The stability of massive stars and its dependence on metallicity and opacity

M. Kiriakidis, K. J. Fricke and W. Glatzel
Universitäts-Sternwarte Göttingen, Geismarlandstr. 11, 3400 Göttingen, Germany

Accepted 1993 February 17. Received 1992 December 10

ABSTRACT

Following a similar investigation based on the LAOM opacity tables, the stability of massive stars is re-examined using the latest available OPAL opacity tables. Two groups of dynamical strange-mode and mode-coupling instabilities are identified, one associated with He ionization and the other with the enhancement of opacity due to heavy elements. In the Hertzsprung–Russell diagram, the stability boundary of the first group closely resembles the observed Humphreys–Davidson limit, while the second group provides a strongly metallicity-dependent instability domain that covers the range of observed luminous blue variables for solar chemical composition. Relations to instabilities on the upper main sequence and in the $\beta$ Cepheids regime are discussed, and the significance of these strong instabilities for stellar and galactic evolution is stressed.

Key words: instabilities – radiative transfer – stars: oscillations – supergiants – stars: variables: other.

1 INTRODUCTION

In a previous paper (Glatzel & Kiriakidis 1993b) we studied the stability of massive stars with respect to infinitesimal non-adiabatic radial perturbations using the Los Alamos (LAOM) opacity tables (Huebner et al. 1977). Within the acoustic spectrum, violent resonance instabilities associated with the He ionization zone were identified whose stability boundaries in the Hertzsprung–Russell diagram (HRD) seemed to be related to the observed Humphreys–Davidson (HD) limit (Humphreys & Davidson 1979). Most of the observed luminous blue variables (LBVs) (Hubble 1926; Hubble & Sandage 1953; Conti 1984), however, lie outside the theoretical instability domain. For further references concerning observations and attempts to explain them, we refer the reader to Glatzel & Kiriakidis (1993b).

Although the details of opacity are not crucial for inferring the existence of the resonance instabilities, the domain of the occurrence of these instabilities in the HRD may depend on opacity. An increase in opacity leads to more extended stellar envelopes, and hence favours resonance instabilities (Gautschy & Glatzel 1990b). We have therefore argued (Glatzel & Kiriakidis 1993b) that a stability analysis based on the new OPAL tables (Iglesias, Rogers & Wilson 1993) that shows an enhancement of opacity due to the contribution of heavy elements will imply a more extended instability domain in the HRD, possibly reducing the remaining differences between the theoretical stability boundary and the HD limit and also providing an explanation for the LBV phenomenon. Meanwhile, an extension of the OPAL tables that allows for the study of massive stars has become available, and the results of a stability analysis based on these tables are presented in this paper.

The main difference between LAOM and OPAL opacities consists of an enhancement due to the contribution of heavy elements in the OPAL tables. Thus – provided the new opacities have an influence at all – the results of a stability analysis will depend on the abundance of heavy elements. A study of stability for different values of metallicity is therefore required.

Using the OPAL opacities, we showed in a previous paper (Glatzel & Kiriakidis 1993a) that main-sequence stars become dynamically unstable above a certain critical mass ($80\; M_\odot$ for solar chemical composition), and that this instability is not due to the well-known $\epsilon$-mechanism. Although these instabilities cannot be associated with He ionization, their appearance as strange modes and mode resonances is similar to that of the instabilities which were discussed on the basis of the LAOM opacities and which are connected with He ionization in the vicinity of the HD limit. Moreover, the heavy element bump in the OPAL opacities has been shown to be responsible for the variability of $\beta$ Cepheid stars through the ordinary $\kappa$-mechanism (Cox et al. 1992; Kiriakidis, El Eid & Glatzel 1992; Moskalik & Dziembowski 1992). Encouraged by this success, Moskalik & Dziembowski (1992) have speculated that the LBV phenomenon may have
the same origin and may operate through an identical mechanism. To a certain degree, this conjecture is in contrast with our finding that, even on the basis of the L놀V opacities, dynamical resonance instabilities in the LBV range exist that cannot be explained by the standard $\alpha$-mechanism. Whether and how the instabilities on the upper main sequence, in the $\beta$ Cepheid range and (caused by He ionization) in the vicinity of the HD limit might be related will be of particular interest in our present study.

The methods used and the stellar models investigated are described in Section 2, and the results of the stability analysis are discussed in Section 3. Some interpretations, concerning modal diagrams and the observed HD limit, are presented in Section 4. Our conclusions follow in Section 5.

