Evaluation and Improvement of Turbulence Parameterization inside Deep Convective Clouds at Kilometer-Scale Resolution

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ABSTRACT

A challenge for cloud-resolving models is to make subgrid schemes suitable for deep convective clouds. A benchmark large-eddy simulation (LES) was conducted on a deep convective cloud with 50-m grid spacing. The reference turbulence fields for horizontal grid spacings of 500 m, 1 km, and 2 km were deduced by coarse graining the 50-m LES outputs, allowing subgrid fields to be characterized. The highest values of reference subgrid turbulent kinetic energy (TKE) were localized in the updraft core, and the production of subgrid TKE was dominated by thermal effects at coarser resolution (2 and 1 km) and by dynamical effects at finer resolution than 500 m. Countergradient areas due to nonlocal mixing appeared on the subgrid vertical thermodynamical fluxes in the updraft core and near the cloud top. The subgrid dynamical variances were anisotropic but the difference between vertical and horizontal variances diminished with increasing resolution. Then offline and online evaluations were conducted for this deep convective case with two different parameterization approaches at kilometer-scale resolution and gave the same results. A commonly used eddy-diffusivity turbulence scheme underestimated the thermal production of subgrid TKE and did not enable the countergradient structures to be reproduced. In contrast, the approach proposed by Moeng, parameterizing the subgrid vertical thermodynamical fluxes in terms of horizontal gradients of resolved variables, reproduced these characteristics and limited the overestimation of vertical velocity.

1. Introduction

In the recent decades, the representation of turbulence in meteorological models has been extensively studied for the atmospheric planetary boundary layer (PBL) (e.g., Deardorff 1972; Mellor and Yamada 1974; Moeng 1984; Stull 1988; Garratt 1994). In comparison, fewer studies have been dedicated to examining turbulence modeling in deep convective clouds at kilometer-scale resolution, despite the fact that deep convection is a crucial forecasting issue, also essential for troposphere–stratosphere exchanges, and that turbulence is a serious aviation hazard. Convective clouds are associated with strong turbulent motions due to heterogeneities within the cloud and between the cloud and its environment (hydrodynamical instabilities, mixing at cloud edges, etc.), strong ascents inside the clouds, evaporative downdrafts and gust fronts associated with strong wind shear, and gravity waves excited at the cloud top when the anvil hits the tropopause. The cauliflower-shaped structures of cumulus and cumulonimbus indicate the presence of numerous turbulent eddies of all sizes. Turbulence effects have an impact on cloud dynamics through the mixing of cloudy air with the surrounding clear air (e.g., Squires 1958; Klaassen and Clark 1985; Grabowski 1993; Grabowski and Clark 1993; Grabowski 2007). Turbulent mixing also affects the cloud microphysics: it enhances the production of ice crystals in deep convective clouds (e.g., Zeng et al. 2011), the cloud water content is decreased when cloudy air is diluted with environmental air (e.g., Burnet and Brenguier 2007). In addition, mixing increases drop collisions (e.g., Benmoshe et al. 2012) and impacts the droplet size distribution (e.g., Pinsky and Khain 1997).

Two main classes of methods exist for representing the turbulence depending on the model resolution: the one-dimensional turbulence scheme (i.e., one-dimensional approximation) where the flow is considered as horizontally homogeneous, with mixing only occurring in the vertical direction (e.g., Mellor and Yamada 1974), or the three-dimensional turbulence scheme, similar to those designed for large-eddy simulations (LES), and known as three-dimensional LES closure.

To better represent turbulence in numerical modeling, LES were developed in the early 1970s for high
Reynolds number in a channel (Lilly 1967; Deardorff 1970), and then applied to meteorological flows (Deardorff 1972; Sommeria 1976). These simulations are based on a fundamental assumption: the chosen resolution Δ must be adapted as the most energetic eddies of typical scale L are resolved by the model and the mean effect of the smallest eddies on the mean flow is parameterized. The most common approach is the eddy-diffusivity method with an additional viscosity (e.g., Smagorinsky 1963), but many other approaches to modeling the subgrid-scale stresses have been developed (e.g., Kosovic 1997; Chow et al. 2005; Matheou and Chung 2014). The nonlocal turbulent transport is thus assumed to be explicitly resolved as the size of the most energetic eddies are larger than the model resolution (L ≫ Δ).

In the PBL, turbulent motions make an important contribution to the vertical transport, which can be decomposed into two parts: a local transport due to small-scale quasi-random motions and a nonlocal transport associated with coherent or quasi-organized structures, which is due to convection (e.g., thermals, plumes, and rolls). For 1D turbulence schemes used in numerical weather prediction (NWP) models or global climate models (GCMs), the local mixing is based on the eddy-diffusivity term where the turbulent flux is expressed as a function of the vertical gradient of the mean variable. The closure of the system is ensured by a mixing length that can be defined as the characteristic size of the most energetic eddies, which remains small in comparison with the model resolution (L ≪ Δ) [e.g., the mixing length developed in Bougeault and Lacarrere (1989)]. The nonlocal mixing initiated by thermals needs to be treated by another approach. The most usual one is the mass flux approach derived from the cumulus convection parameterization (from Arakawa and Schubert 1974), which describes the vertical motion of convective structures. The eddy diffusivity mass flux (EDMF) approach unifies the nonlocal and local turbulence (Siebesma and Teixeira 2000; Houdier et al. 2002; Soares et al. 2004) by combining an eddy-diffusivity term and the mass flux approach in a consistent way. It is worth noting that other approaches exist for taking the nonlocal mixing due to coherent turbulent structures into account. For low-order models, the eddy-diffusivity scheme can be used with an additional countergradient term (Deardorff 1966; Holtslag and Moeng 1991). For high-order schemes, the vertical transport can be represented by the divergence of three-order moments (Canuto et al. 1994, 2001; Tomas and Masson 2006). In a third approach, a transient matrix can be used to describe the nonlocal vertical transport of mass by thermals by explicitly introducing the effect of large eddies (Stull 1984; Romps and Kuang 2011).

At intermediate range (i.e., kilometer and hectometer scales), where the injection of energy for some meteorological phenomena is of the same order as the model resolution (L ≈ Δ), the parameterizations used in LES models are not suitable, nor are those developed for mesoscale models as a part of the turbulent motions are resolved. This gray zone has been termed “terra incognita” by Wyngaard (2004), and some studies have shown that the turbulence schemes have to be adapted at these scales (Honnert et al. 2011; Shin and Hong 2013).

