The evolutionary status of $\beta$ Per

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ABSTRACT

Algod ($\beta$ Per) is a very well-known eclipsing close binary system which was discovered by Goodricke in 1782. Algod has been an intensively studied object ever since, and it has been detected over a very broad range of wavelengths from radio to X-ray. From observations, orbital and physical parameters for Algod have been determined moderately accurately. However, there does not exist an exact evolutionary model which, starting from some initial conditions (period, mass ratio, primary mass), reproduces the present parameters of the system. In this paper, we therefore examine the evolutionary status of $\beta$ Per (Algod). We discuss the system from a non-conservative point of view, concerning mass transfer, and consider mass and angular momentum losses from the system via a stellar wind. We compare the C abundance determination from theoretical models with values known from observational analysis, and construct a theoretical model which agrees reasonably well with the observed one. The computed parameters lie well inside the observational error. During evolution, $\beta$ Per loses about 15 per cent of its initial total mass ($\Delta M \sim 0.81 \, M_\odot$) and 30 per cent of its initial total angular momentum.

Key words: binaries: eclipsing – stars: evolution – stars: individual: Algod – stars: mass-loss.

1 INTRODUCTION

Algod ($\beta$ Per, HD 19356) is the brightest eclipsing binary with deep eclipses. It is the prototype eclipsing semidetached binary, in which one component of the system (initially more massive and more evolved) has filled its Roche lobe and is now transferring material to its companion. The brightest member of the system, Algol A (B8V), is eclipsed every 2.8673 d by its faint companion Algol B, which is a cool low-mass subgiant. This close binary is attended by Algol C (late-type A or early F, luminosity class V) in a 1.862-yr orbit. Algol is an intensively studied object, and it has been detected over a very broad range of wavelengths (from radio to X-ray). Sahade & Wood (1978) have summarized some of the more important studies of Algol, and some more recent work has been cited by Söderhjelm (1980a) and Richards, Mohncacki & Bolton (1986).

One of the key points in all studies of Algol is the interpretation of the optical light curves. Element determinations from photometry have been published in the last two decades by e.g. Hill & Hutchings (1970), Wilson et al. (1972), Grygar & Horak (1974), Guinan et al. (1976), Al-Naimy & Budding (1977), Söderhjelm (1980a), Richards et al. (1988) and Kim (1989). However, a direct determination of the mass ratio ($q = M_1/M_2 = M_B/M_A$), and thence the individual masses of Algol A and B, is not possible without the spectroscopic data for Algol B. Fortunately, the Na D lines of Algol B were detected by Tomkin & Lambert (1978). In Table 1 we present some orbital elements and physical data for the components of the Algol system.

Unfortunately, there are only a few published papers concerning evolutionary models of Algol-type systems, e.g. AS Eri (Refsdal, Roth & Weigert 1974), TV Cas (De Greve, de Landtsheer & Packet 1985), $\beta$ Lyr and V356 Sgr (Ziolkowski 1976, 1985), and none for the prototype of the class, $\beta$ Per [one unpublished study was carried out by Filipowicz (1983), who considered conservative evolution of the mass-losing star]. We have therefore decided to compute an evolutionary model for $\beta$ Per, and to try to determine the initial orbital and physical parameters for it.

2 THE TYPE OF MASS EXCHANGE: CASE A OR EARLY CASE B?

If we want to answer this title question, we must discuss three problems:

(i) the model for the mass transfer;
(ii) the activity of Algol B; and
(iii) the initial masses of the components.
Table 1. Orbits and physical data for the Algol system.

<table>
<thead>
<tr>
<th>Orbits in the Algol system</th>
<th></th>
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<tr>
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<td>2.8673</td>
<td>5</td>
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<td>e</td>
<td>0.2±0.02</td>
<td>0.</td>
<td>5</td>
<td></td>
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<tr>
<td>i [deg]</td>
<td>83±2</td>
<td>81.4±0.2</td>
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Physical data for the components

<table>
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<th>Algol A</th>
<th>Algol B</th>
<th>Algol C</th>
<th>Ref.</th>
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<tbody>
<tr>
<td>Mass [M(_\odot)]</td>
<td>3.70±0.30</td>
<td>0.81±0.05</td>
<td>1.70±0.2</td>
<td>3,4,6</td>
</tr>
<tr>
<td>Radius [R(_\odot)]</td>
<td>2.90±0.04</td>
<td>3.50±0.10</td>
<td>1.60±0.2</td>
<td>3,4</td>
</tr>
<tr>
<td>(T_{\text{eff}}) [K]</td>
<td>12500±500</td>
<td>4500±300</td>
<td>8000±200</td>
<td>2,2,4</td>
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<tr>
<td>log (L/L(_\odot))</td>
<td>2.65±0.075</td>
<td>0.65±0.14</td>
<td>0.97±0.17</td>
<td>calc.</td>
</tr>
<tr>
<td>(M_{\text{1}}) [M(_\odot)/yr]</td>
<td>(\geq 10^{-7})</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Delta \log C)</td>
<td>0.39±0.2</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

References: (1) Cugier & Hardorp (1988); (2) Eaton (1975); (3) Kim (1989); (4) Richards et al. (1988); (5) Söderholm (1980a); (6) Tomkin & Lambert (1978).

