June 1927. *The Equilibrium of the Calcium Chromosphere.* 605

On account of the longitude fluctuation, the correction to longitude at epoch \( \Delta \ell \) cannot be derived separately.

The correction to the assumed mass of Venus (1/408,000) is given by

\[
\frac{\Delta M}{M} = \frac{1}{v} \Delta = -0.0081 \pm 0.0034
\]

corresponding to a value for the reciprocal of the mass of

\( 1/M = 411,300 \pm 1,400. \)

The correction is in the same sense as that derived in the preceding discussion, but of about two-thirds the magnitude.

---

*The Equilibrium of the Calcium Chromosphere.* By P. A. Taylor, Ph.D.

(Communicated by R. H. Fowler, F.R.S.)

1. The object of this paper is to modify Professor Milne’s theory of the equilibrium of the calcium chromosphere† to take account of the curvature of the sun’s surface and the variation of gravitational intensity with distance from the centre of the sun. In other respects the assumptions made will be the same as in Paper II., † pp. 15–19. Certain interesting points arise in the discussion of the chromosphere under these conditions. As might be expected by analogy with the theory of the isothermal gas-sphere, when the gravitation of the chromosphere itself is neglected the density is found to tend to a small positive constant at infinity. It will be found that this approximation is fully justified in the cases with which this paper is concerned, even at the earth’s distance from the sun. Fortunately, the limiting value of the density is not impossibly great. A parameter \( \mu' \) occurs, which plays a very similar rôle to that of \( \mu \), the coefficient of partial support, in Professor Milne’s papers. The solution when \( \mu' = 0 \) is of special type, corresponding to the “fully supported” solution. The relation between \( \mu' \) and \( \mu \) is discussed, and it is found that \( \mu' \) is always slightly less than \( \mu \). Formulæ are found for the density and for the proportion of ions in the excited state, both for \( \mu' = 0 \), and for \( \mu' \neq 0 \). These formulæ, which will be exhibited graphically for certain values of \( \mu' \), will be used in a subsequent paper on the light-intensity of the calcium chromosphere.

I. Fundamental Equations.

2. In accordance with the assumptions already stated or referred to, we suppose that we have only to deal with light of one definite small frequency range \( \Delta \nu \) about the frequency \( \nu \). Let \( I \nu^*(r, \theta) \Delta \nu \) be the intensity per unit solid angle subtended, per unit area of cross-section

† *Monthly Notices, 84, 354; 85, 111; 86, 8; 86, 578.* The last three of these papers will be referred to as Papers I, II, III, respectively.
of beam, of light of this small frequency range $\Delta \nu$ at a point at distance $r$ from the centre of the sun, and in a direction making an angle $\theta$ with the outward-drawn radius vector through the point. Then by considering a small thin cylinder parallel to the radiation considered, we may show that the " $\cos \theta \frac{dI_\nu^*}{dx}$ " of II. (8) becomes replaced by

$$
(\cos \theta \frac{\partial I_\nu^*}{\partial r} - \frac{\sin \theta}{r} \frac{\partial I_\nu^*}{\partial \theta}),
$$

and hence we have in place of II. (8)

$$
\frac{\cos \theta}{\alpha} \left( \frac{\partial I_\nu^*}{\partial r} - \frac{\tan \theta}{r} \frac{\partial I_\nu^*}{\partial \theta} \right) = n_2 \frac{q_1}{q_2} (\sigma + I_\nu^*) - n_1 I_\nu^* .
$$

where $\sigma = 2\hbar \nu^3/\epsilon^2$, $n_1$ and $n_2$ are the numbers of atoms per unit volume in the normal ($\sigma$) and excited ($1\pi$) states respectively, $q_1$ and $q_2$ are the statistical weights of these states, and $\alpha$ is the atomic absorption coefficient.