However, all these issues of turbulent structures have been comparatively little explored in deep convective clouds. Moeng et al. (2010) have shown that no gap exists between resolved and subgrid scales of a cloud-resolving model (CRM) in the energy spectrum related to deep convection and that much of the moisture is transported vertically by small-scale motions. They have also shown that the correlation between resolved and unresolved scales at 1-km resolution strongly affects the unresolved turbulent fluxes. This issue had already been discussed in the turbulence community (e.g., Leonard 1974; Bardina et al. 1980; Chow et al. 2005). From the data analysis of a LES of deep convection, Moeng et al. (2010) proposed a mixed subgrid-scale scheme to represent the subgrid-scale fluxes of scalar variables and momentum in CRMs. For that purpose, they added a term of horizontal gradients of resolved variables to the conventional downgradient eddy-diffusivity scheme as in Bardina et al. (1980). The first part represents the Reynolds terms (which depend only on the subfiltered scales, i.e., the prime variables) whereas the second represents the Leonard term (which depends only on the filtered scales) plus the cross term (which represents the interaction between the filtered scales and the subfiltered scales). In Moeng (2014), the subgrid scheme expresses the vertical fluxes of heat, moisture, and momentum only in terms of the horizontal gradients of these resolved variables, the Reynolds terms exhibiting a spatial distribution very similar to that of the other terms. They have shown that this formulation acts as a closure scheme for an updraft–downdraft representation of subgrid-scale fluxes over the entire deep convection layer. However, as noted by Prein et al. (2015) these promising developments have not yet been tested and used in online simulations. One objective of this paper is to evaluate this parameterization inside deep convective clouds, making some adaptations if necessary.

The weaknesses of the turbulent schemes in CRM for convective clouds have already pointed out a dependence on the resolution. At intermediate range (between 500 and 2 km), Parodi and Tanelli (2010) showed that simulations using LES closure behaved better than the simulations with the 1D turbulence
scheme for a deep convective case in the intertropical convergence zone, and Fiori et al. (2009, 2010) obtained similar results for idealized simulations of supercells. Simulated supercells were also investigated by Adlerman and Droegemeier (2002), who showed a deficit of subgrid turbulent kinetic energy due to poorly resolved gradients at low resolution. Verrelle et al. (2015) also found insufficient turbulent mixing inside convective clouds, more pronounced at coarse resolution with weak thermal production, underlying a lack of entrainment in convective clouds at intermediate range. Lebo and Morrison (2015) found that entrainment and detrainment were suppressed in the simulations of idealized squall lines with horizontal grid spacing larger than 500 m. Moreover, studies of sensitivity to the mixing length (Hanley et al. 2015; Machado and Chaboureau 2015) showed that the use of a small value increased the number of small cells with high area-averaged rain rates, and vice versa. Takemi and Rotunno (2003) investigated the effects of subgrid mixing and numerical filtering in mesoscale simulations of squall lines by examining the sensitivities to the parameters in turbulence-closure schemes as well as the parameters in the numerical filter. More appropriate coefficient values of the Smagorinsky scheme allowed many poorly resolved grid-scale cells to be avoided in simulations of mesoscale cloud systems in a no-shear environment without introducing artificial numerical filters. All the studies mentioned above show that cloud organization is very sensitive to turbulence representation, and that some deficiencies of turbulence schemes exist inside clouds, which are not model specific. Improving turbulence parameterization at kilometer resolution is a fundamental issue as many national weather services use operational convective-scale models [e.g., the Météo-France Applications of Research to Operations at Mesoscale (AROME) model at 1.3-km resolution (Seity et al. 2011; Brousseau et al. 2016), the German version of the Consortium for Small-Scale Modeling (COSMO) at 2.8-km resolution (Baldauf et al. 2011), and the Met Office Unified Model (UM) at 1.5-km resolution (Lean et al. 2008)].

The most common way to evaluate and improve turbulence schemes in convective clouds is to use LES, but the grid spacing required remains an open question. Bryan et al. (2003) underlined the lack of convergence of statistical properties of convective systems with idealized simulations of squall lines with increasing horizontal resolution (from 1000 to 125 m), and showed that about 100-m grid spacing is required to begin to resolve the turbulent eddies in convective systems. In the same order of magnitude, Khairoutdinov et al. (2009) found a convergence for a tropical convective system between a 100-m grid spacing LES and a 200-m grid-spacing simulation. Using an analysis of kinetic energy spectra for an isolated convective system, Ricard et al. (2013) showed also that a 250-m grid simulation resolved an inertial subrange for vertical velocity, reflecting a beginning of convergence. Near convergence in the quantities related to convective clouds was also obtained for the 100-m LES and the 200-m simulation of a Hector thunderstorm (Dauhut et al. 2015).

This study aims to characterize the representation of turbulence inside deep convective clouds at kilometer resolutions using LES as a reference simulation at 50-m resolution and a coarse-graining procedure. This technique captures most of the transport variability by taking successive horizontal means up to a grid resolution to quantify the contribution of the resolved and subgrid terms (Honnert et al. 2011). One of the objectives of this paper is to examine the occurrence of nonlocal turbulence inside convective clouds. Then, an evaluation of the turbulence scheme currently used in the Meso-NH model (Lafore et al. 1998) based on the classical eddy-diffusivity method is conducted and alternative formulations for parameterized fluxes, derived from Moeng et al. (2010) and Moeng (2014) are tested. This is achieved first with an offline evaluation, by using filtered fields from the reference LES to compute the turbulent fluxes, and second with an online evaluation of simulations using 500-m, 1-km, and 2-km horizontal grid spacing.

The study is organized as follows: section 2 describes the methodology used and the LES performed. In section 3, the subgrid turbulent fluxes are characterized at different horizontal resolutions by applying the coarse-graining treatment to the reference LES fields. A first evaluation of the different turbulence formulations is made in section 4, by comparing parameterized fluxes computed from the LES fields with the reference fluxes. The online evaluation of the turbulence parameterization is conducted in section 5 with simulations using hectometer and kilometer horizontal resolutions, and alternative formulations are tested. The results are summarized and discussed in section 6, where some conclusions are drawn.

2. Methodology

a. The Meso-NH model

All the simulations were run with the Meso-NH model, a research model able to simulate atmospheric motions ranging from the large meso-α scale down to the microscale. This Eulerian model uses a gridpoint discretization on a staggered Arakawa C grid with a conformal projection system of horizontal coordinates and a Gal-Chen and Somerville (1975) system of vertical coordinates. The model is nonhydrostatic, and solves an
anelastic system of momentum equations. The governing equations used in this study are based on the pseudo-incompressible system of Durrant (1989). A fourth-order centered advection scheme is used for the momentum components and the PPM advection scheme (Colella and Woodward 1984) for other variables, associated with the forward-in-time time-stepping procedure. A fourth-order explicit horizontal diffusion can be applied on the momentum components to suppress very short wavelength modes. As shown in Ricard et al. (2013), with this dynamical configuration, Meso-NH has a very good effective resolution (about 5–6 \( \Delta x \)) (Skamarock 2004).