2.1 The model for the mass transfer

Because of its brightness, Algol is potentially one of the most important stellar systems for the study of mass transfer and mass loss in interacting binaries. Söderholm (1980a) has published the most complete O-C diagram so far, which shows the behaviour of Algol’s period change over 190 yr. He concluded that the alternate period changes model in Algol-type systems described by Biermann & Hall (1973) does not work in the case of Algol. In this model, the sub-giant secondary periodically suffers a dynamical burst of mass loss leading to an abrupt decrease in the period. The angular momentum stored in the rotation of the primary is then slowly returned to the orbit by tidal interaction, giving a gradual period increase. The only general means available to induce period changes as large as \(10^{-7} P\) is mass flow modified by a variable magnetic field (in fact, we can see a roughly 32-yr periodicity in the O-C diagram). As will be shown in the next section, there is also some evidence for the existence of circumstellar matter. This means that the mean mass transfer rate deduced from period changes depends on the assumed mass and momentum loss from the system. Additionally, as reported by Frieboes-Conde, Herceg & Hog (1970), from time to time some sudden erratic changes of the order of seconds occur (\(\Delta P = 3.5\) s, around 1944, and \(\Delta P = -2.1\) s in 1952). It is clear that the situation is very uncertain, and a new accurate analysis of the O-C diagram is needed. We can estimate a mass transfer rate if we believe that the trend of period changes represents a long-term mass transfer rate. Söderholm’s (1980a,b) conclusion, that the mean mass flow rate from Algol B is unlikely to be less than \(10^{-7} M_\odot\) yr\(^{-1}\), can therefore be accepted only as a zero-order approximation (and is highly uncertain). It is also possible that the activity of the mass-losing star varies on time-scales of millennia or centuries, and this variation is inconsistent with the thermal time-scale of mass transfer predicted by the theoretical model.

On the other hand, spectroscopic observations from the Copernicus satellite (Chen & Wood 1976) showed some additional absorption features in the red wings of the resonance Mg ii lines near 2800 Å. The absorption features were clearly seen only during part of the primary eclipse, in the phase interval 0.9–0.3. Cugier & Chen (1977) and Cugier (1982) have suggested that the additional absorption components originate in a gas stream and in the active region on the Algol A surface. Cugier (1982) has concluded that this region covers about 20 per cent of the Algol A disc, and that its temperature is considerably higher than 20 000 K. He applied the theoretical mass-exchange model of Lubow & Shu (1975, 1976) and showed that from this model the additional absorption components can be visible only in the phase interval 0.9–0.95. Because the observed components are seen in the phase interval 0.9–0.03, the stream is in fact much broader than in the Lubow & Shu model. Cugier evaluated the mass accretion rate as being of the order of \(10^{-8} M_\odot\) yr\(^{-1}\) or greater. However, we must note that the mass accretion rate deduced from stream activities is extremely model-dependent. For U Sge, for example, Olson (1987) and Olson & Bell (1989) gave, for LTE and non-LTE analyses respectively, \(2 \times 10^{-7}\) and \(3–4 \times 10^{-8} M_\odot\) yr\(^{-1}\) for the mass accretion rate.

The recent observations of H\(_\alpha\) profiles by Gillet, Mouchet & North (1989) strongly support the above interpretation of the accretion behaviour in Algol. Gillet et al. have presented high-resolution (0.2 Å) observations of the H\(_\alpha\) line throughout the whole period of the Algol binary system. They found (see their fig. 3) two H\(_\alpha\)-emitting regions, one of which coincides with the stream and the other of which is near phase 0.2. In the first impact region (near phase 0.9–0.03), the hydrogen is completely ionized by the hotspot (temperature higher than 20 000 K).

These spectroscopic observations (Cugier 1982; Gillet et al. 1989) and their interpretation are consistent with the hydrodynamic model of accretion through the Lagrangian point developed by Prendergast & Taam (1974).

