Solution of the field equations for a steady-state cosmology in a closed space

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ABSTRACT
We present equations which describe a steady-state universe in which space is not homogeneous and isotropic, but spherically symmetric around a point near us. We obtain a solution for a closed space with a metric which is smooth everywhere. The vacuum is unstable and continuously creates matter in one hemisphere. This material travels to the other hemisphere where it is absorbed back into the vacuum. The solution is compared to two steady-state models and to the standard model.

Key words: gravitation – relativity – cosmology: theory.

1 INTRODUCTION

This paper presents a new class of solutions of Einstein’s equations in interaction with matter. These solutions lead to a cosmological model of an unusual type, the closed steady-state model (CSSM). Detailed comparison with observation will be undertaken elsewhere; here we motivate the choice of equations, construct the solution, and show that the model is satisfactory at least in zeroth order, i.e. it provides explanations for the Hubble expansion, the cosmic microwave background radiation (CMBR), and the accumulating evidence of evolution in the Universe. These last two points have been particularly troublesome for the original steady-state model (OSSM) (Bondi & Gold 1948; Hoyle 1948), even in its modern version (Hoyle, Burbidge & Narlikar 1993). The remaining zeroth-order requirement is a satisfactory explanation of light-element abundances. This cannot be provided in the model presented here, in which all structure is smoothed out to simplify the solution of the differential equations. We assume that a ‘mini-bang’ mechanism similar to that suggested by Hoyle et al. will ultimately prove satisfactory. Elemental abundances, structure formation, and the fate of galaxies in this model are discussed briefly in Section 4.

We begin by comparing three models, the standard model (SM) (Weinberg 1972, chapter 15), the OSSM, and the CSSM. A central feature of the SM is the Cosmological Principle (CP), which asserts that space is homogeneous and isotropic. In this model, therefore, the components of the metric and curvature tensors $g_{\mu\nu}$ and $R_{\mu\nu}$ are functions of the time alone. The OSSM goes further and eliminates even the time dependence; the idea that $g_{\mu\nu}$ and $R_{\mu\nu}$ should be constants, independent of either space or time, is called the Perfect Cosmological Principle (PCP). Most physicists today believe that the OSSM is contradicted by the observational evidence. But a different steady-state model, the CSSM, suggests interesting new possibilities. A model of this sort was first suggested by Ellis, Maartens & Nel (1978, hereafter EMN) to meet some of the objections to the OSSM, but was rejected by them because it was itself in conflict with observation in other respects. A new version of the CSSM emerges, however, if we follow a different path suggested but not explored by EMN.

The SM can be regarded as derived from the OSSM by lowering the symmetry; metric quantities become time-dependent. Another way of breaking the symmetry of the OSSM is to require metric quantities (and others, such as the matter density) to be functions of a single space coordinate. A theory of this sort is still steady-state since there is no time dependence. But it violates the CP and we have to ensure that at least locally (on a cosmological scale) the CP is approximately preserved. We can do this by requiring spherical symmetry about one point near the current position of our own Galaxy and (as in the OSSM) creating matter at a nearly constant rate in the neighbourhood of that point.

The model considered here results if we suppose in addition that space is closed and spherically symmetric about two antipodal points which we call the Austral Pole (A) and the Boreal Pole (B). Our Galaxy is located quite close to $B$. We are aiming at a version of the CSSM in which all quantities are smooth functions of the radial coordinate; in particular there are to be no singularities at either $A$ or $B$. This is in contrast to the model of EMN in which there is a singularity at $A$. A second major difference appears in the treatment of matter in the model. Matter is treated as static by EMN; the redshift is entirely gravitational. Here, however, we postulate that the matter is in motion with respect to the underlying static metric (see EMN, section 7.5); the redshift has both Doppler and gravitational components. In contrast to the OSSM, however, the receding matter cannot simply disappear into the distance. The space has only a finite, fixed volume so matter has to be
Table 1. Pre-set and computed parameters and functions.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{\text{max}}$</td>
<td>distance between poles, set to unity</td>
</tr>
<tr>
<td>$k$</td>
<td>chemical potential</td>
</tr>
<tr>
<td>$\sigma(0)$</td>
<td>initial value of density $\rho$, set to 0.0036</td>
</tr>
<tr>
<td>$y'(0)$</td>
<td>set to unity; becomes the Hubble constant in familiar units</td>
</tr>
<tr>
<td>$f(r_{\text{max}})$</td>
<td>final value of metric function $f$; set to 0.3</td>
</tr>
<tr>
<td>$\xi(r)$</td>
<td>shape function, determined by three pre-set parameters $n_1$, $n_2$, and $n_3$</td>
</tr>
</tbody>
</table>

Computed parameters and functions:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho(0)$</td>
<td>initial value of density $\rho$</td>
</tr>
<tr>
<td>$c(r)$</td>
<td>vacuum energy function, determined by parameters $e_i$ and $k_i$</td>
</tr>
<tr>
<td>$e(r)$</td>
<td>vacuum energy function, determined by parameters $e_i$ and $k_i$</td>
</tr>
</tbody>
</table>

annihilated at the same rate as it is created, but in a region of the universe far from $B$.

