The accretion curtain model for intermediate polars – I. A kinematical model for radial velocity and velocity dispersion

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Accepted 1992 June 19. Received 1992 June 18; in original form 1991 October 23

ABSTRACT

We present a theoretical model to interpret the radial velocity and velocity dispersion behaviour of the emission lines in intermediate polars. Our assumption is that the emission lines originate in the gas stream which is diverted out of the orbital plane by the dipolar magnetic field of the white dwarf and is funnelled towards the star surface. We have applied the model to the intermediate polars TX Col and EX Hya. Our study suggests that accretion of the white dwarf occurs between 9 and $12 R_w$ from the primary in TX Col and between 4 and $6 R_w$ in EX Hya.

Key words: accretion, accretion discs – binaries: close – stars: individual: TX Col – stars: individual: EX Hya – stars: magnetic fields – novae, cataclysmic variables.

1 INTRODUCTION

The intermediate polars (IPs) and the AM Herculis systems (AMHers) together form a subclass of cataclysmic variables (MCVs) where the magnetic field of the white dwarf plays an important role in the accretion processes (for recent reviews on MCVs see Wickramasinghe 1988; Cropper 1990). In the AMHers, the strength of the magnetic field of the primary ($\approx 10$–$60$ MG) is sufficient to lock it into synchronous rotation with the orbital period ($P_o \approx 80$–$220$ min). Accretion occurs directly onto the white dwarf via an accretion stream, a coupling region, and magnetically confined accretion funnels. The radiation in the optical and IR bands is dominated by strongly circularly and linearly polarized cyclotron emission from the stand-off shocks which occur at the bases of the accretion funnels. Because of phase locking, radial velocity and light variations are seen only at the orbital period.

The IPs cover a wider range in orbital period ($P_o \approx 100$ min–$10$ h). The white dwarf is not phase-locked into synchronous rotation with the orbit and additional variations are seen in light intensity at the spin period of the white dwarf, $P_s$, at the beat period, $P_{\text{beat}} = (P_s^{-1} - P_o^{-1})^{-1}$, and often at both. The spectra of IPs are characterized by prominent emission lines which also show, in addition to the orbital modulation, variations at the spin and/or the beat periods. The radial velocity semi-amplitude can range from a few tens to a few hundreds of kilometres per second and their widths from a few hundreds to a few thousands of kilometres per second. The velocity variations at the spin period are explained by matter which couples on to field lines and corotates with the white dwarf. It is believed that the modulations at the beat period arise from matter fixed in the binary reference system which reprocesses X-rays emitted from accretion shocks on the white dwarf surface.

The IPs generally show no circular polarization. In the one system, BG CMi, where circular polarization has been measured, the field is estimated to be 1–5 MG (Chanmugam et al. 1990; Wickramasinghe, Wu & Ferrario 1991). Wickramasinghe et al. (1991) have argued that the failure to detect circular polarization in most IPs indicates that they have fields that are on average a factor 10 lower than those in the AMHers.

In this paper, we use a modified version of the emission-line model for AM Herculis systems presented by Ferrario, Wickramasinghe & Tuohy (1989, hereafter FWT) to describe the velocity variations in the IPs. Our basic assumption is that the line emission at the spin period arises from two accretion curtains, one above and the other below the orbital plane, where material frozen on to the magnetic field lines flows toward the stellar surface. We show that it is possible to explain the observed variations of radial velocity and velocity dispersion with phase using our model, although this cannot be used strongly to constrain the orbital inclination $i$ and the dipole inclination $\theta$.

The paper is divided into various sections. The accretion pattern in IPs is discussed in Section 2. Our model for the accretion curtain is developed in Section 3 and results for
2 THE ACCRETION PATTERN IN IPs

In systems where magnetic effects are unimportant, the material flows from the inner Lagrangian point and forms a stream which approaches a minimum distance $r_{\text{min}}$ from the primary, around the star and intersects with itself. The matter flow then circularizes, forming a ring of radius $r_{\text{circ}}$ (Lubow & Shu 1975). If magnetic fields are present, the accretion mechanisms become much more complicated and controversial.