2.2 The activity of Algol B

There is strong observational evidence for the presence of a corona around Algol B. Three observational facts, in three different wavelength ranges, support this.

First, consider the radio emission from Algol initially detected by Wade & Hjellming (1972). The first VLBI observations during the two strong outbursts were reported by Clark, Kellermann & Shaffer (1975) and Clark et al. (1976). Results of multi-epoch VLBI observations made by Lestrade et al. (1988) show that the brightness temperature of the radio source in Algol is between \(3 \times 10^8\) and \(5 \times 10^8\) K. The radio emission is from a region the size of which is ~3 times the radius of the K subgiant. Such a hot active coronal region is consistent with gyrosynchrotron emission from energetic electrons with mean energy less than 1 MeV. The inferred magnetic field strength is of order 30 G at the surface of optical depth unity (Lestrade et al. 1988). Two exceptional events occurred during the Lestrade et al. (1988) observations: a high-brightness outburst with corresponding brightness temperature \(3 \times 10^{10}\) K, and a short-duration (15 min), highly polarized (50 per cent) and high brightness temperature (greater than \(5 \times 10^{10}\) K) outburst. A simple physical model which includes all three types of radio emission has been suggested by Lestrade et al. (1988).

Secondly, stars later than spectral type F have deep convective zones which, when combined with rapid rotation (for Algol, 10 times faster than the Sun), are believed to
generate strong magnetic fields via the dynamo effect. It is plausible that there is an ensemble of magnetic coronal loops (radio emission) anchored to the surface of the active star. If this picture is correct some starspots must occur. In fact, Richards (1989) has analysed the 1.2-μm infrared light curve of Algol, and found time-dependent changes in the depths and phases of both eclipses and the asymmetric shape of secondary minimum. She proposed to explain these changes by cyclic variations in the mean temperature of the secondary and by a photometric distortion of the light curve. This type of variability is similar to that seen in the optical light curves of RS CVn binaries. The discovery of similar events for Algol provides evidence of starspot activity in Algol B.

Thirdly, consider the X-ray flare and the quiescent emission from Algol discovered by EXOSAT (van den Oord & Mewe 1989). The flare is distinctly different from the energetic flare observed in RS CVn systems, although the total energies of both flares are comparable (~10^{38} \text{ erg}). It is now generally believed that the X-ray emission from the Algol system comes from an extended corona surrounding Algol B. From the absence of a decrease of X-ray emission during optical eclipse of Algol B, White et al. (1986) concluded that the height of the corona (above the photosphere) must be at least 3 R_\odot, while van den Oord et al. (1989) put this lower limit at 5.5 R_\odot.

We can see that these results are consistent with the corona model predicted from radio observations (Lestrange et al. 1988). We can therefore conclude that Algol B is an active star, losing some of its angular momentum via a magnetic stellar wind.

2.3 The initial masses of the components

As reported by De Greve, Gimenez & Guinan (1992, hereafter DeGG), it is possible to determine the initial range of the primary mass using a general grid of models. We simply need to compare parameters determined from observations (Table 1) with the series of binary evolution tracks prepared by De Greve (1993). Using fig. 1 of DeGG, we can locate the mass-losing star in the mass–luminosity diagram and compare its position with the evolutionary tracks. This comparison gives the following results. The mass-loser is beyond the position of minimum luminosity, climbing up the ascending branch, and the initial mass is smaller than 3 M_\odot. On the other hand, the primary mass cannot be smaller than half of the present total mass (2.25 M_\odot). Because DeGreve's (1993) calculations only deal with early case B mass transfer, and are explicitly taking into account a 50 per cent mass loss from the system, the initial primary mass could even be greater than 3 M_\odot (if we assume more than 50 per cent mass loss and/or very effective angular momentum loss from the system). As DeGG concluded (from inspection of the position of the secondary component on the mass–luminosity and mass–radius diagrams), the secondary mass must be greater than 2 M_\odot, but of course less than the primary initial mass. This gives a value for the initial mass ratio, which must be smaller than 1.5 but greater than 1. If we agree with DeGG's conclusion that the loser is slightly beyond the position of minimum luminosity, we can conclude that the mass-loss rate must be in the range 10^{-7} to a few × 10^{-6} M_\odot yr^{-1}.

2.4 Answer: the type of mass exchange is early case B

Two lines of argument suggest that the mass transfer rate from Algol B is greater than 10^{-7} M_\odot yr^{-1}, but depends somewhat on the mass and angular momentum loss from the system. This is definitely higher than in a nuclear phase of mass exchange, which characterizes the second phase of evolution of systems in case A. This value, together with the present period, suggests early case B mass transfer for the Algol A–B pair. The existence of a well-observed corona (radio, X-ray) suggests the active character of Algol B; the loss of mass and angular momentum from the system must therefore be considered. This means that when we construct evolutionary models we consider a non-conservative scheme of mass transfer.

