Irregular small-amplitude pulsations in yellow supergiant star models

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ABSTRACT
We examine the radial pulsation of hydrodynamic modes for less-massive supergiant stars in an attempt to explain the irregular small-amplitude pulsations observed in yellow supergiant stars (89 Her, for instance), which are believed to occur in the post-asymptotic giant branch. These pulsations are located in the bluer region of the HR diagram, beyond the conventional instability strip of the fundamental mode. It is shown that the higher overtone modes which take on the properties of the strange modes are pulsationally unstable over rather wide regions, which is compatible with observations. Using non-linear hydrodynamic simulations, we demonstrate that the pulsation in the non-linear regime shows chaotic behaviour for models with higher luminosities, for a wide range of parameters. We briefly discuss the transition from regular to chaotic oscillations.

Key words: chaotic phenomena – hydrodynamics – stars: AGB and post-AGB – stars: oscillations – supergiants.

1 INTRODUCTION
F-type supergiant stars have been found which show erratic low-amplitude photometric variations with a time-scale of about 40 d. They were formerly classified as UU Her stars (Sasselov 1984), but it has recently been claimed that UU Her itself shows multi-periodic oscillations and is a pulsating massive supergiant star (Zsoldos & Sasselov 1992). There are suspicions that the UU Her stars are not a homogeneous group. For some of these stars (e.g. 89 Her and HD 161796), there is observational evidence that they are in the post-asymptotic giant branch (post-AGB) stage (Tamura, Takeuti & Zalewski 1992).

The variabilities are assumed to be due to pulsation. The observed effective temperatures, however, indicate that these stars are located in bluer regions beyond the blue edge of the instability strip of classical Cepheids.

Aikawa (1991) has investigated the pulsational properties of less-massive supergiant star models with non-linear hydrodynamic simulations as well as by linear non-adiabatic pulsation analysis. In the present paper, we extend the previous analysis to much wider regions of the HR diagram. We discuss the results of a linear non-adiabatic (LNA) analysis in Section 2, and describe the pulsation behaviour in the non-linear regime in Section 3. We devote Section 4 to discussion of the transition from regular to chaotic oscillations, which are realized for a wide range of parameters. We summarize our results in Section 5.

2 THE LNA ANALYSIS
The values of the stellar parameters of supergiant F stars are only poorly determined. In our present study, therefore, the mass is fixed at 0.8 \( M_\odot \), which is typical of a less-massive Population II star. The effective temperature for HD 161796, a prototype of the pulsating F-type supergiant stars, has been estimated as 6300 K by Fernie (1983) and Fernie & Garrison (1984). There is controversy regarding the luminosities of the yellow supergiant stars. Fernie & Sasselov (1989) have suggested that secular changes in the colours of yellow supergiant stars should be observed, if these stars are in the post-AGB phase and sufficiently luminous, because stellar evolution in the post-AGB phase is rapid and the speed is proportional to the luminosity (Schönberner 1987). We know of a rapidly evolving F-type supergiant star which also shows erratic low-amplitude pulsation, namely FG Sge (Whitney 1978). With the exception of this star, there is no evidence of rapid evolution among pulsating yellow supergiant stars. The stars may be less luminous than generally expected, although Arellano Ferro & Parraro (1990) have recently re-estimated the luminosities of several stars in this class and found them to be more luminous than previously estimated. We therefore simply assume that luminosity is a free parameter, and, in particular, that it is a control parameter of the non-linear dynamical system in the non-linear simulations.
In Aikawa (1991) the effective temperature is fixed at 6300 K, but we extend the previous study to a wider region of the HR diagram. We therefore have model sequences with fixed mass, but the effective temperature as well as the luminosity is varied. The effective temperature is varied within a range of 6000–7500 K appropriate for F-type stars. We vary the luminosity in the range 3500–7000 L_☉.

We construct static and linear non-adiabatic models in the same way as in Aikawa (1991), except for the thermal boundary condition at the stellar surface, which is discussed below. We use the analytical expression for the opacities by Stellingwerf (1975a,b) with Population II composition, X = 0.745 and Z = 0.005, and ignore the effects of convection. The total number of zones in each model is about 90, with finer zoning in the hydrogen ionization zone.