Ghosh & Lamb (1970a, 1979b) and Wang (1987) divided the accretion flow into four regions: an unperturbed disc, an outer transition zone, an inner transition zone and a magnetosphere. They argued that a diamagnetic solution was not viable, and instead had the field threading a large portion of the disc through turbulent diffusion and magnetic field reconnection with embedded small-scale fields in the disc. The structure of the outer transition region is basically the same as that of a standard accretion disc, with the matter essentially in Keplerian motion. In this region, the angular momentum transport switches from viscous stress to magnetic stress. In the inner transition zone, the magnetic stress changes the angular velocity of the material from Keplerian to corotational. In this narrow zone, the magnetic stress disrupts the accretion disc and the matter couples on to the magnetic field lines. Angular momentum balance arguments then yield an inner radius $R_c$ for the disc radius which is given approximately by

$$R_c \approx 0.52 r_{\mu}$$

(Ghosh & Lamb 1979a), where $r_{\mu}$ is the Alfvén radius of the white dwarf for spherical accretion, which is given by

$$r_{\mu} = \frac{27}{R_{\text{wd}}} \left( \frac{\mu}{10^{16} \text{ G cm}^3} \right)^{1/7} \left( \frac{M}{10^{16} \text{ g s}^{-1}} \right)^{-2/7} \left( \frac{M_{\text{wd}}}{M_\odot} \right)^{-1/7} \left( \frac{10^6 \text{ cm}}{R_{\text{wd}}} \right).$$

Here, $\mu$ is the magnetic moment of the white dwarf, $\dot{M}$ the accretion rate, and $M_{\text{wd}}$ and $R_{\text{wd}}$ are the mass and radius of the white dwarf respectively.

Scharlemann (1978), Aly (1980) and more recently Spruit & Taam (1990, hereafter ST) have presented a more complicated picture. They have argued that the manner in which matter couples on to field lines depends on the shape of the field lines near the inner edge of the disc, which in turn depends on assumptions about the conductivity of the disc. They present a model in which the magnetic field near the inner edge of the disc is distorted (‘pinched’) towards the white dwarf due to the presence of induced currents in the disc. The equatorial plane in the disc becomes a minimum of the gravitational potential. In order to reach the surface of the star, the gas, which is forced to flow along magnetic field lines, must have enough thermal pressure or must be able to cross field lines. There are two regions in the magnetosphere where mass can be supported against the gravitational or centrifugal forces. The first region consists of those points on the equatorial plane where the field lines are bent inward by the accretion disc. These field lines support the plasma against the gravitational force. The other region is the ‘lobe’ above the equatorial plane where the field lines are bent outward and support mass against the centrifugal force. In these two regions, the gas frozen on to field lines is in a stable equilibrium for movements along the field, but not for displacements across the field.

ST found that an interchange instability of the Rayleigh-Taylor type could efficiently transport mass across field lines. Through this instability, the plasma can drift into the magnetosphere, which is already corotating with the white dwarf. At a certain distance from the star, the field lines are no longer pinched inward by the disc; the matter becomes unstable to motion off the equatorial plane and starts to flow along magnetic field lines.

The physical mechanisms which determine the size of the annulus, $\Delta R_{\text{coup}}$, over which the matter starts flowing towards the surface of the star along undistorted magnetic field lines, are not well understood. For example, it is not clear whether or not matter penetrating into the magnetosphere consists of small and partially diamagnetic blobs as suggested by Aly & Kuipers (1990). In this model, $\Delta R_{\text{coup}}$ would depend on the time-scales over which these blobs remain diamagnetic in the magnetosphere of the white dwarf before they break up and diffuse via the Kelvin–Helmholtz instability (Aly 1980).

Given the various uncertainties and the lack of consensus between different workers in the field, we have chosen to treat $\Delta R_{\text{coup}}$ as a free parameter. We assume that the region over which the material threads on to undistorted magnetic lines extend from $R_{\text{c1}} = R_c$ to $R_{\text{c2}} = R_c + \Delta R_{\text{coup}}$. In our illustrative model we use $\Delta R_{\text{coup}} = 0.3 R_{\text{c1}}$ but we determine $\Delta R_{\text{coup}}$ from model fits when analysing individual systems.

3 THE MODEL

Our assumption is that the emission lines in MCVs mainly originate from material diverted above and below the orbital plane from the coupling region and which free-falls along magnetic field lines towards the white dwarf. The geometry of the model is the same as that outlined in FWT, but is repeated here for the reader’s convenience.