The equation of radiative equilibrium II. (7) remains the same, viz.—

$$
n_1 \int_0^\pi I_\nu^* \sin \theta d\theta = n_2 \frac{q_1}{q_2} \int_0^\pi (I_\nu^* + \sigma) \sin \theta d\theta .
$$

The integral of the constancy of total net flux (cf. II. (9)) now takes the form

$$
\int_0^\pi I_\nu^* \sin \theta \cos \theta d\theta = \frac{C}{2r^2} .
$$

$C$ being an integration constant (4$\pi^2 \epsilon \Delta \nu$ is the total outward net flux over any sphere concentric with the sun).

Now multiply equation (1) by $\sin \theta d\theta$ and integrate with respect to $\theta$ between the limits $0$ and $\frac{\pi}{2}$. We have

$$
\int_0^{\pi/2} \sin \theta \cos \theta \left( \frac{\partial I_\nu^*}{\partial r} - \frac{\tan \theta}{r} \frac{\partial I_\nu^*}{\partial \theta} \right) d\theta = -an_1 \int_0^{\pi/2} I_\nu^* \sin \theta d\theta + \sigma n_2 \frac{q_1}{q_2},
$$

where

$$
n = n_1 - n_2 \frac{q_1}{q_2} .
$$

Now

$$
\int_0^{\pi/2} \sin^2 \theta \frac{\partial I_\nu^*}{\partial \theta} d\theta = I_\nu^*(r, \frac{1}{2}\pi) - 2 \int_0^{\pi/2} I_\nu^* \sin \theta \cos \theta d\theta .
$$

Therefore

$$
\frac{1}{r^2} \int_0^{\pi/2} \sin \theta \cos \theta \frac{\partial}{\partial r} (r^2 I_\nu^*) d\theta - \frac{1}{r} I_\nu^*(r, \frac{1}{2}\pi) = -an_1 \int_0^{\pi/2} I_\nu^* \sin \theta d\theta + \sigma n_2 \frac{q_1}{q_2} .
$$

Similarly

$$
\frac{1}{r^2} \int_{\pi/2}^{\pi} \sin \theta \cos \theta \frac{\partial}{\partial r} (r^2 I_\nu^*) d\theta + \frac{1}{r} I_\nu^*(r, \frac{1}{2}\pi) = -an_1 \int_0^{\pi/2} I_\nu^* \sin \theta d\theta + \sigma n_2 \frac{q_1}{q_2} .
$$

† This symbol means equation (8) of Paper II. This notation will be used throughout this paper in referring to equations in Professor Milne's papers.
3. Following Professor Milne, we now employ the Schuster-Schwarzschild approximation. We replace $I_r^* (r, \theta)$ by its mean values, $I_r$ and $I_r'$, in the ranges of $\theta (0, \frac{1}{2} \pi)$ and $(\frac{1}{2} \pi, \pi)$ respectively. As an approximation it seems natural to put

$$I_r^*(r, \frac{1}{2} \pi) = \frac{1}{2} (I_r + I_r').$$

Then we have approximately

$$\frac{1}{r^2} \frac{d}{dr} (r^2 I_r) - \frac{1}{r} (I_r + I_r') = 2 \alpha \left[ - n I_r + \sigma n_2 \frac{q_1}{q_2} \right]. \tag{7}$$

and

$$\frac{1}{r^2} \frac{d}{dr} (r^2 I_{r'}) - \frac{1}{r} (I_r + I_r') = 2 \alpha \left[ n I_{r'} - \sigma n_2 \frac{q_1}{q_2} \right]. \tag{8}$$

Equations (2) and (3) become approximately

$$n(I_r + I_{r'}) = 2 \sigma n_2 \frac{q_1}{q_2} \tag{9}$$

and

$$I_r - I_{r'} = C/r^2 \tag{10}$$

respectively. Adding equations (7) and (8) and using equation (10), we find

$$\frac{d}{dr} (I_r + I_{r'}) = - 2 \alpha \sigma C/r^2.$$

Now introduce a new variable $u$, defined by the equation

$$du = - \frac{n \alpha r dr}{r^2} \tag{11}$$

The zero of $u$ is at present left undefined. Then

$$\frac{d}{du} (I_r + I_{r'}) = 2 C,$$

$$I_r + I_{r'} = 2 C (u - u_0) \tag{12}$$

$u_0$ being an arbitrary constant. Therefore, by equation (9),

$$\frac{n_2}{n} = \frac{q_2}{q_1} \frac{C}{\sigma} (u - u_0). \tag{13}$$

4. We now require the modified form of the equation of mechanical equilibrium. We introduce the proper inverse square law of gravitation, but neglect the gravitation of the chromosphere itself; we shall show later that the gravitation of the chromosphere itself is quite negligible at distances from the sun as great as that of the earth. Remembering that $C/r^2$ takes the place of $F_r$ in the equation of net flux, we obtain

