.15. They agree well with the calculated results. We see that when the imposed \( W_c \) increases the ‘observed’ correlation \( \xi_{1/W} \) usually decreases. But for different \( f(A) \) distributions the decrease rate is different.

5 ABSORPTION-LINE EQUIVALENT WIDTH CORRELATION \( \xi_{1/W} \)

5.1 Measurement of \( \xi_{1/W} \)

To compare our calculations with the real observational data we have measured the equivalent width correlation functions for some selected quasars. There are \( N(N-1)/2 \) line pairs for a quasar with \( N \) observed Lyα forest absorption lines. For a high-luminosity QSO there are many Lyα forest lines between Lyβ and Lyγ emission regions and these lines sample a fairly large distance along the line of sight. So high-z QSOs with high-quality observed spectra are ideal objects for our statistical studies. We calculate the proper separation between two clouds with absorption redshifts \( z_1 \) and \( z_2 > z_1 \) by

\[
\frac{2c}{H_0} \frac{1}{1+z} \sqrt{(1+z_1)^{-1/2} - (1+z_2)^{-1/2}},
\]
\]
\[
(5.1)
\]
where \( z = (z_1 + z_2)/2 \). This expression is not valid when \( z_2 - z_1 \) is large. But since we are only interested in detecting correlations at small separations, this approximation does not affect our results. We then regroup the \( N(N-1)/2 \) line pairs into many bins, each of them with a bin size \( \Delta \log(r) = \text{constant} \), and the equivalent width correlation \( 1 + \xi_{1/W} \) is estimated for each bin. It is clear that the results thus obtained depend on the bin size. What we have done is to require that the first bin is large enough to contain about 15 line pairs.
Figure 6. Calculated equivalent width correlation function $\xi_{1/W}$ for the top-hat toy model. Various cut-offs are labelled beside the curves. (a) $f(A) \propto \delta(A-A_0)$; (b) uniform $f(A)$ distribution; (c) $f(A) \propto A^{-1/2}$ with a high-A cut-off; (d) $f(A) \propto A^{-1.7}$ without a high-A cut-off; the open circles are the Monte Carlo simulation results for $W_c=0.0, 0.08$ and 0.15.

For eight selected QSOs we have carried out the $\xi_{1/W}$ measurements. We have excluded all identified absorption systems and the lines shortward of the Ly$\beta$ emission. For each quasar we construct several line samples. These samples are described by three parameters. The first is the rest equivalent width cut-off $W_c$, which excludes those lines with a rest equivalent width $W<W_c$. The second is $\Delta_{lc}$, which removes lines within a distance $r_L$ from the quasar. This is desired to examine the proximity effect. The third parameter is the lower wavelength cut-off $\Lambda_{lc}$, which excludes those lines with $\lambda_{\text{obs}}<\Lambda_{lc}$. Our results for the four quasars with $z_{\text{em}}=3.65$ are shown in Fig. 7. The results for four other quasars are plotted in Fig. 8. The sample parameters and the references to the absorption-line lists are given in Table 2.

5.2 Discussion of individual QSOs

5.2.1 Q0000 - 263 ($z_{\text{em}}=4.104$)

Because of the very high redshift and the high-quality spectra taken by Steidel (1990), this QSO is our prime target for detecting the equivalent width correlation $\xi_{1/W}$. The extrapolation of the mean line density $dN/dz \approx 2.67 \left(1+z\right)^{-2.37}$ (Lu, Wolfe & Turnshek 1991) to $z=4$ predicts $\sim 84$ Ly$\alpha$ forest lines with $W>0.36$ Å for this QSO. Actually 70 are observed between the Ly$\beta$ and Ly$\alpha$ emission regions, slightly lower but not inconsistent with the extrapolation. Our results for $\xi_{1/W}$ are plotted in Fig. 7(a) for seven different line samples. The samples 1, 2, 4 and 6 indeed show a significant signal at the smallest separation. From the figure we also see that the signal is considerably reduced if we exclude those lines (about half of the total lines) at a distance 30 Mpc from the QSO (sample 3).