The origin $O$ of our coordinate system $Oxyz$ is the centre of the white dwarf; the $x$-axis coincides with the line of centres, and the $z$-axis is parallel to the rotation axis of the white dwarf. Consider a second coordinate system $Ox'y'z'$ which is also centred on the white dwarf, and whose $z'$-axis is parallel to the dipole axis and forms an angle $\theta$ with the $z$-axis. The $x'$-axis is in the plane $Oxz'$ (see Fig. 1). If $\theta^*$ is the polar angle of a general point $P$ measured with respect to $Oz'$ and $\psi^*$ is the azimuthal angle with respect to $Ox'$, then the magnetic field line passing through $P$ will have

$$r = r_m \sin^2 \theta^*, \quad \psi = \text{constant}$$

(e.g. Alfvén & Fälthammar 1963), where $r_m$ is the distance from $O$ to the point of intersection of the magnetic field line with the plane $Ox'y'$. If material is coupled in the orbital plane to a field line at a distance $R_c$ from the white dwarf and at a magnetic longitude $\psi^*$, the value of $r_m$ is

$$r_m = R_c(1 + \tan^2 \theta \cos^2 \psi^*).$$

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A particle flowing along a given field line has a velocity \( V \) which satisfies
\[
V = \sqrt{2[\Phi(r) - \Phi(R_e)]},
\]
where \( \Phi(r) \) and \( \Phi(R_e) \) are the potential energy at radius \( r \) and \( R_e \) from the white dwarf respectively.

If \( \phi_s \) is the spin phase and \( i \) the orbital inclination, the unit vector along the line of sight in the reference system \( Ox'y'z' \) is
\[
l = \left( \cos i \cos 2\pi\phi_s, -\sin i \sin 2\pi\phi_s, \cos i \right).
\]

The magnitudes of the radial velocity, \( V_r \), and transverse velocity, \( V_t \), for motion along a given field line are
\[
V_r = \frac{2V \cos \theta^*}{\sqrt{4 \cos^2 \theta^* + \sin^2 \theta^*}},
\]
\[
V_t = \frac{-V \sin \theta^*}{\sqrt{4 \cos^2 \theta^* + \sin^2 \theta^*}},
\]
where \( V \) is defined in equation (4). The radial and transverse unit vectors are given by
\[
e_r = (\cos \theta^* \sin \theta + \sin \theta^* \cos \psi^* \cos \theta, \\
\sin \theta^* \sin \psi^*, \cos \theta \cos \theta^* - \sin \theta \sin \theta^* \cos \psi^*)
\]
\[
e_t = (\cos \theta^* \cos \theta \cos \psi^* - \sin \theta^* \sin \theta, \\
\cos \theta^* \sin \psi^*, -\cos \theta \sin \theta^* - \sin \theta \cos \theta^* \cos \psi^*).
\]

In the \( Ox'y'z' \) reference system, a particle is then at position \( r = re \),

\[
\]

\[
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\]

where \( r \) is given by equation (2) and the components of the velocity are
\[
V_r = V_r e_r, \qquad V_t = V_t e_t,
\]
so that the total infall velocity projected on to the line of sight is
\[
V_l = (V_r + V_t) \cdot l.
\]

The projection of the spin velocity along the line of sight is
\[
V_s = (\omega_s \times r) \cdot l,
\]

where
\[
\omega_s = \left( 0, 0, \frac{2\pi}{P_s} \right).
\]

Therefore, the radial velocity given by these two components is
\[
V_r = V_r + V_s.
\]

The radial velocity is calculated as a density-weighted mean of \( V_r \) over the region where the emission lines are formed.

The orbit velocity component in AM Her Type IPs is always much smaller than the infall velocity, so this was neglected in FWT. In IPs, this component can become very important, since in some systems the average infall velocity is observed to be as low as a few tens of kilometres per second. The orbital component projected on to the line of sight is
\[
V_{\text{orb}} = \frac{2\pi q}{P_o (q + 1)} \left( \frac{M_{\text{out}} (q + 1) P_o^2 G}{4\pi^2} \right)^{1/3} \sin i \cos 2\pi \phi_0,
\]

where \( \phi_0 \) is the orbital phase. Therefore, the total radial velocity is given by
\[
V_{\text{tot}} = \langle V_r \rangle + V_{\text{orb}}.
\]

The quantity \( \langle V_r \rangle \) is calculated over the emission region assuming that the density varies in a manner consistent with the equation of continuity. If \( N_0 \) is the number density on the equatorial plane at a distance \( r = R_c \), we find
\[
N = N_0 \left( \frac{R_c}{r} \right)^{5/2} \left( \frac{4r_m - 3r}{4r_m - 3R_c} \right)^{1/2}.
\]

We have assumed that the profiles of the emission lines are Gaussians, so that \( \sigma_{\text{FWHM}} = 2.354\Delta V \), where \( \Delta V \) is the velocity dispersion obtained from the calculations.

