The fundamental relations of elliptical galaxies

Rafael Guzmán, J. R. Lucey and Richard G. Bower

ABSTRACT
We investigate the basic laws that determine the global structure and metal abundance of elliptical galaxies. The existence of the Fundamental Plane has been considered to imply that the virial theorem is the only structural constraint for giant ellipticals. However, we show that giant ellipticals do not uniformly cover the Fundamental Plane, but are located in a band which is not the result of selection effects. This 'Fundamental Band' implies a second constraint between scalelength and galaxy mass.

On the basis of this result, we present a new framework in which the structure and metal abundance of giant ellipticals are determined by only three fundamental relations: \( M \propto R \langle v^2 \rangle \), \( M \propto R^2 \) and \( Z \propto \langle v^2 \rangle \), where \( M \) is the galaxy mass, \( R \) is the half-mass radius, \( \langle v^2 \rangle \) is the mean square speed of the system's stars and \( Z \) is the average metallicity of the stellar population; \( \xi \) and \( \xi / \xi \) are constants. \( \xi / \xi \) is uniquely determined from the observations. The value of \( \xi / \xi \), however, depends on the assumed scaling laws that relate \( M \) and \( R \) to the observed luminosity and half-light radius. We assume \( M/L \propto M^{\eta} \) and \( R/R_e \propto M^{\lambda} \). The two constants \( \eta \) and \( \lambda \) are mutually constrained by observations, but their values are not uniquely determined. All the wide variety of observed global correlations can be derived as simple combinations of these fundamental relations.

This simple framework provides new insights into the intrinsic differences between giant and dwarf ellipticals. The observed universality of the luminosity–metallicity–velocity dispersion correlations strongly suggests a simple solution within our framework in which \( \xi / \xi \), \( \xi / \xi \) and \( \eta \) adopt the same values for both dwarf and giant ellipticals. In this case, we show that the dependence of \( R/R_e \) on galaxy mass is the only difference between the two galaxy families.

We compare this framework with a theoretical scenario of galaxy formation that combines the hierarchical clustering and the galactic wind models. This picture provides a consistent explanation of the fundamental relations of all elliptical galaxies, assuming \( R/R_e \sim \) constant for dwarf ellipticals while, for giant ellipticals, we find that \( R/R_e \) must be a decreasing function of galaxy mass.

Key words: galaxies: abundances – galaxies: elliptical and lenticular, cD – galaxies: formation – galaxies: fundamental parameters – galaxies: structure.

1 INTRODUCTION

Elliptical galaxies show strong correlations between the global parameters that describe their overall dynamical structure and metal abundance. Early investigations concentrated on monovariate relations, i.e. the galaxy luminosity \( (L) \) was shown to be correlated with effective radius \( (R_e) \) (Fish 1964), central velocity dispersion \( (\sigma) \) (Faber & Jackson 1976), effective surface brightness \( (SB_e) \) (Binggeli, Sandage & Tarenghi 1984) and colour (Sandage 1972) or \( M_g \) line-strength index (Terlevich et al. 1981). These results suggested that elliptical galaxies form a one-dimensional family, with luminosity (or mass) as the main parameter defining the sequence.

Subsequent studies extended this analysis to lower luminosity objects, and found that all the correlations involving effective parameters showed an abrupt change of slope at \( M_b \sim -18 \) mag (Wirth & Gallagher 1984; Kormendy 1985).
The observed trends are the same whether core or global parameters are used (Kormendy 1987). This effect turned out to be one of the major results derived from the study of these global correlations, i.e. there are two fundamentally different kinds of elliptical galaxies, loosely called giants and dwarfs. Giants are the luminous, ordinary ellipticals, characterized by relatively high luminosity-density and relatively low mass-to-light ratio. Despite the similarity in morphological appearance, dwarfs show opposite characteristics. Dwarf ellipticals may represent galaxies swept by galactic winds during their initial starburst phase, leading to the metal-poor, low-density remnants we see today (Dekel & Silk 1986; Yoshii & Arimoto 1987). Although galactic winds could drive out baryons by pressure, they would not affect any non-baryonic dark matter, thus perhaps accounting for the large mass-to-light ratios characteristic of these objects (Dekel & Silk 1986). Alternatively, giants are thought to be descended from protogalaxies massive enough to retain their processed material, thus explaining the high metallicities observed. Dissipative collapse, be it smooth or by gradual merging, leads naturally to higher luminosity densities. However, despite the evidence for different formation processes for the two galaxy families, Caldwell (1983) has shown that the colour–magnitude relation for giants extends smoothly down to the dwarf ellipticals. Also, the correlations between the velocity dispersion and luminosity or Mg2 line-strength index are apparently universal for both galaxy families (Tonry 1981; Bender 1991a). This regularity in the stellar population properties of all ellipticals, independent of their large variety in structure and kinematics, remains both a challenge and an important clue for the development of a consistent picture of galaxy formation.

Recently, a substantial step forward in the understanding of galaxy properties has been made by studying correlations of higher dimensionality. All of the above correlations show a residual scatter which is not accounted for by measurement errors. The observed trends of the residuals in the L–σ relation with other physical variables, such as axial ratio, M/L, surface brightness or colour, hinted at hidden-parameter effects (Terlevich et al. 1981; Tonry & Davis 1981; de Vaucouleurs & Olson 1982; Efstathiou & Fall 1984; Lauer 1985). For giant ellipticals, an improved version of the L–σ relation which incorporates SB2 as a second parameter was found by Dressler et al. (1987) and Djorgovski & Davies (1987), i.e. the Fundamental Plane. Surface brightness may also be incorporated into the colour (or Mg2)–L relation to derive a similar plane relative to the properties of the stellar population, rather than the overall structure, of elliptical galaxies (de Carvalho & Djorgovski 1989). The basic result from these studies is that giant elliptical galaxies appear to form a biperametric, nearly planar family (in logarithmic units). The different monovariate correlations have been interpreted as its oblique projections, while the tilt with respect to the coordinate planes produces the intrinsic scatter (Faber et al. 1987; Djorgovski 1991). The two-dimensionality of this galaxy family suggests that M/L must be a function of only two structural variables and that the virial theorem is the only strong constraint on the structure of giant ellipticals (Faber et al. 1987). For dwarfs, however, no similar bivariate correlation seems to exist (Bender, Burstein & Faber 1992; de Carvalho & Djorgovski 1992).

In this paper, we investigate the fundamental relations of elliptical galaxies, i.e. the basic laws common to all ellipticals from which the wide variety of observed correlations are derived. The scatter and coefficients of these relations imply direct constraints on the structure and stellar populations of present-day ellipticals. In addition, they provide direct information on the physical variables that controlled the early formative processes. Our analysis is largely based on $R_e$, $SB_2$, $\sigma$ and $Mg_2$ measurements listed by Dressler (1984), Lucey et al. (1991) and Guzmán et al. (1992a), for a sample of 66 giant ellipticals in the Coma cluster. In Section 2, we show that giant ellipticals do not uniformly cover the Fundamental Plane. On the basis of this result, we develop, in Section 3, a new framework in which only three fundamental relations are needed to derive all the global correlations of giant ellipticals. This framework is generalized, in Section 4, to provide a common set of fundamental relations for all spheroidal systems. Finally, in Section 5, we discuss the results of our framework within a simple picture of galaxy formation based on the hierarchical clustering scenario and the galactic wind model.

2 THE DISTRIBUTION OF GIANT ELLIPTICALS ON THE FUNDAMENTAL PLANE

The existence of the monovariate correlations indicates that elliptical galaxies must lie on a relatively narrow locus on the Fundamental Plane (see Peacock 1990). The area of the plane covered by ellipticals is, in principle, determined by selection effects of the galaxy sample. Disney (1976) pointed out that correlations such as Fish's law (i.e. $L/R_e^2$ constant) may be caused by selecting objects with a small range of surface brightness, and thus be entirely spurious. If selection effects are indeed responsible for the above correlations, ellipticals should be randomly scattered over the area of the plane limited by these effects.