If the strong correlation signal is due to the enhanced ionizing field near the QSO, we can estimate the intensity of the ionizing field at $z=4$. From Sargent, Steidel & Boksenberg’s (1989) fig. 1 we know that the observed flux at $\lambda_{\text{obs}}=7000$ Å is about 380 $\mu$Jy. Longward of Ly$\alpha$ emission Sargent et al. measured a spectral index $\alpha=1$ ($f_\nu \propto \nu^{-\alpha}$). According to Steidel & Sargent (1987) the true continuum in the Ly$\alpha$ forest region can be represented by the extrapolation of the continuum defined longward of Ly$\alpha$ emission. We thus estimate a Lyman-limit luminosity $L=7 \times 10^{31}$ erg s$^{-1}$ Hz$^{-1}$ for this QSO. The mean intensity of the ionizing field at $v_L$ can be obtained by using $J=L/(4\pi r_{eq}^2)$, where $r_{eq}$ is the
distance from the QSO at which the intensity produced by the quasar is equal to the mean intensity. Adopting \( r_{eq} > 20 \) Mpc, we get \( J_{-22} < 1.2 \) at \( z = 4 \). This is smaller than the value \( \log J = 21 \pm 0.5 \) estimated from the 'proximity' or 'inverse' effect by Bajtlik, Duncan & Ostriker (1988) and Lu et al. (1991) for \( 1.7 < z < 3.8 \). Notice that since \( L \propto H_0^{-2} \) and \( r_{eq} \propto H_0^{-1} \), the estimated \( J \) does not depend on \( H_0 \).

### 5.2.2 Q1208 + 101 (\( z_{em} = 3.811 \))

The spectral resolution for this QSO is low (1.6 Å). Sample 1, which includes all lines, does not show any significant correlation signal for \( \log [r(kpc)] > 2.7 \). Sample 2, which excludes several of the weakest lines, shows a signal in the first bin, and further exclusion of the lines within a distance of 10 Mpc from the QSO (sample 4) reduces the correlation signal.

### 5.2.3 Q2000–330 (\( z_{em} = 3.78 \))

The absorption spectra of this QSO have been extensively discussed in the literature. We notice that with a comparable resolution Steidel (1990) found 119 absorption lines between \( \lambda_{obs} = 5000 \) and \( 5805 \) Å, while Hunstead et al. (1986) observed 145 in the same spectral range! We have used the line list compiled by Hunstead et al. to estimate \( \xi_{1/M} \).

According to Hunstead et al. the line list is highly reliable and essentially complete for \( W_{obs} \geq 1 \) Å, or \( W > 0.25 \) Å. As can be seen from Fig. 7(c) there is a significant correlation signal in the first separation bin for the samples 1, 2 and 4. Exclusion of the lines within a distance of 30 Mpc (sample 5) from the QSO reduces the signal considerably. From Fig. 1 of Sargent et al. (1989) we see that the observed flux at \( \lambda_{obs} = 7000 \) Å is about 350 \( \mu \)Jy. The spectral index longward of Ly\( \alpha \) emission is given by Sargent et al. as \( \alpha = 0.75 \) (corrected for Galactic reddening). This leads to \( L = 6 \times 10^{31} \) erg s\(^{-1}\) Hz\(^{-1}\). Adopting \( r_{eq} > 20 \) Mpc, we get \( J_{-22} < 0.9 \) at \( z = 3.8 \).