Our modelling procedure implicitly assumes optically thin line emission. We expect this assumption to be valid for the higher Balmer lines. However, the Balmer decrement observed in some IPs implies that part of the region where the lower Balmer lines are formed is partially optically thick. Since the number density \( N \) increases rapidly with decreasing \( r \) (equation 15), this region is likely to occur close to the white dwarf surface.

Finally, in order to calculate eclipses by the accretion disc, we need to estimate the distance \( r_t \) from the white dwarf to the inner Lagrangian point. This is obtained by solving
\[
(1 + q)x^5 - (2 + 3q)x^4 + (1 + 3q)x^3 - x^2 + 2x - 1 = 0
\]
(Kopal 1968). Here, \( x = r_t/a \), where \( a \) is the binary separation and \( q = M_{\text{sec}}/M_{\text{wd}} \), where \( M_{\text{sec}} \) is the mass of the secondary star.
4 DISCUSSION

In AM Her, the material escapes from the \( M \) dwarf near the inner Lagrangian point and free-falls towards the white dwarf forming the so-called 'acceleration region' (cf. Liebert & Stockman 1985). Close to some radial distance, the material is captured on to field lines through a coupling region without the formation of a disc. FWT showed that in the ideal case, where there is no dissipation of energy in the coupling region, accretion can occur along any magnetic field line connecting the coupling region to the white dwarf, provided it is wholly contained within the Roche lobe of the white dwarf. Furthermore, they showed that if the coupling occurs sufficiently close to the white dwarf, there will be closed field lines passing through the coupling region which will lie within the critical Roche lobe. Accretion can then occur on to two regions on the white dwarf surface located at both of the foot points of these closed field lines.

In contrast, in IPs, a significant fraction of the energy of the initially free-falling stream is dissipated by viscous forces in a partial accretion disc. Furthermore, ST found that, when the dipole axis was inclined with respect to the spin axis (\( \theta \neq 0 \)), matter located near the inner edge of the disc at first drifts inward across the field, via interchange instability, till it reaches undistorted magnetic field lines. At this point, the plasma starts to flow along the field towards the star. However, accretion on to the pole tilted toward the disc is more likely, since a high potential barrier inhibits the flow towards the other pole. Consequently, accretion can only occur on to one of the two foot points (the 'downhill' foot point) of a closed field line which connects the coupling region to the white dwarf. A difference from AM Her is that the coupling region consists of a complete annulus of the disc extending 360° in rotational azimuth and defined by \( R_c \leq r < R_2 \).

Close to the white dwarf, accretion in IPs is thus likely to occur along field lines which form two 'accretion curtains', one above and the other below the orbital plane. Each of the curtains extends over 180° in magnetic (and rotational) longitude and consists of 'downhill' field lines which connect the coupling region to the surface of the white dwarf.

It is clear that not all the material in the two curtains can give rise to emission lines. For instance, material that is too

Figure 2. Theoretical curves obtained for the radial velocity and FWHM for different values of orbital inclination \( i \), colatitude of the dipole axis \( \theta \), coupling radii \( R_c \), and for fixed values of latitudinal (\( \Theta_f = 40° \) and \( \Theta_r = 0° \)) and azimuthal (\( 0° \leq \psi \leq 360° \)) extension of the line-forming region. Thin line: contribution from upper accretion curtain only. Bold line: contribution from both accretion curtains, with eclipses by the disc taken into account. Top row: \( i = 25°, \theta = 5° \). Bottom row: \( i = 25°, \theta = 25° \). From left to right: \( 3 < R_c < 4, 6 < R_c < 8, 9 < R_c < 12 \) and \( 12 < R_c < 16 \).
close to the shock near the white dwarf surface may have a high degree of ionization caused by the X-ray flux and may not, therefore, contribute to the line intensity. It is not obvious whether those parts of the accretion curtains near the equatorial plane may give rise to emission lines. If ‘lobes’ of matter are present (as predicted by the calculations of ST), then the lower parts of the curtains may be obscured by the matter contained in these lobes.

Accordingly, we cut off in magnetic colatitude the upper accretion curtain (closest to \( \theta^* = 0^\circ \)) by \( \theta^* = \Theta_1^* \) and \( \theta^* = \Theta_2^* (\theta, \psi^*) + \Theta_2 \) and the lower accretion curtain by \( \theta^* = \Theta_1^* (\theta, \psi^*) + \Theta_2 \) and \( \theta^* = 180^\circ - \Theta_1^* \), where \( \Theta_1^* (\theta, \psi^*) \) is the angle between the magnetic axis and the orbital plane. The values given to these parameters depend on each individual system and they are chosen to fit observational data. Thus, four parameters specify the location and extent of each line-emission region: \( R_{c1}, R_{c2}, \Theta_1^* \) and \( \Theta_2 \) (the azimuthal extent of each region is \( \Delta \psi^* = 180^\circ \)).

