On the negative polarization of light scattered by subwavelength regolithic grains

L. O. Kolokolova,¹ M. I. Mishchenko¹* and M. Wolff²

¹The Main Astronomical Observatory of the Ukrainian Academy of Sciences, Goloseevo, Kiev, 252127, Ukraine
²1600 Nelson Avenue, Manhattan Beach, CA 90266, USA

Accepted 1992 July 17. Received 1992 June 29; in original form 1992 April 2

ABSTRACT
Recent laboratory polarimetric measurements of light scattering by layers of alumina grains of different size are used to examine the relevance of the Wolff & Dollfus theory of negative polarization, which has been developed to describe light scattering by regolithic grains of arbitrary size. It is demonstrated that, while reproducing satisfactorily the main polarization characteristics of light scattering by grains much greater than the wavelength, this theory does not describe the negative polarization of light measured by Geake & Geake for subwavelength-sized grains. The smallest grains exhibit a polarization feature that may be called the ‘polarization opposition effect’. This feature requires an alternative explanation and may be due to the so-called weak localization of photons in discrete random media.

Key words: polarization – scattering – minor planets – planets and satellites: individual: Saturn.

1 INTRODUCTION
Measurements of the polarization of scattered light $P$ as a function of the phase angle $V$ have been used for a long time as a remote-sensing technique for studying Solar system bodies without atmospheres. Values of polarization at minimum ($P_{\text{min}}$) and maximum ($P_{\text{max}}$), the corresponding phase angles ($V_{\text{min}}$ and $V_{\text{max}}$), as well as the phase angle at which the polarization changes sign ($V_{c}$) have been found to contain important information about textural and optical properties of the surface (e.g. Geake & Dollfus 1986).

With the development of theories of light scattering by particulate and rough surfaces (e.g. Wolff 1975, 1980; Lumme, Bowell & Zellner 1980; Kolokolova 1990), comparison of observations with results of theoretical computations for more or less realistic scattering models has become possible. Recently, Geake, Geake & Zellner (1984) tested some of these theories against their laboratory data and concluded that the Wolff theory best fitted the measurements and satisfactorily explained all the observed effects for grains that were large compared to the wavelength of light. Much smaller grains may also be present, however, on the surfaces of atmosphereless bodies. To take into account possible small-particle effects, the Wolff theory has recently been extended by Wolff & Dollfus (1990) by introducing a Rayleigh-type scattering in the surface model. Almost simultaneously, an extensive set of laboratory polarimetric measurements has been reported by Geake & Geake (1990). Their measurements cover a wide range of particle sizes down to a tenth of a wavelength, which makes them ideally suited for testing purposes. It is the aim of this paper to compare the Wolff & Dollfus theory with the Geake & Geake data in order to test whether the theory can successfully reproduce polarization correlations that have been observed for very fine grains. We also briefly discuss an alternative explanation of the observed effects.

2 ANALYSIS
The measurements of Geake & Geake (1990) for grains smaller than the wavelength exhibit the following parameter relationships:

$$V_{0} \propto d/\lambda, \quad V_{\text{min}} \propto d/\lambda, \quad P_{\text{min}} \propto \log(d/\lambda),$$

(1)

where $d$ is the mean grain diameter and $\lambda$ is the wavelength. Unfortunately, when varying the mean grain size, Geake & Geake did not measure other structural parameters of the particulate scattering medium, in particular its porosity (i.e. the fraction of the volume occupied by the grains). Therefore it is not clear whether the correlations of equation (1) may contain, in fact, some indirect effects of porosity or other structural parameters. To circumvent this uncertainty, it is

*Present address: NASA Goddard Institute for Space Studies, Hughes STX Corporation, 2880 Broadway, New York, NY 10025, USA.

© Royal Astronomical Society • Provided by the NASA Astrophysics Data System
more convenient to use the correlations
\[ V_{\text{min}} \propto V_0, \quad P_{\text{min}} \propto \log V_0, \quad P_{\text{min}} \propto \log V_{\text{min}} \]  
(see equation (1) and figs 7 and 8 of Geake & Geake 1990).

Our aim is to test whether the Wolff & Dollfus (1990) theory does reproduce the relationships of equation (2). To this end, we have carried out an experimental design which is known as a 2^k-design (for details, see section 3 of Koloklova 1990). This method is especially suited for analysing multiparameter problems and, avoiding excessive computer calculations, enables one to separate essential parameters from non-essential ones and to find correlations between different quantities.

The main parameters of the experimental design are the following:

(i) Factors (or basic model parameters) - in our case, real refractive index, MR, imaginary refractive index, ML, width-to-depth ratio of surface holes, WID, fraction of incident light scattered once, EXT, and mean grain diameter in wavelengths, XTL = d/\lambda (Wolff & Dollfus 1990). The factors are quantities that specify physical properties of the scattering medium and form the input for computing theoretically some characteristics of the scattered light, i.e. optimization parameters.

(ii) Optimization parameters (i.e. quantities that are calculated theoretically for given values of the model parameters and whose behaviour under the influence of changing model parameters is studied) - in our case \( P_{\text{min}}, V_{\text{min}} \) and \( V_0 \).