In Fig. 1, we plot the face-on view of the Fundamental Plane (hereafter FP) for a magnitude-limited sample of 37 ellipticals brighter than $m_r = 15.5$ mag that are located in the central area of the Coma cluster (squares). This data set possesses three important characteristics: homogeneity, equidistant objects and completeness. The best-fitting FP for this sample is

$$-\log R_e - 0.783 \log I_e + 1.130 \log \sigma = -0.042,$$

where $R_e$ is expressed in units of kpc, $I_e$ is the average intensity within $R_e$ in $L_{\odot} \text{pc}^{-2}$, and $\sigma$ is in km s$^{-1}$ [adopting $H_0 = 50$ km s$^{-1}$ Mpc$^{-1}$, $cz_{\text{Coma}} = 7200$ km s$^{-1}$ and $M_{Ie} = 4.78$, so that 1 arcsec is equivalent to 0.7 kpc and $I_e$ is related to $SB_2$ by log $I_e = (26.35 - SB_2)/2.5$. For Fig. 1 we have adopted the following coordinate system:

$$x = (1.890 \log R_e - 0.783 \log I_e + 1.130 \log \sigma)/2.337,$$

$$y = (1.130 \log I_e + 0.783 \log \sigma)/1.375,$$

$$z = (-\log R_e - 0.783 \log I_e + 1.130 \log \sigma)/1.700,$$

where the $x$-axis is perpendicular to the FP and $\{x, y\}$ defines an orthogonal coordinate system on the FP, with the $x$-axis being simply proportional to log $R_e$. We have plotted, in Fig. 1, the five lines that limit the ‘observing window’ of the data.
The fundamental relations of elliptical galaxies

set. This region is partly defined by the sample selection, i.e. \( m_v \leq 15.5 \text{ mag} \) and \( \sigma \geq 100 \text{ km s}^{-1} \). The upper limit in absolute magnitude corresponds to the maximum of the luminosity function for ellipticals (i.e. \( M_v \sim -25 \text{ mag} \)). Surface-brightness limits characteristic of unusually compact and unusually diffuse objects are also considered. Roughly, objects with \( SB_x \geq 17 \text{ mag arcsec}^{-2} \) are too star-like, and those with \( SB_x \geq 23 \text{ mag arcsec}^{-2} \) have too low surface brightness to have been included in this sample; these limits, however, do not significantly affect the results presented below.

Fig. 1 clearly shows that elliptical galaxies do not uniformly cover the FP; rather they are confined to a relatively narrow band that is not a result of selection effects. Within the 'observing window' of the data set, there are no very luminous ellipticals with high surface brightness or galaxies with both low surface brightness and low velocity dispersion. The 29 ellipticals (brighter than \( m_v \sim 14.5 \text{ mag} \)) that lie in the outer area of the cluster are also shown in Fig. 1 (circles). Although these additional ellipticals do not form a magnitude-limited sample, they strengthen the evidence that the distribution of points on the FP is not the result of selection effects alone.

This result can be shown quantitatively, using a simple Monte Carlo simulation. We consider the null hypothesis that there is no intrinsic correlation between galaxy luminosity and surface brightness, i.e. the distribution of galaxies on the FP is only due to selection effects. To test this hypothesis we calculate the probability that the correlation coefficient \( r_c \) of the actual galaxy distribution on the plane may be obtained with a simulated random sample of similar characteristics. A typical galaxy in our simulations has random luminosity and surface brightness selected from our combined data set and a velocity dispersion derived from the equation of the FP, i.e.

\[
\log \sigma = 0.442 \log L + 0.250 \log I_e - 0.390.
\]

(Note that throughout this paper we adopt \( L = 2\pi I_e R_e^2 \).) The corresponding data point on the plane must lie in the region allowed by our selection effects to be finally selected. The surface-brightness limits have been reduced to the maximum and minimum values observed in our Coma sample (i.e. \( SB_x = 18 \) and \( SB_x = 22.5 \text{ mag arcsec}^{-2} \), respectively) to provide a more robust test. Each simulated sample with these characteristics produces a different distribution of data points on the plane for which we compute the correlation coefficient \( r_c \). After \( 10^5 \) simulations we obtained no value of \( r_c \) such as \( |r_c| \geq |r_{\text{null}}| \). The probability that the observed distribution on the FP is due to selection effects alone is thus less than \( 10^{-3} \), and we conclude that this null hypothesis is ruled out.

The position of ellipticals on the FP is adequately described by

\[
y = -0.39 x + \text{constant}
\]

(dashed line in Fig. 1). When this relation is combined with the FP equation given above and \( R_e \), \( I_e \) and \( \sigma \) are eliminated in turn, the following expressions result:

\[
L \propto R_e^{1.2} \quad \text{(cf. Fish 1964)},
\]

\[
L \propto I_e^{1.7} \quad \text{(cf. Faber & Jackson 1976)},
\]

\[
I_e \propto R_e^{0.8} \quad \text{(cf. Kormendy 1977)},
\]

\[
L \propto I_e^{1.5} \quad \text{(cf. Binggeli, Sandage & Tarenghi 1984)}.
\]

The slopes of the derived relations are similar to the values previously reported in the literature. Each of these four monovariate relations is therefore a direct result of both the projection of the FP and the intrinsic distribution of galaxies on the plane.

In Fig. 2, we show the projections of the FP and the selection boundaries on to different planes of observables. The important feature of these plots is that the axes are equally scaled, so that the range of values for each coordinate is always the same (e.g. three logarithmic units, as in Fig. 1). Our observing window is represented by thin solid lines. The thick solid lines outline the boundaries of Fig. 1, and thus allow the tilt of the FP with respect to each plane of observables to be seen. Since the angle between the \( \{ I_e, L \} \) and FP planes is only \( \sim 27^\circ \), this projection provides a good representation of the face-on view of the FP in terms of physically meaningful variables. An almost edge-on view of the FP is shown in the \( \{ \sigma, L \} \) projection. Yet, because of the projection angle, the observed scatter around the Faber-Jackson correlation will be significantly smaller than the actual scatter across the plane.

Fig. 2 allows the correct interpretation of the \( \{ I_e, \sigma \} \) and \( \{ R_e, I_e \} \) projections to be made. In previous work (Faber et al. 1987), the \( \{ I_e, \sigma \} \) plane was considered to show the FP almost face-on. Since the plot of \( I_e \) versus \( \sigma \), scaled according to the range of data points, shows a scatter diagram, elliptical...
galaxies were thought to cover the plane uniformly, and hence to form a biparametric family. The \( \{ R_e, I_e \} \) plane in turn was interpreted as an almost edge-on view of the FP such that the correlation between \( R_e \) and \( I_e \) (Kormendy 1977) would simply reflect the tilt of the FP. As Fig. 2 shows, both the \( \{ I_e, \sigma \} \) and \( \{ R_e, I_e \} \) projections form a similar angle of \( \sim 50^\circ \) with respect to the FP. Thus Kormendy's relation is not the result of the tilt of the FP, but reflects the actual distribution of galaxies on the FP. The relation that describes this distribution (given above) is shown projected on to each plane of observables (dashed lines). This relation provides an accurate description of the observed trends in each projection. The apparently poor correlation between \( I_e \) and \( \sigma \) is simply an artefact of projection effects due to the particular orientation of the galaxy distribution on the FP relative to the \( \{ I_e, \sigma \} \) plane.