Zalewski (1992) has emphasized the effects on the properties of linear non-adiabatic pulsation of using a radiative transfer treatment with spherical geometry instead of the diffusion approximation conventionally used. He has also pointed out that the different effects of these two approximations are attributed to the thermal boundary conditions at the surface boundary, and has suggested using outer boundary conditions obtained with a linear dispersion relation, together with the diffusion approximation, for the modelling of strongly non-adiabatic pulsation. Unfortunately, these boundary conditions cannot be applied directly for non-linear pulsation, but Unno (1989) has formulated a practical solution for the Eddington approximation in spherical geometry. The Hopf function q(τ) in the T−τ relation (see Mihalas 1978) is reduced to 1/3 for spherical geometry. Instead of the T−τ relation with q = 0.577, which is generally used for plane-parallel atmospheres, we use a T−τ relation with q = 1/3 as the thermal condition at the stellar surface in the conventional diffusion approximation for static model construction, in the LNA analysis and in the non-linear hydrodynamic simulations. We note that the absolute values of the linear growth rates for strongly damped and also for unstable modes in linear non-adiabatic pulsation analysis are considerably reduced with the new boundary condition.

The LNA analysis is performed with a modified Castor code. Because non-adiabatic effects on the pulsation properties are so strong, the quasi-adiabatic approximation for initial guesses of the eigenvalues for the LNA equation is not adequate. Eigenvalues sometimes deviate strongly from those of the linear adiabatic equation, and there are modes that have no counterparts in the linear adiabatic (LA) equation (see Shibahashi & Osaki 1981; Saio, Wheeler & Cox 1984; Worrell 1986). We therefore introduce two parameters, ε and µ, to the original LNA equation (Castor 1971) as follows:

\[ \omega^2 X = G_1 X + \epsilon G_2 Y, \]
\[ i\omega \epsilon Y = \epsilon K_1 X + K_2 Y \]

(X not to be confused with hydrogen content). The parameter ε controls the coupling between the acoustic and thermal waves. With ε = 0, the above equation is reduced to the linear adiabatic one, where the eigenfunctions of the equation consist of a complete set, and there is no problem as regards mode identification. With ε = 1, we recover the original LNA equation. Starting with ε = 0 and increasing ε step by step up to 1 (we use 0.02 as the step), we solve the above equation. At ε = 1, we obtain the LNA solutions which have counterparts in the LA equation. With this LNA analysis, we could not find modes that have no counterparts in the LA equation, or that might come from splittings of a solution degenerated in the LA equation; there are still no ambiguities in mode identification. Fig. 1 shows the periods and growth rates (−4πα/ω) as a function of ε for one of the models, namely L = 3600 L_☉, in the model sequence of T_∗ = 6300 K. One of the modes becomes unstable with a remarkable positive growth rate at ε = 1, although other modes become strongly damped. This is a common feature of the LNA properties in our models.

The second parameter µ is introduced for the non-adiabatic reversible (NAR) approximation (Gautschy & Glatzel 1990). The NAR approximation follows from the assumption that the specific heat c_s is zero in the entire stellar envelope; this is equivalent to neglecting the iω term in the LNA equation (i.e., µ = 0). The ordinary acoustic modes have purely real eigenvalues, as in the adiabatic pulsation. Gautschy & Glatzel (1990) have demonstrated, however, that the strange modes have complex conjugate eigenvalues, and so the behaviour of an eigenvalue in the NAR approximation may be a good discriminator of the strange modes from the ordinary acoustic modes.

After obtaining the LNA solutions with the method discussed above (starting from the corresponding adiabatic solution), the parameter µ is then reduced from unity, step by step, until µ = 0, i.e. the solutions in the NAR approximation are found. We use a step of 2 × 10^{-4} to enable us to track smoothly the abrupt changes in the eigenvalues which occur close to µ = 0. Fig. 2 shows the variation of the periods and growth rates with µ for the same model as in Fig. 1. Note that the strongly unstable mode (the third overtone in our definition) in the LNA approximation becomes conjugated to one of the damped modes (the fifth-overtone mode). Other damped modes in the LNA become purely oscillatory. This feature is another common feature of the present models.

