Linearization of a Simple Moist Convection Scheme for Large-Scale NWP Models

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ABSTRACT
A simple Kuo-type convection scheme with an improved closure based on moist enthalpy accession (Kuo symmetric) has been linearized for the tangent-linear (TL) and adjoint (AD) versions of the Global Environmental Multiscale (GEM) model. The nonlinear scheme exhibits a reasonable behavior in terms of heating and moistening rates when evaluated in stand-alone mode over a set of deep convective profiles. A preliminary evaluation of a straightforward linearization in the global TL model has revealed the existence of noise that leads to an unacceptable solution after 12 h of integration. By neglecting several terms in the linearization (detrainment rate and cloud properties), the temporal evolution of humidity analysis increments is improved by including this simplified linearized convection scheme in the TL model. The behavior of the linearized scheme has also been compared favorably to the linearized version of the European Centre for Medium-Range WeatherForecasts (ECMWF) mass-flux convection scheme. When examining the validity of the TL approximation for surface precipitation, it appears that linearization errors are large for both stratiform and convective rainfall (rms errors are about twice the mean absolute perturbed precipitation). These errors are not reduced when considering accumulated rain rates instead of instantaneous quantities. However, the occurrence of “on–off” processes is reduced by a temporal integration of rain. This could make the variational assimilation of accumulated precipitation rates easier. Finally, errors coming from internal nonlinearities are slightly larger than those produced by discontinuities. This confirms the interest for improving the linearity of nonlinear convection schemes for applications in variational contexts.

1. Introduction
The improvement of atmospheric humidity analysis remains a challenging issue in numerical weather prediction (NWP). Better initial humidity fields could be beneficial to the prediction of important weather elements such as precipitation (including severe storms), cloud cover, and near-surface temperature at both meso- and global scales. Recent three-dimensional and four-dimensional variational data assimilation (3D/4DVAR) systems offer the possibility of assimilating new types of satellite and radar data containing information about the atmospheric water cycle (water vapor, cloud/rainwater, and ice contents). Moreover, various space agencies are defining new missions in order to improve the retrieval of these quantities. Therefore it is important to start examining the usefulness of such data for assimilation in NWP models. A central aspect of variational data assimilation schemes is the definition of observation operators that map the model state onto the observation space. When considering quantities like precipitation or cloud water content, the observation operator involves physical parameterization schemes describing moist processes. This is particularly demanding on physical parameterizations because, in a variational framework, the tangent-linear (TL) and adjoint (AD) versions are also required to solve efficiently the minimization problem. The linearization of physical processes for 4DVAR (and also other related applications such as singular vectors and key analysis errors) started 10 yr ago with various levels of success. It has been shown in many studies that modifications (simplifications, smoothing, filtering) need to be performed in order to get useful results from linearized physics (Zupanski 1993; Zupanski and Mesinger 1995; Tsuyuki 1996ab; Sun and Crook 1997; Mahfouf 1999; Janisková et al. 1999; Errico and Raeder 1999; Wu et al. 2000; Treadon et al. 2003). Indeed, all physical processes are characterized by discontinuities and nonlinearities that can significantly reduce the validity range of their linearized versions.

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When considering moist processes, the linearization of large-scale condensation schemes has been done in a rather straightforward manner. So far, most schemes are based on a simple approach in which supersaturations are removed under the constraint of moist static energy conservation. They improve the validity of the TL approximation for humidity in midlatitudes (Mahfouf 1999) and influence significantly the growth of extratropical singular vectors (Coutinho et al. 2004). The situation is less clear for moist convection. A variety of schemes have been linearized for mesoscale and global-scale numerical models, from simple schemes [Kuo-type: Zou et al. (1993); Betts–Miller: Zupanski (1993)] to more complex mass-flux schemes (Treadon 1997; Errico and Raeder 1999; Fillion and Bélair 2004). One important question to ask when designing a linearized moist convection scheme is, for which application(s) will it be used? As explained in the following, the answer to this question has consequences on the choices to be made a priori and a posteriori. Since there is a need to parameterize moist convection when the model grid scale is larger than about 1 km (Bryan et al. 2003), the question of linearized moist convection is relevant for both mesoscale and global-scale 4DVAR applications. Moreover, in its incremental formulation, the 4DVAR problem is usually solved with an NWP model having a coarser grid than that of the nonlinear model.