The existence of the FP has been physically interpreted as the expression of the virial theorem in terms of 'luminosity' variables. This explanation assumes that the luminous part of giant ellipticals is self-gravitating and suggests a weak dependence of the \( M/L \) ratio on galaxy mass, i.e., \( M/L \propto M^{0.2} \) (Faber et al. 1987). The distribution of ellipticals on the FP implies that there is a second constraint on the galaxy structure in addition to the virial theorem. Using the above scaling law for \( M/L \), the observed distribution translates into the relationship between scalelength and galaxy mass, \( M \propto R_e^{1.5} \), or in terms of the binding energy, \( M \propto E_b^{1.5} \) (cf. Fisher 1964; Saito 1979). We note that this additional constraint is not as well-defined as the virial theorem. The rms of the distribution of data points across the FP is \( \sim 4 \) times the rms observed perpendicular to the FP (in log \( L \) units), i.e. elliptical galaxies are distributed neither on a plane nor along a line, but on a band.

3 THE FUNDAMENTAL RELATIONS OF GIANT ELLIPTICALS

In this section, we present a simple framework that describes the global correlations observed for elliptical galaxies. Previous work on this subject treated these separately, and the coefficients (i.e. slopes and zero-points) were fitted to each correlation independently. However, as stated in Section 2, not all the correlations between global parameters are independent. In particular, we have shown that the structural parameters of giant ellipticals are strongly constrained by the two equations that define the 'fundamental band' in the \( \{ R_e, I_e, \sigma \} \) space. The correlations between metallicity and galaxy dynamical quantities pose a different constraint on the stellar content of these galaxies. Our approach aims to identify the minimum number of basic
The fundamental relations of elliptical galaxies

Ellipticals are supported by velocity anisotropy (Davies et al. 1983). Also, although the adoption of $M_g$ as a metallicity scale is justified by Mould's (1978) theoretical analysis, we note that this index is also affected by differences in the age of the stellar population (cf. Burstein et al. 1988; Schweizer et al. 1990; Guzmán et al. 1992a). In our framework we will assume that differences in galaxy kinematics and in the age of the stellar content are second-order effects which are likely to offset the zero-points rather than to alter significantly the slopes of the global correlations.

With the above assumptions we can express the fundamental relations in terms of the 'observable' parameters, and derive the global correlations as linear combinations of these relations. In Fig. 3, we summarize the assumed relations in our framework and their transformation into the different correlations as a function of $\xi, \eta, \eta$, and $\lambda$. The Fundamental Plane [equation (i)] is the expression of the virial theorem (I) in terms of 'luminosity' variables, using the assumed transformations for $M, M/R^2$ and $\langle v^2 \rangle$ (cf. Faber et al. 1987). Applying the same transformation to the second fundamental property (II), we obtain not only the correlation between $L$ and $I_c$ [equation (ii)] but also the $L-R_e$ and $L-I_c$ correlations [equations (iv) and (v), respectively]. These correlations represent the same constraint on scalelength and galaxy mass, but they are expressed in different 'luminosity' variables. Finally, the third fundamental relation (III) translates directly into the observed correlation between $M_{g_2}$ and $\sigma$ [equation (iii)]. Equations (i), (ii), and (iii) are thus equivalent forms of our fundamental relations (I), (II) and (III). The remaining correlations are linear combinations of these three relations. Thus, combining (i) and (ii), we obtain not only the Faber–Jackson relation [equation (vi)], but also the correlation between $R_e$ and $\sigma$ [equation (vii)]. From (i) and (iii) we derive the equivalent expression of the plane defined in de Carvalho & Djorgovski (1989) [equation (viii)]. Finally, the combination of (i), (ii) and (iii) gives the $M_{g_2}$ line index (or colour–magnitude relation [equation (ix)]).

We have explored which values of $\xi, \xi, \eta, \eta$ and $\lambda$ are simultaneously consistent with all nine correlations for the same sample of 66 Coma ellipticals as we used in Section 2. We first calculate for each correlation the two least-squares slopes resulting from minimization in the absissa and ordinate axes (a similar procedure is used for the bivariate correlations [equations (i) and (vii)]). These two values define the allowed range of variation for each slope. The zero-points are not considered in our analysis at this stage. From the nine global correlations listed in Fig. 3, we obtain a system of 11 inequalities and four unknowns, i.e. $\xi, \xi, \eta, \eta$ and $\lambda$. For example, the two regression lines of the $L-R_e$ correlation for our Coma data set have slopes of 1.1 and 1.5. Equation (iv) then implies the following inequality:

$$1.1 \leq \frac{\xi(1-\eta)}{1-\lambda^2} \leq 1.5.$$
THE FRAMEWORK

FUNDAMENTAL PROPERTIES

\[
\begin{align*}
M & \propto R \left( v^3 \right) \quad (I) \\
M & \propto R \zeta \quad (II) \\
Z & \propto \left( v^2 \right)^\frac{1}{2} \quad (III)
\end{align*}
\]

ASSUMPTIONS

\[
\begin{align*}
M/L & \propto M^n \quad \Rightarrow \quad M \propto L^{\frac{n}{1-n}} \\
R/R_e & \propto M^\lambda \quad \Rightarrow \quad R \propto R_e \left( \frac{M}{M^0} \right)^{\frac{1}{1-\lambda}} \\
L & \propto I_e R^2 \quad \Rightarrow \quad M/R^2 \propto L^{\frac{2}{1-n}} \\
isotropy & \Rightarrow \quad \left( v^2 \right) \propto \sigma^2 \\
metallicity index & \Rightarrow \quad \log Z \propto \sigma^2
\end{align*}
\]

GLOBAL CORRELATIONS

\[
\begin{align*}
-\log R_e - \frac{1-\lambda}{1-\xi} \log L_e + \frac{2(1-n)}{1-\xi} \log \sigma & = k_1 \\
\log L_e - \frac{1-\xi}{1-\lambda} \log I_e & = k_2 + \log(2\pi 10^6) \\
M_{g2} - 2\xi \log \sigma & = k_3 \\
\log L_e - \frac{1-\eta}{1-\lambda} \log R_e & = \left( 1 - \frac{1-\eta}{1-\lambda} \right) k_4 + \log(2\pi 10^6) \\
\log L_e + \frac{(1-2\lambda+\eta)(1-\lambda)-(1-\eta)(1-\xi)}{(1-\lambda)(1-\xi)} \log R_e & = \left( 1 - \frac{1-\eta}{1-\lambda} \right) k_2 \\
\log L_e + \frac{1+4\eta-2\lambda}{1-\xi} \log \sigma & = \frac{(1+4\eta-2\lambda)}{2(1-\xi)} k_1 - \frac{3(1-\lambda)(1-\eta)}{2(1-\xi)} k_2 + \log(2\pi 10^6) \\
\log \sigma + \frac{1-\xi}{1-\lambda} \log R_e & = \frac{1+4\eta-2\lambda}{4(1-\xi)} k_1 + \frac{1-\xi}{4(1-\xi)} (k_1 - k_2) k_3 \\
-\log R_e - \frac{1-\lambda}{1-\xi} \log L_e - \frac{1-n}{1-\eta} M_{g2} & = k_1 + \frac{1-n}{1-\eta} k_3 \\
\log L_e - \frac{1-\eta}{1-\xi} \log M_{g2} & = -\frac{1+4\eta-2\lambda}{1-\xi} k_1 - \frac{3(1-\lambda)(1-\eta)}{2(1-\xi)} k_2 - \frac{1-\xi}{1-\xi} k_3 + \log(2\pi 10^6)
\end{align*}
\]

Figure 3. Outline of our framework.

