The Effect of Collisions on Monochromatic Radiative Equilibrium.

By E. A. Milne.

1. In two recent papers* I have dwelt on the circumstance that as we proceed outwards from the interior of a star to the highest levels of the stellar atmosphere we find a transition from local thermodynamic equilibrium to monochromatic radiative equilibrium. It is the purpose of this paper to trace this transition in detail as far as regards the transfer of radiation and to establish an equation already used in a previous paper.†

We shall prove that (at least in the case which it is possible to analyse fully) as the density tends to zero, i.e. towards the outside of a star, absorption and emission correspond to monochromatic radiative equilibrium: whilst as the density tends to infinity, i.e. towards the interior of the star, absorption and emission correspond to local thermodynamic equilibrium. The transition is due to the changing effect of collisions in exciting and de-exciting atoms.

2. It is first necessary to define our terms. The matter in the neighbourhood of a point P will be said to be in local thermodynamic equilibrium at temperature T when its emission of radiation is the same as if it were in strict thermodynamic equilibrium at temperature T, i.e. is the same as if it were enclosed in a uniform-temperature enclosure at temperature T. Then Kirchhoff's law holds strictly. Let \( 4\pi\epsilon_\nu \) be the total emission of radiation within the range \( (\nu, \nu + d\nu) \) during \( dt \), per unit mass. Let \( B_\nu = (2\pi^2/\kappa^3)(\epsilon^{\nu/kT} - 1)^{-1} \) be Kirchhoff's universal function, the intensity of \( \nu \)-radiation in a uniform-temperature enclosure at temperature T. Let \( k_\nu \) be the coefficient of absorption. Then Kirchhoff's law states that

\[
\epsilon_\nu = k_\nu B_\nu . \quad . \quad . \quad . \quad (1)
\]

Now consider the transmission of radiation through material which is in local thermodynamic equilibrium. Let \( I_\nu \) be the intensity of \( \nu \)-radiation in a direction given by the element of length \( ds \). Then by the usual argument the equation of transfer is

\[
\frac{dI_\nu}{\rho ds} = - k_\nu I_\nu + \epsilon_\nu . \quad . \quad . \quad . \quad (2)
\]

using (1), this becomes

\[
\frac{dI_\nu}{\rho ds} = - k_\nu I_\nu + k_\nu B_\nu . \quad . \quad . \quad . \quad (3)
\]

3. We proceed now to monochromatic radiative equilibrium. The matter in the neighbourhood of a point P is said to be in monochromatic radiative equilibrium in respect of \( \nu \)-radiation when it

re-emits all the $\nu$-radiation which it absorbs. The total absorption
per unit mass is

$$k_\nu \int I_\nu d\omega,$$

where $I_\nu$ is the incident intensity of $\nu$-radiation, the integration being
taken over the complete solid angle; $I_\nu$ is of course a function of direction
at $P$. Then the equation of transfer of $\nu$-radiation is found to be

$$\frac{dI_\nu}{\rho ds} = -k_\nu I_\nu + k_\nu \int \frac{I_\nu d\omega}{4\pi} . \quad \cdot \quad \cdot \quad (4)$$

assuming the absorbed radiation is re-emitted uniformly in direction.

Equations (3) and (4) are of course the classical equations of Schuster
and Schwarzschild.

4. We now propose to consider the equation of transfer of radiation
when we remove the restriction to either local thermodynamic equili-
rium or monochromatic radiative equilibrium. We shall impose
only the condition that the state at the point $P$ is a steady state as
regards the distribution of the atoms at $P$ amongst their stationary
states. We shall show, under such further restrictions as seem to be
convenient, that at low densities the equation of transfer tends to the
form (4) and that at high densities it tends to the form (3).

