stars and nebulae that certainly do exist escaped his notice. The fallibility of the observer must not be lost sight of.


1. The Bonn Durchmusterung gives R.A.s and Declinations for 1855.0. Measures of stars on a photographic plate give rectilinear coordinates \((x, y)\) which are connected with standard coordinates \((\xi, \eta)\) by linear relations

\[
\xi = x + ax + by + c \\
\eta = y + dx + ey + f,
\]

and \((\xi, \eta)\) are connected with R.A. and Decl. for some other epoch (say 1900.0) by the relations

\[
\xi = \tan (a - A) \cos q \sec (q - D) \\
\eta = \tan (q - D),
\]

where

\[
\tan q = \tan \delta \sec (a - A),
\]

and \((a, \delta)\) are the R.A. and Decl. of the star,

\[(A, D) \quad ,, \quad ,, \quad ,, \quad ,, \quad plate centre.\]

A circuitous method of comparing the B.D. with the plate is thus to perform the following operations:

1. Bring up the B.D. places to 1900.0.
2. Convert into standard coordinates.
3. Multiply by the plate constants \(a, b, c, \&c.\), to get coordinates comparable with measures on the plate.

The present paper indicates a method of dispensing with operations (1) and (3), and generally performing the comparison very quickly.

2. The places of the Durchmusterung are given to \(0\text{''}1\) in R.A. and \(0\text{'}1\) in Declination. In terms of a réseau interval of \(5\text{'}\) (a unit now in common use for photographic measures) \(0\text{''}1\) on the equator represents \(0\text{''}005\), while \(0\text{'}1\) represents \(0\text{''}02\). It will probably be sufficient accuracy in both coordinates if we calculate to \(0\text{''}01\) of a réseau interval, or in circular measure \(0\text{''}00015\).

For the plates of the Astrographic Catalogue

\(x\) and \(y\) range from \(-02\) to \(+02\) in circular measure,

\(x^2\) and \(y^2\) ,, ,, \(0\) to \(0004\),

\(x^3\) and \(y^3\) ,, ,, \(-00008\) to \(+00008\).
Hence we may neglect powers of the coordinates higher than the second for the present purpose; though it is quite easy to take account of these terms afterwards if so desired.

Now to the second order it will be found that

$$
\xi = (a - \Lambda) \cos \delta, \quad \eta = (\dot{\delta} - D) + \frac{1}{4} (a - \Lambda)^2 \sin 2D.
$$

3. The point of the present method lies in choosing the right value of $\Lambda$. If we increase $\Lambda$ by $\Lambda_0$, we add to $\xi$ and $\eta$ the terms

$$
-\Lambda_0 \cos \dot{\delta}, \quad -\Lambda_0 (a - \Lambda) \sin D \cos D + \frac{1}{4} \Lambda_0^2 \sin 2D,
$$

or if $\Lambda_0$ be of the same order as ($\xi$ and $\eta$), since we can put $\cos \dot{\delta} = \cos D - (\dot{\delta} - D) \sin D$ to the first order, the terms to be added may be written

$$
\Lambda_0 \eta \sin D + \text{const.} \quad -\Lambda_0 \xi \sin D + \text{const.}
$$

which is equivalent to a rotation of the axes through an angle $\Lambda_0 \sin D$.

4. It is tolerably obvious, geometrically, that we can get the equivalence of a rotation of axes, or rather of the whole plate, by shifting the centre in R.A.; especially if we visualise the sphere as projected on a plane, say the tangent plane at the Pole. And the significance of the factor $\sin D$ is also clear from the geometrical standpoint: the rotation $\Lambda_0 \sin D$ vanishes at the equator whatever $\Lambda_0$ may be; in other words, the axes of $\eta$ are all parallel for plates with centres on the equator—we cannot get this rotation there.