Concerning the physical package, the one-moment bulk cloud microphysical scheme with three ice categories (ICE3; Pinty and Jabouille 1998) uses five prognostic hydrometeors (cloud droplets, rain, ice crystals, snow, and graupel mixing ratios) in addition to the water vapor mixing ratio. A subgrid turbulence scheme is used with a 1.5-order closure coming from the system of second-order equations for the turbulent moments and based on a prognostic equation for the turbulent kinetic energy (TKE) (see appendix A). This turbulence scheme (Cuxart et al. 2000), derived from Redelsperger and Sommeria (1986), was developed for both mesoscale applications and LES with the same equations (hereafter referred to as the CBR scheme). The Deardorff mixing length is used for LES resolutions whereas the turbulent mixing length of Bougeault and Lacarrere (1989) is used for CRM and mesoscale model resolutions. The former equals the cubic root of the product of the 3D grid sizes reduced by the thermal stability according to Deardorff (1980). The latter, at any level in the atmosphere, is related to the distance a parcel of air having the initial kinetic energy of the level can travel upward and downward before being stopped by buoyancy effects (see appendix A).

It is worth noting that the 1D version of this turbulence scheme (i.e., neglecting horizontal gradients except for the advection terms in the prognostic equation of TKE) is used in the operational AROME model. In this study, it will be used exclusively in its 3D form.

For the turbulent fluxes, the following diagnostic equations are used:

\[
\bar{u}_i \bar{u}_j = \frac{2}{3} \delta_{ij} e - \frac{4}{15} L C_m e^{1/2} \left( \frac{\partial \tau}{\partial x_j} + \frac{\partial \tau}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial \tau}{\partial x_k} \right),
\]

\[
\bar{u}_i \theta_i = \frac{2}{3} L C_s e^{1/2} \frac{\partial \theta_i}{\partial x_i},
\]

\[
\bar{u}_i r_{np} = \frac{2}{3} L C_h e^{1/2} \frac{\partial r_{np}}{\partial x_i} \phi_i,
\]

where \( u_i \) represents the \( i \)th component of the velocities; \( \theta_i \) is the liquid-ice potential temperature; \( r_{np} \) is the total nonprecipitating water mixing ratio (sum of water vapor, cloud water, and ice); \( \delta_{ij} \) is the Kronecker delta tensor; \( \psi_i \) and \( \phi_i \) are stability functions defined in Cuxart et al. (2000); the Einstein summation convention applies for subscripts \( k \); \( C_s \), \( C_h \), and \( C_m \) are numeric constants from pressure-correlation parameterization (Redelsperger and Sommeria 1981; Cheng et al. 2002); and \( L \) is the mixing length.

The two quasi-conservative variables first introduced by Betts (1973) and Deardorff (1976) in the non-precipitating case and used here in the turbulence scheme are as follows:

(i) the liquid-ice potential temperature is defined as

\[
\theta_i = \theta - \frac{L_v}{C_{ph}} \frac{1}{\pi^2} \frac{1}{r} - \frac{L_s}{C_{ph}} \frac{1}{r},
\]

where \( L_v \) is the latent heat of sublimation of ice, \( L_s \) is the latent heat of vaporization of water, \( C_{ph} \) is the specific heat at constant pressure for moist air, and \( \pi = (P/P_0)(R_d/C_{pd}) \) is the Exner pressure, with \( P_0 \) being the reference value of the pressure (1000 hPa), \( C_{pd} \) is the specific heat at constant pressure of dry air, and \( R_d \) is the gas constant for dry air;

(ii) the total nonprecipitating water mixing ratio is defined as

\[
r_{np} = r_v + r_c + r_i
\]

(i.e., the sum of the mixing ratios of water vapor \( r_v \), cloud water \( r_c \), and ice \( r_i \)).

The vertical flux of virtual potential temperature, needed for the TKE prognostic equation, can be obtained by

\[
\bar{u}_i \bar{\theta}_i = E_\theta \bar{u}_i \bar{\theta}_i + E_{moist} \bar{u}_i r_{np},
\]

where \( E_\theta \) and \( E_{moist} \) are coefficients described in Redelsperger and Sommeria (1986).

In this study, with the exception of the turbulence scheme and the microphysical scheme, ICE3, mentioned above, no other parameterization scheme (deep convection scheme, shallow convection scheme) is used. Moreover, radiation is neglected and a subgrid condensation scheme is also not considered.

b. Large-eddy simulation

Three-dimensional idealized simulations of deep moist convection are performed at very fine resolution without orography or Coriolis force. In this part, the turbulence scheme is used with the Deardorff mixing length.
The initial conditions for potential temperature and humidity are derived from the analytical profiles (see details in appendix B) used by Weisman and Klemp (1984) with a surface mixing ratio of 14 g kg\(^{-1}\), which corresponds to a conditionally unstable environment (CAPE about 2800 J kg\(^{-1}\)). A moderate wind shear is provided to represent an organized thunderstorm with a quarter-circle hodograph lasting a few hours; \(u\) and \(v\) vary from zero at the surface to 6.4 m s\(^{-1}\) at 2 km AGL; between 2 and 6 km \(v\) remains constant and \(u\) varies from 6.4 to 8 m s\(^{-1}\); above 6 km AGL the horizontal wind remains constant in direction and intensity (\(u = 8\) m s\(^{-1}\), \(v = 6.4\) m s\(^{-1}\)). Small perturbations are introduced in the lower troposphere (below 1000 m) with white noise applied to the potential temperature to generate a random convective development. The sensible and latent heat fluxes are prescribed at 200 and 350 W m\(^{-2}\), respectively; the roughness length is fixed at 0.035 m; and a large-scale forcing is prescribed for vertical velocity (ascent of 0.1 m s\(^{-1}\) between 1000 and 5000 m AGL).

First, the model is run on a 3D domain for 165 min. This domain is 120 km \(\times\) 120 km in horizontal area with a vertical extension of 20 km (292 vertical levels). The horizontal grid spacing is 200 m and the vertical grid spacing is 50 m between 0 and 13000 m, then increases progressively above to reach 587 m at 20 km. The lateral boundary conditions are periodic in both horizontal directions. Deep convective systems develop over the domain. Their organization is partly patterned by the vertical wind shear with convective cells of different sizes, stretched in the southwest–northeast direction as shown in Fig. 1a. Then, we select a cumulonimbus cloud at \(t = 165\) min, and an inner domain with 50-m horizontal resolution, centered over this convective system, is run for 30 min in a one-way nesting configuration with the outer domain where the horizontal resolution is 200 m.
Both domains have a 50-m vertical resolution below 13,000 m. The inner domain, large enough to encompass the selected convective cloud, is 30 km × 30 km in horizontal area. This simulation can be considered as a LES, as the grid spacing is finer than 100 m.