\{\eta, \zeta\}, \{\eta, \lambda\} and \{\eta, \xi\} projections. For simplicity, we have restricted these combinations to those solutions with \(\xi \leq 4\). The projections clearly suggest that the space of valid \{\zeta, \eta, \xi, \lambda\} combinations is strongly degenerate. In particular, we note that the locus of possible solutions in the \{\eta, \xi\} projection is mainly determined by the slope of the \(L-\sigma\) relation. Since the value of this slope lies approximately between 3 and 4, the condition

\[
\frac{4}{2n+2} \leq \xi \leq \frac{3}{2n+1}
\]

roughly defines the area of the parameter space allowed by the observations. Alternatively, the linear relation between \(\eta\) and \(\lambda\) is primarily determined from the coefficient of the Fundamental Plane, e.g.

\[
1.5 \eta - 0.5 \leq \lambda \leq 1.3 \eta - 0.3.
\]

Hence, once we adopt a value for one of these parameters, the range of variation for the remaining two is strongly constrained. Obviously, the parameter space in the \{\eta, \xi\} projection is uniquely determined by the range of variation of the \(M_{g2}-\sigma\) slope. In all cases, there is no solution for \(\eta \leq -1\) and \(\eta > 1\). In conclusion, as is clear from Fig. 4, there is a large variety of possible combinations of \{\zeta, \eta, \xi, \lambda\} values consistent with the observed global correlations. This apparent indeterminacy in our framework simply reflects the lack of knowledge about the dark matter distribution in elliptical galaxies.

Particularly interesting is the solution corresponding to \(\lambda = 0\), i.e. \(R \propto R_e\). Hereafter we refer to this solution as the constant collapse factor case. This is equivalent to the assumption that, for giant elliptical galaxies, the mass and light distributions are similar, and it includes the case discussed in Guzmán, Lucey & Bower (1992b), i.e. \(R = R_e\). For our Coma sample

\[
\xi = 1.635, \quad \xi = 0.116, \quad \eta = 0.274.
\]

These values are the mean of the distribution of possible solutions with \(\lambda = 0\). The standard deviations of each distribution are 0.068, 0.021 and 0.013, respectively. Substituting the values of \(\xi, \eta, \xi, \lambda\) into equations (i)-(ix), the slopes for each of the nine correlations are directly obtained (see Table 1). These slopes are very similar to values previously reported in the literature for the individual correlations. Finally, the resulting slopes of our fundamental relations (i), (ii) and (iii) are used to derive the mean values of the zero-points \(k_1, k_2, k_3, k_4\) from our data set. The zero-points of the
remaining six correlations are then calculated, using the linear combinations given in Fig. 3. We show, in Fig. 5, the linear fits to the different observed correlations for the Coma sample as listed in Table 1. The first two top panels show the edge-on and almost face-on views of the FP. The third top panel shows the $M_{B_2}$-$\sigma$ relation. As noted previously, these three correlations represent our three fundamental relations. The remaining panels simply show different combinations of these relations. Clearly, our framework provides an excellent description of the observed correlations for giant elliptical galaxies.

The Coma data set constitutes a homogeneous population of cluster ellipticals. A more heterogeneous sample of $\sim 400$ cluster and field ellipticals is given in Faber et al. (1989). In Fig. 6, we show the correlations listed in Table 1 superimposed on this data set and zero-pointed to their $B$-band Coma subsample. Again, our fits are in excellent agreement with the observed trends in each plot. The significantly larger scatter observed for this sample is partly due to the larger measurement errors of the new data set and the uncertainty introduced in the distance estimates. The small offsets of the adopted Coma zero-point relative to the mean zero-point observed in the $M_{B_2}$ correlations are probably due to age differences in the stellar content, related to the environment, affecting the $M_{B_2}$ index (see Guzmán et al. 1992a).

Although the fits shown in Table 1 correspond to the particular case discussed above, there are, in fact, a whole host of other valid $(\zeta, \xi, \eta, \lambda)$ combinations. If the assumption that $\lambda = 0$ is relaxed, we can choose, for example, the following combination from the solutions shown in Fig. 4:

$$\zeta \sim 3, \quad \xi \sim 1/10, \quad \eta \sim -1/5 \quad \text{and} \quad \lambda \sim -2/3.$$

This value of $\xi$ defines a different mass-radius relation for giant ellipticals. When this solution is substituted into equations (i)-(ix), however, coefficients almost identical to those shown in Table 1 are obtained. This case will be discussed further in Section 5.

We have therefore demonstrated that the slopes and zero-points of all global correlations observed for giant ellipticals can be fitted simultaneously in a consistent manner, assuming these correlations to be simply linear combinations of only three fundamental relations, but this set of basic laws is not unique. The precise functional form, i.e. the $(\zeta, \eta, \lambda)$ values, will depend on the assumed mass distribution in elliptical galaxies.

### Table 1. The observed correlations.

<table>
<thead>
<tr>
<th>Eq.</th>
<th>Global Correlation</th>
<th>$(ZP)$</th>
<th>rms</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>$-\log R_e - 0.785 \log L + 1.140 \log \sigma$</td>
<td>$-0.026$</td>
<td>$0.073$</td>
</tr>
<tr>
<td>(ii)</td>
<td>$\log L + 1.460 \log R_e$</td>
<td>$0.143$</td>
<td>$0.377$</td>
</tr>
<tr>
<td>(iii)</td>
<td>$M_{B_2} - 0.232 \log \sigma$</td>
<td>$-0.029$</td>
<td>$0.018$</td>
</tr>
<tr>
<td>(iv)</td>
<td>$\log L - 1.187 \log R_e$</td>
<td>$0.987$</td>
<td>$0.153$</td>
</tr>
<tr>
<td>(v)</td>
<td>$\log L + 0.813 \log R_e$</td>
<td>$3.069$</td>
<td>$0.153$</td>
</tr>
<tr>
<td>(vi)</td>
<td>$\log L - 3.739 \log \sigma$</td>
<td>$2.051$</td>
<td>$0.258$</td>
</tr>
<tr>
<td>(vii)</td>
<td>$\log R_e - 0.317 \log R_e$</td>
<td>$2.076$</td>
<td>$0.097$</td>
</tr>
<tr>
<td>(viii)</td>
<td>$-\log R_e - 0.785 \log L + 4.913 M_{B_2}$</td>
<td>$-1.250$</td>
<td>$0.124$</td>
</tr>
<tr>
<td>(ix)</td>
<td>$\log L - 16.115 M_{B_2}$</td>
<td>$6.065$</td>
<td>$0.352$</td>
</tr>
</tbody>
</table>

### 4 GENERALIZATION: THE FUNDAMENTAL RELATIONS OF SPHEROIDAL SYSTEMS

Although the conventional morphological classification divides the spheroidal systems into giant ellipticals, compact ellipticals, dwarf ellipticals, dwarf spheroidals and bulges, these objects can be grouped in two different galaxy families according to their structural properties. Giants, bulges and compacts define a major single sequence in which the effective surface brightness decreases with both luminosity and effective radius, while dwarf ellipticals and dwarf spheroidals show the opposite behaviour (cf. Kormendy 1977; Binggeli, Sandage & Tarenghi 1984; Wirth & Gallagher 1984; Kormendy 1985). Hereafter we refer to these two families simply as 'giants' and 'dwarfs'. The structural and stellar population properties of a relatively homogeneous sample in which all types of spheroidal systems are well represented have recently been studied by Bender, Burstein & Faber (1992, 1993, hereafter BBF). Their analysis is based on the distribution adopted by each galaxy type in a new, physically motivated, coordinate system which they term the $x$-space. They conclude that the giants sequence is characterized by a declining role for gas versus stars with increasing mass. The reason for the dwarfs sequence remains unknown, although...
mass loss may be a major factor. BBF's results are discussed further at the end of this section.