5. We proceed to use this principle to get an equivalent for precession. Bringing up the star places from 1855° to 1900° in the ordinary way is a rather complex numerical process. Viewed as the comparison of two plates of the same stars at different epochs, it merely means that one plate is rotated in its own plane with respect to the other, through a definite angle. To find this angle let us suppose two stars on the axis of $\eta$ at 1900°, so that $\xi = 0$ for both; and let one be at the centre of the plate, the other $\Delta$ south of it, so that $\eta = 0$ for the first and $\eta = -\Delta$ for the second. The problem is to choose $\Lambda$, the adopted R.A. of plate centre for 1855°, so that the value of $\xi$ for the two stars may be, not necessarily zero, but the same for both.

We can then reduce both values to zero by subtracting an appropriate constant for the plate, which is a simple matter.

6. If $a$ be the common R.A. of the stars for 1900°, the precessions for the two stars are

$$
m + n \sin a \tan D
$$

$$
m + n \sin a \tan (D - \Delta).
$$

[We are considering differential effects, and the secular variation, even for forty-five years, does not matter.]
March 1900. *Comparing the Bonn Durchmusterung etc.* 429

Thus the R.A.s for 1855° will differ by

\[ 45\alpha \sin \alpha [\tan D - \tan (D - \Delta)] \]

Write this \[ \beta \sin \Delta \tan D \]

where \[ \beta = 90\alpha \sin \alpha \cosec 2D \]

Denote the R.A. of the central star for 1855° by \( A_1 \); that of the southern star by \( A_2 \); and that of the appropriate plate centre by \( A_3 \). Then we have to find \( A_3 \), so that

\[ (A_2 - A_3) \cos (D - \Delta) = (A_1 - A_3) \cos D \]

or

\[ A_2 [\cos D - \cos (D - \Delta)] = A_1 \cos D - A_2 \cos (D - \Delta) \]

\[ = A_1 [\cos D - \cos (D - \Delta)] + (A_1 - A_2) \cos (D - \Delta). \]

Thus \[ A_3 = A_1 + \beta \]

where from above the value of \( \beta \) is

\[ \beta = 90\alpha \sin \alpha \cosec 2D. \]

7. It will check this analytical result to look at the question geometrically.

Let \( E \) be the pole of the ecliptic, \( P \) be the pole of the equator, \( S \) a star, of coords. \((\alpha, D)\).

Precession carries \( P \) to \( R \), in the direction perpendicular to \( EP \), and towards the First Point of \( Aries \).

Thus \[ \angle SPR = \alpha. \]

Further \[ PR = 59'' \cdot 2 \sin \omega \times t \]

\[ = 45\alpha, \text{ as in } \S\ 6. \]

Now \( PS \) and \( RS \) are the directions of the axis of \( y \) for the two plates, which are inclined to one another at

\[ \angle PSR = PR \sin \alpha \cdot \cosec SP. \]

\[ = 45\alpha \cdot \sin \alpha \cdot \sec D. \]

We wish to correct this by choosing a plate centre differing in R.A. by the quantity \( \beta \) of the last paragraph. And by \( \S\ 3 \) or
§ 4 the effect of this change of R.A. is to rotate the plate through an angle $\beta \sin D$.

Hence

$$\beta \sin D = 45^\circ \sin a \sec D$$

or

$$\beta = 90^\circ \sin a \cosec 2D.$$

8. It may be a convenience to others to give here a table of the coefficients of $\sin a$ for different declinations, and the rule of signs.