At this point we make the very natural assumption that the matter of the galaxies does not appear out of nothing but results from the conversion of a different form of energy. We call this simply the vacuum without attempting to relate it in detail to the vacuum of particle-physics theories. A distinctive feature of this model is then a counterflow, with energy of different kinds moving in opposite directions so that the overall flux of energy and momentum is zero.

To build a viable CSSM from these general ideas we have first to solve a boundary-value problem for a coupled set of non-linear ordinary differential equations made up of the Einstein equations of general relativity, suitable creation/annihilation equations, and (as we shall see) equations for the energy of the vacuum. We start integrating these equations from $B$, our local neighbourhood where we know the conditions. We continue integrating as far as $A$ and require our functions to behave smoothly at that point. It is not obvious that the equations possess any solution of this type, but in fact they do. The solutions are not unique; they depend on the values given to a few input parameters and the form given to a function $n(r)$, the creation function.

To demonstrate the existence of this solution and show that it satisfies the zeroth-order conditions is only the first step in creating a successful CSSM. Some of the first-order questions that remain are discussed in Section 5.

2 THE BOUNDARY-VALUE PROBLEM

2.1 Metric conventions and coordinate systems

We follow the conventions of Adler, Bazin & Schiffer (1965): index 0 corresponds to time, while indices 1, 2, 3 correspond to $r$, $\theta$, $\phi$ in standard spherical polar coordinates. The metric signature is $(+ - - -)$; units are such that $G = c = 1$. The origin of coordinates can be either at $A$, in which case we use $q$ as the radial coordinate, or at $B$, when we use $r$. The metric tensor can be chosen diagonal and is given the form $g_{\mu\nu} = diag(f,-1,-h,-h\sin^2\theta)$, where $f$ and $h$ are functions of $r$. The space can be visualized by suppressing the coordinates $t$ and $\theta$ and looking at the surface $\theta = \pi/2$. This surface turns out to be surprisingly close to an ordinary 2-sphere. The origin of coordinates is at one pole (usually $B$), radial distances are measured from the pole along meridians of longitude, and the metric coefficient $h(r)$ is proportional to the square of the radius of the circle of latitude at coordinate $r$. In particular we anticipate that $h$ will behave like $q^2$ near $A$ and like $r^2$ near $B$. This coordinate system will be referred to as the Preferred Frame (PF). Our velocity with respect to this frame can be easily measured because the CMBR is expected to be at rest in it (see Section 3.2). The equations given below possess a scaling symmetry so we can without loss of generality choose the maximum value of $r$ ($r_{\text{max}}$) the distance from $B$ to $A$ measured around the ‘sphere’ to be unity (see Table 1); $q$ is then equal to $1 - r$.

2.2 Energy-momentum tensor and field equations

Matter is represented as pressure-free dust travelling with four-velocity $X^\mu$; its energy-momentum tensor is $T^{\mu\nu} = \sigma X^\mu X^\nu$. The four-vector $X^\mu$ satisfies $g_{\mu\nu}X^\mu X^\nu = 1$. Because the motion is purely radial we write $X^0 = x$, $X^1 = y$, $X^2 = X^3 = 0$, so that $f x^2 - y^2 = 1$.

We now have to decide how to represent the energy-momentum tensor of the vacuum. This is normally incorporated in the cosmological constant, but a more general form is needed in the present model because we no longer have a homogeneous space. A simple form for this tensor would be $C^{\mu\nu} = \text{diag}(c/f,0,e,e\sin^2\theta) + d g^{\mu\nu}$, where $c$, $d$, and $e$ are all functions of $r$. The last term is analogous to the usual cosmological term; the first term is new and appears because our model provides a natural frame of rest (the PF, in fact) and consequently two four-vectors, one along $r$ and the other along $r$ (the latter is not defined at the two poles). But this form for $C^{\mu\nu}$ is not general enough. The vacuum acts as source and sink for the matter. If we are to preserve overall energy-momentum conservation (as we must if we wish to use the Einstein equations) the vacuum needs to carry energy from place to place, rather like the nineteenth-century ‘ether’. We therefore give it a four-velocity $A^\mu$, with components $(a,b,0,0)$, and regard it as a fluid with pressure and internal stress. The non-zero components of $C^{\mu\nu}$ now become: $C^{00} = (ca^2 + d/f)$; $C^{11} = (cb^2 - d)$; $C^{22} = (e - d/h)$; $C^{33} = (e - d/h)\sin^2\theta$; $C^{0i} = \text{cab}$. In the limit $b \to 0$ we regain the previous form for $C^{\mu\nu}$.