### 5.2.4 Q0055–269 (\( z_{em} = 3.653 \))

Only 43 Ly\( \alpha \) forest lines are listed by Steidel (1990). Sample 1 shows fairly large fluctuations in correlation and produces a strong signal for the first separation bin. This correlation signal disappears when the seven weak lines (lines 40 to 48) within a distance of 5 Mpc from the QSO are removed. Fig. 1
of Sargent et al. (1989) shows that the observed flux at $\lambda_{\text{obs}} = 7000$ Å is about 170 $\mu$Jy. The spectral index longward of Ly$\alpha$ emission is given by Sargent et al. as $\alpha = 0.47$. This leads to $L = 3 \times 10^{31}$ erg s$^{-1}$ Hz$^{-1}$. Adopting $r_{\text{sd}} = 5$ Mpc we get $J_{-22} = 8$ at $z = 3.65$.

5.2.5 Q0837+109 ($z_{\text{em}} = 3.326$)

The 0.8 Å resolution spectrum by Sargent, Boksenberg & Steidel (1988) reveals 88 Ly$\alpha$ forest lines. Samples 1, 4 and 5 display a significant correlation signal at the smallest separation. Careful examination shows that the signal is largely produced by the three very weak lines (16, 17 and 18) at $\lambda_{\text{obs}} \sim 4919$ Å (see the sample 6 result). All these three lines have $W < 0.1$ Å.

5.2.6 Q0142−100 ($z_{\text{em}} = 2.727$)

The 0.8 Å resolution spectrum by Sargent et al. (1988) shows 115 Ly$\alpha$ forest lines between the Ly$\beta$ and Ly$\alpha$ emission regions. We did not find any significant correlation at a scale $\log[r(\text{kpc})] > 2.5$ (see Fig. 8b).

5.2.7 Q2206−199 ($z_{\text{em}} = 2.559$)

This QSO was selected for the correlation study because Pettini et al. (1990) have observed it with an extremely high resolution ($\sim 0.09$ Å). There are 74 Ly$\alpha$ forest lines in their published line list. There are gaps in the spectrum which are not covered by the observations, but this does not affect the small-scale correlation. The extremely high resolution enables us to explore the correlation at very small separations (but see Section 5.3 below). Marginal signals are detected at small separations, caused by very weak lines with $W < 0.1$ Å (see Fig. 8c).

5.2.8 Q1247+267 ($z_{\text{em}} = 2.039$)

This QSO was selected because it has a relatively low redshift. It is also a very bright quasar and the spectrum by Sargent et al. (1988) has a very high signal-to-noise ratio. There are 47 Ly$\alpha$ forest lines listed by Sargent et al. Sample 1, which includes all these lines, shows a significant signal for the smallest separation bin (see Fig. 8d). After removing the seven lines (42 to 48) within a distance of 20 Mpc from the
Table 2. Absorption-line samples for measuring $\xi_{1/W}$.

<table>
<thead>
<tr>
<th>Object</th>
<th>Sample</th>
<th>$W_\alpha$ (Å)</th>
<th>$r_{\text{eq}}$ (Mpc)</th>
<th>$\lambda_{em}$ (Å)</th>
<th># of lines</th>
<th>Pair # in bin 1, 2</th>
<th>Refs.</th>
</tr>
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<tbody>
<tr>
<td>Q0000-263</td>
<td>S1</td>
<td>140</td>
<td>17, 40</td>
<td>1</td>
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<tr>
<td>$z_{em} = 4.104$</td>
<td>S2</td>
<td>20</td>
<td>16, 41</td>
<td></td>
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<td></td>
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<tr>
<td>$r_{\text{eq}} = 1.2A$</td>
<td>S3</td>
<td>30</td>
<td>17, 33</td>
<td></td>
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<td></td>
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<tr>
<td>S4</td>
<td>0.16</td>
<td>124</td>
<td>15, 38</td>
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</tr>
<tr>
<td>S5</td>
<td>0.2</td>
<td>112</td>
<td>20, 36</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>S6</td>
<td>0.16</td>
<td>20</td>
<td>5515</td>
<td>62</td>
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<td>S7</td>
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<td>20</td>
<td>77</td>
<td>17, 37</td>
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<tr>
<td>Q1208-111</td>
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<td>62</td>
<td>15, 44</td>
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<tr>
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<td></td>
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<tr>
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<td>14, 32</td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>Q2147+267</td>
<td>S1</td>
<td>47</td>
<td>16, 47</td>
<td>3</td>
<td></td>
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<tr>
<td>$z_{em} = 2.039$</td>
<td>S2</td>
<td>0.1</td>
<td>39</td>
<td>17, 29</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$r_{\text{eq}} = 0.8A$</td>
<td>S3</td>
<td>20</td>
<td>40</td>
<td>16, 37</td>
<td></td>
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</tr>
</tbody>
</table>