**Rule.**—To form directly from the *Durchmusterung*, Epoch 1855°, standard coordinates for 1900°: if $\Lambda_0$ be the R.A. for 1855° of the actual plate centre, add to $\Lambda_0$ the quantity $K \sin \Lambda_0$, where $K$ is given in the following table in seconds of time:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^\circ$ or $90^\circ$</td>
<td>...</td>
<td>$15^\circ$ or $75^\circ$</td>
<td>240</td>
<td>$30^\circ$ or $60^\circ$</td>
<td>139</td>
</tr>
<tr>
<td>$1^\circ$</td>
<td>$89^\circ$ (57)</td>
<td>$16^\circ$</td>
<td>$74^\circ$</td>
<td>226</td>
<td>$31^\circ$</td>
</tr>
<tr>
<td>$2^\circ$</td>
<td>$88^\circ$ (29)</td>
<td>$17^\circ$</td>
<td>$73^\circ$</td>
<td>214</td>
<td>$32^\circ$</td>
</tr>
<tr>
<td>$3^\circ$</td>
<td>$87^\circ$ (19)</td>
<td>$18^\circ$</td>
<td>$72^\circ$</td>
<td>204</td>
<td>$33^\circ$</td>
</tr>
<tr>
<td>$4^\circ$</td>
<td>$86^\circ$ (14)</td>
<td>$19^\circ$</td>
<td>$71^\circ$</td>
<td>194</td>
<td>$34^\circ$</td>
</tr>
<tr>
<td>$5^\circ$</td>
<td>$85^\circ$ (11)</td>
<td>$20^\circ$</td>
<td>$70^\circ$</td>
<td>186</td>
<td>$35^\circ$</td>
</tr>
<tr>
<td>$6^\circ$</td>
<td>$84^\circ$ (10)</td>
<td>$21^\circ$</td>
<td>$69^\circ$</td>
<td>179</td>
<td>$36^\circ$</td>
</tr>
<tr>
<td>$7^\circ$</td>
<td>$83^\circ$ (8)</td>
<td>$22^\circ$</td>
<td>$68^\circ$</td>
<td>173</td>
<td>$37^\circ$</td>
</tr>
<tr>
<td>$8^\circ$</td>
<td>$82^\circ$ (6)</td>
<td>$23^\circ$</td>
<td>$67^\circ$</td>
<td>167</td>
<td>$38^\circ$</td>
</tr>
<tr>
<td>$9^\circ$</td>
<td>$81^\circ$ (4)</td>
<td>$24^\circ$</td>
<td>$66^\circ$</td>
<td>162</td>
<td>$39^\circ$</td>
</tr>
<tr>
<td>$10^\circ$</td>
<td>$80^\circ$ (2)</td>
<td>$25^\circ$</td>
<td>$65^\circ$</td>
<td>157</td>
<td>$40^\circ$</td>
</tr>
<tr>
<td>$11^\circ$</td>
<td>$79^\circ$ (1)</td>
<td>$26^\circ$</td>
<td>$64^\circ$</td>
<td>152</td>
<td>$41^\circ$</td>
</tr>
<tr>
<td>$12^\circ$</td>
<td>$78^\circ$</td>
<td>$27^\circ$</td>
<td>$63^\circ$</td>
<td>149</td>
<td>$42^\circ$</td>
</tr>
<tr>
<td>$13^\circ$</td>
<td>$77^\circ$</td>
<td>$28^\circ$</td>
<td>$62^\circ$</td>
<td>145</td>
<td>$43^\circ$</td>
</tr>
<tr>
<td>$14^\circ$</td>
<td>$76^\circ$</td>
<td>$29^\circ$</td>
<td>$61^\circ$</td>
<td>141</td>
<td>$44^\circ$</td>
</tr>
</tbody>
</table>

The method cannot be advantageously employed for plates near the equator or the poles.

**Example.**—To compare the B.D. with a plate taken with centre at $14^h$ 40$^m$, $+37^\circ$ at the Epoch 1900°.

The R.A. of the plate centre for 1855° is $14^h$ 38$^m$ 5$^s$.

Take from above table the quantity for $+37^\circ$, viz. 125$^s$. Multiply this by $14^h$ 40$^m$, which is $-\sin 40^\circ$ or $-643$, getting $-80^s$ or $-1^m$ 20$^s$. Add this algebraically to $14^h$ 38$^m$ 5$^s$, getting $14^h$ 36$^m$ 45$^s$ = $\Lambda'$ say.