(iii) Response functions which connect the optimization parameters with the factors.

In our calculations, we used response functions of the form
\[ Y = a_0 \left[ 1 + \sum_{i=1}^{k} a_i x_i + \sum_{i=k+1}^{C_k} a_i x_i x_{i+k} + \sum_{i=k+1}^{C_k} a_i x_i x_{i+k+1} + \ldots \right], \]
where \( C_k \) is the number of combinations of \( k \) things taking \( n \) at a time. Here, \( Y \) is an optimization parameter, the \( x_i \) are factors, and \( k \) is the number of factors (in our case \( k = 5 \)). The coefficients \( a_i \), if known, may be used to evaluate the influence of each factor or product of factors on the optimization parameter. For example, the larger the influence of the factor \( x_i \) on \( Y \), the larger should be the corresponding coefficient \( a_i \). If \( a_i > 0 \), the optimization parameter increases with the factor \( x_i \), while if \( a_i < 0 \) it decreases with \( x_i \).

To determine initially unknown coefficients \( a_i \) for each of the optimization parameters, we proceeded as follows. For each factor \( x_i \), two values were chosen: \( x_i^h \) (high level) and \( x_i^l \) (low level). For \( k = 5 \) factors, \( 2^5 = 32 \) values of the optimization parameter \( Y \) were computed from the Wolff & Dollfus theory and a system of \( 2^5 = 32 \) equations (3) was thus obtained. By solving this system numerically, the unknown coefficients \( a_i \) were determined.

The choice of factor levels is very important. The range of factor levels should be real and should contain only those factor values for which the optimization parameter exists. The high and low levels should be nearly equidistant from the most probable value of the factor. We found that, in some cases, these requirements almost uniquely determined the choice of the factor levels. For example, it appeared that, for \( EXT > 0.2 \), the polarization could sometimes be positive for all phase angles. We therefore used the pair of \( EXT \) levels \([0.001, 0.2] \). For \( MR \) we used a constant value, \( MR = 1.77 \), corresponding to alumina. For \( ML \) and \( XTL \) we used the pairs \([0, 0.01] \) and \([0.1, 1] \), respectively. For \( WID \) we chose the pair \([0.5, 8] \) and replaced this factor by its logarithm, \( L\text{WID} = \ln(W\text{ID}) \). Thus the equidistant value of the factor \( L\text{WID} \) is at \( W\text{ID} = 2 \), which corresponds to the mean inclination of surface microfacets of about 45° (e.g. Lumme et al. 1985).

Our calculations have resulted in the following response functions, in which only the most significant terms are included (they determine \( P_{\text{min}} \) with an accuracy of 0.05 per cent and \( V_{\text{min}} \) and \( V_0 \) with an accuracy of 0.5%):

\[ V_0(\text{deg}) = 17.5(1 - 0.31 \; EXT - 0.80 \; L\text{WID} + 0.25 \; EXT \times L\text{WID}), \]
\[ V_{\text{min}}(\text{deg}) = 7.5(1 - 0.27 \; EXT - 0.80 \; L\text{WID} + 0.21 \; EXT \times L\text{WID}), \]
\[ P_{\text{min}}(\text{per cent}) = 1.10(-1 - 0.04 \; M\text{I} + 0.40 \; E\text{XT} + 0.98 \; L\text{WID} + 0.16 \; X\text{TL} - 0.39 \; E\text{XT} \times L\text{WID} - 0.15 \; X\text{TL} \times L\text{WID} + 0.06 \; X\text{TL} \times E\text{XT} + 0.06 \; X\text{TL} \times L\text{WID} \times E\text{XT})]. \]
Here, the factors are replaced by dimensionless encoded variables as follows:
\[ x \rightarrow (2x - x^h - x^l)/(x^h - x^l), \]
Thus \( x \in [-1, 1] \).

One easily sees that these response functions (4–6) are consistent with only the first of the three correlations of equation (3). Indeed, even the signs of the coefficients before the factors \( E\text{XT} \) and \( L\text{WID} \) are different for \( V_0 \) and \( V_{\text{min}} \), on the one hand, and \( P_{\text{min}} \), on the other. We must therefore conclude that the Wolff & Dollfus theory should not be used in calculations for particles smaller than the wavelength.

At the same time, it should be noted that we have carried out an analogous experimental design for coarser grains and have found that the Wolff & Dollfus theory successfully reproduces the trends
\[ V_0 \rightarrow \text{constant}, \quad V_{\text{min}} \rightarrow \text{constant}, \quad P_{\text{min}} \rightarrow \text{constant}, \]
which were observed by Geake & Geake (1990) for grains that are much greater than the wavelength. This result supports the conclusion of Geake et al. (1984) that Wolff’s explanation of negative polarization works well for coarser-grained surfaces.