We summarize the results of the LNA analysis as follows. The fundamental mode is pulsationally stable for the present range of stellar parameters, but one of the higher modes is always unstable, with remarkably large positive growth rates. Some of the higher overtone modes have large negative growth rates, and the pulsation periods of these modes are close to that of the excited mode.

Fig. 3 shows the cumulative work integrals as a function of the mass coordinate - ln(1 - q), where q = M/M_∗, for the pair of modes which are complex conjugate in the NAR approximation for the same model as in Figs 1 and 2. The two modes are complex conjugate in the NAR approximation, and the origin of the instability of the unstable mode is not related directly to the conventional κ- and γ-mechanisms (Gautschy & Glatzel 1990). While it is thus questionable to calculate the work integrals of pulsational instability for these modes, these integrals might suggest the physical origins of the instability of those modes. The solid lines labelled with numbers in the figure (3: third-overtone mode; 5: fifth-overtone mode) represent the two modes in the LNA pulsation, while the dotted lines represent those in the NAR approximation. Since the two modes are complex conjugate in the NAR approximation, the cumulative work integrals in the NAR approximation are perfect mirror images. The LNA work integrals also have this property to some extent.
Figure 1. The variation of the pulsation periods (in days) and the growth rates of lower modes as a function of $\epsilon$, the degree of coupling between the acoustic and thermal waves. The numbers attached to the lines denote the pulsation mode (0: fundamental; 1: first overtone, and so on). The model parameters are $M = 0.8 \, M_\odot$, $L = 3600 \, L_\odot$, and $T_{\text{eff}} = 6300 \, K$.

Figure 2. The variation of the pulsation periods (in days) and the growth rates as a function of $\mu$. The cases of $\mu = 0.0$ and 1.0 correspond to the original NAR and the fully LNA approximations respectively. Other notations and model parameters are the same as in Fig. 1.
Note that the hydrogen ionization zone has a remarkable effect, causing the modes to be strongly unstable and also strongly damped. The thick solid line represents the work integral of the limit cycle, which is realized in the non-linear pulsation of this model. Note also that the driving of the pulsation at the hydrogen ionization zone is remarkably reduced, while the damping in the inner region is increased, compared with those in the LNA approximation.

Finally, we note that the fundamental mode becomes unstable at much higher luminosities than those of the present models with the present range of effective temperature. Many authors (e.g. Fadeyev 1982, 1984; Takeuti, Nakata & Aikawa 1985), however, have shown that the fundamental mode pulsators in less-massive supergiant stars have quite large amplitudes, in contrast to the small observed amplitudes in the yellow supergiant stars. Christy (1966) has shown that first-overtone pulsators may have smaller amplitudes than the fundamental mode pulsators in hydrodynamic models of RR Lyrae variables. We therefore expect that higher overtone modes may be responsible for the small amplitudes observed in yellow supergiant stars.

3 NON-LINEAR MODELS

Non-linear hydrodynamic simulations are performed for six model sequences with a fixed mass, $M = 0.8 \, M_\odot$, as in the LNA analysis. Each sequence is specified by the effective temperature, which is selected in the range $6000$–$7200 \, K$ with a step of $300 \, K$. Luminosity is varied as a control parameter in the range $3500$–$7000 \, L_\odot$ for each sequence. A large number of non-linear models with different luminosities are constructed for a $T_{\text{eff}} = 6300 \, K$ model sequence to determine the features of the transitions from regular to irregular oscillations which occur when luminosity is varied. Non-linear models for other model sequences are only calculated for luminosities with a step of $500 \, L_\odot$.

The non-linear hydrodynamic pulsation code, TORUS (Simon & Aikawa 1986) is used as in Aikawa (1991). The code has a facility for dynamical re-zoning, probing the temperature variations with time at the hydrogen ionization zone. We use the Lagrangian formulations of the hydrodynamical equations. The basic assumptions used in the static and LNA models are also adopted in the non-linear calculations. The Richtmyer–Morton formula for artificial viscosity with a cut-off (Stellingwerf 1975a) is used with the coefficient $C_O = 4$. The static models used for the LNA analysis are perturbed with a small-amplitude oscillation in the form of the third-overtone adiabatic eigenfunction. The total number of zones is fixed during simulations for each model and is the same as for the corresponding LNA models.