For these two reasons, and using the DeGG results as guidelines, we have looked for a solution of the Algol problem by considering the evolution in early case B non-conservative mass exchange.

3 ASSUMPTIONS AND INITIAL CONDITIONS

We have investigated evolutionary models for the β Per (Algol) system. Our calculations were carried out under the following assumptions.

(i) The primary reaches its critical Roche lobe (R_{\text{crit}}) when the core hydrogen content X_c has been reduced to 0 (evolving across the Hertzsprung gap – early case B) and the primary radius is 5, 50 or 100 per cent greater than the maximum main-sequence radius (R_{\odot}) for a given stellar mass. These choices determine the initial separation and hence the initial period P_0 for all our systems. In our theoretical calculations we used the following notation: primary – Algol B; secondary – Algol A.

(ii) All evolutionary calculations were carried out using a standard Henyey-type code (Paczynski 1970). Both components of the binary systems were calculated simultaneously. Partial ionization of hydrogen and helium and partial dissociation of hydrogen molecules were taken into account for the outer layers of the components. We calculated the nucleosynthesis (Fowler, Caughlan & Zimmerman 1975; Harris et al. 1983) of the elements H, 3He, 4He, 12C, 14N and 18O in the centres of both components.

During the evolution of our models, the mass is transferred between the two components on a thermal time-scale. For this phase, the mass flow rate was calculated by means of a procedure which maintains the stellar radius R_1 equal to the mean Roche lobe radius R_{\text{crit}} with a relative accuracy of 10^{-3}.

(iii) Only non-conservative evolution was considered for most models. The evolution of the primary filling its Roche lobe and the change of the semimajor axis of the system were computed under the additional assumption that the mass and orbital angular momentum are lost by a stellar wind. The formalism which we adopted is as follows. We have introduced an arbitrary parameter f_1 describing the loss of mass from the binary system (Ziółkowski 1985), and define

\[ M = M_1 f_1, \quad M_2 = -M_1 (1 - f_1), \]

where M is the mass-loss rate from the system, M_1 is the rate of mass flow from the primary and M_2 is the accretion rate
on to the secondary component. The matter leaving the
system will carry off its intrinsic orbital angular momentum
on a time-scale given by the formula

$$ \frac{d \ln J}{dt} = f_2 \frac{M_2 M}{M_1(M_1 + M_2)} \text{ yr}^{-1}, $$

where $M_1$ and $M_2$ are the masses of the primary and sec-
ondary components respectively. We have introduced the free
parameter $f_2$, which describes the efficiency of the orbital
angular momentum loss from the system. We must note that
the physically arbitrary parameter in equation (2) (when $f_2 \neq 1$) is chosen so as to have the convenient result that the
period varies as a unique function of the current masses (for
given initial primary mass $M_{1,0}$, mass ratio $q_0 = M_{1,0}/M_{2,0}$
and period $P_0$), and is therefore independent of the mass-
loss-rate history. It is more convenient to rewrite equations
(1) and (2) in the form

$$ \Delta \log J = f_2 (f_1 \Delta \log M_1 - \Delta \log M). $$

From standard orbit theory, we know that

$$ \Delta \log P = \Delta \log M + 3 \Delta \log \left( \frac{J}{M_1 M_2} \right), $$

where $\Delta$ represents changes in system parameters between
initial and final stages. The last two equations make clear
how one should go from the present conditions to possible
initial conditions, for various $P_0$, $q_0$, $f_1$, and $f_2$.

In our calculations we used several values of $f_1$ (Table 2),
but $f_1$ is kept constant during the mass transfer. A change in
$f_1$ necessitates a change in $f_2$, but $f_2$ is also kept constant
during a given calculation.

(iii) The importance of the different mechanisms of
mixing is discussed in more detail by Sarna (1992). We
included two of them in our numerical code: the accretion
effect and mixing induced by an inverted gradient of molecular
weight (thermohaline mixing - convection).