The stochastic nature of moist convection can make its linearization questionable, but there have been a number of encouraging results on the use of linearized schemes in the NWP context. Mahfouf (1999) showed that the temporal evolution of humidity analysis increments in the Tropics was improved when including a simplified linearized mass-flux convection scheme in the TL version of the European Centre for Medium-Range Weather Forecasts (ECMWF) model. Mahfouf and Rabier (2000) showed that tropical wind forecast errors as well as precipitation spin down were reduced when accounting for diabatic processes in the inner loop of 4DVAR. Barkmeijer et al. (2001) demonstrated the importance of linearized diabatic processes (moist convection being the dominant one) to produce realistic tropical singular vectors associated with the development of tropical cyclones. The Jacobians of the ECMWF mass-flux scheme have been used successfully for precipitation retrievals within a 1DVAR context by Marécal and Mahfouf (2000). However, the linearization of mass-flux convection schemes is tedious because of the complexity of the nonlinear (NL) versions, and it can produce expensive numerical codes that are not always useful, as experienced by Errico and Raeder (1999) with the Hack scheme. Marécal and Mahfouf (2003) compared the ECMWF mass-flux scheme and the Betts–Miller adjustment scheme for the assimilation of a single observation of precipitation in the ECMWF 4DVAR. The Betts–Miller scheme being more linear than the mass-flux scheme, a better convergence of the incremental formulation was achieved with this scheme. The benefit of linearizing a more realistic scheme (i.e., one that describes cloud microphysical processes) lies in the possibility of assimilating cloudy and rainy satellite radiances as well as radar reflectivities. This has been demonstrated in a recent feasibility study by Moreau et al. (2004). Indeed simple schemes that bypass the cloud stage cannot provide realistic hydrometeor profiles (rainwater and cloud water contents) that are required to model radiances and reflectivities. Simple convection schemes can only be used for the assimilation of surface precipitation observations (Zou and Kuo 1996; Zupanski et al. 2002).

Despite the interest of rather complex TL/AD convection schemes for the assimilation of cloudy radiances and reflectivities, the present study focuses on the linearization of a simple scheme for a number of reasons. First, Mahfouf (1999) and Janisková et al. (1999) have shown that linearized parameterization schemes can be much simpler than their NL versions and still be beneficial. The breakdown of the TL approximation in some situations will lead to large errors whatever the realism of a parameterization scheme (principally when on-off switches are activated). For the time propagation of a cost-function gradient, a more realistic scheme may not produce better results than a simpler one if the breakdown of the TL approximation is dominated by on-off switches or by internal nonlinearities of the complex scheme. Recent studies on moist singular vectors (Coutinho et al. 2004) have demonstrated the dominance of stratiform precipitation in modulating the scale and amplitudes of these growing modes over midlatitudes in winter. In that context, the need for a complex linearized convection scheme is not justified. Starting from the controversial conditional instability of the second kind (CISK) theory of Charney and Eliassen (1964), there have been many theoretical and numerical studies on the cooperative role of cumulus and large-scale circulation on the development of tropical cyclones with rather crude descriptions of moist convection (Smith 1997). There are some common ingredients to any cumulus parameterization that can be easily examined and understood with simple schemes. For example, all linearized schemes use the so-called traditional linearization approach in which triggering is defined by the trajectory (Zou 1997). Errors in the TL approximation can therefore be split into a component
coming from on-off switches and a component due to internal nonlinearities. The linearization of a simple convection scheme is also a useful starting point to better understand the links between precipitation (or heating rates) and atmospheric precursors (in terms of dynamical and thermodynamical initial conditions). This is a crucial point for the relevance of performing variational assimilation of observations on condensed water. The study of Errico et al. (2003) is currently the only one that has used the adjoint technique to examine such links.

A simple Kuo-type convection scheme has been linearized for various applications in the Global Environmental Multiscale (GEM) model (Côté et al. 1998a,b) that is used operationally at the Canadian Meteorological Centre (CMC). The NL scheme is described in section 2 and its behavior briefly evaluated. The linearized version of this scheme is then derived in section 3 and various simplifications are justified. The simplicity of the NL scheme has allowed a precise identification of the sources of nonlinearities and their removal. The methodology proposed for simplifications leads to TL/AD schemes that are inexpensive and suitable to any NL moist convection scheme. The validity of the TL approximation is evaluated in section 4 by examining the temporal evolution of finite-size perturbations over a 24-h period. Conclusions are given in section 5 along with plans for future experiments that will be undertaken regarding the assimilation of precipitation with the TL/AD versions of this moist convection scheme.

2. The nonlinear convection scheme

a. Description

The nonlinear convection scheme has been developed by C. Girard (Malhotra et al. 1998) following ideas from Bougeault (1985) in which the description of a Kuo-type scheme is improved by the knowledge gained from mass-flux schemes. The heating and moistening rates produced by cumulus convection are expressed as follows:

\[
\begin{align*}
\frac{\partial T}{\partial t}_{\text{conv}} &= \frac{1}{C_p} \left[ -\frac{\partial s}{\partial t}_{\text{LS}} + K(s_c - s) \right], \\
\frac{\partial q}{\partial t}_{\text{conv}} &= \left[ -\frac{\partial q}{\partial t}_{\text{LS}} + K(q_c - q) \right],
\end{align*}
\]