In Fig. 7, we show the different global correlations for BBF's sample. Dwarfs (filled symbols) show an abrupt change of slope with respect to the characteristic trend for giants (open symbols) in all the correlations involving the effective parameters. Despite these differences, it is important to stress that all types of spheroidal systems follow the same \( L-\sigma \) relation (cf. Tonry 1981). In addition, Bender (1991a) has recently shown that the \( M_g-\sigma \) correlation is universal for all spheroidal systems. In order to understand the intrinsic differences and similarities between the two galaxy families we have analysed BBF's sample using the same framework as adopted for Coma ellipticals, i.e. under the assumptions specified in Fig. 3 we express the global correlations in terms of four constants \( (\xi, \xi, \eta, \text{ and } \lambda) \) which are determined from BBF's data set.

For luminous ellipticals, bulges and compacts, the observed similarity in the global correlations suggests that the same relations as we presented in the previous section may also be representative of both compacts and bulges. The dashed lines superimposed on this sample of giants in Fig. 7 are, in fact, the same correlations as listed in Table 1, zero-pointed to the mean values derived for this data set. Clearly, our framework provides an excellent fit to the data for each correlation. Hence we conclude that the same fundamental relations that describe the correlations observed for giant ellipticals are equally valid for characterizing the structure and metal abundance of both compact ellipticals and bulges of disc galaxies.

Dwarf ellipticals and dwarf spheroidals, nevertheless, show significant differences compared to giants in most global correlations. We have thus studied which fundamental relations are required in our framework to derive these different correlations for BBF's dwarfs sample. This data set consists of \( L, R_e, I_e, \sigma \) and \( M_g \) measurements for 17 objects. Following the same procedure as that used for giants, we have obtained the set of \( \xi, \xi, \eta \) and \( \lambda \) values consistent with
all the global correlations for this sample (Fig. 8). The projections in the \(\{\eta, \xi\}\) and \(\{\eta, \zeta\}\) planes are consistent with those shown in Fig. 4 (dashed lines), since the allowed area is determined mainly by the slopes of the \(L-\sigma\) and \(M_{B_0}-\sigma\) correlations (see Section 3), which are universal for both giants and dwarfs. The smaller variation in \(\xi\) observed for dwarfs simply reflects a better determination of the \(M_{B_0}-\sigma\) slope for this sample. The projections of the parameter space involving \(\lambda\) do show, however, significant changes between both galaxy families.

As in Section 3, we discuss first the solution corresponding to the assumption of constant collapse factor. If \(\lambda = 0\), then

\[
\xi = 3.298, \quad \zeta = 0.097 \quad \text{and} \quad \eta = -0.231,
\]

where, as before, these values are the means of the distributions of valid solutions. The standard deviations of each distribution are 0.164, 0.006 and 0.047, respectively. Substituting these values into equations (i)–(ix) shown in Fig. 3, we obtain our fits to BBF’s dwarfs data set. These fits are shown in Fig. 7 (solid lines) superimposed on BBF’s sample of dwarfs. As before, the adopted zero-points are simply the mean zero-points derived from the data for each correlation. In all cases, the fits provide an excellent representation of the observed distribution of data points.

Thus, for the constant collapse factor case (i.e. \(\lambda = 0\)), the fundamental relations of giant and dwarf ellipticals can be summarized as follows:

<table>
<thead>
<tr>
<th>Dwarfs</th>
<th>Giants</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M \propto R_{e} \langle \nu^2 \rangle)</td>
<td>(M \propto R_{e} \langle \nu^2 \rangle)</td>
</tr>
<tr>
<td>(M \propto R_{e}^{2.5})</td>
<td>(M \propto R_{e}^{1.6})</td>
</tr>
<tr>
<td>(Z \propto \langle \nu^2 \rangle^{0.1})</td>
<td>(Z \propto \langle \nu^2 \rangle^{0.1})</td>
</tr>
</tbody>
</table>

Hence, if mass and light are similarly distributed in all spheroidal systems, then dwarf and giant galaxies have intrinsically different relations between mass and scalelength. Also, since consistency with observations yields \(M/L \propto M^{-0.23}\) for dwarfs and \(M/L \propto M^{-0.27}\) for giants, they would show opposite variations of the mass-to-light ratio with galaxy mass. We note that the universality of the \(L-\sigma\) relation implies a strange conspiracy between the coefficients of both
relations (i.e. $\xi$ and $\eta$) to keep this slope, defined in our framework as $[2\xi(1-\eta)]/(\xi-1)$, constant. The constant collapse factor case thus requires a non-trivial relation between the structure and the stellar population of elliptical galaxies which seems difficult to understand.

The observed universality of the $M_{B2}-\sigma$ correlation suggests, on the other hand, a different, simpler interpretation. The existence of this relation implies a less complete conversion of the protogalactic gas into stars in smaller systems. Hence the same physical process that drives this
relation will also cause a variation in $M/L$ with galaxy mass. The observed constancy in the $M_{B_2}-\sigma$ slope, i.e. $\xi_{\text{dwarfs}} \sim \xi_{\text{giants}}$, then suggests $\eta_{\text{dwarfs}} \sim \eta_{\text{giants}}$. In this case, the universality of the $L-\sigma$ relation would simply reflect a common mass–radius relation for dwarf and giant ellipticals, i.e. $\xi_{\text{dwarfs}} \sim \xi_{\text{giants}}$. As pointed out in Section 3, our fitting procedure ensures that any set of $\{\xi, \eta, \xi, \lambda\}$ values represented in Fig. 8 can equally describe the observed distribution of points in Fig. 7. In particular, we note that it is possible to find solutions with precisely the same $\{\xi, \eta, \xi\}$ values for all spheroidal systems if $\lambda$ takes different values for dwarfs and giants. The universality of both the $L-\sigma$ and $M_{B_2}-\sigma$ correlations can be thus interpreted in our framework to imply that the only difference between the two galaxy families is simply the dependence of the effective collapse factor on galaxy mass. This new interpretation is fully discussed in the following section.

Obviously, our framework should be able to describe any other representation of the structure and stellar content properties of spheroidal systems. In particular, BBF have proposed a new coordinate system to study the structural characteristics of spheroidal systems, i.e. the $\kappa$-space. Fig. 9 shows BBF's parametrization of the edge-on and face-on views of the FP (i.e. the $\kappa_1-\kappa_3$ and $\kappa_1-\kappa_2$ planes) equivalent to figs 2a and 2b of their paper. These three variables are defined in terms of the observed parameters as follows:

$$\kappa_1 = \log(R_0 \sigma^2)/\sqrt{2},$$
$$\kappa_2 = \log(R_0^{-1} I_2 \sigma^2)/\sqrt{6},$$
$$\kappa_3 = \log(R_0^{-1} I_3 \sigma^2)/\sqrt{3}.$$

Our fundamental relations (I) and (II) then imply that these variables will be related by

$$\kappa_3 = \sqrt{2} \frac{\eta - \lambda}{3 - 1 - \lambda} \kappa_1,$$

$$\kappa_2 = \sqrt{6} \frac{2(\xi - 2) - \xi(2\eta - 5\lambda)}{\xi(1 - \lambda)} \kappa_1.$$

Substituting the values of $\xi$, $\eta$ and $\lambda$ as specified above, we obtain the fits to the observed distributions of giant and dwarf galaxies in Fig. 9. As expected, the resulting fits can successfully describe the position of data points in both the $\kappa_1-\kappa_2$ and $\kappa_1-\kappa_3$ planes.

We have therefore shown that only three fundamental relations are required to describe all the global correlations of all spheroidal systems. The value of $\xi$, that characterizes the mass–radius relation depends on the assumed transformation between $R$ and $R_0$ for dwarf and giant ellipticals; thus our framework cannot provide purely from the observations a unique expression of these fundamental relations. However, it naturally suggests a simple solution, in which the only difference between dwarf and giant ellipticals is the scaling law for the effective collapse factor.