$$\frac{dp}{dr} = - (n_1 + n_2) \frac{mg a^2}{r^2} + \frac{n \alpha C \Delta v}{cr^2} \tag{14}$$

where $p$ is the pressure, $m$ the mass of the atom, $g$ the intensity of
gravitation at the sun’s surface, \(a\) the radius of the sun, and \(c\) the velocity of light.

Since

\[
\begin{align*}
    n_1 + n_2 &= n + n_2 \left[ 1 + \left( \frac{q_1}{q_2} \right) \right] \\
    &= n \left[ 1 + \left( 1 + \frac{q_1}{q_2} \right) c^2 \right] \left( u - u_0 \right)
\end{align*}
\]

we may write equation (14) in terms of \(u\)—

\[
\frac{dp}{du} = \frac{mg\alpha^2}{a} \left[ I - \frac{\pi \alpha C \Delta \nu}{mg\alpha^2} + \frac{C}{\sigma} \left( 1 + \frac{q_2}{q_1} \right) (u - u_0) \right].
\]

The integral of this is

\[
p - p_0 = \frac{mg\alpha^2}{a} \left( I - \frac{\pi \alpha C \Delta \nu}{mg\alpha^2} \right) (u - u_0) + \frac{mg\alpha^2 c^2}{2\sigma a} \left( 1 + \frac{q_2}{q_1} \right) (u - u_0)^2
\]

where \(p_0\) is an integration constant. Since the zero of \(u\) was left undefined, the constant \(u_0\), which occurs in equation (12), is quite arbitrary. We choose \(u_0\) to satisfy the quadratic equation in \(u_0\)—

\[
p_0 = \frac{mg\alpha^2}{a} \left[ I - \frac{\pi \alpha C \Delta \nu}{mg\alpha^2} \right] u_0 - \frac{mg\alpha^2 C}{2\sigma a} \left( 1 + \frac{q_2}{q_1} \right) u_0^2
\]

Equation (16) now takes the simpler form—

\[
p = \frac{\mu' mg\alpha^2}{a} u + \frac{mg\alpha^2 C}{2\sigma a} \left( 1 + \frac{q_2}{q_1} \right) u^2
\]

where

\[
\mu' = I - \frac{\pi \alpha C \Delta \nu}{mg\alpha^2} - \frac{C u_0}{\sigma} \left( 1 + \frac{q_2}{q_1} \right).
\]

\(\mu'\) is a convenient parameter which plays a very similar rôle in this work to that of \(\mu\) in Professor Milne's papers. As we shall see later, it is not exactly the coefficient of partial support. Nevertheless the form of equation (18) shows that \(\mu' = 0\) will give rise to a solution of special form, just as \(\mu = 0\) does in Professor Milne's work. We now proceed to a consideration of this special case.

II. Case \(\mu' = 0\).

5. Equation (18) becomes

\[
p = \frac{mg\alpha^2 C}{2\sigma a} \left( 1 + \frac{q_2}{q_1} \right) u^2
\]

To find the density law, we have (see II. (27))

\[
p = 2 (n_1 + n_2) k T_0,
\]

where \(T_0\) is the temperature of the chromosphere. Now the statement that \((n_2/n)\) is small compared with unity is in accordance with the conditions of the problem, for, conducting the equations on the assumption
that \((n_2/n)\) is small, we do obtain small values of \((n_2/n)\) in the solution.†