We consider the case of an assembly containing atoms of a given
species supposed capable of two and only two stationary states. The
assembly may contain any number of atoms of other species, also any
number of ions and electrons. We refer to the state of lower energy
as the normal state (state 1), that of higher energy as the excited state
(state 2). Let $n_1$ be the number of atoms per cm.$^3$ in state 1, $n_2$ the
number in state 2, at the point $P$. Let $A_{2\rightarrow1}$, $B_{1\rightarrow2}$, $B_{2\rightarrow1}$ be the
usual Einstein probability coefficients, defined with reference to the
intensity of radiation (not the density).* If $q_1$, $q_2$ are the statistical
weights of the states 1 and 2, we have the Einstein relations

$$\frac{A_{2\rightarrow1}}{B_{1\rightarrow2}} = \frac{2\hbar \nu^3}{c^2} \frac{q_1}{q_2} \quad \cdot \quad \cdot \quad \cdot \quad (5)$$

$$\frac{B_{2\rightarrow1}}{B_{1\rightarrow2}} = \frac{q_1}{q_2} \quad \cdot \quad \cdot \quad \cdot \quad (6)$$

It is convenient to abbreviate by writing

$$\sigma = \frac{2\hbar \nu^3}{c^2} \quad \cdot \quad \cdot \quad \cdot \quad (7)$$

* This means that, for instance, $B_{1\rightarrow2}I_\nu dt$ is the probability that an atom will
absorb during $dt$, under the influence of isotropic radiation of intensity $I_\nu$. Einstein
and most other writers have used a coefficient which we may call $B_{1\rightarrow2}'$, such that
$B_{1\rightarrow2}' dt$ is the probability in question, where $q_\nu$ is the density of $\nu$-radiation.
For isotropic radiation, $q_\nu = 4\pi \nu I_\nu/c^2$, so that $B_{1\rightarrow2} = (4\pi/c) B_{1\rightarrow2}'$. It appears to
the writer that $B_{1\rightarrow2}'$ is much more convenient to use than $B_{1\rightarrow2}$ when we wish
to consider extensions to non-isotropic fields.
Let $\alpha(\nu)$ be the atomic absorption coefficient for an atom in the state $i$. Then it is easily shown * that approximately
\[ \int \alpha(\nu) d\nu = \frac{B_{1 \rightarrow 2}\nu}{4\pi} \]
the integral being taken through the narrow range of frequencies within which the atom can absorb. To simplify our work we shall represent this as uniform absorption within an interval $\Delta\nu$, and we shall accordingly write
\[ \alpha, \Delta\nu = \frac{B_{1 \rightarrow 2}\nu}{4\pi} \]
where $\alpha$, is the mean atomic absorption coefficient in $(\nu - \frac{1}{2}\Delta\nu, \nu + \frac{1}{2}\Delta\nu)$.

We next introduce further probability coefficients $\dagger b_{1 \rightarrow 2}, a_{1 \rightarrow 2}$ to take account of the effect of collisions in exciting and de-exciting atoms in the states $1$ and $2$ respectively. Let $b_{1 \rightarrow 2} dt$ be the probability that an atom of the kind in question, in state $1$, is excited to state $2$ by a collision with any atom, ion, or electron present in the assembly, during $dt$. Let $a_{2 \rightarrow 1} dt$ be the probability that an atom in state $2$ is de-excited to state $1$ by a collision, during $dt$. In the first type of collision, known as "collisions of the first kind" or inelastic collisions, the energy of excitation $\nu$ is derived from the kinetic energy of the colliding particles. In the second type of collision, known as "collisions of the second kind" or super-elastic collisions, the colliding particles rebound with additional kinetic energy $\nu$.

The coefficients $A_{2 \rightarrow 1}, B_{1 \rightarrow 2}, B_{2 \rightarrow 1}$ are atomic constants independent of temperature. The coefficients $a_{2 \rightarrow 1}, b_{1 \rightarrow 2}$ are, on the contrary, functions of the temperature. Further, they depend on the total number of particles present, and on their kinds. For a given composition, they will be proportional to the total density. As regards dependence on the temperature, there is strictly speaking no temperature, since we are considering a state of the most general kind, exposed to any kind of external radiation. There will, however, be a certain distribution of velocity amongst the particles present, and this distribution we may assume to be of Maxwellian form. In that case it will mimic the distribution in thermodynamic equilibrium at some temperature $T$. This $T$ is, as regards the assembly under consideration, merely a parameter characterising the velocity-distribution. It is, however, the "temperature" of the assembly as measured by a thermometer whose bulb was enclosed in a perfectly conducting sheath; for such a thermometer, being screened from radiation by the perfectly

* M.N., 85, 118, 1924. The same relation has been given by Tolman (Phys. Rev., 23, 693, 1924) and others.