This LES generates a well-developed cumulonimbus with cauliflower-shaped structures that seem very realistic. At $t = 175$ min (i.e., 10 min after the beginning of the simulation with the two nested domains), we can identify the same convective structures inside the selected area as in the outer domain but with finer features revealing jagged cloudy boundaries for the 50-m grid-spacing domain (Fig. 1b). Vertical cross sections through the convective system show that the cumulonimbus extends vertically over the troposphere from 1- to 12-km altitude. The width of the main updraft is a few kilometers (4–6 km), with maximum values of vertical wind speed more than 24 m s$^{-1}$, located around 8 km AGL (Fig. 1c). At the cloud top, strong subsidence is present, likely related to hydrodynamic and thermodynamic instabilities resulting from the inflow of drier environmental air into the cloud, which interacts with hydrometeors during the so-called entrainment process (e.g., de Rooy et al. 2013). Strong values of vorticity highlight the turbulent structures inside the cloud, the largest eddies exceeded 1 km in size (Fig. 1d). The subgrid TKE is weak (most of the values are between 0 and 16 m$^2$ s$^{-2}$) as the most energetic eddies are resolved (Fig. 2a), which is consistent with the LES regime. The resolved TKE, computed as $1/2 \sum_{\mathbf{u}} (\mathbf{u} \cdot \mathbf{u} - \langle \mathbf{u} \rangle^2)$ (where $\langle \cdot \rangle$ is the horizontal mean over the LES domain), represents more than 95% of the total TKE (resolved and subgrid TKE).

This LES is used as a reference simulation to characterize the turbulent fluxes of dynamical and thermodynamical variables of the model at $t = 175$ min, when it is considered to represent a mature deep convective cloud.

**Fig. 2.** Vertical cross sections along the line shown in Fig. 1b of the subgrid TKE (m$^2$ s$^{-2}$) of the (a) LES and computed from the LES by box filtering at different resolutions: (b) 500 m, (c) 1 km, and (d) 2 km. The cloud boundaries are represented as the sum of cloud and ice water mixing ratios above 0.001 g kg$^{-1}$ (black contours).
c. The coarse-grained turbulent fluxes

The reference LES fields are filtered to characterize the subgrid turbulent fluxes within the convective cloud at coarser horizontal resolutions (500 m, 1 km, and 2 km). For that purpose, the fields are coarse grained by using a box mean (respectively 10 × 10, 20 × 20, and 40 × 40 gridpoint averaging on the LES fields) as in Honnert et al. (2011), Shin and Hong (2013), and Moeng (2014).

The subgrid turbulent fluxes of reference at coarser Δx resolution are computed in each box of size Δx^2 as follows:

\[
\overline{u^i u^j}_{\text{ref}} = \frac{1}{2} \sum_{i=1}^{3} \sum_{j=1}^{3} \overline{u_i u_j}_{\text{ref}} \Delta x, \quad (7)
\]

\[
\overline{u^i S^j}_{\text{ref}} = \overline{(u_i - \overline{u_i}) (s - \overline{s})}_{\text{ref}} + \overline{f^{ui} f^{jx}}_{\text{ref}}, \quad (8)
\]

where \(u_i, u_j\) are the wind components (\(u, v, w\)), \(s = \theta_t\), or \(r_{np}\) (i.e., the liquid-ice potential temperature, or the total nonprecipitating water mixing ratio).

The term \(\overline{a}_{\Delta x} = 1/Q \sum_{q=1}^{Q} a_q\) represents the mean value of \(a\) over a box of size Δx^2 for Q points of the LES; \(f^{ui} f^{jx}\) and \(f^{ui} f^{jy}\) represent the subgrid turbulent fluxes at each point of the LES.

Then, the reference subgrid TKE at coarser Δx resolution is computed as

\[
\overline{\varepsilon}_{\text{ref}}^{\Delta x} = \frac{3}{2} \Delta x \sum_{i=1}^{3} \sum_{j=1}^{3} \overline{u_i u_j}_{\text{ref}} \Delta x, \quad (9)
\]

The turbulence can be generated by shear and friction, known as dynamical production, or by buoyancy, known as thermal production (prognostic equation of TKE given in appendix A).

The subgrid dynamical production (DP) of reference is expressed in terms of reference dynamical fluxes and wind gradients coarse grained at Δx resolution:

\[
\overline{\text{DP}}_{\text{ref}}^{\Delta x} = -\frac{3}{2} \sum_{i=1}^{3} \sum_{j=1}^{3} \overline{u_i u_j}_{\text{ref}} \frac{\partial \overline{\varepsilon}_{\text{ref}}^{\Delta x}}{\partial x_j}. \quad (10)
\]

The subgrid thermal production (TP) of reference is obtained by a combination of vertical fluxes of heat and total nonprecipitating water with the expressions \(E_{\theta}\) and \(E_{\text{moist}}\) coarse grained at Δx:

\[
\overline{\text{TP}}_{\text{ref}}^{\Delta x} = \frac{g}{\theta_{\text{v0}}} \left( \overline{E_{\theta} w}^{\Delta x}_{\text{ref}} + \overline{E_{\text{moist}} w}^{\Delta x}_{\text{npref}} \right), \quad (11)
\]

where \(g\) is the acceleration due to gravity, and \(\theta_{\text{v0}}\) is the virtual potential temperature of the reference state.

d. Turbulence parameterizations

Two turbulence schemes were evaluated inside the clouds at 2-km, 1-km, and 500-m grid resolution. The first one was the K-gradient formulation used in the current Meso-NH (Cuxart et al. 2000) in its 3D form (called Kgrad) [Eqs. (1)–(3)]. The second one considered a formulation of the vertical subgrid fluxes based on the product of horizontal resolved gradients as proposed by Moeng (2014) (called Hgrad).

In this second parameterization, the vertical thermodynamical turbulent fluxes are given by

\[
\overline{w s'} = C_{\Delta x} \left( \frac{\partial \overline{\theta}}{\partial x} \frac{\partial \overline{s}}{\partial x} + \frac{\partial \overline{w}}{\partial y} \frac{\partial \overline{s}}{\partial y} \right), \quad (12)
\]

with \(s = \theta_t\) or \(r_{np}\), and Moeng (2014) proposed \(C_{\Delta x} = 5(\Delta x^2/12)\).

See appendix C for the formulas using the LES fields filtered at Δx resolution.