References: (1) Steidel 1990; (2) Hunstead et al. 1986; (3) Sargent et al. 1988; (4) Pettini et al. 1990.

5.3 Discussion

We have measured $\xi_{1/W}$ for eight selected QSOs. In four cases, i.e., Q0000-263, 2000-330, 0055-269 and 1247+267, we have detected a positive correlation signal in the smallest separation bins, which seems to have been produced mainly by the lines near the QSO emission redshifts. In one case, Q0837+109, the correlation signal is caused by the three very weak lines.

It is well known that there are many uncertainties involved in measuring equivalent widths of QSO absorption lines: (i) the noise due to counting statistics and sky background; (ii) the subjective decomposition of blended lines; and (iii) the uncertainty in the continuum level in the Lyα forest region. Our measurements of $\xi_{1/W}$ rely critically on the quality of measurements of the weak lines. Because of the uncertainties, however, these lines are usually the least reliable, especially in the case of low resolution and low signal-to-noise ratio spectra. We also know that the measured redshift of an absorption line does not necessarily represent its position in space. At $z = 4$ a line-of-sight peculiar velocity of 100 km s$^{-1}$ corresponds to a separation of 180 kpc. Such velocity dispersion, as well as poor spectral resolution, affects our correlation results at small separations. But these uncertainties alone are not enough to explain why the correlation is produced by the lines near QSO emission redshifts. One possibility is that the spectral signal-to-noise ratio systematically declines towards the blue end because the detector quantum efficiency decreases at shorter wavelengths. Also, when absorption lines are close enough to the quasar redshift to be superposed on the blue wing of the Lyα emission line, the signal-to-noise ratio is boosted significantly. Thus more weak lines near $z_{em}$ may be included in the line list and this leads to the observed $1/W$ correlation. This explanation can be tested by measuring the simulated spectra with inhomogeneous continuum signal-to-noise. Another possible explanation is that the observed correlation signal may be produced by the enhanced ionizing field near the QSO. In this case the inferred $J_{21}$ is 0.1 at $z = 4$ from Q0000-263 (see Section 5.2.1), which is smaller than the value estimated by Bajtlik et al. (1988) and Lu et al. (1991) for lower redshifts. This low $J_{21}$ value, however, is consistent with the calculated quasar contribution (see Table 1). It is interesting to note that recently Dobrzycki & Bechtold (1991) have discovered a large ($\sim 10$ Mpc) void in the spectrum of QSO 0302-003. From the assumption that the void is due to the proximity effect generated by the observed nearby quasar, they derived $J_{21}$ = 0.1 at $z = 3.2$. Unfortunately this result can be further complicated by the possible quasar anisotropic emission and luminosity variation on a time-scale of a few $\times 10^7$ yr.