The Einstein field equations are now (Adler et al. 1965, equation 9.80)

$$G^{\mu\nu} = -8\pi\left(Z^{\mu\nu} + C^{\mu\nu}\right).$$

(1)

Only four index combinations give independent, non-trivial equations: 00, 11, 22 and 01.

2.3 Creation equations

In constructing the equations that govern the creation of matter, we are guided by the observation that the proper motions of galaxies are generally small. This means that matter should be created at coordinate $r$ with the velocity appropriate to the galaxies at that coordinate. The simplest equation that ensures this is

$$\left(Z^{\mu\nu}\right)_{,\nu} = nX^\mu,$$

(2)

where $n$ is a scalar function of $r$. This 'creation function', $n(r)$, is constructed with the help of a pre-set auxiliary function $\xi(r)$ (the 'shape function') which resembles a step function that
starts positive and decreases sharply to negative values at a
close value of $r$. $n(\rho)$ is then defined in this paper as $\sigma_0 \zeta(\rho)$,
where $\sigma_0$ is a constant that is determined as soon as we fix the
value of $y'(0)$ (or, effectively, the Hubble constant). This form
for $n(\rho)$ gives a constant creation rate in the region around $B$,
a quasi-exponential rise in $\sigma$ as we approach the annihilation
region, and a quasi-exponential fall as soon as $\zeta(\rho)$ becomes
negative.

Equation (2) yields two non-trivial equations, for $\mu = 0$
and $\mu = 1$. These equations and the four Einstein field equa-
tions form the simplest set which will produce the kind of
solution we are seeking. Rather than developing that solution
at once, however, we prefer to introduce a slightly more com-
plicated set of equations, in which the above set appears as
a special case.

2.4 Primary radiation

The creation of matter out of the energy of the vacuum is
reminiscent of a process which is thought to occur, in the SM,
at a time soon after the Big Bang as the result of a symmetry-
breaking transition. Before this time all fields are massless;
after it, they acquire the masses we now see. In a steady-state
theory such a transition, if it happens at all, must happen
continuously, and both forms of matter must be present to-
gether. To allow for this possibility we introduce the primary
radiation, with energy-momentum tensor $\mathcal{W}_{\mu\nu}$. This has the
form appropriate to blackbody radiation travelling with four-
velocity $U^\mu$, so $\mathcal{W}_{\mu\nu} = \rho(4U^\mu U^\nu - g^\mu\nu)/3$. We write $U^0 = u,$
$U^1 = v,$ and $U^2 = U^3 = 0$, so that $fu^2 - v^2 = 1$. The primary
radiation has no direct connection with the CMBR, which
contributes so little to the total energy-momentum tensor that
we can omit it from our equations. Ordinary matter, described
by $Z_{\mu\nu}$, is appropriately called secondary matter.

With the primary radiation included, the Einstein equa-
tions read

$$G_{\mu\nu} = -8\pi(\mathcal{W}_{\mu\nu} + Z_{\mu\nu} + C_{\mu\nu}).$$

These equations are written out in the appendix. As before,
only the index combinations 00, 11, 22 and 01 give non-trivial,
independent equations.

2.5 Latent heat of the vacuum

Tolman (1926) considered a model containing radiation and
matter which could interconvert. He found, not surprisingly,
that the equilibrium concentration of matter was negligi-
ble in conditions like those in our neighbourhood. To produce
an appreciable concentration of matter, most of the energy
has to come from the vacuum in the form of latent heat;
this, of course, recognized in the standard treatments of the
symmetry-breaking transition. To represent the energy balance
of the phase change we introduce the equation

$$(C_{\mu\nu} + \kappa Z_{\mu\nu})_{,\sigma} = 0,$$

which has the informal interpretation that, when one $Z$
particle, of mass $m$, is created, the vacuum releases an amount of
energy $E_0$ (the latent heat); the ratio $E_0/m$ is the quantity $\kappa$
in equation (4). $\kappa$ could in general be a function of $r$ but here we
always assign it a constant value. Equation (4) is written out
in the appendix. Only the index values $\mu = 0$ and $\mu = 1$ give
non-trivial equations.