3. Characterization of turbulence from the reference LES

a. Reference subgrid TKE

Figure 2 shows the vertical distribution of subgrid TKE within the convective system from the reference LES for the three coarser resolutions (2 km, 1 km, and 500 m) obtained by the box mean from Eq. (9). For the resolutions considered, the high values of TKE are localized inside the main updraft. The values reach over 60 m^2 s^-2 whereas they are weaker, ranging between 0 and 8 m^2 s^-2, on the cloud edges. The TKE increases when the horizontal grid spacing increases, which was to be expected from the subgrid/resolved partition. With 500-m horizontal grid spacing, the turbulence is partly resolved and the TKE must be weaker than with 2-km grid spacing where the turbulence is essentially subgrid. It is worth remembering that the current turbulence schemes are generally not able to represent larger subgrid TKE at coarser resolution inside clouds as shown by Adlerman and Droegemeier (2002) and Verrelle et al. (2015). The vertical distribution of subgrid TKE depends on the resolution, with large values also located in the upper part of the cloud at 2-km resolution whereas the strong values are found only under 7 km AGL for 500-m resolution. Moreover, smaller structures are discernible at 500-m resolution while the TKE field is more uniform at 2-km resolution.

b. Reference subgrid thermal and dynamical productions

Inside the cloud, the subgrid thermal production [from Eq. (11)] is greater than the subgrid dynamical production [from Eq. (10)] between the cloud base and 9 km AGL for 2- and 1-km resolutions, whereas at 500-m resolution the two source terms of turbulence have almost equal magnitudes (Fig. 3). Above 9 km AGL, the
dynamical production dominates, regardless of the resolution, which indicates that the turbulence production is mainly related to wind shear likely due to gravity waves in the upper part of the cloud.

The subgrid thermal production between 4 and 9 km AGL increases significantly with decreasing resolution. The thermal production steers the increase of subgrid TKE at coarse resolution. In return, the dynamical production is greater at fine resolution than at coarse resolution, due to the stronger gradients at fine resolution. The production of subgrid turbulence becomes more three-dimensional at fine resolution as a result of the increase of dynamical production with increasing resolution (Honnert and Masson 2014).

At LES resolution, the subgrid thermal production is seen to be very weak whereas there is a strong subgrid dynamical production.

In addition, the subgrid dynamical production of reference can be negative at 2- and 1-km resolutions, meaning that an energy transfer from the subgrid part to the resolved part could occur in agreement with Moeng (2014)’s findings.

The sum of dynamical and thermal productions is quite similar regardless of the resolution, revealing a balancing effect between the two productions. Yet the subgrid TKE increases substantially with decreasing resolution (as shown just above), which indicates that the other terms, such as dissipation or turbulent/advection transport (see appendix A), contribute to the variation of the subgrid TKE with the resolution.

c. Reference subgrid dynamical variances

Whatever the coarse-grained resolution, the reference variance for the vertical velocity [from Eq. (7)] is greater than the reference variances for the horizontal velocities as shown on the mean vertical profiles averaged inside the cloud in Fig. 4. Thus, the dynamical variances are anisotropic, but the gap between the vertical and horizontal variances narrows with increasing resolution. In other words, the turbulence becomes more isotropic when approaching the LES resolution. That is indeed the case for the subgrid dynamical variances of the LES, which have the same magnitude (Fig. 4a). The highest anisotropy is produced at the 2-km box filtering resolution with a ratio of up to 2 around 5-km height. This is in line with Lane and Sharman (2014) who showed a strong anisotropy of dynamical variances in the main convective region from the LES of a squall line, with a maximum obtained at 2.5-km scale considering the spectral properties of velocity perturbation. Moreover, in our case, at coarser resolutions the subgrid variances are greater, which is consistent with the subgrid TKE increase.

d. Reference subgrid thermodynamical fluxes

Vertical cross sections of heat vertical flux \(w^\theta_l\) [from Eq. (8)] (Figs. 5a–c) show that this flux is mainly negative in the lower part of the cloud up to about 5 km AGL for the three resolutions considered (2 km, 1 km, and 500 m). Above, the flux becomes positive in the core of the updraft, while the vertical gradient of \(\theta_l\) is essentially positive. According to Deardorff (1966), if the product between the flux and the vertical gradient (i.e., \(w^\theta_l \times (\partial \theta_l / \partial z)\)) is positive, then the vertical flux is countergradient. The countergradient areas are located in the updraft and at the cloud top on both sides of the updraft (Figs. 5d–f). The horizontal size of these areas increases with decreasing resolution. Hence, the updraft may well be the cause of a nonlocal turbulence generated by coherent structures, which leads to countergradient fluxes. These countergradient areas, well known for convective boundary layers (Deardorff 1966),
have not yet been studied extensively for deep convective clouds. One of the aims of this study was to reproduce these countergradient areas in deep convective clouds with the turbulence schemes at kilometer scales, knowing that the formulation of the turbulent diffusion based on $K$ gradient is unsuitable as the flux and the gradient are necessarily of opposite sign.

The liquid-ice water potential temperature (here noted $\theta_l$) is a conservative thermodynamic variable for nonprecipitating convection. This variable is conservative under liquid to vapor, vapor to ice, and liquid to ice phase changes (Tripoli and Cotton 1981). However, the presence of countergradient areas clearly indicates that this variable is not conservative everywhere, in particular in precipitating areas with hydrometeors like snow, graupel, or rain. To further investigate these countergradient features, $\overline{w_0\theta_l}$ can be separated into three components. From the definition of $\theta_l$ [Eq. (4)], the vertical flux of liquid potential temperature can be written in the following form:

$$\overline{w_0\theta_l} = \overline{w_0\theta'} - \frac{L_i}{C_{ph}} \frac{1}{\pi} \overline{w_0r_{np}} - \frac{L_s}{C_{ph}} \frac{1}{\pi} \overline{w_0r_{mp}}. \tag{13}$$

Figure 6 shows that the vertical flux of potential temperature is mostly positive inside the convective part of the cloud. Clearly, as expected, $\overline{w_0r_{np}}$ is positive and contributes to the negative part for $\overline{w_0\theta'}$ below 5 km AGL. The contribution of $\overline{w_0r_{mp}}$ is more complex as positive values alternate with negative values above 5 km AGL inside the convective updraft. In particular, negative values of $\overline{w_0r_{mp}}$ inside the core of the updraft and in the anvil part at cloud top, together with positive values of $\overline{w_0\theta'}$, reinforce the positive flux $\overline{w_0\theta'}$.

Vertical cross sections of total nonprecipitating water vertical flux ($\overline{w_0r_{np}}$) show that this flux is mainly positive.
from the base to the top inside the cloud, except in the anvil on both sides of the updraft. The magnitude of the flux increases with decreasing resolution (Fig. 7). Countergradient areas are also present, but they are smaller than those for the heat vertical flux (not shown). Moreover, they are reduced to the updraft base and cloud top on both sides of the updraft, a situation that is certainly associated with the cloud-top entrainment/detrainment processes.