Despite the good description that our framework provides of the correlations observed for this sample, our conclusions on the properties of dwarfs are not as robust as the results we derived for giants. As BBF point out, this dwarfs sample may be affected by selection effects. Although the derived correlations between photometric parameters show a reasonably good agreement with those found in the literature for larger samples (cf. Binggeli & Cameron 1991), no definite results can be drawn until larger data sets (especially velocity dispersion and $M_{B_2}$ measurements) of magnitude-limited samples of dwarf galaxies are obtained.

5 IMPLICATIONS FOR GALAXY FORMATION

The observational framework presented in the previous sections is based on three fundamental relations for ellipticals. Two determine the galaxy structure, i.e. the virial theorem and the mass–radius relation. The third links the metal abundance of the stellar population to the galaxy potential well. A coherent theory of galaxy formation must be able to explain these three relations in a consistent manner. The theoretical picture we discuss here combines the hierarchical clustering and the galactic wind models, following the Dekel & Silk (1986) analysis of the origin of dwarf galaxies.

![Figure 9](https://example.com/figure9.png)

**Figure 9.** The $\kappa_1-\kappa_3$ and $\kappa_1-\kappa_2$ planes for BBF's data set. Symbols have the same meaning as in Fig. 7. The dashed lines and solid lines represent the trends implied by our framework for giants and dwarfs, respectively.
picture of galaxy formation may provide a simple, consistent explanation of the fundamental relations we have presented.

5.1 On the formation of giant elliptical galaxies

Previous studies on the scaling laws for galaxies have used the observed correlations to probe their formation within the hierarchical clustering scenario (Faber 1982; Djorgovski 1992). This is based on dissipationless gravitational clustering of dark haloes, coupled with dissipative settling of gaseous matter within the halo cores (White & Rees 1978). These haloes form in a hierarchical fashion. At each epoch, there is a certain scale of density fluctuations where the rms fluctuation amplitude is just entering the non-linear regime of growth. This scale therefore provides a characteristic mass \( M_h \) that is just beginning to collapse at each epoch. If the spectrum of the density fluctuations is assumed to be a simple power law with spectral index \( n \), then this scale is related to the redshift at formation (Peebles 1980),

\[
M_h \propto (1 + z)^{-6(n+3)}.
\]

Each epoch has an associated characteristic density \( \rho \propto (1 + z)^3 \).

Since the final virialized density of the haloes forming at this epoch is some constant multiple of the background density,

\[
\frac{M_h}{R_h^3} \propto \rho,
\]

there is a characteristic relation between the masses of the objects formed and their radii:

\[
M_h \propto R_h^{6(n+5)}.
\]

For the standard CDM model, the effective value of the spectral index in the relevant mass range of giant ellipticals is \( n = -2 \) (Blumenthal et al. 1984). Thus the characteristic relation predicted between mass and radius is

\[
M_h \propto R_h^3.
\]

This relation and the virial theorem

\[
\frac{M_h}{R_h^3} \propto \langle v^2 \rangle_h
\]

constitute the two basic laws that determine the structure of massive dark haloes independently of galaxy type.

In this scenario, the structural properties of visible galaxies are simply presumed to relate directly to the dark haloes in which they are embedded. This is, however, a non-trivial assertion. The main assumptions required to contrast the theoretical predictions with the observed relations are briefly summarized below (see Faber 1982 for a detailed description). Initially, all protogalaxies are considered to have the same baryonic-to-dark matter ratio. The baryonic component will be more densely concentrated than the dark matter halo due to dissipative processes. In spheroidal systems, dissipation ends when the baryonic matter turns from gas into stars, thus preventing further collapse. After the onset of star formation, the galaxy luminosity will be simply proportional to the mass in stars \( M_\odot \). Strictly, this is only valid if the typical age and stellar initial mass function are roughly the same for all galaxies; this is generally accepted to be the case for ellipticals. Under these assumptions, the structure of elliptical galaxies can be directly linked to the predicted properties of their dark haloes in terms of two quantities: the collapse factor \( (R_h/R_e) \) and the dark-matter-to-light ratio \( (M_h/L) \). These free parameters in the model simply reflect the lack of any a priori ideas of the amount of dissipation and gas loss during the early formative processes of the galaxy.

The conventional approach is to assume that the collapse factor is constant, i.e., \( R_h \propto R_e \). This is based on the idea that the mean gas density at the onset of star formation is a constant fraction of the halo density (Mathews 1972), an assertion that can be justified by consideration of the instability and fragmentation of the gas cloud during its collapse. Since star formation is presumed to halt the collapse of baryonic matter, the half-light radius is then directly proportional to the characteristic radius of the surrounding halo. Within this radius, the luminous matter in giant ellipticals is presumed to be self-gravitating. Under these assumptions, Faber (1982) showed that the halo variables can be directly related to the global parameters of visible galaxies (i.e. the half-light radius \( R_e \), the mean square stellar velocity \( \langle v^2 \rangle \) and the virial mass \( M_v \) within \( R_e \)). Thus the two structural scaling laws for giant ellipticals arise in a natural way in this scenario. Indeed, the results of this approach are entirely equivalent to the constant collapse factor case discussed in Section 3. In our framework, the structural relations for this case are

\[
M \propto R_e^{\xi} \langle v^2 \rangle; \quad M \propto R_e^{\xi \cdot 3}.
\]

While these relations are in rough agreement with the theoretical predictions, a more detailed analysis reveals that the conventional approach does not provide an adequate description. In particular, there are two relevant problems. First, it is important to note that, since the luminous matter is assumed to be self-gravitating, the dark-halo properties do not enter into the virial relation; their effect is only to define the parameter \( R_e \) in the mass–radius relation indirectly (through Mathew's gas-cloud instability criterion). Whereas CDM predicts a slope \( \xi = 2 \) for this relation, we find that in our framework \( \xi = 1.6 \). These two values are inconsistent at the level of over 5 standard deviations (see Section 3). There are a number of effects that might account for small changes in this slope. In particular, the assumption that the luminous part of the galaxies is self-gravitating may not be entirely correct. We will return to this point later.

Secondly, Faber et al. (1987) showed that consistency with the observed Fundamental Plane of elliptical galaxies would imply a small but significant increase of \( M/L \) with galaxy mass within the effective radius. For the case with constant collapse factor, our framework yields \( M/L \propto M^{\xi \cdot 3} \). A possible explanation is that the stellar content varies with galaxy mass (i.e. \( M_\odot/L \) is not constant). This variation is not likely to be explained in terms of age differences, since it requires the stellar populations of less massive ellipticals to be younger, whereas according to the hierarchical clustering scenario these systems formed earlier. The required change in \( M_\odot/L \) thus suggests a variation of the IMF with galaxy mass. This explanation would lead us to a highly speculative area, which is beyond the scope of this paper. A second possibility is to consider that more massive haloes are less...
efficient at forming stars. However, if this is the case, why is
the metal abundance higher in these systems? So far in this
analysis we have not made use of the third fundamental
relation established in our framework. Specifically, the high-
metallicy characteristic of more massive ellipticals (strictly
deeper potential depth) implies that a greater fraction of the
initial gas content has been converted into stars. The only a
priori assumption we have made about the initial baryon-to-
dark matter ratio is that it is similar in all systems. Therefore,
if $M/L$ is constant, we conclude that $M/L$ must be a
decreasing function of galaxy mass, which is contrary to the
observed scaling law derived above. Note that the effect of
metallicity on the integrated light of the galaxy also cannot
totally explain the observed trend in $M/L$. As noted by
Dressler et al. (1987), the increasing bolometric correction
with galaxy mass would account for a slope of only $\sim 0.07$
in the above scaling law for $M/L$ in the $B$-band. Furthermore,
Djorgovski & Santiago (1992) have shown that, although this
slope varies slightly with the effective bandpass wavelength,
it is always significantly different from zero.