The geometry that we have just presented for the two curtains describes a symmetric situation where for each mass element with \( \{ \theta^*, \psi^*, R_c \} \), there is a mass element with \( \{ 180^\circ - \theta^*, 180^\circ + \psi^*, R_c \} \) which makes an equal and opposite contribution to the radial velocity. However, in general we do not expect total ‘radial velocity cancellation’, since while the accretion curtain above the orbital plane is always visible, the other may be partially or completely eclipsed by the disc, depending on phase. In the remainder of this paper, we shall often use the term ‘velocity cancellation’ when similar quantities of matter are blueshifted and redshifted, yielding a total radial velocity average value close to zero.

In our model, we take full account of eclipses by the disc but neglect eclipses of one curtain by the other, since these are optically thin. We assume that the disc extends from \( R_{c1} \) to \( 0.7 R_*, \) the outer edge being determined by the tidal effects. This is probably an upper limit for the extension of a disc in quiescence [see, for example, the theoretical work of Whitehurst (1988) and Whitehurst & King (1991) and the observations of Z Cha by O’Donoghue (1986) and of HT Cas by Wood, Horne & Vennes (1992)].

We have developed several models to investigate the behaviour of the radial velocity and velocity dispersion curves caused by a change in the viewing angle, dipole inclination, location of the coupling region and angular distance from the magnetic poles. The results are illustrated in Figs 2–9. In each radial velocity panel, we present the total contribution from both curtains with eclipses taken into account (bold line) and the contribution from the accretion curtain which is above the orbital plane (thin line). The curves shown

![Figure 3](https://academic.oup.com/mnras/article-abstract/260/1/149/1010875)

**Figure 3.** As Fig. 2, but for \( i = 25^\circ \) and \( \theta = 45^\circ \) (top row) or \( \theta = 65^\circ \) (bottom row).
have been obtained using $\Theta^i = 40^\circ$, $\Theta_z = 0^\circ$, $M_{\text{wd}} = 0.7 \, M_\odot$, $M_{\text{sec}} = 0.3 \, M_\odot$, $P_0 = 3.75 \, \text{h}$ and $P = 22.5 \, \text{min}$ ($P_0 = 0.1 \, P$, as it is observed in most IPs). With the above values, we have $R_{\text{wd}} = 7792 \, \text{km}$ (obtained by assuming Nauenberg’s 1972 mass–radius formula), $a = 109 \, R_{\text{wd}}$ and $r_c = 64 \, R_{\text{wd}}$, so that the partial accretion disc extends up to $45 \, R_{\text{wd}}$ from the centre of the white dwarf.

For the simple case of accretion along a single field line with emission occurring over an arc length between $r$ and $r + \Delta r$, the velocity $V$ and dispersion $\Delta V$ (neglecting spin and orbital contributions) are given approximately by

$$V \approx V_n \cos \beta,$$
$$\Delta V = \frac{1}{2} V_n \frac{|\Delta r|}{r} \cos \beta$$

(FWT), where $V_n$ is the free-fall velocity at radius $r$ and $\beta$ is the angle that the tangent to the field line makes with the line of sight. These expressions show that if $\beta$ does not cross $90^\circ$, the velocity dispersion and the radial velocity will have extrema at $\beta_{\text{min}}$ and $\beta_{\text{max}}$. If, on the other hand, the viewing geometry is such that $\beta$ crosses $90^\circ$, the velocity dispersion curves will have additional local minima at these phases (two per spin cycle). In this simple approximation, both the velocity dispersion and the radial velocity will be zero at these minima.

In IPs, each accretion curtain consists of field lines which extend over $180^\circ$ in magnetic longitude $\psi^i$. However, we find that the single field line approximation can still be used to understand qualitatively the main effects, provided we select a field line that is centrally placed in the curtain (i.e. passing through the median value of $\psi^i$). Additional effects which must be considered are the sinusoidal variations of the velocity at the orbital period and the changes that are introduced by the spin term (equation 11) at the spin period. The latter are seen as asymmetries in the radial velocity and velocity dispersion curves and result from the fact that the spin term gives velocity variations that are $90^\circ$ out of phase with the velocity variations caused by flow along a field line.