Therefore approximately

\[
p = 2nkT_0 \quad \ldots \quad (21)
\]

\[
n = \frac{na^2C}{4\sigma kT_0^2} \left( \frac{1}{1 + \frac{q_2}{q_1}} \right) u^2
\]

\[
= -\frac{r^2}{a} \frac{du}{dr}
\]

by equation (11). This integrates and gives

\[
u = \frac{\lambda' r_0}{a^4} \frac{r}{r - r_0} \quad \ldots \quad (22)
\]

where

\[
\lambda' = \frac{4\sigma kT_0a^2}{mgC[1 + (q_2/q_1)]}
\]

and \(r_0\) is an integration constant. Therefore

\[
n = \frac{\lambda' (r_0)^2}{a^2a^2 \frac{r^2}{(r - r_0)^2}} \quad \ldots \quad (24)
\]

As we should expect by analogy with the theory of the isothermal gas sphere, \(n\) tends to a small positive constant at infinity.

Equation (17) for \(u_0\) contains the integration constant \(p_0\). As this has not been determined, we are free to determine \(u_0\) in terms of the physical conditions of the problem. We use the condition that \(I_{\nu'}\) is zero for very large \(r\) (e.g. at the earth's distance from the sun). Combining equations (10) and (12), we have

\[
I_{\nu'} = C[u - u_0 - (1/2r^2)].
\]

Therefore

\[
u_0 = \text{limiting value of } [u - (1/2r^2)] = \frac{\lambda' r_0}{a^4}.
\]

Substituting in equation (13), we get

\[
\frac{n_2}{n} = \frac{C\lambda' r_2}{\sigma a^4 q_1} \frac{r_0}{r - r_0} \quad \ldots \quad (25)
\]

6. For light of a neighbouring frequency \(\nu'\) which is not absorbed,

\[
I_{\nu'} = (I_{\nu'})_0(a/r)^2,
\]

where \((I_{\nu'})_0\) is the photospheric value of \(I_{\nu'}\), i.e. \(\sigma[\exp (hv/kT) - 1]\).‡

Also, if \(\kappa\) denotes the ratio of the residual intensity in the line to the intensity in the neighbouring continuous spectrum,

\[
\kappa = \frac{(I_{\nu'/I_{\nu}})_{r \text{ large}}}{C/r^2}
\]

\[
= \frac{\sigma(a/r)^2[\exp (hv/kT) - 1]}{C/r^2}
\]

† See fig. 2.
‡ Paper I., p. 122; the photosphere is supposed to radiate like a black body at temperature \(T\).
since $I_\nu'$ is small compared with $I_\nu$ when $r$ is large. Therefore

$$C = \frac{\kappa \sigma a^2}{[\exp(h\nu/kT) - 1]}.$$  \hspace{1cm} (26)

By equations (10), (12), and (22) we have

$$I_\nu = C \left[ \frac{1}{2r^2} + \frac{\lambda r_0}{a^4} \frac{r}{r - r_0} - \frac{\lambda' r_0}{a^4} \right].$$

As in Paper I,† we assume that when $r = a$, $I_\nu$ has the photospheric value $\sigma[\exp(h\nu/kT) - 1]$. Thus

$$\frac{1}{\kappa a^2} = \frac{1}{2a^2} + \frac{\lambda r_0}{a^4} \left[ \frac{a}{a - r_0} - 1 \right]$$

$$\left( \frac{r_0}{a} \right)^2 + \left( \frac{2 - \kappa}{2\lambda'} \right) \frac{r_0}{a} - 1 = 0.$$  \hspace{1cm} (27)

Now

$$\frac{(2 - \kappa)a}{2\lambda'} = \frac{amq(2 - \kappa)[1 + (q_2/q_1)]}{8kT_0[\exp(h\nu/kT) - 1]}$$

$$= 412,$$

with the values $k = 1.372 \times 10^{-16}$, $T_0 = 5000^\circ$, $m = 40.07 \times 1.651 \times 10^{-24}$ gm., $g = 2.73 \times 10^4$ gm. sec.$^{-2}$, $a = 6.966 \times 10^{10}$ cm., $\kappa = 0.11$, $(q_2/q_1) = 3$, $(c/\nu) = 3.950 \times 10^{-5}$ cm., $c = 3.10^{10}$ cm. sec.$^{-1}$, $h = 6.55 \times 10^{-27}$ erg. sec., $T = 6000^\circ$.