2 METHODS AND MODELS

The construction of stellar models, together with the basic assumptions and parameters adopted and the methods used to test the stability of the models with respect to infinitesimal radial perturbations (namely, the linear non-adiabatic analysis), has been extensively discussed in previous papers (Ober, El Eid & Fricke 1983; Kiriaiodis 1987; Gautschy & Glatzel 1990a,b; Glatzel & Kiriaiodis 1993b). For the general theory of stellar pulsations and stability, we refer the reader to Cox (1980) and Unno et al. (1989). Contrary to a previous investigation (Glatzel & Kiriaiodis 1993b) of the stability of massive stars, in which opacities were taken from the L놀V tables (Huebner et al. 1977), the present study is based on the opal library (Iglesias et al. 1993; Rogers & Iglesias 1992).

For different initial chemical compositions and initial masses – mass loss was accounted for by adopting the empirical mass-loss rate given by Lamers (1981) – stellar evolution was followed from the ZAMS into the shell-burning stage to effective temperatures below $\log T_{\text{eff}} \approx 4$. Consideration of different initial chemical compositions is prompted by the strong dependence of the opal opacities on metallicity $Z$.

For the initial chemical composition $(X, Y, Z) = (0.746, 0.25, 0.004)$ – adequate for Small Magellanic Cloud (SMC) stars – the HRD of the calculated evolutionary tracks is given in Fig. 1. The curves are labelled with the corresponding initial masses, which have been chosen to resolve and cover the domain of instability. Counterparts of Fig. 1 for the solar initial composition $(X, Y, Z) = (0.7, 0.28, 0.02)$ and enhanced initial metallicity $(X, Y, Z) = (0.7, 0.27, 0.03)$ are shown in Figs 2 and 3, respectively. In these HRDs the positions of several LBVs and the location of the observed HD limit (dashed line) are indicated.

Some characteristic quantitative data on the evolutionary tracks shown in Figs 1–3 for selected stellar models are listed in Table 1 as a function of the time $t$ elapsed since hydrogen ignition on the ZAMS. Note that masses decrease during the evolution. As a consequence, the ashes of CNO burning appear on the surface for initial masses above $80 M_\odot$ ($Z = 0.004$) and $60 M_\odot$ ($Z = 0.02$). The properties and the significance of density inversions that are connected with convection in He and H ionization zones and which occur below $\log T_{\text{eff}} = 3.85$ have been discussed extensively in a previous investigation (Glatzel & Kiriaiodis 1993b). Density inversions can also be associated with the convection zone caused by the excess opacity due to heavy elements around $\log T \approx 5.3$ that is present in the opal tables. Their domain of existence is indicated in Fig. 3. We note that instability and density inversions are not related, i.e. no density inversions are found for a wide range of unstable models and some stable models exhibit density inversions. These findings are in contrast with the explanation proposed by Maeder (1988, 1993). As to the He ionization zones, the new density inversions are inevitable, provided that mixing length theory for convection is considered to be valid.

Figure 1. Hertzsprung–Russell diagram containing the evolutionary tracks of eight stars (dotted lines) with the initial chemical composition $(X, Y, Z) = (0.746, 0.25, 0.004)$ and the initial masses indicated. Unstable phases are denoted by solid lines, and thick lines correspond to dynamical growth rates ($\sigma_1 < -0.1$). Together with the observed positions of some LBVs, the location of the HD limit is shown as a dashed line.
3 RESULTS

For each stellar model, the stability analysis yields an infinite set of complex eigenfrequencies $\sigma$ corresponding to the various possible modes of radial pulsations of the star. Real ($\sigma_r$) and imaginary ($\sigma_i$) parts of the eigenfrequencies will be given in units of the inverse of the global free-fall time $t_{ff} = \sqrt{3GM/R^3}$. $\sigma_r > 0$ corresponds to a damped mode, and $\sigma_i < 0$ indicates instability. In the diagrams, the associated real parts are plotted as tiny ($\sigma_r > 0$) or thick ($\sigma_r < 0$) dots. The results will be presented in modal diagrams, where $\sigma_r$ and $\sigma_i$ of the lowest order eigenmodes are given as a function of the effective temperature along an evolutionary track, i.e. a stellar model will be identified by its effective temperature along the track. Ambiguities in this representation (due to the core contraction phase) are removed by connecting data points according to their evolutionary order.

3.1 Low-metallicity $Z = 0.004$

Modal diagrams for evolutionary tracks corresponding to an initial metallicity of $Z = 0.004$ and initial masses of 60, 80 and 100 $M_\odot$ (see Fig. 1) are shown in Figs 4, 5 and 6 respectively.
Table 1. Some quantities as a function of time $t$ (in units of $10^6$ yr) for six stars with different initial masses and chemical compositions.