First, we consider the properties of the accretion curtain which is seen most directly by the observer. The results in Figs 2–9 (thin line) show the effects of changing the orbital inclination $i$ while keeping the dipole inclination $\theta$ and disc parameters ($R_{c1}$, $R_{c2}$) fixed. In general, the oscillations in radial velocity at the spin period show a larger excursion.
towards negative than positive values, since the line emitting material in the accretion curtain which is facing the observer is mostly moving towards the observer at all phases. The radial velocity amplitude decreases as $i$ decreases, reaching zero for face-on viewing of the disc ($i=0$), as expected. We also note that the radial velocity oscillates symmetrically about zero as $i \rightarrow 90^\circ$.

Next, we consider the effects of changing the dipole inclination $\theta$ for fixed $i$ and disc parameters. At low values of $\theta$, $\beta$ does not cross $90^\circ$ and the phase of maximum velocity dispersion corresponds to the phase of the largest negative value of radial velocity in a given spin cycle. For sufficiently high values of $\theta$ ($\geq 30^\circ$), $\beta$ crosses $90^\circ$ twice and secondary minima appear in the velocity-dispersion curves on either side of the maximum. These minima are not always placed symmetrically in phase above the maximum because of the contributions from the spin term as discussed previously.

Figs 2–9 also show the effects of changing $R_{el}$ while keeping the other parameters fixed for the main accretion curtain. As $R_{el}$ is increased, both the velocity and the velocity amplitude decrease, since the emitting volume is shifted away from the white dwarf to regions of lower free-fall velocity. However, in general, these changes are small. Larger changes occur when allowance is made for the fact that, at large $R_{el}$, the second accretion curtain, which lies below the orbital plane, will become (partially) visible.

The second accretion curtain, which is located below the orbital plane, can play a dominant role in determining the velocity characteristics of the system if it can be seen either through the central hole of the truncated disc, or from below the disc, or both. This is illustrated in the series of models in Figs 2–9, where the bold line, corresponding to the radial velocity variations from both accretion curtains, should be compared directly with the thin line, corresponding to the curtain located above the orbital plane only.

Consider first the models with $i=88^\circ$ (Fig. 9) which show a dramatic reduction in velocity amplitude as $R_{el}$ is increased from $3R_{wd}$ to $6R_{wd}$. At $R_{el}=3R_{wd}$, the second accretion curtain is only just visible through the hole in the centre of the partial accretion disc. Therefore, the radial velocity modulations from the curtain above the orbital plane coincide with the total radial velocity, and the two curves shown overlap for $R_{el}<6$ and $\theta \leq 45^\circ$. As $R_{el}$ is increased, both curtains extend to greater heights above and below the orbital plane and the second curtain becomes visible also from below the disc. As a result, there is now almost total cancel-
lation in radial velocity. The velocity dispersion remains high, although reduced in magnitude in comparison to the model with $R_t = 3 R_{wd}$. These effects are illustrated in Figs 10(a)–(c), where we show the visible portions of the two funnels for a series of models with $i = 85^\circ$ and $\theta = 45^\circ$. The diagrams for $\phi > 0.5$ are mirror images of those for $\phi < 0.5$ when they are reflected about the line of projection of the spin axis on to the plane of the sky, and so are not shown.

Velocity cancellation is also very effective at low viewing angles ($i \leq 40^\circ$), since most of the second accretion curtain is visible at all phases through the hole of the truncated disc. This is illustrated in the models in Fig. 11, which were obtained using $i = 25^\circ$, $\theta = 5^\circ$, $\Theta_1 = 10^\circ$ and $\Theta_2 = 0^\circ$.

The curtain models with $i = 45^\circ$ and $65^\circ$ show a smaller reduction in velocity amplitude, since the second curtain can never be seen from below the orbital plane and only partially through the hole of the truncated disc. Therefore the velocity cancellation is not as complete.

We have also investigated the radial velocity and FWHM behaviour caused by changing the angles $\Theta_1^*$ and $\Theta_2^*$, whilst keeping the other parameters fixed (not shown). We have found that for decreasing values of $\Theta_1^*$, both the mean radial velocity and the velocity dispersion increase. These results can be explained as follows. The velocity of the matter along a given magnetic field line is very low when it is close to the equatorial plane, but increases as $\theta^*$ decreases. On average, as the angle $\Theta_1^*$ diminishes, increasingly large portions of the emitting gas move closer to the white dwarf and reach the maximum height above the orbital plane. These effects lead to higher radial velocities. If both curtains are visible, the mean radial velocity could still be very low, but the velocity dispersion would be higher. This effect is of particular importance when locating the line-emission region, since very high values for the velocity dispersion suggest that the emitting volume is located closer to the white dwarf and that a large fraction of the material has reached the maximum height above the plane. The same effects result from increasing $\Theta_2^*$ at fixed $\Theta_1^*$ when we progressively exclude, from the emitting volume, the low radial velocity components arising from material close to the equatorial plane.