Thus equation (27) has a large negative root, $(r_0/a) \approx -412$, and a root nearly unity, $1 - \delta$ (say), where

$$1 - 2\delta - 412\delta = 0,$$

$$I - (r_0/a) = \delta = 1/414 = 0.00242.$$  \hspace{1cm} (28)

Now in order that $n$ shall be a finite decreasing function of $r$ for $r > a$, we must have $< (r_0/a) < 1$.

Therefore we must take the root given by equation (28).

Since $(r_0/a)$ differs from unity by so little, we may put $(r_0/a) = 1$, except where the difference between $(r_0/a)$ and a quantity of order unity is concerned. Equations (24) and (25) take the forms

$$n = \frac{\lambda'}{a_2^2} \frac{r^2}{(r - r_0)^2}.$$  \hspace{1cm} (29)

$$\frac{n_1}{n} = \frac{C\lambda q_2}{\sigma^2 q_1} \frac{r_0}{r - r_0}.$$  \hspace{1cm} (30)

By equation (19), since $\mu' = 0$, we have

$$\frac{\pi aC\Delta \nu}{mga^2c} = I - \frac{C \lambda r_0}{\sigma a^4} \left[ 1 + \frac{q_2}{q_1} \right]$$

$$= I - \frac{4kT_0}{mga}.$$  \hspace{1cm} (31)

† p. 126.
approximately. Now \( \frac{4kT_0}{mga} = 2.18 \cdot 10^{-5} \). Neglecting this in comparison with unity, we have

\[
a\Delta \nu = \frac{mgc}{\pi \kappa \sigma} \left[ \exp \left( \frac{h \nu}{kT} \right) - 1 \right]
\]

by equation (26), and relation II. (6) gives

\[
\tau = \frac{(q_2/q_1)c \nu}{4mgc[\exp (h \nu/kT) - 1]}
\]

This is the same value of \( \tau \) (i.e. \( 1.8 \cdot 10^{-8} \) secs.) as that obtained by Professor Milne in Paper II.† Supposing that \( \Delta \nu \) corresponds to a line width of \( \Delta \lambda \) Angstroms, we find

\[
\begin{align*}
a' &= 5.38 \cdot 10^{-14}/\Delta \lambda \\
\frac{\lambda'}{a^2a} &= 5.36 \Delta \lambda \\
\frac{C \nu q_2}{a^2q_1} &= 1.635 \cdot 10^{-5}
\end{align*}
\]

7. We now investigate the mass of the chromosphere in this, the worst case with which we shall deal. If \( \rho \) is the density, we have approximately

\[
\rho_0 = mn = \frac{m \lambda'}{a'^2a} \left( \frac{r^2}{(r - r_0)^2} \right)
\]

\[
\pi \int_a^r \rho r^2 dr = \frac{4 \pi m \lambda'}{a'^2a} \int_a^r \frac{r^4 dr}{(r - r_0)^2}
\]

\[
= \frac{4 \pi m \lambda'}{a'^2a} \left[ \frac{3r^3 + r_0^2r + 3r_0^2r + 4r_0^3 \log_e (r - r_0) - (r_0^4/[r - r_0])}{r} \right]
\]

\[
= \frac{4 \pi m \lambda'}{a} \left[ \frac{3[(r/a)^3 - 1] + (r/a)^2 - 1 + 3[(r/a) - 1] + 1/[1 - (a/r)]}{1 - (r_0/a)} \right]
\]

approximately, provided that \( r/a \) is not nearly unity. We find that this expression has the value \( 2.64 \cdot 10^{-13} \Delta \lambda \times \) (sun’s mass) when \( (r/a) = 1000 \), that is, the gravitation of the chromosphere itself is quite negligible at several times the earth’s distance from the sun.