<table>
<thead>
<tr>
<th>$t$</th>
<th>$Z$</th>
<th>$\frac{M}{M_0}$</th>
<th>log $\frac{L}{L_0}$</th>
<th>log $T_{\text{eff}}$</th>
<th>log $T_e$</th>
<th>log $\rho_e$</th>
<th>$X_e$</th>
<th>$X_\text{c}$</th>
<th>$X_\text{s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>0.004</td>
<td>60.0</td>
<td>5.68</td>
<td>4.70</td>
<td>7.50</td>
<td>0.28</td>
<td>0.746</td>
<td>0.746</td>
<td></td>
</tr>
<tr>
<td>2.563</td>
<td>55.6</td>
<td>5.70</td>
<td>4.64</td>
<td>7.65</td>
<td>0.44</td>
<td>0.358</td>
<td>0.746</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.977</td>
<td>49.9</td>
<td>5.88</td>
<td>4.66</td>
<td>7.83</td>
<td>0.50</td>
<td>0.000</td>
<td>0.746</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.988</td>
<td>49.7</td>
<td>5.95</td>
<td>3.84</td>
<td>8.29</td>
<td>2.52</td>
<td>0.000</td>
<td>0.746</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| 0.000 | 0.004 | 80.0 | 5.91 | 4.73 | 7.61 | 0.21 | 0.746 | 0.746 |
| 2.246 | 73.1 | 6.01 | 4.65 | 7.66 | 0.38 | 0.349 | 0.746 |
| 3.455 | 64.3 | 6.08 | 4.39 | 7.84 | 0.92 | 0.000 | 0.745 |
| 3.465 | 64.1 | 6.13 | 3.83 | 8.20 | 2.12 | 0.000 | 0.745 |

| 0.000 | 0.004 | 100.0 | 6.08 | 4.74 | 7.61 | 0.16 | 0.746 | 0.746 |
| 2.208 | 90.9 | 6.16 | 4.66 | 7.67 | 0.32 | 0.355 | 0.746 |
| 3.125 | 78.5 | 6.23 | 4.36 | 7.83 | 0.83 | 0.000 | 0.720 |
| 3.149 | 77.9 | 6.27 | 3.81 | 8.16 | 1.88 | 0.000 | 0.714 |

| 0.000 | 0.02 | 20.0 | 4.63 | 4.54 | 7.53 | 0.61 | 0.700 | 0.700 |
| 5.155 | 19.3 | 4.77 | 4.49 | 7.56 | 0.64 | 0.354 | 0.700 |
| 8.274 | 18.5 | 4.89 | 4.41 | 7.69 | 1.03 | 0.000 | 0.700 |
| 8.710 | 17.6 | 5.00 | 3.85 | 8.26 | 2.99 | 0.000 | 0.700 |

| 0.000 | 0.02 | 40.0 | 5.35 | 4.64 | 7.57 | 0.39 | 0.700 | 0.700 |
| 2.807 | 37.6 | 5.46 | 4.57 | 7.60 | 0.42 | 0.353 | 0.700 |
| 4.545 | 34.3 | 5.56 | 4.40 | 7.75 | 0.91 | 0.000 | 0.700 |
| 4.679 | 34.1 | 5.63 | 3.80 | 8.27 | 2.65 | 0.000 | 0.700 |

| 0.000 | 0.02 | 60.0 | 5.71 | 4.67 | 7.50 | 0.28 | 0.700 | 0.700 |
| 2.299 | 55.4 | 5.80 | 4.50 | 7.61 | 0.32 | 0.347 | 0.700 |
| 3.569 | 48.3 | 5.88 | 4.29 | 7.78 | 0.84 | 0.000 | 0.700 |
| 3.578 | 48.1 | 5.93 | 3.81 | 8.10 | 1.91 | 0.000 | 0.699 |

The corresponding evolutionary tracks in the HRD are shown in Figs 1 and 2, and the modal diagrams in Figs 4–9. $Z$: initial mass fraction of heavy elements. $M$ and $L$: mass and luminosity in solar units. $T_{\text{eff}}$, $T_e$, and $\rho_e$: effective temperature, central temperature and central density in cgs units. $X_e$, $X_\text{c}$: the mass fraction of hydrogen at the centre and on the surface, respectively.

Along the track corresponding to $M_\infty = 60 M_\odot$ all modes are stable. The real parts of the eigenfrequencies, however, exhibit wave-like distortions which may be interpreted as being the result of the crossing of at least two sets of modes, where the resonances have unfolded into avoided crossings. Due to the behaviour of the imaginary parts, even the two resonances between $4.1 < \log T_{\text{eff}} < 4.3$ and $2 < \sigma < 3$ must be classified as avoided crossings. The set of modes whose eigenfrequencies are essentially independent of the effective temperature may be identified with the ordinary acoustic modes; the additional set(s) whose eigenfrequencies strongly decrease along the evolutionary track will be called `strange modes' in the following.