If now $(a, \epsilon)$ be the coordinates of any star for 1855°, and we form

$$\tilde{x} = (a - \Lambda') \cos \epsilon, \quad \eta = (\tilde{\epsilon} - 37^\circ) + \frac{1}{4} (a - \Lambda')^2 \sin 74^\circ$$
as in § 2 (where the angles are expressed in circular measure), then \( \xi \) and \( \eta \) only differ by a constant from standard coordinates for \( 1900^\circ. \)

9. We have hitherto supposed the plate correctly oriented for \( 1900^\circ. \) Any error of orientation may be corrected in the same way, by again altering the assumed R.A. of plate centre. If \((x, y)\) denote measures on the plate, and \((\xi, \eta)\) standard coordinates, an error of orientation is indicated by the relations

\[
\begin{align*}
\xi &= x - by \\
\eta &= y + bx,
\end{align*}
\]

\( b \) being (in circular measure) the angle of rotation.

If this is compensated by a change \( A_3 \) in the plate centre, we must have by § 3 or § 4

\[
\pm b \times 13751 = A_3 \sin D,
\]

assuming that \( A_3 \) is expressed in seconds of time.

The following table indicates the value of \( A_3 \) for different values of \( b \) and different centres:

<table>
<thead>
<tr>
<th>( b )</th>
<th>Dec. = 10°</th>
<th>20°</th>
<th>30°</th>
<th>40°</th>
<th>50°</th>
<th>60°</th>
<th>70°</th>
<th>80°</th>
<th>90°</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>79</td>
<td>40</td>
<td>28</td>
<td>21</td>
<td>18</td>
<td>16</td>
<td>15</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>0.002</td>
<td>158</td>
<td>80</td>
<td>55</td>
<td>43</td>
<td>36</td>
<td>32</td>
<td>29</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td>0.003</td>
<td>238</td>
<td>121</td>
<td>82</td>
<td>64</td>
<td>54</td>
<td>48</td>
<td>44</td>
<td>42</td>
<td>41</td>
</tr>
<tr>
<td>0.004</td>
<td>317</td>
<td>161</td>
<td>110</td>
<td>86</td>
<td>72</td>
<td>64</td>
<td>55</td>
<td>56</td>
<td>55</td>
</tr>
<tr>
<td>0.005</td>
<td>396</td>
<td>201</td>
<td>138</td>
<td>107</td>
<td>90</td>
<td>80</td>
<td>73</td>
<td>70</td>
<td>69</td>
</tr>
<tr>
<td>0.006</td>
<td>475</td>
<td>241</td>
<td>165</td>
<td>128</td>
<td>108</td>
<td>95</td>
<td>88</td>
<td>83</td>
<td>83</td>
</tr>
<tr>
<td>0.007</td>
<td>554</td>
<td>281</td>
<td>193</td>
<td>150</td>
<td>126</td>
<td>111</td>
<td>102</td>
<td>97</td>
<td>97</td>
</tr>
<tr>
<td>0.008</td>
<td>634</td>
<td>322</td>
<td>220</td>
<td>171</td>
<td>144</td>
<td>127</td>
<td>117</td>
<td>111</td>
<td>110</td>
</tr>
<tr>
<td>0.009</td>
<td>713</td>
<td>362</td>
<td>248</td>
<td>193</td>
<td>162</td>
<td>143</td>
<td>131</td>
<td>125</td>
<td>124</td>
</tr>
<tr>
<td>0.010</td>
<td>792</td>
<td>402</td>
<td>275</td>
<td>214</td>
<td>180</td>
<td>159</td>
<td>146</td>
<td>139</td>
<td>138</td>
</tr>
</tbody>
</table>

**Rule of Signs.**—If \( b \) denotes a rotation in the direction above indicated, so that

\[
\begin{align*}
\xi &= x - by, \\
\eta &= y + bx,
\end{align*}
\]

\((x, y)\) denoting measures, and \((\xi, \eta)\) standard coordinates, then the adopted R.A. of the plate centre is to be **increased** by the above quantity before deriving \((\xi, \eta)\); and we shall then get

\[
\xi = x + \text{const.}, \quad \eta = y + \text{const.}
\]