5 APPLICATION TO TX COL

In AM Her, the orbital and dipole inclinations can be determined by polarimetry, so that models of the type discussed in this paper can be used to locate the line-emission regions.
unambiguously. In IPs, these two quantities are also free parameters so that the models are not as strongly constrained.

The system TX Col ( = 1H0542 - 407) was discovered by Tuohy et al. (1986) to be the optical counterpart of an HEAO-1 X-ray source. Their EXOSAT X-ray data showed the presence of a strong and nearly sinusoidal pulsation at a period near 32 min, which was interpreted as the spin period of the white dwarf. A second period at 6.2 ± 0.5 h was attributed to the orbital rotation of the system. Another period near 35 min was explained as a beat phenomenon between the aforementioned periodicities caused by X-rays reprocessed in a region fixed in the orbital frame of reference.

In 1989, Buckley & Tuohy reported time-resolved spectroscopic and photometric observations of TX Col confirming the presence of all the periodicities detected in the X-rays. Buckley & Tuohy (1989) fitted the emission lines with a single Gaussian profile FWHM = 1200 km s⁻¹, although for their higher signal-to-noise ratio data, it was necessary to introduce a second broad Gaussian component with FWHM = 2500 km s⁻¹ in order to fit the wings of the Balmer lines. We identify this component as originating from material located much closer to the white dwarf than most of the material that produces the lines. Their radial velocity study showed sinusoidal velocity variations at the spin period which were characterized by a very low velocity semi-amplitude of $K \approx 20$ km s⁻¹ superimposed on the orbital modulation of $K \approx 50$ km s⁻¹.

Our model has been applied to the data for TXCol obtained by Buckley & Tuohy (1989). The low values of the semi-amplitude of the spin component and the absence of an eclipse by the secondary star suggest low orbital inclinations. Our best-fitting model is shown in Fig. 12. In this model, $M_{wd} = 0.7$, $M_{sec} = 0.5$, $i = 25 \pm 5°$, $\theta = 10° \pm 5°$, $\Theta_1 = 40°$, $\Theta_2 = 15°$ for the curtain located above the orbital plane and $\Theta_1 = 45°$, $\Theta_2 = 10°$ for the other curtain. Furthermore, we have used $R_{c1} = 9 R_{wd}$ and $R_{c2} = 12 R_{wd}$. The uncertainties are about ±2° for $\Theta_1$ and $\Theta_2$ and of about 2 $R_{wd}$ for the threading radius. The different values of $\Theta_1$ and $\Theta_2$ for the two curtains may be explained by a small difference in the magnetic field strength of the two polar caps caused by a dipole which is slightly offset from the centre of the white dwarf. The model is less sensitive to location and extent of the threading radius, although values of $R_{c1} \leq 6 R_{wd}$ have to be excluded, since they give velocity dispersions which are too high.
6 APPLICATION TO EX HYDRAE

EX Hydrae is an IP with an orbital period of 98 min and a white dwarf spin period of 67 min. This system is particularly interesting, since: (i) it is one of the few IPs with spin and orbital periods of comparable length; (ii) it is the only IP with an orbital period below the catalytic variable period gap; and (iii) it is the only eclipsing IP. Point (iii) is very important to our analysis since knowledge of the orbital inclination places a strong constraint on the radial velocity modelling.

Hellier et al. (1987) and Katschuk et al. (1987) conducted a radial velocity study of EX Hya. Their analysis showed that all the emission lines were very broad with wings that extended up to ±3500 km s\(^{-1}\), thus giving FWHM of \(\approx 3700\) km s\(^{-1}\). Unfortunately, no radial velocity curves modulated at the spin period are in the literature, although from fig. 6(a) of Hellier et al. (1987), it appears that the amplitude of these variations must be very low (\(\leq 40\) km s\(^{-1}\)).