With increasing mass, the avoided crossings in the core-burning phase become more pronounced, indicating a reduced coupling strength between ordinary and strange modes (see Figs 5 and 6). (For a discussion of the concept of coupling, we refer to Glatzel 1987.) As a consequence, higher order strange modes are more clearly discernible and the interpretation of the modal diagrams as being the result of the crossing of several sets of modes becomes more obvious. Similarly, in the shell-burning phase higher order strange modes appear (Fig. 6) and the lowest order resonances show not only reduced coupling strength but also the opposite coupling scheme, i.e. they have unfolded according to the instability band scheme. Some of the resonance-driven instability bands are strong enough to overcome the intrinsic damping of the modes, leading to an instability of the corresponding stellar model (see Figs 5 and 6).

Since instability is caused by resonances between different modes, different modes are responsible for instability in different evolutionary phases (see Fig. 6). In particular, if the single resonance instabilities are not sufficiently strong, they need not overlap and may enclose stable gaps. Thus instability does not always occur in a continuous fashion. During evolution, stable and unstable phases having several simultaneously unstable modes may alternate.

This situation is qualitatively identical to the results of the analogous study based on the older LNL opacities. In particular, the modal diagrams for initial masses of 60 and 80 $M_\odot$ are very similar (compare Figs 4 and 5 of this paper to figs 2 and 3 of Glatzel & Kiriakidis 1993b). In both studies the resonances that become unstable first with increasing mass involve the lowest order strange mode and the second and third ordinary acoustic overtones in the shell-burning phase. Quantitatively, when compared with the previous investigation, the blue edge of the domain of instability is shifted towards lower effective temperatures, while the instabilities have become slightly stronger (e.g. no stable gaps occur along the 100-$M_\odot$ track of the present study). With respect to these (quantitative) differences, we note that the investigations compared do not refer to the same metallicity, and deviations are to be expected. On the other hand, we have argued in Glatzel & Kiriakidis (1993b) that results based on the opal opacities, at least for low metallicities, should be similar to those based on the LNL tables, since the difference between them decreases with increasing metallicity. The present study confirms this conjecture, and, moreover, proves that new features due to the contribution of heavy elements in the opal opacities are not responsible for the instabilities in the upper right corner of the HRD above the HD limit.

Unstable phases along the various evolutionary tracks in Fig. 1 are indicated by solid lines; thick lines denote models with $\eta < -0.1$ for at least one mode. Note that the blue edge of the theoretical instability domain coincides with the blue edge of the observed HD limit, while the lower luminosity limit, in particular below $\log T_{\text{eff}} \approx 4.1$, is not consistent with the observed HD limit. Of the four LBVs entered in Figs 1 and 2, only η Car is predicted by our study to be unstable.

3.2 Solar composition $Z = 0.02$

The main difference between the LNL and OPAL opacities consists of the contribution of heavy elements. The proper treatment of this contribution in the OPAL calculations leads to an appreciable opacity enhancement around $\log T \approx 5.5$. Note that a small opacity bump around this temperature is also present in the LNL tables. Owing to its origin, this
opacity enhancement depends strongly on the metallicity $Z$. Consequently, all opacity-dependent effects will also be extremely sensitive to $Z$. For this reason, we study the resonance-driven instability discussed here for various values of $Z$. For solar composition ($Z = 0.02$) and evolutionary tracks corresponding to initial masses of 60, 40 and 20 $M_\odot$ (see Fig. 2), modal diagrams are shown in Figs 7–9.

With respect to the real parts of the eigenfrequencies, the modal structure of an object that initially has 60 $M_\odot$ and $Z = 0.02$ (Fig. 7) is qualitatively identical to that of its counterpart with 100 $M_\odot$ and $Z = 0.004$ (Fig. 6). In both the core- and shell-burning phases we observe multiple mode crossings between ordinary acoustic and strange modes, with an almost identical pattern for the two objects. In the shell-burning phase we meet the same situation even with respect to the stability properties (i.e. the imaginary parts): the dominant instabilities involve the lowest order strange mode and the lowest four ordinary acoustic modes.