(iv) The standard Population I chemical composition
($X = 0.7$, $Z = 0.03$) opacity tables (Huebner et al. 1977)
and mixing-length theory (as in Paczyński 1969) were used
throughout the calculations. For elemental abundances, we
adopted the Kurucz (1979) values for C, N and O:

$$ X_C = 0.00397, X_N = 0.00143, X_O = 0.00964. $$

As initial parameters for our model, we have the initial
primary mass ($M_{1,0}$), the mass ratio ($q_0 = M_{1,0}/M_{2,0}$), the
period ($P_0$) and $f_1$, $f_2$. As observational constraints for our
evolutionary model, we have the current mass transfer rate
($\geq 10^{-7} M_0$ yr$^{-1}$), the mass ratio ($q_0 = 0.22 \pm 0.02$),
the period ($P = 2.8673$ d) and, additionally, the carbon abun-
dance on the secondary surface, $\Delta \log C = -0.39 \pm 0.2$
(Cugier & Hardorp 1988). The free parameter of our model is
$f_2$. This means that, in general, we can find many solutions
(with respect to $f_2$), i.e. we can construct many different
evolutionary models reproducing the present parameters of
$\beta$ Per.

4 RESULTS

4.1 Non-conservative early case B

It is well known that during the evolution of a binary system
the total mass and the total orbital angular momentum are
not conserved. However, we have no reliable observational
or theoretical clues that would permit us to describe quanti-
tatively the mass loss and angular momentum loss from close
binary systems. For this reason, when we consider the case of
non-conservative evolution we have to introduce an arbitrary
parameter $f_1$ describing the loss of mass from the system (see
equation 1). The loss of orbital angular momentum is des-
cribed by equation (2) with scaling factor $f_2$. We kept
values of $f_1$ and $f_2$ constant during the mass transfer. This
means that we are looking for some mean value of mass and
angular momentum loss from the system during its lifetime.
This problem was reviewed in more detail in Ziołkowski
(1985).

The models calculated in this paper describe the evolution
of a few systems with different initial total masses, periods and
mass ratios. The computational strategy (looking for a
consistent model) was as follows. We initially fixed values of
$M_{1,0}$ and $f_1$ to calculate $M_{1,0}$. Next, we calculated the evolu-
tionary track for the primary component and compared the

![Figure 1. The characteristic evolutionary tracks in the HR diagram are shown for three systems with different initial mass ratios and secondary component masses equal to 2.1 $M_\odot$. The evolutionary track for the secondary component (dotted line) is calculated for the best-fitting primary: $M_{1,0} = 3.19 M_\odot$, $f_1 = 0.33$, $f_2 = 1.93$. The other two tracks for initial primary masses of 4.01 and 2.94 $M_\odot$ are for the free parameters $f_1 = 1.57$ and 2.35, respectively. The big dots on the tracks show the position for a primary mass of 0.81 $M_\odot$ and a secondary mass of 3.7 $M_\odot$. The big crosses show the error box for the primary (bottom) and secondary (top) components. The thin solid line labelled ZAMS shows the zero-age main sequence.](https://academic.oup.com/mnras/article-abstract/262/2/534/1161725)
theoretical position with the observational error boxes on the Hertzsprung–Russell (HR) diagram (see Fig. 1) and the period–mass diagram (Fig. 2a) for a value of $f_1$ calculated from equations (3) and (4). If the position did not agree, we changed $f_1$ and $M_{1,0}$ and repeated the above procedure. If the position did agree, we calculated the secondary component's evolutionary track and compared it with the error box. When the position of the secondary lay outside the observed parameter range, we changed the initial secondary mass ($M_{2,0}$) and tried again.

For a very broad range of the secondary mass, $1.7 \leq M_{2,0}/M_\odot \leq 2.5$, the position of the primary is inside the error box (Figs 1–4), and it always is inside for some range of $f_1$. The value of $f_1$ increases with decreasing $M_{2,0}$. The crucial point is therefore the secondary evolution. The following two factors can exclude models:

(i) during mass accretion, the secondary radius increases so rapidly (for example, see Kippenhahn & Meyer-Hofmeister 1977) that its inner Roche lobe ($R_{\text{cr,2}}$) is filled and a contact configuration is formed before inversion of the mass ratio takes place;

(ii) the final position of the secondary is outside the error box (Figs 1 and 2b). This is the case for $M_{2,0} < 2.3 M_\odot$.

We must remark that the first factor is only formal (in fact, we do not know how to model such a situation). It is also possible that a real system might go through a contact phase and emerge semidetached (see for example De Greve 1989).

Only for the above range of the secondary initial mass do we have agreement with observational data. Our best-fitting model is presented in Figs 3–5 and in Table 2. The computed parameters for different stages of the close binary system's evolution are presented in Table 3(a). From Figs 3 and 4 and our tables, we see that all the computed orbital and physical parameters lie inside the observational errors. In addition, in Fig. 5 the chemical evolution of carbon on the stellar surface for both components is presented. During the semidetached evolution, the best-fitting model loses about 15 per cent of its initial total mass ($\Delta M = 0.81 M_\odot$) and 30 per cent of its initial total orbital angular momentum. The mass of the convective envelope is $0.1 M_\odot$, and it extends over 50 per cent of the radius of the primary component. The mass of the helium core is equal to $0.07 M_\odot$.