III. Case \( \mu' \neq 0 \).

8. We now proceed to investigate the case when \( \mu' \) is not zero. As before, equation (21), we have approximately

\[ p = 2nkT_0 \]

Therefore, by equation (18),

\[ n = Pu(I + Qu) \]  

where

\[ P = \frac{\mu' m a^2}{2akT_0} \]

\[ Q = \frac{C}{2\mu' \sigma} \left( I + \frac{q_1}{q_1} \right) \]

By equation (11)

\[ du = -\frac{Pa u(I + Qu)dr}{r^2}, \]

\[ \frac{Pa}{r} = \log \frac{u}{I + Qu} + \text{const.} \]

\[ u = \frac{r}{A \exp(-P_a/r) - Q} \]

where \( A \) is an integration constant. Therefore

\[ n = AP \exp(-P_a/r) / [A \exp(-P_a/r) - Q] \]

and

\[ \frac{n_2}{n} = \frac{Cq_2}{\sigma q_1} \left[ \frac{1}{A \exp(-P_a/r) - Q} - \frac{1}{A - Q} \right] \]

since \( u_0 \) is the limiting value of \([u - (1/2r^2)]\) as before.

9. Now by equations (10), (12), and (36),

\[ I_x = C \left[ \frac{1}{2\alpha^2} + \frac{1}{A \exp(-P_a/r) - Q} - \frac{1}{A - Q} \right] \]

and when \( r = a, I_x = \sigma / [\exp(hv/kT) - 1] \). Moreover, as when \( \mu' = 0, C = \kappa \sigma a^2 / [\exp(hv/kT) - 1] \). Therefore

\[ \frac{1}{\kappa a^2} = \frac{1}{2\alpha^2} + \frac{1}{A \exp(-P_a/a) - Q} - \frac{1}{A - Q} \]

\[ f(A) \equiv A^2 - \left\{ \frac{2\kappa a^2}{2 - \kappa} \left[ \exp(P_a/a) - 1 \right] + Q \left[ \exp(P_a/a) + 1 \right] \right\} \]

\[ + Q^2 \exp(P_a/a) = 0 \]

Now

\[ f(\pm \infty) = +\infty \]

\[ f(Q \exp(P_a/a)) = - \left[ \frac{2\kappa a^2}{2 - \kappa} \right] Q \exp(P_a/a) \left[ \exp(P_a/a) - 1 \right] < 0 \text{ for all } \mu' \]

\[ f(Q) = - \left[ \frac{2\kappa a^2}{2 - \kappa} \right] Q \left[ \exp(P_a/a) - 1 \right] < 0 \text{ for all } \mu'. \]

Thus equation (39) in \( A \) always has two real roots, one less than \( Q \), and one greater than \( Q \exp(P_a/a) \). To decide which is the correct value of \( A \), we use the criterion that \( n \) is a decreasing function of \( r \). By differentiating equation (37) we get

\[ \frac{dn}{dr} = -\frac{AP^2 a}{r^2} \left[ A \exp(-P_a/r) + Q \right] \exp(-P_a/r). \]
June 1927. The Equilibrium of the Calcium Chromosphere.

This is to be negative for \( r > a \). Therefore \( A > Q \exp(Pa/a) \), that is, \( A \) is the greater of the two roots of equation (39).

We find the following numerical values:

\[
P_a = 6.39 \times 10^{16} \mu', \quad (Pa/a) = 9.175 \times 10^4 \mu', \quad Q = 2.54 \times 10^{18} / \mu',
\]

\[
\frac{2\kappa a^2}{(2 - \kappa)} = 5.65 \times 10^2, \quad (Cq_2/\sigma q_1) = 3.81 \times 10^{18} \quad (40)
\]

10. In this paragraph \( \mu' \gg 10^{-4} \) throughout, and then \( \exp(Pa/a) \) is large compared with unity, and we may write the equation for \( A \)

\[
A^2 - \beta A + \gamma = 0 \quad . \quad . \quad . \quad (41)
\]

where

\[
\beta = \left[ \frac{2\kappa a^2}{(2 - \kappa)} + Q \right] \exp(Pa/a)
\]

\[
\gamma = Q^2 \exp(Pa/a).
\]

Also \( (\gamma/\beta^2) \) is very small compared with unity, and the greater of the roots of equation (41) in \( A \) is \( \beta \) approximately.