The result that instabilities and mode patterns in the shell-burning phase are qualitatively identical both for different metallicities and for the different opacity tables (see Glatzel & Kiriakidis 1993b) may be understood on the basis of the interpretation of the modal diagrams below $\log T_{\text{eff}} \approx 4.4$ given in Glatzel & Kiriakidis (1993b). In this temperature range, He i or He i/H ionization zones are responsible for the occurrence of strange modes and the associated instabilities. As opacities for the relevant densities and temperatures only weakly depend on metallicity and do not differ

![Figure 4](https://academic.oup.com/mnras/article-abstract/264/1/50/1075506/54.M.Kiriakidis.K.J.Fricke.and.W.Glatzel)
very much between the two opacity tables, similar results are to be expected. The Z-dependent opacity bump around log $T = 5.3$ that is due to the contribution of heavy elements lies well below the He and H ionization zones, and, being screened by them, has only an indirect influence on the stability properties of the star. The modal structure in the H-ionization range (below log $T_{\text{eff}} = 3.9$), which shows fast variation of the eigenfrequencies with effective temperature, has not been fully resolved; the modal diagrams may not be complete in this range, although we believe that the dominant instabilities have been identified. A detailed study of this regime is necessary, and will be presented elsewhere. To achieve the required high resolution in $T_{\text{eff}}$ an investigation based on envelope models seems to be more adequate than that using the coarse grid of complete evolutionary models.

The mode labelled 'SEC' in Figs 4–7 is found to be strongly damped and therefore of academic interest only; it requires special consideration. In addition to the high damping rate, its frequency decreases when followed from low effective temperatures to the beginning of the shell-burning phase. This behaviour has been found previously in a study of extreme helium stars (Gautschy & Glatzel 1990b) for strange modes of type II (defined there), which were shown to belong to the secular spectrum. In order to investigate the conjecture that the modes labelled 'SEC' are also strange modes of type II and therefore of thermal origin (secular modes), we have artificially reduced the acceleration term in the equation of motion. Even for vanishing acceleration, i.e. without the restoring force necessary for the existence of acoustic modes, we were able to identify counterparts of the
SEC modes. Thus we have to classify them as oscillatory secular modes. For a detailed discussion of the thermal and secular spectrum, in particular with respect to the occurrence of strange modes of type II, we refer to Gautschy & Glatzel (1991).

Following the interpretation of Glatzel & Kiriakidis (1993b), strange modes and mode crossings above $\log T_{\text{eff}} = 4.4$, i.e. essentially in the core-burning phase, are associated with the opacity bump around $\log T = 5.3$. Thus we expect a strong dependence of this phenomenon on metallicity. In fact, the stability properties of the stellar models are very sensitive to metallicity, although for the frequencies $\sigma_i$ there is no qualitative difference between results based on different opacity tables and different $Z$ in this phase (cf. Figs 6 and 7 and fig. 3 of Glatzel & Kiriakidis 1993b). Contrary to the analysis based on the LATEL opacities and on models with low initial metallicities, the two lowest order strange modes (and their resonances with the ordinary acoustic modes) are associated with strong instabilities. These disappear and give way to the previously discussed instabilities caused by He II ionization if the star has become cool enough to allow for a He II ionization zone, i.e. for the 60-$M_\odot$ object around the core-contraction phase.

The dependence on mass of the resonance and strange-mode instabilities associated with the heavy element bump of opacity is similar to that discussed for their counterparts connected with He and H ionization zones. Both the range of effective temperature for unstable models and the strength of the instabilities increase with mass. For an initial mass of 60-$M_\odot$, instability sets in during the early core-burning phase immediately after the star has left the ZAMS (see Fig. 7). An object having 80-$M_\odot$ initially is already unstable on the
ZAMS, and remains so throughout its evolution. Thus the instability of main-sequence models (for $Z = 0.02$ above 80 $M_\odot$) discussed in Glatzel & Kiriakis (1993a) is the continuation on the main sequence of the strange-mode instabilities associated with the enhancement of the opacity due to heavy elements around $\log T = 5.3$ discussed here. Any interpretation given here and in the following therefore also applies to the situation described in Glatzel & Kiriakis (1993a), and vice versa. As a consequence of the fact that we are dealing with the same physical phenomenon, modal diagrams of an evolving model as a function of effective temperature (see e.g. Fig. 7) and of main-sequence stars as a function of mass (see e.g. fig. 8 of Glatzel & Kiriakis 1993a) are very similar.

Modal diagrams for stars with initial masses of 40 and 20 $M_\odot$ are shown in Figs 8 and 9 respectively. For $M_{\infty} = 40 M_\odot$, instabilities connected with He ionization have disappeared, and instability bands have turned into a sequence of avoided crossings (cf. Figs 7 and 8 in the range $3.9 < \log T_{\text{eff}} < 4.4$). The growth rate of the only unstable mode, the lowest order strange mode, which is associated with the heavy element opacity bump, is reduced, and its coupling to the ordinary acoustic modes (in the form of avoided crossings) becomes stronger, thus weakening the impression of the crossing of two sets of modes. Apart from very low effective temperatures (H ionization), no indication of mode crossings and additional strange modes is found in the modal diagram for $M_{\infty} = 20 M_\odot$ (Fig. 9). The frequencies are regularly spaced and all modes are stable, except for the fundamental mode and the first overtone of the ordinary acoustic spectrum around the core-contraction phase.