3 DISCUSSION
The results of the preceding section demonstrate that the Wolff & Dollfus theory, though containing Rayleigh scatterers, does not accurately describe the negative polarization of light produced by subwavelength-sized grains. This apparently may be explained by the fact that the theory does not take into account effects of coherence which may be important near the backscattering direction. The negative polarization observed for the smallest grains is characterized by the so-called ‘polarization opposition effect’. Specifically, polarization, being zero at zero phase angle, becomes
appreciably non-zero and sometimes reaches its maximum negative value (i.e. $P_{\text{min}}$) at a phase angle of a degree or even a few tenths of a degree. From laboratory-controlled experiments and theoretical computations (e.g. Wolf et al. 1988; van der Mark, van Albada & Lagendijk 1988), it is known that such grains also exhibit a photometric opposition effect of roughly the same angular width, which is due to the so-called weak localization of photons in discrete random media (or coherent backscattering mechanism). It may therefore be reasonable to suggest that this weak localization of photons is likely to explain the polarization opposition effect as well.

Both sharp photometric and polarimetric opposition effects have been observed at visible wavelengths for Saturn’s rings (Franklin & Cook 1965; Dollfus 1979; Johnson et al. 1980). This led Johnson et al. to the conclusion that both phenomena are due to light-scattering processes that occur within optically active regolithic layers of individual ring particles and are not due to multiple reflections of light and mutual shadowing by different particles. Recently, Mishchenko & Dlugach (1992) have shown that weak localization is a likely explanation of the photometric opposition effect of Saturn’s rings. Specifically, they assumed that macroscopic particles of Saturn’s rings are covered with a layer of small water ice grains, and demonstrated that both the width and amplitude of the opposition effect are consistent with theoretical calculations of weak localization for effective grain radii of a few tenths of a micron. We therefore suggest that the same submicron-sized ice grains produce the polarization opposition effect via the coherent backscattering mechanism.

Unfortunately, the vector theory of weak localization is still far from being completed and does not enable one to calculate quantitatively the angular profiles of the photometric and polarization opposition effects for realistic scattering models. Nevertheless, both laboratory-controlled experiments for latex particles (van Albada, van der Mark & Lagendijk 1988) and theoretical calculations for point-like scatterers (van Albada & Lagendijk 1987) demonstrate that weak localization can result in both photometric and polarization opposition effects. In the measurements of van Albada et al. and the calculations of van Albada & Lagendijk, the scattering medium was illuminated by a fully linearly polarized, perpendicularly incident beam of light, and the intensity of the reflected light was measured for the copolarized and cross-polarized components in two scattering planes, the first one being parallel (hereafter subscript $\parallel$) and the second one being perpendicular (hereafter subscript $\perp$) to the incident polarization. Weak localization results in backscattering peaks in the intensity profiles for both the copolarized and cross-polarized components. It was found that, in the vicinity of the backscattering direction, the cross-polarized intensity is the same for both scattering planes (i.e. $I_{\text{cross-polar,}} \parallel = I_{\text{cross-polar,}} \perp$), while the intensity profile for the copolarized component is spatially anisotropic. Specifically, whereas in exactly the backscattering direction (i.e. at zero phase angle) $I_{\text{copolar,}}(0) = I_{\text{copolar,}}(0)$, at a small phase angle $\nu$ the component $I_{\text{copolar,}}(\nu)$ becomes appreciably greater than the component $I_{\text{copolar,}}(0)$, where $\nu$ is roughly equal to the half-width at half-maximum of the opposition peak. It is obvious that, in the case of unpolarized incident light, this would result in the so-called polarization opposition effect.

A transparent qualitative explanation of the spatial anisotropy was given by van Albada et al. (1988). According to their explanation, the spatial anisotropy and, therefore, the polarization opposition effect can be observed only for the smallest, nearly Rayleigh-scattering grains. For larger grains, the effects should disappear. This was indeed observed by van Albada et al. (1988) and Geake & Geake (1990).

Finally we note that, in addition to discrete random (power-like) media, the coherent backscattering enhancement has also been observed for random rough surfaces (e.g. O’Donnell & Mendez 1987; Kim et al. 1990). As was pointed out by Muinonen (1990), the existence of the negative polarization branch in the polarimetric phase curve follows from the measurements of O’Donnell & Mendez as well, although they used different terminology. However, it is not clear if coherent backscattering by random rough surfaces can produce the polarization opposition effect. In the measurements of O’Donnell & Mendez and Kim et al., the characteristic angular width of the effects observed in the backscattered light is about a few degrees rather than several tenths of a degree.

ACKNOWLEDGMENTS

We thank J. E. Geake for useful comments which helped us to improve the presentation of the paper. MIM is grateful to R. Smoluchowski for a useful discussion and acknowledges support from P. C. Day, H. T. Johnson, M. Cotman and J. H. Tietjen.

REFERENCES

Dollfus A., 1979, Icarus, 37, 404
Johnson P. E., Kemp J. C., King R., Barbour M. S., 1980, Nat, 283, 146
Kolokolova L. O., 1990, Icarus, 84, 305
Lumme K., Karttunen H., Irvine W. M., 1985, Earth, Moon & Planets, 33, 19
Muinonen K., 1990, PhD thesis, University of Helsinki
Wolf M., 1975, Appl. Opt., 14, 1395
Wolf M., 1980, Icarus, 44, 780