In conclusion, we find that the conventional approach (i.e.
$R_H/R_e \sim$ constant) cannot provide a simple explanation of
the structural relations of giant ellipticals. In addition, the
required variation in $M/L$ directly contradicts the observed
metallicity–velocity dispersion relation.

5.2 Towards a global picture of galaxy formation?

The standard model discussed above requires two free
parameters (i.e. $R_H/R_e$ and $M_H/L$) to describe the structure
of visible galaxies. Since there are are only two observational
structural constraints [i.e. correlations (i) and (ii) of Fig. 3],
this theoretical picture, as it stands, lacks any real predictive
good correlation between metal abundance and velocity dispersion [correlation (iii) of Fig. 3] adds a new, power.

The most widely accepted explanation of this correlation is
based on the galactic wind model (Larson 1974). After
the onset of star formation, both the frequency of supernovae
explosions and gas metal enrichment in spheroidal systems
increase very rapidly. When the thermal energy of the gas
heated by supernovae explosions exceeds the galaxy binding
energy, the gas will be driven out of the system and the
formation of new stars is halted thereafter. As a result of
the decreasing binding energy of the gas, a wind occurs much
earlier in less massive systems, i.e. the time at which this
galactic wind blows is an increasing function of the initial gas
mass $M_0$. As a consequence, both the average metallicity and
the $M_0/M_{10}$ fraction also increase with $M_0$. On the assump-
tions that both the initial baryon-to-dark matter ratio and the
stellar mass-to-light ratio are constant (i.e. $M_0 \propto M_H$ and
$M_S \propto L$), the variation of $M_H/L$ can thus be determined
consistently with the metallicity–velocity dispersion relation
by incorporating the galactic wind model in the hierarchi-
cal clustering scenario discussed above. Hence there is no need
for any a priori ideas on the collapse factor, and the assump-
tion $R_H/R_e \sim$ constant of the conventional approach can be
relaxed. The scaling law for the collapse factor thus remains
as the only unknown to be determined from the observa-
tions.

In order to estimate the dependence of the mass-to-light
ratio on galaxy mass, we have used Arimoto & Yoshii's
(1987) galactic wind model. On the assumptions of a star
formation rate per unit mass proportional to the fractional
gas mass and a universal IMF, this model has successfully
reproduced the chemical and photometric properties of
spheroidal systems, i.e. the colour–magnitude diagram. Due
to the uncertainties associated with the transformation
between the [Fe/H] metallicity indicator that Arimoto &
Yoshii modelled and the $M_0$ index used in this paper (see
Worthey, Faber & González 1992), our analysis is not based
on a direct comparison with the metallicity–velocity disper-
sion relation of our framework. Note, however, that the
mass–metallicity relation derived in their model is in rough
agreement with the empirical relation given by Mould (1984)
and the different colour–magnitude diagrams observed by
Persson, Frogel & Aaronson (1979). From table 2 of
Arimoto & Yoshii's paper, we derive the following average
relation:

$$M_H/M_S \propto M_0^{-1/5}$$

Arimoto & Yoshii's model is only concerned with gas
protoclouds. In the picture of galaxy formation that we are
discussing here, the star-forming gas cloud is considered to
be embedded in a dark halo. Since $M_0 \propto M_H$ and $M_S \propto L$, we
obtain

$$M_H/L \propto M_0^{-1/5}$$

This scaling law thus provides the predicted variation of
$M_H/L$ when galactic winds are considered to be the physical
process driving the observed metallicity–velocity dispersion
relation. It is important to stress at this point that the
metallicity–velocity dispersion relation is universal. This
suggests that the process of gas loss should be equally effi-
cient in the whole range of galaxy masses from dwarfs to
giants. The above scaling law thus applies to all elliptical
galaxies.

The formation of dwarf galaxies as a result of galactic
winds has been discussed by Dekel & Silk (1986). They find
that the observed $L-R_c$, $L-\sigma$ and $L-z$ relations for
dwarfs can be produced inside dominant haloes with a mass–radius
relation that resembles the predictions of the CDM standard
model. The observational evidence for dark matter in dwarfs
is compelling (see Ashman 1992, and references therein).
We have therefore used a similar theoretical picture to study the
properties of dwarf ellipticals in contrast with our obser-
vational framework. If we assume that the dark halo
dominates the gravitational potential, then the halo prop-
erties are directly related to the global parameters of visible
galaxies, i.e. $M_0 \propto M_H$, $R_H \propto R$ and $(\nu^2)_{\text{HI}} \propto (\nu^2)$. In this case, the
above condition on the galaxy mass-to-light ratio is, in fact,
equivalent to considering $\eta - 1/5$ in our framework. Fig. 8
shows that this value constrains $\xi$ and $\lambda$ to be $2.5 \leq \xi \leq 4.0$
and $-0.1 \leq \lambda \leq 0.1$, respectively. We recall here the results
obtained in our discussion of the constant collapse factor
case for dwarf ellipticals. For $\lambda = 0$ we found that $\xi = 3.298$
$\cdot (1.0 = 0.164)$ and $\eta \approx -0.231 (1.0 = 0.047)$. This value of $\eta$
is remarkably similar to the theoretical prediction. Thus our
framework suggests that a plausible solution consistent with
the galactic wind model is simply $\xi = 3$, $\eta = -1/5$ and $\lambda = 0$.
For this case, the two structural relations for dwarf ellipticals are

$$M \propto R(\nu^2); \quad M \propto R^3.$$
This mass–radius relation is in good agreement with the theoretical relation predicted by CDM in the relevant mass range of dwarf ellipticals (Dekel & Silk 1986). Hence the observations naturally allow a solution for the $\xi$ and $\eta$ values consistent with both the CDM hierarchical clustering and the galactic wind models. More interestingly, the implied constancy in the collapse factor (i.e. $R_\odot \approx R_{\ast}$) agrees very well with Mathews’ (1972) original idea that passage over a critical threshold induces star formation in spheroidal systems and thus halts the collapse of the baryonic matter. Therefore our framework clearly suggests that dwarf ellipticals may have simply formed as a direct result of the initial density fluctuations in a CDM universe, in agreement with Dekel & Silk’s result.

In Section 4, we noted that the universality of the $M_{\ast}/\sigma$ and $L/\sigma$ correlations naturally suggests a solution within our framework, in which both dwarfs and giants differ only in the scaling law for the collapse factor. This solution indirectly assumed that the luminous matter in all spheroidal systems is not self-gravitating. In contrast to the widely accepted evidence for dwarfs, the presence of dark matter in giant ellipticals is still controversial (see reviews by Faber 1982 and Ashman 1992). However, in Section 5.1, we showed that the assumption of self-gravitation (i.e. $R = R_{\ast}$) in giant ellipticals failed to provide a consistent agreement between the theoretical predictions and the observed fundamental relations in our framework. A possible way to resolve this disagreement is to assume that giant ellipticals are also embedded in a dark halo that determines the galaxy gravitational potential. In this case, if $\lambda$ is not a priori assumed to be equal for all ellipticals, then we can adopt the same scaling laws as predicted by the hierarchical clustering and galactic wind models for dwarfs to describe the structure and stellar content of giant ellipticals (i.e. $\xi \sim 3$ and $\eta \sim -1/5$). This value of $\xi$ corresponds to a spectral index of the density fluctuations $n = -3$. This is not in complete agreement with the canonical value derived in the CDM standard model on this galaxy scale (i.e. $n = -2$), but such a flat spectrum remains a plausible alternative, especially if CDM is to be reconciled with the recent COBE observations showing that the CDM standard model does not have enough power on large scales (Efstathiou, Bond & White 1992). This discrepancy might be resolved by simply shifting the CDM spectrum to slightly larger scales which would result in a more negative power spectrum on galaxy scales.