To summarize, the coarse-graining applied to the LES has provided the reference fields for the different grid resolutions and has highlighted some differences according to the resolution. One of the main characteristics is the presence of countergradient fluxes, which is a challenge to reproduce with turbulent schemes at these kilometer resolutions.

4. Offline evaluation

In this part, we diagnostically evaluate the two different turbulence parameterizations presented above (Kgrad and Hgrad) using the LES filtered fields in the turbulence schemes formulations [using Eqs. (C1)–(C4) in appendix C]. These diagnosed turbulent fluxes are then compared with the turbulent fluxes of reference, coarse grained at the same resolution [using Eqs. (7) and (8)].

Figure 8 shows the vertical profiles of thermodynamical turbulent fluxes for the tested parameterizations compared to the reference fluxes at the 1-km resolution, while Fig. 9 presents vertical cross sections of the same parameterized fields. The K-gradient formulation does not represent the vertical heat flux properly inside the convective clouds: the mean reference flux is positive between 5 and 7.5 km AGL while the parameterized flux remains negative. In particular, the countergradient structures, localized at the cloud top around the updraft (not shown) for the reference flux (Fig. 5) are not reproduced with the parameterized flux (Fig. 9a). By construction, with the K-gradient formulation, the flux cannot be positive when the vertical gradient is positive. This is analogous to the well-known deficiency of K-gradient formulation in convective boundary layers.

In the same way, the vertical flux of total non-precipitating water is underestimated by the parameterized flux based on the K-gradient approach inside the convective updraft, in particular between 2 and 6 km AGL (Fig. 8b). Moreover, the countergradient structures, localized at the cloud top around the updraft (not shown) for the reference flux (Fig. 5) are not reproduced with the parameterized flux (Fig. 9a).
shown), associated with negative turbulent flux for the reference (Fig. 7b), are not reproduced with this scheme, leading mean fluxes to be overestimated above 10 km AGL (Fig. 9b). The parameterization also tends to create small patches of negative flux in the lower part of the cloud below 5 km AGL, which are absent from the reference (Fig. 7b). This becomes more marked with increasing resolution and coincides with countergradient areas of the reference (not shown).

The formulation based on horizontal gradients (Moeng 2014) gives a better representation of positive flux of $w\theta_l$ within the convective cloud, which is associated with countergradient structures. The change of sign of the vertical heat flux is well replicated. There is also a clear improvement in the flux at the cloud top (Figs. 8a,b). The vertical distribution of $w\theta_{np}$ is also improved with larger values (Fig. 8b). Moreover, there are no patches of negative values in the updraft (Fig. 9d). Sensitivity to the value of coefficient $C_D$ is strong: using a coefficient $C_D$ equal to $7D^2/12$ instead of the value $5D^2/12$ recommended by Moeng (2014) gives better results for both vertical fluxes (Figs. 8a,b).

The formulation of thermodynamical vertical fluxes derived from horizontal gradients as proposed by Moeng (2014) gives good agreement with the reference and consequently a larger thermal production of TKE than using the $K$-gradient formulation. In this section, only results for the 1-km resolution are shown but similar results have been obtained for the 2-km and 500-m resolutions.

As a first step, the diagnostic evaluation has shown the limitation of the classical $K$-gradient approach to represent the vertical thermodynamical fluxes inside convective clouds. A better representation can be obtained by using vertical fluxes based on products of horizontal gradients as proposed by Moeng (2014). Some
adaptations to the initial formulation are proposed: the $H$-gradient approach is applied only to thermodynamic fluxes as the main issue is related to a poor representation of countergradient areas associated with the thermal production of the subgrid TKE.

5. Online evaluation

The $H$-gradient approach is now evaluated online, by running the convective case at kilometer resolutions.

a. Characteristics of the simulations

The simulation domain (120 km $\times$ 120 km horizontally and 20 km vertically) and the initial conditions are the same as for the simulation at 200-m resolution. The duration is 4 h, the horizontal resolution is 1 km, and the vertical resolution is 100 m up to 13 km AGL and progressively stretched above. The control simulation (CTRL), using the CBR turbulence scheme everywhere in the 3D form with the Bougeault and Lacarrere (1989) (noted BL) mixing length, is evaluated in the clouds by referring to the LES fields coarse grained at the grid resolution. Then, different sensitivity tests are conducted in order to improve the turbulent fluxes. At CRM resolution, some studies have shown a deficit of subgrid TKE (Adlerman and Droegemeier 2002; Verrelle et al. 2015) inside convective systems. First, as turbulent mixing issues are often addressed by changing...
the mixing length (Hanley et al. 2015; Machado and Chaboureau 2015), sensitivity to the mixing length is studied by testing the Deardorff mixing length (DEAR) and by doubling the Bougeault and Lacarrere (1989) mixing length inside the clouds (2BL) in order to assess whether the mixing length, as the closure of the CBR turbulence scheme, can solve the problems. Second, the $H$-gradient approach (Hgrad) with the adaptations is tested. It is applied only to the vertical thermodynamical fluxes above the boundary layer (an average height of 1000 m is used), the $K$-gradient formulation remaining for other dynamical fluxes and for vertical thermodynamical fluxes in the PBL. Indeed, in this idealized configuration using a horizontally homogeneous basic state, the $H$-gradient approach based only on horizontal gradients, cannot overcome the convective inhibition in the lower troposphere, so the use of vertical gradients for turbulent heat and moisture fluxes is necessary to couple the atmospheric flow to the surface in order to destabilize the PBL through the heating from the surface and eventually trigger the convective activity. Then, sensitivity to the coefficient $C_D$ is studied. Additionally, the new scheme is also tested at other horizontal resolutions (2 km and 500 m) and compared with the $K$-gradient simulations at the same resolutions. The time step is the same for all simulations and small (0.2 s) as the temporal integration scheme for horizontal and vertical gradients of the turbulent scheme is chosen explicit. Table 1 summarizes the characteristics of all the runs.

### b. CTRL run

For the CTRL run, a convective boundary layer forms during the first hour of simulation (not shown). Cumulus develop after 45 min, from the top of the boundary layer,
located at about 1 km AGL. After 65 min, the tops of the most developed clouds reach 5 km AGL. At about 70 min, the deep convective clouds reach above the tropopause located at about 10 km AGL. This spinup period is followed by a drop of the convective activity (between 80 and 110 min) before a second period of deep convection that initiates a more stationary phase. In the remainder of the paper, vertical profiles from the different runs are compared during this stationary phase (at $t = 175$ min) with the LES profiles obtained in the previous part and coarse grained at the kilometer resolutions. For this comparison, the profiles are averaged over clouds (i.e., $r_c + r_r \approx 10^{-3} \text{g kg}^{-1}$) at the mature stage (i.e., reaching 8 km AGL) in order to select well-developed convective clouds similar to the LES ones.