† R. H. Fowler, Phil. Mag., 47, 257, 1924. Cf. also Eddington, Internal Constitution of the Stars, p. 339 (1926), whose analysis suggested the present treatment. The results are, however, somewhat different. Eddington assumed monochromatic radiative equilibrium in the continuous spectrum adjacent to an absorption line. The present treatment will show that this assumption is not justifiable, as the continuous spectrum must be produced in a region of higher density than the absorption line.
conducting and therefore perfectly reflecting sheath, would simply measure the intensity of the molecular bombardment to which it was subjected.

According to the principle of detailed balancing, if the assembly were in thermodynamic equilibrium we should have super-elastic collisions exactly balancing inelastic collisions. The condition for this is

\[ n_1 b_{1 \rightarrow 2} = n_2 a_{2 \rightarrow 1} \quad \quad \quad \quad (10) \]

But in thermodynamic equilibrium at temperature T we have by Boltzmann's relation

\[ \frac{n_2}{n_1} = \frac{q_2 e^{-h\nu/kT}}{q_1} \quad \quad \quad \quad (11) \]

Hence in thermodynamic equilibrium at temperature T we have

\[ \frac{b_{1 \rightarrow 2}}{a_{2 \rightarrow 1}} = \frac{q_2 e^{-h\nu/kT}}{q_1} \quad \quad \quad \quad (12) \]

This equation must, however, subsist in virtue of the velocity distribution of the particles present. Hence, although we have proved it in the first instance only for thermodynamic equilibrium, it must also hold whenever the velocity-distribution may be described by a parameter-temperature T. The ratio \( b_{1 \rightarrow 2} / a_{2 \rightarrow 1} \) is of course independent of the density.

We accordingly take (12) as holding in general.

As regards the separate dependence of \( b_{1 \rightarrow 2} \) and \( a_{2 \rightarrow 1} \) on the temperature, it is clear on general grounds that \( b_{1 \rightarrow 2} \) will largely depend on the numbers of particles of different kinds endowed with a kinetic energy exceeding \( h\nu \). Thus \( b_{1 \rightarrow 2} \) will contain a factor \( e^{-h\nu/kT} \); its other factors, and also \( a_{2 \rightarrow 1} \) will vary much more slowly with the temperature.*

5. Now consider the transfer of radiant energy through the system. Take a thin slab, of area \( dS \) and thickness \( ds \). The number of quanta of \( \nu \)-radiation entering the slab in directions inclined within a solid angle \( d\omega \) normal to the slab is

\[ \frac{I_\nu d\omega dS}{h\nu}, \]

and the number leaving in the same direction is

\[ (I_\nu + dI_\nu) d\omega dS \]

\[ \frac{h\nu}{h\nu}. \]

The excess of the latter number over the former, in the steady state, must be equal to the excess of emissions of \( \nu \)-quanta from the matter inside the slab in directions included in \( d\omega \) over the number of similar absorptions. The number of such emissions is

\[ n_2 d\omega dS(A_{2 \rightarrow 1} + B_{2 \rightarrow 1} I_\nu) d\omega / 4\pi, \]

* Cf. R. H. Fowler, loc. cit.
Apr. 1928. on Monochromatic Radiative Equilibrium.

and the number of such absorptions is

\[ n_1 ds dS B_{1\rightarrow 2} I_\nu d\omega / 4\pi. \]

Hence

\[ \frac{dI_\nu \Delta \nu}{hvd\sigma} = \frac{i}{4\pi} [n_2(A_{2\rightarrow 1} + B_{2\rightarrow 1} I_\nu) - n_1 B_{1\rightarrow 2} I_\nu]. \]

Using (5), (6), (7), and (9), this gives

\[ \frac{dI_\nu}{a_\nu d\sigma} = -n_1 I_\nu + \frac{q_1}{q_2} n_2 (\sigma + I_\nu). \]  

(13)

This is the equation of transfer of radiation.