The values of $\xi$ and $\eta$ for both dwarfs and giants are thus determined according to the theoretical predictions. From Fig. 8 we then deduce that, unlike the result found for dwarfs, $\lambda \sim -2/3$ for giant ellipticals, i.e. the collapse factor must decrease with galaxy mass. Hence in more massive systems the luminous matter is apparently less concentrated relative to the dark matter than in less massive systems. This is the same effect as that shown by Navarro (1992) in his simulations of dissipative galaxy formation in a CDM universe. As he suggests, a possible interpretation of the origin of this effect is that more massive systems have, on average, undergone more numerous mergers after the bulk of their stars were formed. These mergers would tend to mix stars and dark matter more thoroughly in large systems, resulting in the stars being less concentrated with respect to the dark matter in the more luminous ellipticals. This may prove to be a plausible interpretation, given the growing evidence for merger signatures in giant ellipticals, e.g. shells and distorted isophotes (Schweizer et al. 1990) and kinematically decoupled cores (Bender 1991b). The idea that the final mergers leading to larger galaxies in this family were more stellar (and less gaseous) is also consistent with the observed trends of other properties with galaxy mass, including bulge-to-disc ratio, radio properties, rotation, degree of velocity anisotropy and peculiar kinematics (BBF). Indeed, from the analysis of the structural properties alone, BBF reach a conclusion similar to the result we find here, i.e. CDM predictions can be made to fit the data for giants only if dissipation decreases significantly with galaxy mass. In our study, however, the required variation in the collapse factor is derived in the context of a global, common picture of galaxy formation consistent with both the structure and stellar population properties of all spheroidal systems.

In conclusion, if both dwarfs and giants are dark-matter-dominated, the relations,

$$M \propto R^{\xi}, \quad M \propto R^{3}, \quad Z \propto (v^{2})^{1/10}, \quad M/L \propto M^{-1/5}$$

constitute a set of scaling laws common to all spheroidal systems. These relations are in good agreement with a theoretical picture of galaxy formation based on hierarchical clustering in a CDM universe and the galactic wind model. Our framework then implies that the only difference between dwarfs and giants is the scaling law for the collapse factor. For dwarfs: $R/R_{\ast} \sim \text{constant}$, whereas for giants: $R/R_{\ast} \propto M^{-2/3}$. These relations suggest different formative processes for each galaxy family. The derived constancy of the collapse factor for dwarfs is in excellent agreement with Mathews’ gas cloud instability criterion. This result suggests that dwarf ellipticals originated directly from the initial density fluctuations. Thus part of the scatter observed around the global correlations may be caused by a spread in the formation epoch of these galaxies. However, a different process is needed to explain the required variation between the mass and light distributions within giants. Mergers appear as a plausible alternative. Although the simplicity of our framework makes this picture appealing, we note that our results are far from conclusive. In particular, the question of how the mass–radius and metallicity–velocity dispersion relations can be preserved in the merging scenario may have no simple answer. Theoretical models that specifically address the influence of mergers on the observed properties of giants and larger, homogeneous data sets for dwarfs are needed before we can be certain that these results provide real clues to a better understanding of the process of galaxy formation.

6 Conclusions

We have analysed the well-known correlations between global parameters (i.e. $L$, $R_{\ast}$, $I_{\ast}$, $\sigma$ and $M_{\ast}$) for elliptical galaxies. The main results presented in this paper can be summarized as follows.

(1) We have shown that the Fundamental Plane of giant elliptical galaxies is not uniformly populated. Giant ellipticals are located in a relatively narrow band which is not the result of selection effects. This implies a second constraint between scalelength and galaxy mass, in addition to the virial theorem. The observed $L/R_{\ast}$, $L/I_{\ast}$, $L-\sigma$ and $L-R_{\ast}$ correlations are simply the projections of this band on to the different coordinate planes.
(2) We have developed a simple framework to describe the structure and stellar content properties of giant ellipticals. In our framework there are only three fundamental relations: $M \propto R \eta \lambda$, $M \propto L \eta \lambda$, and $Z \propto \eta \lambda$. The coefficient $\xi$ is uniquely determined from the observations. If $M_{20}$ is adopted as a metallicity indicator, then $\xi = 0.11$. The coefficient $\xi$ depends, however, on the assumed variations of the mass-to-light ratio and the effective collapse factor with galaxy mass. We have assumed that $M/L \propto M^{0.4}$ and $R/R_e \propto M^{0.3}$. The conventional approach is to assume that the effective collapse factor is constant, i.e. $\lambda = 0$ (or, equivalently, giant ellipticals are baryon-dominated). In this case, we find that $\xi = 1.6$ and $\eta = 0.27$, although there are many other valid solutions for the $[\xi, \eta, \lambda]$ values. All the observed correlations for giants are consistently derived as linear combinations of these fundamental relations.

(3) We have demonstrated that this framework can also successfully describe the global correlations of dwarf elliptical galaxies. Our framework provides new insights into the differences and similarities between the two galaxy types. In particular, the observed universality of the luminosity–metallicity–velocity dispersion correlations strongly suggests a simple solution within our framework in which $\xi$, $\eta$ and $\lambda$ adopt the same values for both dwarf and giant ellipticals. In this case, the dependence of $R/R_e$ on galaxy mass is the only difference between the two galaxy families.

(4) We have discussed the results of our observational framework within a simple picture of galaxy formation based on the hierarchical clustering scenario and the galactic wind model. For giant ellipticals, we have shown that this theoretical picture is not consistent with the fundamental relations derived under the assumption of self-gravitation (i.e. $R = R_e$). Theory and observations can be reconciled in a simple way if both dwarf and giant ellipticals are assumed to be dominated by dark matter. For dwarf ellipticals, this theoretical picture provides a consistent explanation of the observed fundamental properties, provided that $R/R_e$ is constant. This result suggests that these systems may have simply formed as a direct result of the initial density fluctuations in a CDM universe. For giant ellipticals we find that $R/R_e$ must decrease with galaxy mass, i.e. the luminous matter in giant ellipticals is apparently less concentrated inside their dark haloes with increasing mass. This effect suggests a different origin for giant ellipticals, in which mergers may have played a determinant role.

ACKNOWLEDGMENTS

RG thanks Richard Larson for enlightening discussions on the scaling laws of elliptical galaxies during the International Winter School held at the Canary Islands. Shaun Cole, Mike Fitchett and Alfonso Aragón Salamanca provided helpful comments. We especially thank Julio Navarro, Steve Zepf and Ian Smail for their suggestions, which greatly improved the original version of this paper. We also thank the referee for constructive comments. This work has been financially supported by SERC. RGB acknowledges the receipt of a SERC/Royal Society NATO fellowship. We thank the Starlink project for the provision of computing facilities. Finally, RG gratefully acknowledges Esperanza Carrasco and Alberto Carramiñana for their support during the early stages of this project.

REFERENCES

Bender R., Burstein D., Faber S. M., 1993, preprint (BBF)
Blumenthal G. R., Faber S. M., Primack J. R., Rees M. J., 1984, Nat, 311, 527
Disney M. J., 1976, Nat, 263, 573
Elstathiu G., Fall S. M., 1984, MNARS, 206, 453
Elstathiu G., Bond J. R., White S. D. M., 1992, MNARS, 258, L1
Larson R. B., 1974, MNARS, 169, 229

© Royal Astronomical Society • Provided by the NASA Astrophysics Data System
746  R. Guzmán, J. R. Lucey and R. G. Bower