If we set $\kappa = 0$ we regain the situation considered by
Tolman (1926). Equation (4) reduces to $(C_{\mu\nu})_{,\sigma} = 0$, from
which it follows that $b = 0$. The vacuum here plays a purely
structural role and does not exchange energy with the other
fields. If we set $\kappa = 1$, however, we can use the relation
$(W_{\mu\nu} + Z_{\mu\nu} + C_{\mu\nu})_{,\sigma} = 0$, which is a consequence of the Einstein
equations, to infer that $(W_{\mu\nu})_{,\rho} = 0$, from which it follows that
$v = 0$. For this choice of $\kappa$ we can set $\rho(0) = 0$ and regain
the original ‘minimal set’ of equations as a special case.

In practice we should set $\kappa$ to a value slightly less than
unity; this will ensure that most of the energy in the secondary
matter is derived from the vacuum while allowing for a small
contribution from the primary radiation. For the remainder
of this paper we will use $\kappa = 0.99$.

2.6 Integrating the principal equation set

The eight equations derived from equations (2), (3) and (4),
together with the subsidiary conditions $f_a^2 - b^2 = 1$, $f_a^2 - v^2 =
1$ and $f_a^2 - y^2 = 1$, are the equations for the integration.
We consider $c$ and $e$ to be arbitrary functions which can be
adjusted to get a good fit to the boundary conditions. The
integration then yields the eleven functions $a$, $b$, $d$, $f$, $h$, $u$, $v$, $\rho$, $x$, $y$ and $\sigma$. Near $B$ most of these functions have expansions in
even powers of $r$; the only exceptions are the three velocities
$b$, $v$ and $y$, which have expansions in odd powers. Near $A$ all
functions have expansions in even powers of $q$.

The initial value of $\sigma$ is taken to be 0.0036, which is 3
per cent of the ‘closure density’ in the static Einstein universe,
$3/8\pi$. The recession velocity is fixed by requiring $y'(0) = 1$, i.e.
the Hubble constant is unity in our units. These are simply
reasonable choices; others are possible.

To integrate the equations, starting from $B$, we use a com-
bination of series expansion and implicit Runge-Kutta. Just as
with Bessel’s equation there is an unwanted singular solution
that inevitably appears as we approach $A$. Consequently we
stop the forward integration as soon as $\sigma$ becomes negli-
gible (typically $< 10^{-6}$) and start a backward integration from
$A$ to meet the forward solution. This backward integration is
comparatively simple; the creation equations can be discarded,
v = 0, and $u = 1/\sqrt{f}$. As first noted by Tolman (1949, pp.
312–319) $\rho$ is then proportional to $1/f^2$. At the meeting point
of the forward and backward solutions we compare $f$, $h$, $f'$
and $h'$; the sum of the squares of the differences is an adequate
measure of the goodness of fit.

The computation reduces to this: embed the integration in a
minimization program, and vary the available parameters
until all boundary conditions are met. The usual procedure is
to treat the value of $f$ at $A$ as a fixed input parameter (see
Table I) but let the computer determine the initial value of $\rho$.
The functions $c$ and $e$ can be represented in a number of ways.
A Fourier series is a natural choice, but, if we are concerned
with finding just one solution rather than a whole range of
them, there is a more efficient parametrization. Expand $c$ in the
form

$$c = \sum_i c_i f^i + \sigma \sum_i j_i r^{k_i}$$

where $c_i$ and $j_i$ are constant coefficients. Expand $e$ in a similar
way but write $h$ rather than $f$ in the first sum and use expansion
coefficients $e_i$ and $k_i$. These series may not provide a complete
set of functions but they do give enough shape to $c$ and $e$ to

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enable a fit to be obtained. The sums proportional to $\sigma$ take account of the fact that strong variation is to be expected in the region where $\sigma$ is changing rapidly. When $\sigma$ has declined to zero, on the other hand, the remaining sums are in a form that permits the $\mu = 1$ component of equation (4) (see appendix, equation (A9)) to be integrated immediately for the function $d$.

Two terms in each of the sums in equation (5) and the corresponding expansion for $e$, and the initial value of $\rho$, a total of nine variable parameters (see Table 1), are enough to achieve the fit shown in Figs 1(a)–(f), where the final value of $f$ has been chosen to be 0.3. The familiar problem in the SM of determining the value of the cosmological constant (Weinberg 1989) appears here in a new light. The functions $c$, $d$ and $e$ in the present model are largely determined by the global requirement of a metric free of singularities.

In obtaining this fit we allowed the computer considerable freedom in adjusting the parameters, and the initial values were simply given reasonable values. No attempt was made to constrain the parameters to satisfy the classical cosmological tests, as described, for example, by Sandage (1988).