6. By way of parenthesis, we shall now verify that (13) reduces to (3) when the state reduces to strict thermodynamic equilibrium. The two will be identical provided

\[ \rho k_\nu = a_\nu \left( n_1 - n_2 \frac{q_1}{q_2} \right) \]  

(14)

and

\[ \rho k_\nu B_\nu = a_\nu \frac{q_1}{q_2} n_2 \sigma. \]  

(15)

Equation (14) provides the relation connecting the mass-absorption coefficient for the whole set of atoms under consideration and the atomic absorption coefficient. Dividing (15) by (14) we get

\[ B_\nu = \frac{n_2 (q_1 / q_2)}{n_1 - n_2 (q_1 / q_2)} \sigma. \]  

(16)

But in thermodynamic equilibrium (11) holds. Substituting in (16), we get

\[ B_\nu = \frac{\sigma}{e^{hv/kT} - 1}. \]  

(17)

This is precisely Planck's law, as it should be. Thus (13) includes (3) as a special case. On our assumptions, the mode of transfer of radiation is completely known when \( n_1 \) and \( n_2 \) are known as functions of position.

7. We now write down the condition that the state is steady. We simply state that the number of transitions \( 1 \rightarrow 2 \) is equal to the number \( 2 \rightarrow 1 \). This gives

\[ n_1 \left[ B_{1\rightarrow 2} \int I_\nu \frac{d\omega}{4\pi} + b_{1\rightarrow 2} \right] = n_2 \left[ A_{2\rightarrow 1} + B_{2\rightarrow 1} \int I_\nu \frac{d\omega}{4\pi} + a_{2\rightarrow 1} \right]. \]  

(18)

Write

\[ \epsilon = \frac{b_{1\rightarrow 2}}{B_{1\rightarrow 2}}, \]

and use relations (5), (6), and (12). Then (18) becomes

\[ \left( n_1 - n_2 \frac{q_1}{q_2} \right) \int I_\nu \frac{d\omega}{4\pi} - n_2 \frac{q_1}{q_2} \sigma + \epsilon \left[ n_1 - n_2 \frac{q_1}{q_2} e^{hv/kT} \right] = 0. \]

(19)
From what has gone before, the coefficient $\epsilon$ is proportional to the total density, and depends on the temperature principally through a factor $e^{-h\nu/kT}$.

8. From (19) we find

$$\frac{n_2(q_1/q_2)}{n_1 - n_2(q_1/q_2)} = \frac{[I_\nu(d\omega/4\pi) + \epsilon]}{\sigma + \epsilon(e^{h\nu/kT} - 1)}$$

Introduce this in (13). It becomes

$$\frac{dI_\nu}{a_\nu ds} = \left(n_1 - n_2 q_1^2\right)\left[I_\nu + \frac{[I_\nu(d\omega/4\pi) + \epsilon]}{1 + (e^{h\nu/kT} - 1)(\epsilon/\sigma)}\right]$$

Introduce the optical thickness $\tau_\nu$ for $\nu$-radiation, defined by

$$a_\nu\left(n_1 - n_2 q_1^2\right)ds = d\tau_\nu = k_\nu ds$$

Further, write

$$\eta = \frac{\epsilon}{B_\nu(\tau)} = \frac{b_{1\rightarrow 2}}{B_{1\rightarrow 2}} e^{h\nu/kT} - 1$$

Then (21) becomes

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + \frac{[I_\nu(d\omega/4\pi) + \eta B_\nu(\tau)]}{1 + \eta}$$

This is the form taken by the equation of transfer when account is taken of the existence of a steady state.

9. We have seen that approximately $\epsilon \propto \rho e^{-h\nu/kT}$. Hence from (23), except for very low temperatures, $\eta$ is approximately independent of temperature and proportional to the total density. Hence if we apply (24) to a star we see that $\eta$ is zero at the boundary and increases towards the interior.