At intermediate range (i.e., 500 m–2 km), the Meso-NH model generates vertical velocity that is too strong inside convective clouds, together with subgrid TKE that is too weak (Verrelle et al. 2015). This is confirmed in Figs. 10b and 10c for the CTRL simulation at 1-km horizontal resolution compared with the reference profiles obtained from the LES. Dynamical production seems correct (Fig. 10d) but the thermal production is too weak compared with the LES (Figs. 10e,f). Moreover, the positive area between 5 and 7.5 km AGL for the vertical turbulent heat flux, associated with the countergradient structures, is not represented. Different tests are now addressed in order to improve the turbulence scheme inside convective clouds.

c. Sensitivity to the mixing length

First, the impact of the mixing length in the clouds is assessed, as a modification of the mixing length could be expected to resolve some of the issues. The mixing length found with Deardorff formulation (noted DEAR) is larger than the BL mixing length below 9 km AGL, and in particular below 5 km AGL (Fig. 10a). As a result, the TKE increases slightly in the lower part of the clouds (Fig. 10c) due to a slight increase of both thermal and dynamical turbulent fluxes, but the thermodynamic fluxes remain largely underestimated (Figs. 10e,f).

When the value of the BL mixing length in the clouds is doubled (noted 2BL), its value increases more than twice compared with the CTRL run as there is feedback between the mixing length and the TKE (Fig. 10, in orange). The averaged TKE in the clouds increases by up to a factor of 5 everywhere, but with overestimated values above 6 km AGL. The dynamical production is much larger but too strong compared with the reference LES. Here $\omega^r_{np}$ is improved with larger values but this flux is also too strong above 6 km AGL. Even though the heat flux values are too negative, $\omega^r_{th}$ is also improved below 5 km AGL, but it remains negative above 5 km AGL and does not represent the positive flux associated with countergradient structures. Moreover, the mean vertical velocity is even stronger inside the convective clouds (Fig. 10b).

Although the magnitude of turbulent fluxes and TKE increases with increasing mixing length, it is not possible to represent the countergradient areas and the vertical velocity remains overestimated when only the mixing length is modified. Modifying the mixing length does not appear to be the way to improve the representation of in-cloud eddies.

d. Evaluation of the H-gradient approach

The $H$-gradient approach as presented above, and using $C_{Hr}$ equal to $3\Delta x^2/12$, reproduces the bipolar
structure (negative values below 5 km AGL and positive values between 5 and 7.5 km AGL) of the vertical turbulent heat flux with only a slight underestimation (Fig. 11, in red). Unlike the $K$-gradient formulation, the vertical heat flux based on the product of horizontal gradients as proposed by Moeng (2014) is able to represent the countergradient structures inside the convective clouds. This confirms the results obtained with the offline evaluation. Moreover, the turbulent vertical flux of total nonprecipitating water is remarkably well represented. There is a slight overestimation in comparison with the CTRL run (Figs. 11d and 10d). The correct shapes of the vertical profiles of the dynamical production of subgrid TKE reinforce the decision to target the improvement of the thermal production of the subgrid TKE as a priority. The new scheme reproduces the characteristic shape of the vertical profile of TKE in comparison with the reference LES, even though its magnitude remains underestimated. This is a clear improvement compared with the CTRL run (Figs. 11c and 10c).

A sensitivity study to the coefficient $C_{Dx}$ was also conducted. The previous results were obtained with a coefficient $C_{Dx}$ equal to $3\Delta x^2/12$. With a larger coefficient, $C_{Dx}$ equal to $7\Delta x^2/12$, corresponding to the best value of the offline evaluation, the magnitudes of the thermal fluxes are too large compared with the reference LES. There is positive feedback with a larger TKE and a larger mixing length. However, the vertical velocity still remains too strong compared with the reference LES and is quite similar that of the HGRAD_C3 run except above 6 km AGL, where there is a very slight reduction (Fig. 11b). The differences of coefficient between offline and online evaluations are probably due to a dynamical interaction between parameterized fluxes and resolved variables during the online runs. We also tested two other values: $5\Delta x^2/12$ and $4\Delta x^2/12$ (Fig. 11). With $5\Delta x^2/12$, both fluxes are too strong, whereas with $4\Delta x^2/12$, $\overline{wT}$ is well represented but $\overline{wr_{np}}$ is still slightly overestimated. One hypothesis is that this coefficient could be linked to the scales of the turbulent structures, which differ between the two thermodynamical fluxes. Therefore, as a perspective, two distinct coefficients could be necessary for the fluxes.

This new scheme was also tested with 500-m and 2-km horizontal grid spacing (Fig. 12). At these horizontal
resolutions, using the same coefficients $C_A$ ($3\Delta x^2/12$ or $4\Delta x^2/12$), the new scheme, unlike the $K$-gradient simulations, also reproduces the countergradient areas (Fig. 12). Moreover, it represents the magnitude of the vertical fluxes and the TKE better (shown only for the vertical fluxes), even though this magnitude is underestimated for the vertical heat flux and the TKE at 2-km resolution. Also, vertical velocity is reduced, which is more realistic compared with the reference LES (not shown). At 500-m resolution, it should also be noted that the $K$-gradient run (CTRL_500m) performs better for the vertical heat flux below 5 km AGL than at 1- and 2-km resolutions (Figs. 12a, b, and 10e), as is also the case for $\overline{w'r'}_w$ (Figs. 12c, d, and 10f) and the subgrid TKE (not shown).

We obtained quite similar results when applying the $H$-gradient approach inside the clouds only, above the PBL, instead of throughout the free troposphere, as the subgrid TKE is mainly increased inside the clouds (not shown). Furthermore, cloud cover and precipitation are slightly increased compared to the CTRL runs when the Hgrad approach is used, as the subgrid mixing is enhanced. This is an issue for a further study, in relation with the future introduction of subgrid condensation scheme.

6. Summary and conclusions

In this study, we have assessed the limitations, inside convective clouds, of the commonly used eddy-diffusivity scheme that relates subgrid vertical turbulent fluxes to vertical gradients of the resolved variables.

First, an idealized LES of deep convection with 50-m grid spacing was used as a reference simulation to characterize the turbulence at coarser horizontal resolutions (500 m, 1 km, and 2 km). Turbulent fluxes of reference were obtained by using box filtering. The subgrid TKE increased with coarser resolutions inside convective clouds, as expected. It appeared clearly that the subgrid thermal production was larger than the subgrid dynamical production from the cloud base to 9 km AGL for 1-km and 2-km horizontal resolutions, whereas at 500-m horizontal resolution both sources of TKE had similar magnitudes. Above 9 km AGL, the dynamical production dominated regardless of the resolution. The thermal production controlled the increase.
of TKE at coarser resolution. In contrast, the subgrid dynamical production was larger at higher resolution, making the turbulence more three dimensional. The dynamical variances were anisotropic but the gap between vertical and horizontal variances diminished with increasing resolution. The thermodynamical turbulent fluxes had countergradient features, indicative of non-local turbulence, localized inside the main updraft and near the cloud top for the vertical heat flux, and only near the cloud top for the vertical moisture flux.