Due to the continuous connection of this instability to the strange-mode instability at higher masses, one way to interpret this instability is to regard it as a trace of the unstable
fundamental strange mode associated with the heavy element opacity bump, where the strange mode is only weakly unstable and coupling with the ordinary acoustic spectrum occurring through avoided crossings is very strong. On the other hand, the instability is continuously connected with the ordinary $\kappa$-mechanism instability – also caused by the heavy element opacity bump – of $\beta$ Cepheid stars with masses below 20 $M_\odot$ (see Cox et al. 1992; Kiriakidis et al. 1992; Moskalik & Dziembowski 1992; Gautschy & Saio 1993), and so may alternatively be interpreted in the light of this connection. In other words, at the metallicities considered here there is a continuous transition from weak standard $\kappa$-mechanism instabilities at low masses to strong strange-mode instabilities at high masses, both being caused by the heavy element opacity bump.

We have indicated unstable stellar models in the HRD for $Z = 0.02$ (Fig. 2) in the same way as for low metallicities. Compared with the case of low metallicity (Fig. 1), the domain of unstable models has significantly expanded, both to lower luminosities and to higher effective temperatures. Objects with initial masses higher than 80 $M_\odot$ (should they exist) are unstable throughout their evolution. The blue edge of the instability domain that has a smaller slope than the main sequence crosses the main sequence at log $L/L_\odot = 5.94$ and log $T_{\text{eff}} = 4.69$. As luminosity decreases, the width of the instability region shrinks, its continuation for masses below 20 $M_\odot$ being the $\beta$ Cepheid strip. For initial masses between 40 and 20 $M_\odot$ the red edge of the instability domain lies, almost independent of the effective temperature, at log $T_{\text{eff}} = 4.35$, and drops below log $T_{\text{eff}} = 3.7$ when the initial mass exceeds $50 M_\odot$. We note that for 40 $M_\odot < M_{\text{in}} < 60 M_\odot$ the red edge of the instability domain is not well defined. Along the evolutionary tracks in this range, unstable phases may be separated by stable gaps (see also Fig. 1 and
The stability of massive stars

Figure 9. As Fig. 7, but for \(M_\odot = 20M_\odot\).

(0.75, 0.65) to (1.0, 0.7)

fig. 1 of Glatzel & Kiriakiidis 1993b), this being an immediate consequence of the fact that the instability is caused by the superposition of various resonance instabilities of different modes. These do not necessarily overlap, particularly close to the stability boundaries. Nevertheless, it is remarkable that the ragged lower luminosity limit of the theoretical instability domain for low effective temperatures closely follows the corresponding part of the observed HD limit.

In contrast to the case of low metallicity, where the blue edge of the instability domain coincides with the HD limit, a corresponding identification is not immediately obvious from Fig. 2 for solar composition. When considering the origin of the instabilities, however, we find that even in the case \(Z = 0.02\) the blue edge of the HD limit approximately follows the position at which resonance instabilities connected with He II ionization set in, i.e. where instabilities caused by the heavy element opacity bump and He II instabilities alternate. The transition between the two types of instability (i.e. around the blue edge of the HD limit) is associated with a minimum of the growth rate (see the 80-M_\odot track in Fig. 2 and the modal diagram for \(M_\odot = 60M_\odot\) in Fig. 7).

With respect to the LBVs in Fig. 2 we note that according to our results all of them should be unstable, provided that they have approximately solar composition.

4 INTERPRETATIONS

4.1 A comment on the origin of strange modes

The way in which the origin of strange modes might be understood in terms of a modified sound velocity has been described in a previous paper (Glatzel & Kiriakiidis 1993b). (For alternative explanations see Shibahashi & Osaki 1981 and Zalewski 1992.) There we argued that edges and bumps
in the run of the sound velocity through the star divide the star into several acoustically decoupled zones, to each of which belongs a discrete spectrum of acoustic modes whose frequencies change in the course of evolution. As the dependences of the frequencies on stellar parameters are not identical, this leads to multiple crossings between the different sets of modes, which unfold into either avoided crossings or instability bands.

The discontinuities and peaks necessary for this explanation to work could not, however, be identified in the run of either the adiabatic or the isothermal sound velocity. We therefore proposed that a redefinition of the sound velocity based neither on adiabatic nor on isothermal changes of state, which is more appropriate to the highly non-adiabatic stellar envelopes considered, could provide the desired behaviour of the sound velocity. For the models investigated here, which are based on the opal opacities rather than on the laol tables in previous studies, we have re-examined the run of the adiabatic sound velocity. The result is shown in Fig. 10, where the adiabatic sound velocity $\upsilon_S$ is given as a function of radius for three stellar models – identified by their effective temperatures – in the core-burning phase of
the object with $M_\text{e} = 60 M_\odot$ and $Z = 0.02$. The adiabatic sound velocity is no longer a monotonically decreasing function, and an inversion is associated with the heavy element opacity bump.