As $\eta \to 0$, (24) tends to the form

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + \int I_\nu d\omega$$

This is identical with (4), the equation of transfer for monochromatic radiative equilibrium.

As $\eta \to \infty$, (24) tends to the form

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + B_\nu(\tau)$$

This is identical with (3), the equation of transfer for local thermodynamic equilibrium at temperature $T$. In (24), we recall, $T$ is simply the parameter-temperature fixing the velocity-distribution.

We infer that as we penetrate a star from the boundary inwards, we begin with monochromatic radiative equilibrium and pass continuously to local thermodynamic equilibrium. The higher the density, that is, the more the atoms are bombarded by other atoms, the more the atoms emit according to Kirchhoff's law.

10. Yet another interpretation can be given to (24). Following
Schuster, let us suppose the material to possess an absorption coefficient $k_\nu$ and a scattering coefficient $s_\nu$. Then the equation of transfer is

$$\frac{dI_\nu}{ds} = -(k_\nu + s_\nu)I_\nu + s_\nu \int I_\nu \frac{d\omega}{4\pi} + k_\nu B_\nu . \quad (27)$$

This is identical with (24) if we put

$$\eta = \frac{k_\nu}{s_\nu}, \quad d\tau_\nu = (k_\nu + s_\nu)ds . \quad (28)$$

Thus our result is equivalent to the statement that near the boundary of the star the dominant feature is scattering without change of frequency, whilst the further we penetrate the star the more we encounter regions where absorbed radiation is re-emitted in frequencies depending on the local temperature, i.e., on the local velocity-distribution.

II. It is of interest to attempt a rough numerical evaluation of the quantity $\eta$. To see its order of magnitude, let us confine attention to the electronic collisions.

Let $N_e$ be the number of electrons per unit volume, $m$ the mass of the electron. In a Maxwellian distribution at temperature $T$, the number of electrons moving with velocity between $c$ and $c + dc$ with directions inside $d\omega$ is

$$N_e \left( \frac{m}{2\pi kT} \right)^{3/2} e^{-\frac{mv^2}{kT}} dvd\omega .$$

Hence the number with velocities exceeding $v_0$ colliding with a stationary sphere of radius $r$ in time $dt$ is

$$4\pi N_e \left( \frac{m}{2\pi kT} \right)^{3/2} \pi r^2 dt \int_{v_0}^{\infty} e^{-\frac{mv^2}{kT}} v^2 dv .$$

Putting $\frac{1}{2}mv_0^2 = \hbar v$, this comes to

$$N_e r^2 \left( \frac{8\pi kT}{m} \right)^{1/2} \left( 1 + \frac{\hbar v}{kT} \right) e^{-\frac{\hbar v}{kT}} dt .$$

Let the fraction $f$ of these be effective in exciting atoms from state 1 to state 2. Then we have

$$b_{1\rightarrow 2} = fr^2 N_e \left( \frac{8\pi kT}{m} \right)^{1/2} \left( 1 + \frac{\hbar v}{kT} \right) e^{-\frac{\hbar v}{kT}} .$$

For $B_{1\rightarrow 2}$ we may adopt the value $^{8}$

$$B_{1\rightarrow 2} = \frac{4\pi}{c} \frac{\pi e^2}{m \hbar v} ,$$

where $e$ is the charge on the electron. This is equivalent to adopting the theoretical value for the mean life of the excited atom.

We have then from (23)

$$\eta = \frac{me^3}{8\pi^2 r^2 e^2} fr^2 N_e \left( \frac{8\pi kT}{m} \right)^{1/2} \left( 1 + \frac{\hbar v}{kT} \right) \left( I - e^{-\frac{\hbar v}{kT}} \right) . \quad (29)$$

We observe that, in accordance with the general line of argument above, \( \eta \) varies only slowly with \( T \) and is proportional to the electron density.

From this formula let us find the partial electron pressure \( P_e = Ne kT \) when \( \eta \) is unity.