We show in Fig. 13 the best fit to some data for EX Hya obtained by Hellier et al. (1987). The model has \(M_{\text{wd}} = 0.95\), \(M_{\text{sec}} = 0.13\), \(i = 87 \pm 3^\circ\), \(\theta = 3 \pm 2^\circ\), \(\Theta_1 = 35 \pm 5^\circ\), \(\Theta_2 = 25^\circ \pm 5^\circ\), \(R_{c1} = 4.5 \pm 0.5 R_{\text{wd}}\) and \(R_{c2} = 5.5 \pm 0.5 R_{\text{wd}}\). The low-amplitude velocity variations modulated at the spin period place a strong constraint on the values of the orbital inclination and colatitude of the magnetic dipole, since lower values of \(i\) or higher values of \(\theta\) will give radial velocity oscillations that are too high. The value of the inner edge of the threading region is also fixed by the very high velocity dispersion observed, so that values of \(R_{c1} > 5 R_{\text{wd}}\) are not likely to occur.

We remark that the value of \(i\) employed by us to interpret the data is somewhat higher than that calculated by Hellier et al. (1987) through a study of the X-ray eclipse of EX Hya. In fact, according to the table published by Chanan, Middleton, 

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et al. (1976), which tabulates mass ratio as a function of the eclipse half angle and orbital inclination, our inclination value leads to an eclipse duration of \(\approx 7\) min FWHM against the 2.59 min found by Rosen et al. (1991) through Ginga observations of the eclipse of EX Hya.

In order to explain this discrepancy, we need either to assume that the radial velocity variations are much larger than they appear to be in the published spectroscopic data of EX Hya (e.g. \(\geq 200\) km s\(^{-1}\)), or that the accretion disc in
EX Hya is optically thin and both accretion curtains are always visible at all phases. In the latter case, cancellation effects would account for the very low amplitude of the velocity variations at the spin period. The second explanation is likely to be the correct one since it is consistent with the low accretion rate in EX Hya deduced by Rosen, Mason & Córdova (1988) (= 3 x 10^{15} g s^{-1}). This suggests that, at least during quiescent states, the accretion disc in EX Hya is marginal and may only consist of a tenuous and optically thin accretion ring around the white dwarf.

If we now employ the stellar masses deduced by Hellier et al. (1987), namely \( M_{\text{wd}} = 0.78 \ M_\odot \) and \( M_{\text{sec}} = 0.13 \ M_\odot \), and we take \( M = 3 \times 10^{15} \text{ g s}^{-1} \), we obtain from equation (1) an upper limit for the magnetic moment of the white dwarf \( \mu = 6 \times 10^{31} \text{ s}^{-1} \), which yields a magnetic field \( B_\ast = 0.15 \text{ MG} \).

7 CONCLUSIONS

We have presented a theoretical model for the radial velocity and velocity dispersion behaviour in IPs which assumes the presence of a partial accretion disc and a dipolar magnetic field, the field determining the accretion pattern beyond a certain distance from the white dwarf. The line-emitting material is located in two accretion curtains, one above and the other below the orbital plane, each extending 180° in rotational azimuth. In these accretion curtains, the gas is frozen on to magnetic field lines and flows toward the surface of the white dwarf.

The behaviour of the radial velocity and velocity dispersion curves depends critically on whether only one or both accretion curtains are visible at any given phase. We can have a view of the second funnel either through the hole in the middle of the disc at low inclinations, or from below the disc when the system is seen close to edge on, or via both at some intermediate viewing angle. Velocity cancellations occur when the second curtain is either wholly or partially visible. This suggests that systems showing small radial velocity amplitudes and large dispersions are either eclipsing binaries or systems seen nearly face-on.

We have applied our model to two IPs: TX Col and EX Hya. We have shown that we can provide an acceptable fit to the radial velocity and velocity dispersion data of these
Figure 10. Visible portions of the two accretion curtains obtained using $i = 88°$, $\theta = 45°$, $\Theta_F = 0°$ and $\Theta_2 = 0°$ at four different phases for different values of coupling radii: (a) $R_C = 3R_{\text{adv}}$; (b) $R_C = 6R_{\text{adv}}$; (c) $R_C = 9R_{\text{adv}}$. The diagrams for $\phi > 0.5$ are mirror images of those shown.
systems based on our model. However, the location of the line-emitting region is not strongly constrained unless either $i$ or $\theta$ is known from other considerations.

We have found that the material accretes on to the white dwarf between 9 and 12 $R_{\text{wd}}$ from the primary in TX Col and between 4 and 6 $R_{\text{wd}}$ in EX Hya.

ACKNOWLEDGMENTS

We wish to thank Simon Rosen for a critical reading of the manuscript and Rob Whitehurst for useful comments. Lilia Ferrario acknowledges the UK SERC for a Research Associateship and the Australian Research Grants Scheme for the award of an ARC Research Associateship.

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