Therefore

\[
A = \left[ \frac{2\kappa a^2}{(2 - \kappa)} + Q \right] \exp(Pa/a) \quad . \quad . \quad (42)
\]

With this value of \( A \) we find that

\[
\frac{Cu_0}{\sigma} \left( 1 + \frac{q_2}{q_1} \right) < 2.1 \times 10^{-8}.
\]

Therefore, by equation (19), unless \( \mu' \) is very nearly unity, the value of \( a\Delta \nu \) is \( (1 - \mu') \) times its value when \( \mu' = 0 \) (equation (31)). Therefore we have

\[
\alpha = 5.38 \times 10^{-14} (1 - \mu') / \Delta \lambda, \quad P = 1.19 \times 10^{30} \mu' \Delta \lambda / (1 - \mu'),
\]

\[
PQ = 3.015 \times 10^{47} \Delta \lambda / (1 - \mu') \quad . \quad (43)
\]

We now know all the constants in the formulae for \( n \) and \( n_e/n \).

IV. General.

11. We now proceed to connect the parameter \( \mu' \) with \( \mu \), the true coefficient of partial support. We may write equations (14) and (15)

\[
\frac{(n_1 + n_2)mga^2}{r^2} = \frac{nmga^2}{r^2} \left[ 1 + \frac{C}{\sigma} \left( 1 + \frac{q_2}{q_1} \right) (u - u_0) \right] = -\frac{dp}{dr} + \frac{n\pi aC\Delta \nu}{cr^2}.
\]

Now the left-hand side of this equation represents the weight of the atoms in unit volume. Therefore, if at distance \( r \) from the centre of the sun a fraction \( [1 - \mu(r)] \) of the atoms are supported by radiation pressure, we have

\[
\frac{n\pi aC\Delta \nu}{mga^2} = [1 - \mu(r)] \left[ 1 + \frac{C}{\sigma} \left( 1 + \frac{q_2}{q_1} \right) (u - u_0) \right].
\]

Now \( \mu \) is the limiting value of \( \mu \) \( r \), i.e. the value of \( \mu \) \( r \) when \( u = u_0 \). Therefore by equation (19)

\[
\mu = \mu' + \frac{Cu_0}{\sigma} \left( 1 + \frac{q_2}{q_1} \right) \quad . \quad . \quad . \quad (44)
\]
When $\mu' = 0$, by equation (22), $u_0 = (\lambda' r_0 / a^4) = (\lambda / a^3)$ approximately. Therefore

$$\mu = \frac{C\lambda'}{\sigma a^3} \left( 1 + \frac{q_2}{q_1} \right) = 2 \cdot 18 \cdot 10^{-5} . . . (45)$$

Fig. 1.—Density in atoms per cubic centimetre.

When $\mu' \neq 0$, by equation (36) $u_0 = 1/(A - Q)$, and

$$\mu = \mu' + \left[ \frac{C}{\sigma} \left( 1 + \frac{q_2}{q_1} \right) / (A - Q) \right] . . . (46)$$

Since the last term in this equation is always positive, $\mu'$ is always less than $\mu$.

When $\mu' \geq 10^{-4}$, the last term in equation (46) is less than $2 \cdot 10^{-8}$, and therefore $\mu = \mu'$ to a close approximation.

Curves for $n$ (or $[n_1 + n_2]$) and $(n_2/n)$ (or $[n_2/(n_1 + n_2)]$) are drawn in
figs. 1 and 2 for $\mu' = 0$, $10^{-4}$, $10^{-3}$ respectively. For purposes of comparison, curves have also been drawn from Professor Milne's formulæ † when $\mu = 0$, formulæ obtained neglecting the curvature of the sun and the inverse square law of gravitation. In order conveniently to include a large range, log $n$ and log ($n_2/n$) have been plotted against sec.$^{-1}$ ($r/a$), the scales being adjusted to read $n$, ($n_2/n$), and ($r/a$)

directly.‡ We may note that in none of these cases is ($n_2/n$) greater than $6.74 \cdot 10^{-3}$, thus justifying the assumption that ($n_2/n$) is small. Also we note that the limiting densities as given by our formulæ at great distances from the sun are $(5.56, 0.0474, 5.5 \cdot 10^{-36}) \times \Delta \lambda$ atoms per

† The value of $x_0$ quoted in Paper I., p. 126, becomes halved approximately when account is taken of the statistical weights of the states.