Non-linear calculations are performed with $2 \times 10^5$ time-steps as a first run for each model. Although we use variable step-size in the simulation, the first run may cover the time interval of about $5000$–$10000 \, \text{d}$, depending on model parameters. For each model, we perform a second run with the same number of steps, and check the change in pulsation behaviour with the aid of power spectra for the time variation of photospheric magnitude at two epochs, with $400$ days of data at the end of both runs. We confirm that there are no essential changes in the two spectra, and so conclude that the pulsation behaviour in the non-linear regime settles into a steady state, and that the irregular behaviour that appears in some models is not transient.

We confirm that, for a wide range of effective temperature, models with lower luminosities show regular pulsation, while models with higher luminosities show irregular pulsation, in the non-linear regime. Moreover, the amplitudes of regular and irregular pulsations are small; this is in sharp contrast to those for hydrodynamic pulsation models for red giant stars. For the $T_{\text{eff}} = 6000 \, K$ sequence, the irregularity in higher luminosity models is not as prominent as in other sequences. For other sequences with higher effective temperatures and higher luminosities, we can see well-developed irregular oscillations.

Fig. 4, for example, shows the time variations of the magnitude at the photosphere in the non-linear regime during the last $400 \, \text{d}$ of the second run for models in the $T_{\text{eff}} = 6300 \, K$ sequence. The luminosities are given next to the curves, which are shifted vertically. No cosmetic treatment is applied to the curves.

Burki, Mayor & Rufener (1980) have reported time variations of the velocity observed from photospheric lines of HD 161796. The velocity variations have time-scale components of about $10 \, \text{d}$, much less than the time-scale of the observed photometric variations. In our models, the linear pulsation periods of the driving mode are about $10 \, \text{d}$. In the chaotic pulsation, however, subharmonic components of these pulsation periods are strongly enhanced, and so variations with a time-scale of about $20 \, \text{d}$, in addition to those with a time-scale of $10 \, \text{d}$, are prominent. It is suggested that a large contribution from the subharmonic components must cause the longer time-scale photometric variations observed.

4 TRANSITION FROM REGULAR TO IRREGULAR PULSATION

A large number of models were constructed for the sequence with $T_{\text{eff}} = 6300 \, K$ to find the transition route from regular to irregular oscillations. Two systematic routes of the transition have been found for hydrodynamic pulsation models (Aikawa 1987; Buchler & Kovács 1987; Buchler, Goupil & Kovács 1987). These routes have been identified as the period-doubling cascade and the intermittency, two universal routes of the transition from regular motion to chaos found in simple dynamical systems (e.g. Schuster 1988).

Fig. 5 shows a Feigenbaum tree of the sequence $T_{\text{eff}} = 6300 \, K$. The abscissa is the luminosity of models in the sequence (a control parameter of the dynamical system), and the ordinate is the discrete values of a dynamical quantity at the Poincaré section (Schuster 1988). In our study, we pick up values of the magnitude at the photosphere at maximum expansion. Data in the non-linear regime are compiled, and so about $1000$ data points are obtained, but for clearness only $200$ points for each model are plotted. There is no essential difference in the Feigenbaum tree compared to the limited data.

Fig. 5 demonstrates that, when the control parameter (in our case the luminosity) is varied, complicated transitions from regular to irregular oscillations are involved in the sequence. For a rather wide range of the control parameter in the middle of the sequence (from $3.57$ to $3.67$ in units of $L/L_\odot$), a period $2$ oscillation (i.e. where the dynamical system returns to the same position at the Poincaré section, having

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passed through another, different position at the Poincaré section) is clearly recognized. The period 4 oscillation, which may be caused by a subharmonic bifurcation of the period 2 oscillation, occurs at higher luminosities. At even higher luminosities, we observe band structures due to chaotic oscillations. The chaotic band becomes wider for higher luminosities, indicating well-developed chaos.