10. If the plate has a scale value different from that adopted
for \((\xi, \eta)\), this can be allowed for in a variety of ways. Thus, in forming
\[
\xi = (a - A) \cos \delta
\]
if we wish to form instead
\[
(1 + p)\xi = (a - A) (1 + p) \cos \delta,
\]
we can do so indirectly by writing \((\delta + \epsilon)\) for \(\delta\), where
\[
\cos (\delta + \epsilon) = (1 + p) \cos \delta
\]
or \(\epsilon = -p \cot \delta\).

It must be remembered that great refinements are not necessary for the comparison of B.D. with our measures, and the mean scale value for a number of plates will probably be near enough for all, in spite of refraction and aberration. At Oxford the focal length of the instrument is nearly one per cent. too great, so that measures made with a standard réseau of 5\textsuperscript{mm} spaces have all to be diminished in the ratio 1\textsuperscript{0078} to 1. Recently, this troublesome correction (which ought to have been avoided by greater care on the part of the maker of the object-glass) has been obviated by measuring with a réseau of 5\textsuperscript{mm}\textsuperscript{04} spaces, so that the scale value is now approximately correct. Suppose then that we have made tables for the more recent plates, as below.

**Table for \((a - A) \cos \delta\) in Réseau intervals.**

<table>
<thead>
<tr>
<th>Dec' = 25° 0'</th>
<th>10'</th>
<th>20'</th>
<th>30'</th>
<th>40'</th>
<th>50'</th>
<th>26° 0'</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a - A) m s</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 10</td>
<td>0'453</td>
<td>0'453</td>
<td>0'452</td>
<td>0'451</td>
<td>0'451</td>
<td>0'450</td>
</tr>
<tr>
<td>20</td>
<td>0'906</td>
<td>0'905</td>
<td>0'904</td>
<td>0'903</td>
<td>0'901</td>
<td>0'900</td>
</tr>
<tr>
<td>30</td>
<td>1'360</td>
<td>1'358</td>
<td>1'356</td>
<td>1'354</td>
<td>1'352</td>
<td>1'350</td>
</tr>
<tr>
<td>40</td>
<td>1'813</td>
<td>1'810</td>
<td>1'853</td>
<td>1'850</td>
<td>1'848</td>
<td>1'845</td>
</tr>
<tr>
<td>50</td>
<td>2'266</td>
<td>2'263</td>
<td>2'259</td>
<td>2'257</td>
<td>2'254</td>
<td>2'250</td>
</tr>
<tr>
<td>1 0</td>
<td>2'719</td>
<td>2'716</td>
<td>2'711</td>
<td>2'708</td>
<td>2'704</td>
<td>2'700</td>
</tr>
<tr>
<td>10</td>
<td>3'172</td>
<td>3'168</td>
<td>3'163</td>
<td>3'159</td>
<td>3'155</td>
<td>3'150</td>
</tr>
<tr>
<td>20</td>
<td>3'626</td>
<td>3'621</td>
<td>3'615</td>
<td>3'610</td>
<td>3'606</td>
<td>3'600</td>
</tr>
</tbody>
</table>

&c. &c. &c.

the equivalent of the seconds and tenths being found from a supplementary table, and added to the number taken from above before contracting to two figures.

To use this table for comparing the B.D. with measures on
the old plates with defective scale value, we need only substitute for the headings,

\[
25^\circ 0', 25^\circ 10', 25^\circ 20', 25^\circ 30', 25^\circ 40', \text{ &c.}
\]

the values \(25^\circ 56', 26^\circ 6', 26^\circ 15', 26^\circ 25', 26^\circ 34', \text{ &c.}\)

and the tables are then adapted to the old plates.

For the formation of \(\eta = (\zeta - D) + \frac{1}{4}(a - A)^2 \sin 2D\), the second term will not generally need correction, and the first term can be multiplied by the proper factor by means of a small table.

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