To be sure that the above solution is correct, we have tried to construct another model for $M_{2,0} = 2.5 M_\odot$ and $f_1 = 0.4$, but with longer initial periods, $P_0 = 2.786$ d (primary radius

![Figure 2](https://academic.oup.com/mnras/article-abstract/262/2/534/1161725)

**Figure 2.** The period as a function of primary mass (a) and the radius–mass diagram (b) for the three systems shown in Fig. 1. The best fit for the primary component is for $M_{1,0} = 3.19 M_\odot$, $f_1 = 0.33$ and $f_2 = 1.93$ (solid line). The radius of the secondary component (dotted line in b) for the best-fitting primary lies outside the error box. The upper dotted line in (b) shows the equivalent Roche lobe radius for the secondary component. The crosses on the right-hand side (near 0.81 $M_\odot$) and the left-hand side (near 3.7 $M_\odot$) show the observational error boxes for the primary and secondary, respectively.

![Figure 3](https://academic.oup.com/mnras/article-abstract/262/2/534/1161725)

**Figure 3.** As Fig. 1, but for the best-fitting model: $M_{1,0} = 2.81 M_\odot$, $M_{2,0} = 2.5 M_\odot$, $P_0 = 1.61$ d, $f_1 = 0.40$ and $f_2 = 1.82$. 

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is 50 per cent greater than $R_{\text{MS}}$) and 4.29 d (100 per cent). As a consequence of these longer periods we must lose more angular momentum, and a greater $f_2$ (see equations 3 and 4) is needed; specifically 2.35 and 2.79 respectively. As an effect of the increase of $f_2$, the time of mass transfer decreases (more rapid mass exchange) and the secondary is less evolved than we need (marginally inside the error box). In both cases, the primary components have parameters inside the error box (see Fig. 6), but the mass flow rates ($|M_1|$) are rather high. In the case $f_2 = 2.79$, the primary loses its mass in a catastrophic way on the hydrodynamical timescale, because a thick convective envelope is formed. In addition, the period always decreases (Fig. 6a). For $f_2 = 2.35$, when the system reaches Algol parameters, $|M_1| = 1.9 \times 10^{-6}$ $M_\odot$ yr$^{-1}$. If we further decrease the initial secondary mass, for example to $M_{2,0} = 2.1$ $M_\odot$ ($f_1 = 0.33$), we must increase $f_2$ and, for system periods of 2.84 d (primary radius 50 per cent greater than $R_{\text{MS}}$) and 4.38 d (100 per cent), we have $f_2 = 2.57$ and 3.07 respectively. Now, because the systems additionally have greater $q_0$, the $|M_1|$ are even greater than in the previous case ($M_{2,0} = 2.5$ $M_\odot$).

The carbon abundances in all cases discussed above are inside the errors ($\Delta \log C = -0.39 \pm 0.2$), and therefore we must conclude that carbon abundance is not a very sensitive factor with which to verify models in practice. In fact, much more accurate observations are needed. We therefore conclude that the best-fitting model has the following initial conditions:

$$M_{1,0} = 2.81 M_\odot, \quad q_0 = 1.124, \quad P_0 = 1.61 \text{ d}, \quad f_1 = 0.40, \quad f_2 = 1.82,$$

with about 20 per cent error for all initial parameters. The above errors are somewhat arbitrary, and illustrate the influence of the observing errors and our free parameter $f_2$ on our initial conditions. However, having one additional free parameter, we are able to reproduce the present state of $\beta$ Per quite accurately. We found that satisfactory models are possible only for a strip in the $f_2$-$M_{2,0}$ plane. This strip was found to lie along the line $f_2 = -0.26 M_{2,0} + 2.47$ with 20 per cent error. The evolutionary track for the best-fitting model (corresponding to the values of the parameters $M_{1,0} = 2.81 M_\odot$, $M_{2,0} = 2.5 M_\odot$, $P_0 = 1.61$ d, $f_1 = 0.4$ and $f_2 = 1.82$) is shown in Figs 3–5 and Table 3(a). The agreement with the observational data (see Tables 1 and 2) here is good. However, we must note that, due to observing errors, $f_2$ can be greater than the value of 1.82, for example 2.1, and such a model can also fit the orbital and physical data for $\beta$ Per satisfactorily. A system with a longer initial period (2.786 d) needs a greater value of $f_2$ (2.35), which leads to a high mass...
Table 2. Comparison of theoretical and observational data for the secondary component in Algol.