According to our model, the peak in sound velocity should give rise to the occurrence of a strange-mode phenomenon even in the adiabatic approximation. A modal diagram of the lowest order neutrally stable adiabatic eigenfrequencies for the evolutionary track corresponding to $M_\text{e} = 60 M_\odot$ and $Z = 0.02$ is shown in Fig. 11. In fact, in the core-burning phase a strange-mode phenomenon similar to that found in the exact non-adiabatic treatment (cf. Fig. 7) is present. Note that in the adiabatic approximation oscillatory modes have to be neutrally stable, and mode crossings can unfold only into avoided crossings. The dashed line in Fig. 11 denotes the inverse of the sound traveltime between the photosphere and the position of the heavy element opacity bump, which is identical to the location of the inversion of the sound velocity in Fig. 10. As expected, this inverse time-scale resembles the run of the lowest order strange mode in Fig. 11, thus supporting the idea that a consideration of the sound velocity may provide an explanation of the strange-mode problem. For strange modes associated with the He ionization, the consideration of a redefined sound velocity is still necessary. No strange modes are present in the adiabatic approximation here (see Fig. 11).

### 4.2 Some comments on the metallicity dependence of the instability domain

For completeness, we have also performed a stability analysis of the stellar models along the evolutionary tracks with initial metallicities $Z = 0.03$ (in Fig. 3, the corresponding HRD, unstable phases are indicated as in Figs 1 and 2). Apart from slightly different stability boundaries, the result turns out to be identical to that for $Z = 0.02$ discussed earlier. We have also tested the stability of stellar models on an evolutionary track corresponding to $M_\text{e} = 90 M_\odot$ and $Z = 0.01$. The track (instability is indicated in the usual way) is inserted in the HRD for $Z = 0.02$ (Fig. 2). The initial mass was chosen in order to obtain stellar models close to the observed position of the object R127. A metallicity of $Z = 0.01$ is thought to be appropriate for the Large Magellanic Cloud to which R127 belongs. From Fig. 2 we deduce that the unstable phases for $Z = 0.01$ are consistent with R127 being an LBV. On the other hand, a $Z = 0.004$ for the Large Magellanic Cloud would not be sufficient to cause R127 to become unstable.

With respect to the $Z$-dependence, it is important to note that two different effects are responsible for the instability of massive stars: the strange-mode instabilities that are due to the heavy element opacity bump (present only in recent opacity tables), and which are thus extremely sensitive to metallicity; and resonance instabilities associated with He ionization which only weakly depend on metallicity.

In the HRD, instabilities due to He ionization are located in the region above a boundary which is approximately defined by the observed HD limit. While the blue edge of the domain of existence of these instabilities coincides with the corresponding part of the HD limit [this is particularly obvious when they are the only instabilities present ($Z = 0.004$; Fig. 1)], its lower luminosity limit depends weakly on metallicity, decreasing with growing metallicity and thus approaching the corresponding part of the HD limit. This is likely to be an indirect effect caused by the overall increase of opacity with metallicity which leads to more extended stellar envelopes and thus favours resonance instabilities. For very low metallicities, our stability analysis indicates the possibility that stable objects above the HD limit could in principle be observed.

The heavy element opacity bump induces instabilities in a domain below the HD limit which is extremely sensitive to the metallicity $Z$. This domain does not exist at all for low metallicities ($Z = 0.004$, appropriate for Small Magellanic Cloud stars) and covers the whole range of the observed LBVs for solar composition, extending from the ZAMS to the HD limit above $\log L/L_\odot \approx 5.95$ and continuing as the $\beta$ Cepheid strip for low luminosities. For intermediate metallicities, which have not been investigated in detail, the domain is likely to define an at least partially separate instability strip between the ZAMS and the HD limit (see the case $Z = 0.01$, $M_\text{e} = 90 M_\odot$ presented in Fig. 2 which exhibits a stable gap between the two types of instability). The strong metallicity dependence established for the instabilities discovered suggests that in an interpretation of the observations of LBVs and related objects different metallicities should be considered separately. When stability boundaries are determined observationally, their metallicity dependence should be taken into consideration.

Within the framework of our results, the HD limit - which is approximately identical to the theoretically determined stability boundary of the He-ionization-induced instabilities - could be interpreted in two ways. The first possibility is that the instability domain that is common to all objects essentially independently of metallicity is determined by the He-ionization-driven instability located above the HD limit. If objects with different metallicities are not distinguished observationally, the stability boundary thus determined will be the HD limit. The second possibility is that the final result of an instability caused by the heavy element opacity bump is finite amplitude variability, while He-ionization-driven resonance instability precludes the existence of stellar models in this part of the HRD. If this is the case, we should not observe stars above the HD limit.