A striking feature of Fig. 1(a) is the form of the function $h$, which is remarkably close to the function expected for a spherical space, $m = \sin^2(\pi r)/\pi^2$. This is surprising in view of the complicated transitions which are occurring in the space, but it remains true over a wide range of input parameter values.

In the region around $A$ the secondary matter has all disappeared so we have just the primary matter, in the form of blackbody radiation at rest in the PF. In the region around $B$, on the other hand, it is the secondary matter that shows a simple behaviour. From equation (2) we can show easily (see appendix) that $x = 1/f$. The same relation is obtained from the geodesic equation, so the secondary matter (the galaxies) is in free fall as we would expect for a pressure-free dust.

We can now see why it is reasonable to expect a solution to exist. The forward and backward solutions are compared in the region where only the primary radiation remains. Klein (1947) has solved the Einstein equations for this situation, a spherically symmetric configuration of radiation. He was considering stars and did not extend his solution to cosmological scales; nevertheless our solution is essentially his, with the addition of our vacuum-energy terms.

2.7 Redshift versus distance

The function $z$ plotted in Fig. 1(f) is the conventional $z$ used by astronomers, computed as follows. Consider an observer stationary in the PF at coordinate $r$. In this neighbourhood choose new (barred) coordinates so that the metric is Lorentzian, i.e. $g_{00} = 1$, $g_{11} = -1$. This gives $t = \int f \, d\bar{t}$, $\bar{x} = \sqrt{f} \, x$, $\bar{y} = y$. The observer will see secondary matter moving with speed $\beta = y/\bar{x} = y/x/\sqrt{f}$. Since $x = 1/f$ and $y = \sqrt{f} x^2 - 1 = \sqrt{(1 - f)/f}$, we find $\beta = \sqrt{1 - f}$. This speed results in a Doppler shift of $\sqrt{(1 + \beta)/(1 - \beta)}$. Light travelling to $B$ is further redshifted by the factor $1/\sqrt{f}$; the final redshift factor is equal to $1 + z$. $y$ and $z$ are both linearly dependent on $r$ for small $r$.

3 ZERO-ORDER SUCCESS

We consider in order the three points mentioned in the introduction.
3.1 Hubble expansion

The linear dependence of \( z \) on \( r \) shown in Fig. 1(f) demonstrates the Hubble expansion. Contrast this with the model of EMN in which this linear dependence could not be obtained because the matter was assumed to be stationary.

3.2 CMBR, zeroth order

EMN suggested that the CMBR might emanate from the singularity that exists at \( A \) in their model. The present model has no such singularity but the origin of the CMBR may be similar. Around the annihilation region, wherever it may be, there is likely to be a cloud of dust thrown up like spray from a waterfall. This will conceal the annihilation region from our direct gaze; we will see instead the radiation from the surface of the cloud. If the cloud is cool enough this radiation could be the CMBR. Both the dust cloud and the CMBR will be at rest in the PF. The temperature of the CMBR will vary according to the Tolman relation (Tolman 1949, pp. 312–319); the energy density is proportional to \( 1/f^2 \) so the temperature will be proportional to \( 1/\sqrt{f} \).

The mechanism of thermalization by whiskers suggested by Hoyle et al. (1993) has been criticized on the grounds that the whiskers, which fill all space, will make space opaque in the range of radio wavelengths where we know space is actually very transparent. In the present model we do not have this problem. The thermalizing medium is the dust cloud which is localized in a part of space far away from us.

3.3 Evolution, zeroth order

The OSSM could not accommodate evolution because it was based on the PCP. Space in the CSSM, however, is not homogeneous, so there is no reason to suppose that conditions in regions far from us are the same as those in our neighbourhood. In the SM we assume that our view of distant objects tells us about conditions in the remote past. In the CSSM there is no evolution in time; instead we have a gradual change in conditions as we travel out from our location near the Boreal Pole.

4 ZEROTH-ORDER QUESTIONS

The remaining zeroth-order observations which need to be explained are the elemental abundances and the formation of structure such as stars and galaxies. The model presented in this paper cannot provide such an explanation because we have intentionally treated secondary matter as smoothed out to simplify the equations. But for the sake of completeness we give here a short discussion of these important questions, relying heavily on the work of Hoyle et al. (1993, hereafter HBN).

4.1 Elemental abundances

One of the most persuasive features of the SM is the natural explanation it provides for the elemental abundances. HBN maintain, however, that the observed abundances can also be reproduced by a continuous-creation model in which particles of the Planck mass are created and rapidly decay into baryons and leptons. We will assume here that this claim is correct and introduce a similar mechanism for particle creation in the CSSM. When primary matter is converted to secondary, most of the energy is contributed by the vacuum in the form of latent heat. This too can create a number of secondary particles, and we will suppose that the final elemental abundances can be made to fit observation, as they did for HBN.