<table>
<thead>
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<th>Theoretical models</th>
<th>Algol A</th>
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<tr>
<td>( P_0 ) [days]</td>
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</tr>
<tr>
<td>( q_0 = M_{1,0}/M_{2,0} )</td>
<td>2.050</td>
</tr>
<tr>
<td>( M_{2,0}/M_0 )</td>
<td>1.7</td>
</tr>
<tr>
<td>( f_1 )</td>
<td>0.23</td>
</tr>
<tr>
<td>( L_0 )</td>
<td>2.03</td>
</tr>
<tr>
<td>( \Delta t ) [yrs]</td>
<td>2.51E8</td>
</tr>
<tr>
<td>( \Delta t ) [yrs]</td>
<td>1.04E6</td>
</tr>
<tr>
<td>( X_{C2} )</td>
<td>0.68</td>
</tr>
<tr>
<td>( R_2/R_0 )</td>
<td>2.50</td>
</tr>
<tr>
<td>( T_{eff,2} ) [K]</td>
<td>13400</td>
</tr>
<tr>
<td>(</td>
<td>M</td>
</tr>
<tr>
<td>( \Delta i ) [deg]</td>
<td>0.54</td>
</tr>
</tbody>
</table>

Notes: *contact configuration formed - secondary mass = 3.51 \( M_\odot \); \( \Delta t \) = duration of the mass exchange; \( X_{C2} \) = central hydrogen abundance; \( \Delta \log C \) = carbon abundance in comparison to the cosmic value.

Table 3. (a) Non-conservative evolution of 2.81 + 2.50 \( M_\odot \) (case early B). (b) Non-conservative evolution of 3.3 + 1.21 \( M_\odot \) (case AB).

(a)

| Phase  | Age [10^6 yrs] | \( P \) [days] | \( |M|_1 \) [M_\odot/yr] | \( M_{\text{Algo B}} \) -primary (loser) | \( \log L/L_\odot \) | \( \log T_\odot \) | \( R/R_\odot \) | \( \log M/M_\odot \) | \( \log L/L_\odot \) | \( \log T_\odot \) | \( R/R_\odot \) | \( X_c \) | \( M_{cc}/M_\odot \) |
|--------|---------------|---------------|-----------------|---------------------------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|--------------|----------------|
| ZAMS   | 0.000         | 1.610         | 0.00            | 2.81                           | 1.693          | 4.036          | 1.994          | 2.50            | 1.491          | 3.999          | 1.669         | 0.700         | 0.47          |
| TAMS   | 4.353         | 1.610         | 0.00            | 2.81                           | 1.869          | 3.944          | 3.777          | 2.50            | 1.607          | 3.954          | 2.675         | 0.305         | 0.31          |
| RLOF   | 4.518         | 1.610         | 9.71E-8         | 2.81                           | 1.994          | 3.965          | 3.932          | 2.50            | 1.614          | 3.949          | 2.737         | 0.294         | 0.31          |
| M_{max} | 4.525         | 1.463         | 1.74E-6         | 2.29                           | 1.214          | 3.791          | 3.382          | 2.81            | 1.936          | 4.012          | 2.968         | 0.351         | 0.44          |
| Limin  | 4.540         | 2.248         | 3.71E-7         | 2.01                           | 0.576          | 3.652          | 3.239          | 2.58            | 2.221          | 4.085          | 2.933         | 0.413         | 0.55          |
| Mobt   | 4.549         | 2.867         | 2.87E-7         | 0.81                           | 0.609          | 3.645          | 3.482          | 2.70            | 2.274          | 4.098          | 2.942         | 0.411         | 0.56          |
| End of comp. | 4.579     | 5.775         | 4.48E-8         | 0.48                           | 0.828          | 3.650          | 4.382          | 3.90            | 2.369          | 4.122          | 2.934         | 0.438         | 0.67          |