Second, the diagnostic evaluation of the current turbulence scheme, with respect to LES coarse-grained fields, showed that the thermal production was largely underestimated inside convective clouds. Moreover, the countergradient features of $w^0T^1$ and $w^0r^1_{np}$ were not reproduced as the current $K$-gradient formulation is not appropriate. An alternative formulation, parameterizing the vertical thermodynamical fluxes on products of horizontal gradients as proposed by Moeng (2014), gave a better representation of thermal production of turbulence with countergradient features inside convective clouds. Compared with Moeng (2014), the best fit of the vertical heat and moisture fluxes was obtained with $C_{Dx}$ equal to $7\Delta x^2/12$.

Third, an online evaluation conducted with the same idealized framework using simulations with 500-m, 1-km, and 2-km horizontal grid spacing, having reached a near-equilibrium deep convection regime after 120 min, behaved in a similar fashion to the LES. Although the magnitude of turbulent fluxes and TKE increased, it was not possible to represent the countergradient areas and the vertical velocity remained too strong when only the

![Figure 12](http://journals.ametsoc.org/mwr/article-pdf/145/10/3947/4776661/mwr-d-16-0404_1.pdf)
The horizontal gradient approach now needs to be evaluated on real cases of deep convection as the online evaluation has only been conducted on an idealized case. Moreover, a specificity is that the new scheme is only applied above the boundary layer. This will impose to diagnose the boundary layer height to apply the $H$-gradient only above this level. Further work will evaluate the interaction of this scheme with other parameterizations: a subgrid condensation scheme that could remain necessary at kilometer scale for deep clouds and a shallow convection scheme, which also deals with nonlocal turbulence but in the boundary layer through the mass flux approach.

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APPENDIX A

TKE Evolution Equation and Mixing Lengths

The prognostic equation for the turbulent kinetic energy $e$ is

$$\frac{\partial e}{\partial t} = -\frac{1}{\rho_{d0}} \frac{\partial (\rho_{d0} e u_i u_j)}{\partial x_j} - \frac{g}{\theta_{v0}} \frac{\partial \theta_{v}}{\partial x_j} + \frac{1}{\rho_{d0}} \frac{\partial}{\partial x_j} \left[ C e^{1/2} \frac{\partial}{\partial x_j} \right] - C e^{3/2} L, \tag{A1}$$

where $e$ evolves from advection (first term on right-hand side), dynamical production (second term), thermal production (third term), turbulent transport (fourth term), and dissipation (fifth term). Here $u_i$ represents the $i$th component of the velocities; $\theta_{v}$ is the virtual temperature; $\rho_{d0}$ and $\theta_{v0}$ are the density and virtual temperature profiles of the reference state, respectively; $g$ is the acceleration due to gravity; $L$ is the mixing length; and $C_{2m}$ and $C_e$ are closure constants. Bars denote means and primes denote turbulent components.

The term $L$ can be

- the Deardorff mixing length defined as

$$L = \text{Min} \left( \Delta x \Delta y \Delta z \right)^{1/3}, \tag{A2}$$

with $N$ the Brunt–Väisälä frequency.
the Bougeault–Lacarrre mixing length defined as
\[ L = \left[ \frac{(l_{\text{up}})^{2/3} + (l_{\text{down}})^{2/3}}{2} \right]^{-3/2}, \] (A3)

with the distances \( l_{\text{up}} \) and \( l_{\text{down}} \) as follows:
\[ \int_{z}^{z+l_{\text{up}}} \frac{g}{\theta_{x0}} [\theta_v(z') - \theta_v(z)] \, dz' = e(z), \]
\[ \int_{z-l_{\text{down}}}^{z} \frac{g}{\theta_{x0}} [\theta_v(z) - \theta_v(z')] \, dz' = e(z), \]
where \( l_{\text{down}} \leq z \). (A4)

**APPENDIX B**

**Initial Conditions of the Simulations**

The analytical expressions of the environmental potential temperature and relative humidity, used in initial conditions of the simulations, are defined as in Weisman and Klemp (1984). They are given by
\[ \bar{\theta}(z) = \begin{cases} \theta_0 + (\theta_{tr} - \theta_0) \left( \frac{z}{z_{tr}} \right)^{5/4}, & z \leq z_{tr} \\ \theta_{tr} \exp \left[ \frac{g}{C_p T_{tr}} (z - z_{tr}) \right], & z > z_{tr} \end{cases} \]
\[ \bar{T}(z) = \begin{cases} 1 - \frac{3}{4} \left( \frac{z}{z_{tr}} \right)^{5/4}, & z \leq z_{tr} \\ 0.25, & z > z_{tr} \end{cases} \] (B1, B2)

where \( \theta_0 \) is the surface potential temperature; and \( \theta_{tr}, T_{tr} \), represent the potential temperature and actual temperature, respectively, at the tropopause. In our study, \( z_{tr} \) the height at the tropopause is 10 km.

**APPENDIX C**

**Parameterized Fluxes Using the LES Fields Filtered at \( \Delta x \)**

For the \( K \)-gradient approach,
\[ \overline{w^r_{\ell}} \equiv \left[ \frac{1}{3} L \sqrt{\varphi_{\text{ref}}} \frac{\partial \varphi}{\partial z} \right] \phi_{\ell} \], (C1)
\[ \overline{w^r_{np}} \equiv \left[ \frac{1}{3} L \sqrt{\varphi_{\text{ref}}} \frac{\partial \varphi_{np}}{\partial z} \right] \psi_{\ell} \], (C2)

where \( L \) is the Bougeault–Lacarrère mixing length computed from LES fields coarse grained at \( \Delta x \).

For the \( H \)-gradient approach,
\[ \overline{w^r_{\ell}^h} \equiv C_D \left( \frac{\partial \varphi_{\ell}^h}{\partial x} \frac{\partial \varphi_{\ell}^h}{\partial y} + \frac{\partial \varphi_{\ell}^h}{\partial y} \frac{\partial \varphi_{\ell}^h}{\partial y} \right), \]
\[ \overline{w^r_{np}^h} \equiv C_D \left( \frac{\partial \varphi_{np}^h}{\partial x} \frac{\partial \varphi_{np}^h}{\partial y} + \frac{\partial \varphi_{np}^h}{\partial y} \frac{\partial \varphi_{np}^h}{\partial y} \right). \]

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