Creation takes place, for HBN, in explosive events of various scales, from active galactic nuclei (AGNs) to superclusters of galaxies. We will suppose here, again following HBN, that AGNs and the jets of material that frequently accompany them are major sites of creation, so that the outflowing matter is not merely a fraction of the matter falling into an accretion disc but is for the most part genuinely new material. The jets will have approximately 'primordial composition', i.e. hydrogen with the canonical admixture of He and smaller amounts of other elements.

4.2 Structure formation

In the model of HBN, creation tends to occur explosively around compact high-mass objects because C-field quanta are attracted there. A similar mechanism can be invoked in the CSSM. We suppose that there is a potential barrier which normally prevents interconversion of primary and secondary matter, but that this barrier is reduced in AGNs. Conditions on the two sides of the barrier normally favour the conversion of primary to secondary material, but in the annihilation region we have to suppose that the pressure of secondary material increases to a level at which the reverse transition takes place. This process cannot be adequately represented in our simple model because we assume the secondary matter has zero pressure. The shape of the creation function, \( n \), determines the switch from creation to annihilation. In a more realistic model the secondary matter would possess pressure and an equation of state, and the creation function would be determined from knowledge of the potential barrier and chemical kinetics. The annihilation region would possibly include the point \( A \) instead of being confined to a band around \( A \) as in the present paper.

Now consider the evolution of a cluster of galaxies near the Boreal Pole. The new matter emerging from AGNs will probably be absorbed into the galaxies of the cluster until one or more of the AGNs becomes so massive and the creation rate so large that the pressure in the cluster increases. The gas may escape from the cluster or the galaxies themselves may be scattered. In either case there appears a large cloud of (mainly) primordial material ready to condense into a new cluster. Subsequent galaxy formation will occur in a similar way to the SM, the principal difference being that there may be a significant amount of dust, molecules, and heavy elements remaining from the original cluster. The creation rate in this cloud will fall to zero temporarily as new galaxies form.

During this whole process the galaxies will be falling towards the annihilation region near the Australl Pole. We would be unlikely to find a galaxy of extreme age in our neighbourhood near the Boreal Pole. Even though, in the simple theory, material exactly at this Pole will not accelerate, in practice the general turbulence associated with the creation process will almost certainly nudge the material into its free fall.

The fact that galaxies age as they fall does not imply that
5 FIRST-ORDER QUESTIONS

Our immediate tasks in this paper are now accomplished. There remain a large number of first-order questions. Only when at least the most important of these have been successfully addressed will the CSSM become a serious rival to the Standard Model. In this section we list some of these problems, without making any claim to completeness.

5.1 Theoretical issues

One of the most distinctive features of the SM is the effective way in which it separates the region in which known physics can be expected to apply from the ones (near the Big Bang or the Big Crunch) where new physics must be introduced. This constraint on most of our theorizing prevents us from unnecessary invention. In the CSSM, however, we have to anticipate the appearance of new physics at any time and at many different places. Some of the questions listed here may not have answers within physics as we know it now; nevertheless it is clearly wise not to use new physics unless we are forced to.

5.1.1 Our location

An essential element in the CSSM is the assumption that we are located quite close to the Borealis Pole. This assumption is supported by the fact that our velocity through the CMBR (and hence through the PF) is small compared to the velocity of light. Such a location is nevertheless improbable, as was pointed out by EMN. These authors used the Anthropic Principle to increase the probability. A galaxy created near $B$ will be at 'dead centre' and will have no tendency to fall towards $A$ in any direction. We show in the appendix that the time taken for a galaxy to fall from coordinate $r$ to the annihilation region involves an integral which is logarithmically divergent as $r \to 0$. Galaxies old enough for life to have developed might therefore be more likely near $B$. On the other hand, the model has been constructed to simulate the OSSM in the neighbourhood of $B$, and there is a large region of space in which the age distribution of galaxies is almost independent of position. The problem of our apparently privileged position therefore remains a real one.

5.1.2 Stability

We cannot be sure that the CSSM will be stable, especially since the original Einstein static model was shown to be unstable. A useful investigation would be to examine the stability of a limiting case of the CSSM in which there is no secondary matter, only primary radiation and the vacuum energy. The resulting model resembles the electromagnetic geons considered by Wheeler (1962), but generalized to an entire universe.

5.1.3 Entropy

In the CSSM the total entropy must remain constant. Since it surely increases in much of the universe there must be regions where it decreases. Here new physics will be needed unless some way of understanding this effect can found in the context of general relativity. Entropy, to some physicists at least, presents a serious problem in the context of the SM also. Penrose (1989, p. 344) has estimated that the probability that the Universe started with the necessary low value of entropy is of order one part in $10^{10^{123}}$. That this number is rarely cited as a major objection to the SM is an indication of how successfully that model divides amenable problems from those (the initial conditions) that are considered to be beyond our grasp.