We find

\[
(P_e)_{\eta=1} = \frac{8\pi^n e^2}{mc^2} \cdot \frac{1}{f(\frac{mkT}{8\pi})(1 + \frac{h\nu}{kT})^{-1}(1 - e^{-h\nu/kT})^{-1}}
\]

The value of \( f \) for ionisation collisions has been studied by R. H. Fowler (loc. cit.). It appeared that the effective target area was of the order of the cross-section of the atom. Let us assume something of the same kind is true for excitation collisions, and put as regards order of magnitude \( f = 1 \), \( r = 1.5 \times 10^{-8} \text{ cm} \). We will further take \( T = 6000^\circ \), and take for \( \nu \) the mean frequency of the H and K lines of Ca\(^+\). With \( e = 4.774 \times 10^{-4} \) E.S.U., \( m = 9.00 \times 10^{-28} \) gram, \( h = 6.55 \times 10^{-27} \) we find

\[
(P_e)_{\eta=1} = 1.46 \times 10^3 \text{ dyne cm.}^{-2} = 1.46 \times 10^{-3} \text{ atmos.}
\]

If \( P_e \) in the reversing layer of a star is of the order of \( 10^{-4} \) atmos., it would follow from this rough calculation that in the reversing layer of a star \( \eta \) is about \( \frac{1}{16} \). As we have neglected the effects of collisions other than electronic collisions, \( \eta \) will be rather higher than this. If, however, we adopt a value of the order \( \eta = \frac{1}{10} \) for the reversing layer of a star, it would follow that the reversing layer is very close to monochromatic radiative equilibrium. In the photospheric layers we shall have a transition from monochromatic radiative equilibrium to local thermodynamic equilibrium as we go inwards; for \( P_e \) = \( 10^{-2} \) atmos. we should have \( \eta = 10 \), which practically corresponds to local thermodynamic equilibrium.

12. Equation (24) has been deduced on special assumptions concerning the atom in question, namely, that it exists in only two stationary states. A difference will be introduced if cyclic processes of the type \( 1 \rightarrow 3, 3 \rightarrow 2, 2 \rightarrow 1 \) occur, since this will result in radiation being transformed in frequency. On the other hand, the parallelism of (24) with Schuster’s form (27) suggests that (24) has a wider validity than its mode of derivation indicates; it contains in fact no explicit reference to the stationary states of the atom, except in the parameter \( \eta \).

13. If we assume (24) to be of universal validity for all frequencies we have potentially an equation from which we can deduce the emergent intensity in the whole spectrum, both inside the absorption lines and between them, once we know \( T \) and \( \eta \) as functions of the optical depth. Schuster’s famous investigation* corresponds in effect to adopting a definite gradient for \( T \) and taking \( \eta = \) constant. If our arguments are right, this cannot give a good representation of the true state of affairs, since as we have seen \( \eta \) increases inwards from zero to some very large number. Unfortunately it does not seem possible at present to make progress with (24) by using a variable \( \eta \), say \( \eta \sim \rho \). What,

however, I have attempted to do in two recent papers is to assume
$\eta = 0$ on one side of a certain plane, $\eta = \infty$ on the other. This
can be at best only a crude representation, but the solution obtained
in this way may be expected to exhibit some of the features of the true
solution.

According to the above rough numerical estimate, monochromatic
radiative equilibrium sets in at $P_e = 10^{-3}$ atm., as we pass outwards
from the photosphere. In the two recent papers mentioned I have
assumed demarcation between $\eta = 0$ and $\eta = \infty$ might be taken as
dividing the chromosphere and upper reversing layer from the lower
reversing layer and photospheric layers. It would now appear that
I took this plane too high up, at too low a pressure, and some revision
* may be required. Selective radiation-pressure may be appreciable at
greater depths in the reversing layer than I estimated.