How do the measured $\xi_{1/W}$ results compare with our theoretical $\xi_{1/W}$? There are some difficulties involved in doing such a comparison. We must exclude the proximity effect produced by the known QSOs. We also need to know the relationship between $J$ and $W$. For $W = 0.2$ A and a Doppler parameter $b = 30$ km s$^{-1}$, the optical depth at the line centre is about 1.5. The approximation that $W \ll J$ begins to break down for such lines, and for stronger lines. This means that the saturation effect must also be included. But as has been clearly demonstrated by Pettini et al. (1990), many Lyα forest lines discussed in this paper will no doubt be decomposed into several weaker components at higher spectral resolution. So it may still be reasonable to assume that $W \ll J$. Another problem has to do with the way we have measured $\xi_{1/W}$. Since the Lyα forest lines in a high-$z_{em}$ quasar span a relatively large redshift range (for a $z_{em} = 4$ quasar, the lines at $z = 3.2$ are also sampled), the measured $\xi_{1/W}$ only represents roughly the situation at the mean redshift. From Fig. 4 we see that $\xi_1(r)$ increases very rapidly with $z$, so it should be very valuable to compile a large absorption-line sample involving many QSOs and close pairs of gravitationally lensed quasars to study the $z$ evolution of $\xi_{1/W}$. Now we still have the problem of dealing with the cut-off $W_c$. Equations (4.4)-(4.9) are complicated enough. Monte Carlo simulation seems to be the best way to handle this problem. Work along this line is being pursued currently. Until this has been done a thorough explanation of $\xi_{1/W}$ is impossible.

Bearing in mind all the problems mentioned above, we now try to draw some preliminary conclusions from the $\xi_{1/W}$ measurements. First let us consider the possibility that the observed QSOs were indeed the dominating sources at early epochs. If the observed $1/W$ correlation is produced by the proximity effect, then we know that the intrinsic $f(A)$ distribution is favourable enough for us to detect the general...
(not near a known QSO) J correlation. Sample 6 of Q0000−263, which excludes the lines within a distance of 20 Mpc from the QSO, does show a significant $\xi_{1/w}$ signal at the smallest separation. To check whether the magnitude is also consistent with our $\xi_{J}$ calculation we need to know the exact shape of the $f(A)$ distribution. Until this check is done our current results cannot rule out the possibility that $J$ was dominated by the QSOs and had a small value $J_{-21} \sim 0.1$ at $z \sim 4$. Now let us examine the possibility that galaxies dominated $J$ at high $z$. In this case we do not expect to see any $J$ correlation, except near a known powerful QSO (the proximity effect). This is because at the present epoch the mean space density of the $L^*$ galaxies is roughly $10^{-2}$ Mpc$^{-3}$, which is much larger than the QSO comoving space density. For a $J_{-21} \sim 1$ at $z \sim 4$ the observed $1/W$ correlation cannot be explained by the enhanced ionizing field near the QSOs. But it may be explained by the non-uniform signal-to-noise ratio effect.

6 CONCLUSIONS

Using Markoff’s method we have derived the joint probability distribution of intensities of the ionizing field at two different locations. The intensity correlation function $\xi_{J}$ has been calculated for randomly distributed QSOs as the main ionizing sources, taking into account the absorption produced by QSO absorption-line systems. We have shown that the Ly$\alpha$ forest line clouds can be used as intensity indicators to reveal $\xi_{J}$ at high $z$. We have also shown that, because of the equivalent width cut-off effect, the measured equivalent width correlation $\xi_{1/w}$ is usually smaller than $\xi_{J}$. We have measured $\xi_{1/w}$ for eight selected QSOs and have found, in some cases, significant $1/W$ correlations produced mainly by the lines near the QSO emission redshifts. Such correlations may be generated by the non-uniform spectral signal-to-noise, or may be caused by the enhanced ionizing field near the QSOs. If the latter explanation is correct, then from the affected wavelength range in Q0000−263 we conclude that $J_{-21} \leq 0.1$ at $z \sim 4$. We have pointed out some difficulties involved in comparing the observed $\xi_{1/w}$ with the calculated $\xi_{J}$. We need superb observational data and simulated spectra to examine the non-uniform signal-to-noise ratio effect. With the up-coming Keck Telescope we expect that better data for high-$z$ QSOs will become available very soon. The method we have described in this paper is of potential applicability in setting important constraints on the dominating ionizing sources at early epochs.

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