5.1.4 Vacuum energy

We would like to be able to relate the vacuum energy-momentum tensor to the ideas of particle physics. The problem of the cosmological constant is an old one, and it takes a particularly complicated form in the CSSM because the vacuum energy has to vary with position in such a way as to meet boundary conditions at the two poles. It also has to react to changes so as to make the whole model stable.

5.2 Observations

5.2.1 CMBR, first order

We do not know whether the surface of the dust cloud that radiates the CMBR can remain cool enough. Very energetic processes are going on inside the cloud. Galaxies and radiation are being annihilated and converted into the latent heat of the vacuum. Even if we settle this problem adequately, we still have to account for the uniformity of the CMBR, and the small departures from uniformity.

5.2.2 Evolution, first order

There are many data about evolutionary effects as a function of redshift. These have to be reinterpreted in terms of a variation with $r$ rather than $t$. The CSSM is flexible in that we are (at present, at least) free to choose the shape of the creation function, $n$, to fit the data. A constant value in our neighbourhood will simulate the OSSM, and we can expect minimal evolution. In the present paper we have set $n \propto \sigma$ which should produce more young galaxies at large distances, in keeping with the Butcher-Oemler effect (Butcher & Oemler 1978a,b) and the Lyman $z$ clouds.

5.2.3 Age distribution of galaxies

Because the CSSM resembles the OSSM in our local region, it is not surprising that we can derive (see appendix) an age distribution for local galaxies which is the same as that discussed by Bondi (1952, pp. 149, 166), namely an exponential distribution with a time constant equal to one-third of the Hubble time. Now we do see apparently young galaxies in our vicinity (Arp et al. 1990), and also dim ones (Edmuns 1993), but not enough of them to match an exponential distribution.
This has in the past been used as an argument against steady-state theories. The current observational situation is discussed by HBN.

In the CSSM, in contrast to the OSSM, the age distribution of galaxies will be a function of $r$ and will depend on the shape of the creation function; see Section 5.2.2, above. The two models agree, of course, for small $r$.

### 5.2.4 Inhomogeneity

The feature that most clearly distinguishes the CSSM from the SM is perhaps the spatial inhomogeneity of the CSSM. At large distances the radial direction is distinguished from the transverse directions. This may have observable consequences. As an example we will discuss the phenomenon of relativistic beaming found in a number of radio sources (Cohen 1990). Beamed sources are thought to be aligned quite closely with the line of sight. In the SM they are believed to be part of a larger parent population with random orientations which can often be detected by their isotropic emission (say, from radio lobes). Astronomers have sometimes been worried by an apparent surplus of beamed sources, but so far these crises have been averted by enlarging the parent population (Barthel 1989). If we eventually find that we have enlarged the parent population as far as we can and we still have too many sources beamed towards us, we will be obliged to face the possibility that the sources are not randomly oriented in the sky but are preferentially aligned along the line of sight.

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### APPENDIX A

We list here the equations used to set up the boundary-value problem. We also outline the integration process, and derive the flux equation and the expected age distribution for galaxies. In the following equations, primes denote differentiation with respect to $r$, with the exception of $f'$ and $h'$. These are regarded as independent variables linked by the suplementary equations

$$\frac{df}{dr} = f' \quad \text{and} \quad \frac{dh}{dr} = h'. \quad (A1)$$

This is the normal procedure for reducing second-order equations to first-order ones for numerical integration.

Einstein field equations, for index combinations 00, 11, 22 and 01, are

$$3(2f^2h' + 2fh'' - h(f')^2) = 16\pi f^2h(3y^2\sigma + 3f^2x^2\sigma + 8f\rho - 2\rho + 6he + 3f^2d - 6 + 3b^2c), \quad (A2)$$

$$3(-4f^2hh' + 2f^2(h')^2 - 2f^2fh'' + h(f')^2) = 16\pi f^2h(3y^2\sigma + 3f^2x^2\sigma + 8\rho - 2\rho - 6he + 3f^2d + 6 + 3b^2c), \quad (A3)$$

$$3(2fh'' + f'h' - 4f) = 16\pi f(-3\sigma - 2\rho - 2f^2d - 6 + 3b^2c), \quad (A4)$$

$$3xy\sigma + 4w\rho + 3abc = 0. \quad (A5)$$

Creation equations, for indices 0 and 1, are

$$2f^2xy\sigma + 2fhx'\sigma + 2f^2hx'\sigma + 2fhx'\sigma + 2fhx'\sigma + 2fhx'\sigma + 2fhx'\sigma + 2fhx'\sigma + 2fhx'\sigma + 2fhx'\sigma = 2fhx', \quad (A6)$$

$$2fhy'\sigma + 2fhx'\sigma + 2fhx'\sigma + 2fhx'\sigma + 2fhx'\sigma + 2fhx'\sigma + 2fhx'\sigma + 2fhx'\sigma + 2fhx'\sigma + 2fhx'\sigma = 2fhx', \quad (A7)$$