### 5 CONCLUSION

The stability of massive stars has been re-examined on the basis of standard physics and the opal opacity tables (Iglesias et al. 1992). A variety of strange-mode and resonance instabilities can be identified, and the associated domain of instability in the HRD is determined. We emphasize that these instabilities are not related to the classical instability mechanisms (the $\kappa$- and $\epsilon$-mechanisms).

With respect to their origin, the instabilities fall into two groups. One of these is associated with He ionization, and the corresponding domain of instability is confined approximately by the observed Humphreys–Davidson limit. As these instabilities only weakly depend on chemical composition, they exist for all cases (for low metallicities they are the only ones present) and their range of instability is common to all instability domains. The mechanism and origin of the resonance-driven instabilities that are due to He ionization have already been discussed with respect to the
HD limit in a previous analysis based on the LAO1 opacities (Glatzel & Kiriakidis 1993b). For the He-ionization-driven instabilities, the agreement of the theoretical stability boundaries and the observed HD limit is closer for the OPAL opacities than for the LAO1 tables.

In addition to instabilities caused by He ionization, the opacity enhancement due to heavy elements present in the latest OPAL tables (Iglesias et al. 1993) is responsible for strongly metallicity-dependent strange-mode instabilities. These instabilities are found to be identical to instabilities on the upper main sequence discussed in a previous investigation. For high masses these instabilities provide dynamical growth rates during the whole evolution of an object from the ZAMS up to the HD limit, where they give way to the resonance instabilities due to He ionization. The corresponding instability domain covers the whole range of observed LBVs for solar chemical composition. This domain extends as a narrow strip to low luminosities and masses, becoming identical here to the β Cepheid strip. Although the continuously connected common instability domain for β Cepheid stars and LBVs, and the common origin of the instabilities -- namely the heavy element opacity bump -- seem to suggest that the instability mechanisms are identical, we emphasize that this is not the case. β Cepheids are unstable due to the κ-mechanism, while LBV variability is driven by strange-mode instabilities. The two mechanisms can be separated by applying the non-adiabatic reversible (NAR) approximation, which excludes the κ-mechanism but not mode resonances nor strange modes (for a test in this direction, see Glatzel & Kiriakidis 1993a and Kiriakidis et al. 1992).

The extreme sensitivity of the stability boundaries to metallicity suggests an observational study of this point. The interpretation of existing data should at least take into account the possibility that the LBV phenomenon is metallicity-dependent. The observationally determined HD limit -- which is according to our study the absolute theoretical stability boundary -- could be a result of disregarding the chemical composition.

The present study has been restricted to the investigation of radial perturbations. If non-radial oscillations were also taken into account, further instabilities could in principle expand the domains of instability determined here. However, for strange-mode instabilities in extreme helium stars and perturbations with a low harmonic degree ℓ, the results of a study in this direction were almost identical to those for the case ℓ = 0 (Glatzel & Gautschy 1992). In general, high-ℓ perturbations were less unstable than low-ℓ ones. Thus, although we cannot exclude this possibility, we do not consider a significant change of our results for ℓ ≠ 0 to be very likely.

The final result of the violent instabilities discovered cannot be anticipated from a linear analysis and deserves further investigation. For this purpose, we are currently performing numerical calculations of the non-linear evolution of the instabilities. Low growth rates seem to lead to non-linear pulsations, while high values of |σ|−0.1, corresponding to dynamical time-scales |σ|−0.1 are found to be associated with ejections of the outer layers of the star that correspond to a pulsationally driven wind with mean mass-loss rates up to 10−3 M⊙ yr−1. Thus the resonance and strange-mode instabilities may imply an upper mass limit for stellar objects (rather than instabilities driven by the κ-mechanism; see, however, Appenzeller 1970). The quantitative determination of upper mass limits and mass-loss rates as a function of metallicity, due to these instabilities, will provide very important input into models of the chemical and photometric evolution of galaxies.

We emphasize that the origin of instability and variability -- together with the wind that may be the final result -- lies well below the photosphere of the star. Moreover, the theoretical instability domain calculated on the basis of standard stellar evolution physics using up-to-date opacities agrees reasonably well with the range of observed LBVs, indicating that not only stellar evolution but also stability calculations are consistent with the observations.

ACKNOWLEDGMENTS

Financial support for the DFG under grants GI 127/2-1 (WG) and GI 127/4-1 (MK) is gratefully acknowledged. The numerical computations have been carried out using the IBM 3090-300E of the GWDG at Göttingen.

REFERENCES