‡ The curves for $n$ have been drawn for the case $\Delta \lambda = 1$; for any other value of $\Delta \lambda$ the vertical scale must be considered raised or lowered through a suitable distance.
cubic centimetre according as $\mu' = 0, 10^{-4}, 10^{-3}$ respectively. These
correspond to densities of $(3.68 \cdot 10^{-22}, 3.14 \cdot 10^{-24}, 3.6 \cdot 10^{-58}) \times \Delta \lambda$
gm cm$^{-3}$ respectively.† We are tempted to try and link up with
Professor Eddington’s theory of interstellar matter. Unfortunately,
however, our theory may break down at great distances from the sun,
because there a large proportion of the calcium atoms may become
doubly ionized,‡ and thus lose the support of radiation pressure. That
is to say, beyond a certain distance from the sun the density may fall
off more rapidly than is indicated by the formulæ of this paper.§

---

**The Light-Intensity of the Calcium Chromosphere.**

By P. A. Taylor, Ph.D.

*(Communicated by R. H. Fowler, F.R.S.)*

1. The results of the preceding paper are applied to the calculation
of H and K radiation as observed in a telescope pointed near the limb
of the sun in terms of the height above the limb of the point towards
which the telescope is directed. The final integration can only be
carried out numerically. This numerical integration is performed
for $\mu' = 0, 10^{-4}$ (i.e. $\mu = 2.18 \cdot 10^{-5}, 10^{-4}$). Comparison of the final
result with observation indicates that the chromosphere is not exactly
“fully supported”; the coefficient of partial support is probably of
order of magnitude $10^{-4}$ (i.e. in the limit of great heights about $10^{-4}$
of the weight of the atom is balanced by the pressure gradient, the
remainder of the weight being balanced by the radiation pressure).

† Such densities would have no effect on the earth’s motion. In order to obtain
cosmolonogical effects, H. Jeffreys has to introduce a resisting medium of density
$5 \cdot 10^{-14}$ or $4 \cdot 10^{-15}$ g cm$^{-3}$ (The Earth, chap. iv.). The density for $\mu' = 0$
is rather high for interstellar matter (see A. S. Eddington, Proc. Roy. Soc., A, 111
(1926), pp. 426, 428), but is not impossible, I think, for interplanetary matter.
§ Equations (9) (12), and the observed value of the residual intensity deter-
mine $X = \int (na/r^3)dr$, or $Y = \int (a/r^4)4\pi r^2dr$, the integrals being taken from the
photosphere to the earth’s orbit. If $n$ refers to Ca$^+$ atoms only, $Y$ remains un-
changed if Ca$^{++}$ atoms are also present, for the mechanical relations are not used in
determining $Y$. Also $Y$ is always less than the total number of Ca$^+$ atoms, and if
most of the chromosphere is near the sun, $Y$ is approximately equal to the total
number of Ca$^+$ atoms. That is to say, the observed residual intensity demands
the presence of a certain amount of Ca$^+$, irrespective of how it arises. At great
heights, where low densities prevail, there will be a continuous formation of Ca$^+$
by expulsion of electrons by radiation from Ca$^+$ atoms, and if the rate of recapture
of electrons by collision is less than the rate of expulsion of electrons, there will be
a net continuous formation of Ca$^{++}$ atoms; and presumably, in order to maintain
a steady state, Ca$^{++}$ atoms will fall back into the photosphere, and Ca$^+$ atoms will
be expelled to take their place. The degree of ionization in this steady state will
not necessarily be the same as in an equilibrium distribution of the same density
and temperature. It is hoped to investigate these points in a subsequent paper.