(b)

| ZAMS   | 0.000         | 1.387         | 0.00            | 3.30                           | 1.969          | 4.084          | 2.184          | 1.21            | 0.159          | 3.748          | 1.283         | 0.700         | 0.02          |
| RLOF   | 2.679         | 1.387         | 1.62E-7         | 3.30                           | 2.154          | 3.997          | 4.032          | 1.21            | 0.166          | 3.749          | 1.298         | 0.705         | 0.02          |
| M_{max} | 2.681         | 0.685         | 7.66E-6         | 2.44                           | 0.406          | 3.699          | 2.172          | 2.07            | 1.152          | 3.938          | 1.702         | 0.689         | 0.33          |
| Limin  | 2.682         | 0.726         | 2.99E-6         | 1.89                           | 0.290          | 3.688          | 1.975          | 2.62            | 1.561          | 4.013          | 1.927         | 0.692         | 0.46          |
| Mobt   | 4.402         | 2.867         | 2.96E-9         | 0.81                           | 0.684          | 3.667          | 3.479          | 2.37            | 2.323          | 4.070          | 3.566         | 0.242         | 0.43          |
| End of comp. | 4.821     | 3.588         | 1.12E-9         | 0.73                           | 0.759          | 3.660          | 3.871          | 3.78            | 2.437          | 4.062          | 4.203         | 0.011         | 0.28          |

Notes: ZAMS = zero-age main sequence; TAMS = terminal-age main sequence; RLOF = Roche lobe overflow; \( M_{cc} \) = mass of the convective core.

Figure 6. As Fig. 4, but for different values of \( f_1 \): 1.82 (best-fitting model); 2.35 \( (P_0 = 2.786 \text{ d}) \) and 2.79 \( (P_0 = 4.29 \text{ d}) \). The upper and lower dotted lines (b) show the equivalent Roche lobe radius and the secondary radius, respectively, for the intermediate case (solid line in a and b).
4.2 An almost-conservative case AB

To be sure that the exclusion of case AB mass transfer (see arguments in Section 2) is well grounded, we have calculated the evolution of the almost-conservative case for the following initial parameters: $M_{1,0} = 3.3 \, M_\odot$, $M_{2,0} = 1.21$ and $P_0 = 1.387 \, \text{d}$ (the period is 15 per cent longer in comparison with the conservative case). An almost-conservative case means that, when the convective envelope has developed in the primary, the loss of angular momentum by the magnetic stellar wind is turned on [we used the Stepien formula (Sarna 1992)]. The results of this computation are presented in Figs 7 and 8 and in Table 3(b). The position of the primary is correct in both HR (Fig. 7) and radius–mass (Fig. 8b) diagrams. However, there is a big problem with the position of the secondary. If we calculate the normal reaction to accretion (Kippenhahn & Meyer-Holmeister 1977), a contact configuration is formed after transferring $\sim 0.04 \, M_\odot$ (dash-dotted line in Fig. 8b). It is difficult to exclude such a possibility, but in fact we do not know how to model such a situation. Because the initial mass ratio is rather high ($\sim 2.73$), this system can even overflow its outer critical surface and evolve through a common-envelope phase (Paczynski 1976), or come into normal contact and afterwards return to a semidetached state (De Greve 1989). If we consider the secondary component as a star increasing its radius in agreement with nuclear evolution (a rather unphysical assumption), the final radius is too large ($R_2 = 3.5 \, R_\odot$). For these two reasons, we cannot exclude this model. Fortunately, during the evolution when the system shows the Algol parameters, the mass transfer rate is $\sim 3 \times 10^{-9} \, M_\odot \, \text{yr}^{-1}$ (the nuclear time-scale of the mass exchange). For this last reason alone, therefore, we can exclude the hypothesis that Algol evolves from case AB mass transfer.

5 CONCLUSIONS

The results of our evolutionary investigations can be summarized as follows.

(i) Conservative models have not explained well the present parameters of Algol.

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**Figure 7.** As Fig. 1, but for the system with $M_{1,0} = 3.3 \, M_\odot$, $M_{2,0} = 1.21 \, M_\odot$, $P_0 = 1.387 \, \text{d}$ and case AB mass transfer.

**Figure 8.** As Fig. 2, but for the system from Fig. 7. In (b) are presented two different reactions of the secondary component: the dash-dotted line with the accretion effect, and the dotted line (parallel to the ZAMS) without this effect (see text for explanation).
(ii) Only non-conservative evolution gives reasonably good agreement with observational parameters. From computations, we have determined the initial period and mass ratio to be around 1.6 d and 1.2, respectively. During evolution, the system loses 15 per cent of its initial total mass (0.81 M☉) and 30 per cent of its initial total angular momentum. However, due to uncertainties in the orbital and physical data for β Per, the determination of initial parameters can be affected by an error as large as 20 per cent.

(iii) Our model of β Per agrees extremely well with one by De Greve et al. (1992). From a general grid, De Greve et al. found that initial masses are 2.7 and 2.4 M☉, and the mass loss from the system is 31 per cent (f = 0.31) of the transferred mass (12 per cent of the total initial mass).

(iv) The carbon abundance is not a very sensitive factor with which to exclude models, since the [C/H] observational error is still 50 per cent.

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