14. It ought to be mentioned that as we have confined ourselves
in this paper to the emission and absorption of radiation by transitions
between stationary states, the analysis only refers to the atoms pro-
ducing the core of the absorption line. It has no bearing on the question
of wings and line-widths. We have simply shown that at reversing
layer pressures, and lower pressures, collisions do not interfere with
monochromatic radiative equilibrium, whilst at photospheric pressures
the effect of collisions is to cause a redistribution of intensity in a given
frequency so as to make it approximate to that given by Kirchhoff's
law. In discussing the formation of a Fraunhofer line, we are bound
to take into account both regions. For the photospheric layers them-
selves will play some part in "reversing" the line. Any calcium, for
instance, present in the layers which give an appreciable continuous
spectrum must begin to form an absorption line, owing to its absorption
coefficient being higher than the general coefficient of absorption.
Above the photospheric layers the general coefficient of absorption
decreases to zero, and we have layers of gas transparent except in line
frequencies; here the lines are deepened.

15. If these suggestions correspond in any way with actuality,
they have a general bearing on the theoretical calculation of the profiles
of absorption-line energy-curves. Dr. Minnaert,† in his recent important
work on the profiles of absorption-curves in the sun's Fraunhofer spec-
trum, has compared his results with calculations based on the formula

$$I_\nu = \int_0^\infty \epsilon_\nu e^{-k_\nu \tau} d\tau.$$  

This formula is true generally, if $\epsilon_\nu$ denotes the coefficient of emission
of $\nu$-radiation. But we are only entitled to take $\epsilon_\nu = B_\nu$ (Planck's
function) in the region in which local thermodynamic equilibrium
holds. Outside these layers, $\epsilon_\nu$ is no longer given by $B_\nu$. In the centre
of the line it is given by

$$\epsilon_\nu = \left( \frac{n_2(q_1/q_2)}{n_1 - n_2(q_1/q_2)} \right) \sigma$$

for the case of an atom with only two stationary states; \( n_2/n_1 \) may in the outer layer "correspond" to much lower temperatures than those predicted by the theory of radiative equilibrium for the integrated radiation, and so \( \varepsilon_r \) may be very different from \( B_r(\tau) \). In the wings of the line other considerations such as those brought forward by Stewart * and Unsöld † will be required.

16. The estimate \( P_e = 10^{-3} \) atmos. for the layer in which \( \eta = 1 \) is probably, as already mentioned, an overestimate of pressure. In this connection it may be borne in mind that the collision radius of an atom for excitation by similar atoms may be much larger than its radius as calculated from the kinetic theory of gases. For example, line-broadening for mercury atoms has been found by Orthmann and Pringsheim ‡ to correspond to a cross-section some 100 times the gas-kinetic cross-section. On the other hand, line-weakening (de-excitation of excited atoms) seems to correspond to a cross-section only some 10 times the gas-kinetic cross-section. It is possible that the estimate \( P_e(\eta=1) = 10^{-3} \) atmos. should be reduced to \( 10^{-4} \) or \( 10^{-5} \) atmos.

17. In conclusion, it may be mentioned that the transition from local thermodynamic equilibrium to monochromatic radiative equilibrium has an important bearing on the change of residual intensity between centre and limb. When strict monochromatic equilibrium holds, the central residual intensity should be cut down by a factor 2/5 between centre and limb, whereas if local thermodynamic equilibrium held right up to the boundary, as Schwarzschild pointed out, the line at the limb should fade into the continuous background. The fact that lines do not disappear at the limb shows that some measure of monochromatic equilibrium holds for all lines, as predicted by the theory. This is simply the selective scattering inferred by Schwarzschild.

Possibly a better model of a stellar atmosphere could be constructed by fitting on to one another, continuously, three layers, namely, layers for which \( \eta = 0, \eta = 1, \) and \( \eta = \infty \), corresponding to increasing depths.

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Note on the Theory of the Outer Layers of a Pulsating Star.
By Dr. J. J. M. Reesinck.

(Communicated by the Secretaries.)

1. In *M.N.*, 87, 414, 1927, the author examined on a very much simplified model the outer layers of a pulsating star. In *M.N.*, 87, 539, 1927, Professor A. S. Eddington gave a much more exact treatment of the same problem. In a postscript he compares the two investigations, but I think that it may be of some interest to examine

* *Astrophys. Journ.,* 59, 35, 1924.