Vacuum equations, for indices 0 and 1, are

$$abc + kxy\sigma = 0, \quad (A8)$$

$$2fhd' = 4fhhb'c + 2fhhb'c + 2fhhb'c + 2fhhb'c + f'h'c - 2fhh'e + k(2f'h'\sigma + 2f'h'y^2\sigma + 4f'h'y\sigma + 2fhx'\sigma + f'h'x') - 3fh'h' - 6hf' + 12f. \quad (A9)$$

From equations (A2), (A3) and (A4) we can derive the algebraic equation:

$$0 = 32\pi f^2h(3\sigma^2 + 4\rho + \rho + 3b^2c - 3d) - 3f(h')^2 - 6hf'h' + 12f. \quad (A10)$$

From equations (A6) and (A7) we can derive the simple differential equation for $x$:

$$\frac{dx'}{x} + \frac{f'}{f} = 0, \quad (A11)$$

from which it follows that $x = 1/f$.

A convenient set of variables for the integration is $f, h, f', h', h, b$ and $d$, and the equations to be integrated are (A1), (A2), (A4), (A6) and (A9). The remaining variables can be found algebraically.
If we multiply equation (A6) by $2\pi\sqrt{f}$ we can integrate it between radii $r_1$ and $r_2$ to obtain a flux equation:

$$(4\pi f^{3/2} h x y \sigma)_{1} - (4\pi f^{3/2} h x y \sigma)_{2} = 4\pi \int_{r_1}^{r_2} f^{3/2} n h x \, dr$$

(A12)

or, since $x = 1/f$,

$$(4\pi h y \sigma \sqrt{f})_{1} - (4\pi h y \sigma \sqrt{f})_{2} = \int_{r_1}^{r_2} h n \sqrt{f} \, dr .$$

(A13)

Now $4\pi h$ is the area of the spherical surface at radius $r$, so we have the natural interpretation: $y \sqrt{f}$ is the flux of particles and $n \sqrt{f}$ the creation rate.

If $t$ denotes coordinate time in the PF, and $s$ the invariant interval, then

$$ds^2 = f \, dr^2 - dr^2, \quad \frac{dt}{ds} = x, \quad \frac{dr}{ds} = y .$$

(A14)

The coordinate time taken for a galaxy to fall from $r_1$ to $r_2$ is given by

$$t = \int_{r_1}^{r_2} \frac{x}{y} \, dr .$$

(A15)

The proper time $\tau$, however, is just the interval $s$:

$$\tau = s = \int_{r_1}^{r_2} \frac{dr}{y} .$$

(A16)

Such integrals have to be computed numerically. In the limiting case of small $r_2$, however, an analytic form is available and the flux equation can be used to derive an age distribution for galaxies in the region near the Boreal Pole. We can set $f = x = 1$, $t = \tau$ and $y = Hr$, where $H$ is the Hubble constant. The galaxies passing through the sphere at radius $r_2$ have a range of ages. Those with age in the interval $t$ to $t + dt$ will all have been created between the radii $r_1$ and $r_1 + dr_1$, where $dr = dr_1/(Hr_1)$ and

$$t = \int_{r_1}^{r_2} \frac{dr}{y} = \int_{r_1}^{r_2} \frac{dr}{Hr} = \frac{1}{H} \ln \left( \frac{r_2}{r_1} \right) ,$$

(A17)

so that

$$r_1 = r_2 \exp (-tH) .$$

(A18)

The creation rate is essentially constant over this small volume, so the fraction of the galaxies with ages in the prescribed interval is proportional to the volume of space in which they were created, $4\pi r_1^2 dr_1$. In terms of the age this fraction becomes $4\pi H r_2^3 \exp (-3Ht) \, dt$. The normalized age distribution will then be $3H \exp (-3Ht) \, dt$, just as for the OSSM (Bondi 1952). We should not expect this age distribution to correspond too closely to reality. It has been deduced in the context of an idealized model in which secondary matter has been smoothed out. The main feature, that galaxies of a wide range of ages should be seen at any point, will surely survive in a more realistic model, but the precise form of the distribution will probably be different. A brief discussion of this topic is given in Section 5.2.3.

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