There is another chaotic band in the lower luminosity part of the Feigenbaum tree. A period 5 oscillation is observed at the lowest luminosity, but, with a very slight shift of the parameter towards higher luminosities, it is replaced by simple periodic oscillations. For higher luminosities, there is a small interval with chaotic bands between the region of the limit cycles and the period 2 oscillation.

One-dimensional return maps are constructed to enable us to see more clearly the transition to chaotic oscillations at higher luminosities. Fig. 6 shows a series of one-dimensional return maps for the higher luminosity part of the model sequence. Two successive values of the magnitude at the Poincaré section, \( x(i) \) and \( x(i+1) \), are plotted alternately as one-dimensional return maps. The value of the control parameter (i.e. the luminosity) is given on each plot. We can see the period 2 oscillation for lower luminosities (e.g. \( L = 4985 \) and \( 4971 \, \text{L}_\odot \)). For higher luminosities, we recover well-developed chaotic oscillations. However, the plots for intermediate luminosities suggest a much more complicated transition from regular to irregular oscillations than those in simple dynamical systems. The unimodal nature of the first return maps represents the transition due to the period-doubling cascade (Buchler & Kovács 1987), and the feature of a 'ghost' limit cycle is a sign of intermittency (Aikawa 1987). Fig. 6 also suggests that the transition has more complicated features, which indicate a termination of the period-doubling cascade with an interruption by other routes. This is a sign that higher degrees of freedom are involved in the transition.

Fig. 7 displays three-dimensional plots of three successive values of magnitude at the Poincaré section for the \( T_{\text{eff}} = 4985 \) and 5000 K models. Compared with the corresponding plots in Fig. 6, we can see much clearer structures in Fig. 7: a noisy period 9 oscillation is recognized in the \( T_{\text{eff}} = 4985 \) K model, and this noisy periodic oscillation is
deformed to chaotic oscillation, as shown in the 3D plot for the $T_{\text{eff}} = 5000$ K model. This reminds us that even in simple dynamical models the universal routes are sometimes interrupted by periodic oscillations and by other types of route (Arnesodo et al. 1983), and it is suggested that higher degrees of freedom of the systems are involved in those interrupted transitions (e.g. Goupil, Auvergne & Serre 1991). Thus the transition in the present sequence may be caused by similar situations where the universal routes are interrupted, although the nature of the transition is still not clear.

5 CONCLUSIONS

We have shown that pulsation models of less-massive F-type supergiant stars have an unstable mode which has the properties of a strange mode for a wide range of stellar parameters.

Non-linear hydrodynamic simulations demonstrate that higher luminosity models show chaotic pulsation in a wide region of the HR diagram, while regular pulsation is realized in lower luminosity models. The transition from regular to irregular oscillation is more complicated than the universal routes of transition known from simple dynamical systems, and it is suggested that the irregular oscillations in higher luminosity models may be high-dimensional chaos.

Since it may be the case that the behaviour of a strange mode depends on the treatment of radiative transfer in the modelling, it is desirable to perform non-linear simulations for small-amplitude pulsators in less-massive supergiant stars.
Figure 6. A series of the one-dimensional return maps for the model sequence $T_{\text{eff}} = 6300$ K. Only the higher luminosity part of the sequence is drawn, and the luminosity is given on each map. The data are the same as in Fig. 5, except that the full data obtained in the second run are plotted.

Figure 7. Three-dimensional plots of the Poincaré section data for the $L = 4985$ and 5000 $L_\odot$ models of the $T_{\text{eff}} = 6300$ K model sequence. Three successive data are plotted in a three-dimensional space. We can see a noisy periodic oscillation with a period of 9 for $L = 4985$ $L_\odot$, but this periodic oscillation becomes divergent in the model with $L = 5000$ $L_\odot$. 
with better approximations of radiative transfer. It is also desirable to develop better numerical treatments of hydrodynamical modelling[s], such as the use of the artificial tensor viscosity (Tscharnoeter & Winkler 1979) instead of the traditional Richtmyer–Morton formula.

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