(1) (2)

where \( s \) is the dry static energy and \( q \) is the specific humidity (the corresponding quantities with subscript “c” are in-cloud values). The subscript LS stands for “large scale,” corresponding to dynamical tendencies, plus tendencies from radiation and vertical diffusion. The first terms can be interpreted as the cloud-induced subsidence when vertical advection dominates the large-scale dynamics (as is often the case in the Tropics). Indeed, in convective regions the in-cloud mass flux \( \omega \) almost compensates the large-scale velocity \( \omega \) (i.e., \( \omega = \omega \)), thus,

\[-\left( \frac{\partial s}{\partial t} \right)_{\text{LS}} = \omega \frac{\partial s}{\partial p}.
\]

(3)

The same approximation also holds for \( q \). A major simplification with respect to the scheme described by Bougeault (1985) is that it does not require an additional closure equation for the mass flux \( \omega \). This approximation has also been suggested by Emanuel (1991) who recognized that despite a great deal of effort expended to specify the mass flux in cumulus clouds for parameterization schemes its exact specification is somewhat superfluous. Indeed, when the convection scheme is turned on (i.e., the atmosphere is unstable to parcel ascent) the mass flux will be determined by the large-scale forcing under a quasi-equilibrium assumption that is supported by observations (Arakawa and Schubert 1974). This scheme is called “Kuo symmetric” since the same formulation is used for heating and moistening rates, whereas in a Kuo “standard” scheme the large-scale tendencies do not appear in the convective heating rate.

The terms \( K(s_c - s) \) and \( K(q_c - q) \) respectively in (1) and (2) describe the detrainment of cloudy air characterized by \( (s_c, q_c) \) with the environment. As in Kuo (1965) and Bougeault (1985), the scheme uses a relaxation toward a single cloud profile with a time constant \((1/K)\) independent of altitude.

Conservation of moist static energy \( h = s + Lq \) over the convective column (defined between cloud base \( p_b \) and cloud top \( p_t \)),

\[
\int_{p_b}^{p_t} \left( \frac{\partial h}{\partial t} \right)_{\text{conv}} dp = 0,
\]

(4)

allows the estimation of \( K \):

\[
K = \frac{\int_{p_b}^{p_t} \left( \frac{\partial h}{\partial t} \right)_{\text{LS}} dp}{\int_{p_b}^{p_t} (h_c - h) dp}.
\]

(5)

In particular situations where the large-scale forcing is dominated by moisture convergence \( M_r \), the dry static energy and the humidity changes are approximated as

\[
\left( \frac{\partial s}{\partial t} \right)_{\text{LS}} \approx 0 \quad \text{and} \quad \left( \frac{\partial q}{\partial t} \right)_{\text{LS}} \approx M_r.
\]

(6)
Equations (1) and (2) reduce to the initial Kuo (1965) formulation. As in Bougeault (1985), the cloud profiles $T_c$ and $q_c$ are determined from a bulk convective updraft starting at the lowest model level. The moist adiabatic construction includes the entrainment of environmental air with a rate of $5 \times 10^{-5} \text{s}^{-1}$. Convective regions are identified where the updraft has a positive buoyancy measured by the excess of virtual temperature $T_w - T_v$. The scheme is activated in regions where a positive accession of moist static energy at the lowest level (trigger function) is diagnosed. The vertical layers where convection modifies the environment are characterized by

$$\left( \frac{\partial h}{\partial t} \right)_{LS} > 0 \quad \text{and} \quad \left( \frac{\partial s}{\partial t} \right)_{LS} < 0,$$

which indicates the existence of both large-scale moistening and cooling. The role of deep convection can be seen as counteracting the large-scale destabilization of the atmosphere by redistributing heat and moisture on the vertical. Simplicity is the main advantage of this scheme together with the lack of important tuning parameters [such as the moistening parameter $b$ in Kuo (1974) and Anthes (1977)].

Surface rainfall rate is simply diagnosed as

$$R_c = \frac{C_p}{L} \int_{p_b}^{p_1} \left( \frac{\partial T}{\partial t} \right)_{\text{conv}} dp \quad \frac{\text{mm day}^{-1}}{\text{g}},$$

Even though it describes the main sink of moisture produced by deep moist convection on the large scale, a number of additional physical processes are not accounted for, such as rainfall evaporation, moist downdrafts, or detrainment of liquid/ice water. This scheme is currently used at CMC in the Ensemble Prediction System and for climate integrations in research mode, with a more sophisticated description of rainfall processes including microphysics (Mailhot et al. 1998). For applications in the TL/AD versions of the GEM model, the time scales of interest range from 6 to 48 h and spatial scales between 100 and 300 km. For smaller horizontal scales alternative convection schemes like the Kain–Fritsch scheme could be required (Fillion and Bélair 2004).

Kuo-type schemes have been strongly criticized by Raymond and Emanuel (1993), the main reason being that moist convection is caused by a conditional instability and not by a large-scale water supply. The use of both large-scale tendencies for $s$ and $q$ (instead of $q$ only in the original Kuo schemes) allows this scheme to respond to a destabilization of the atmosphere by the large-scale environment. For example, this scheme can replicate the solution of a radiative-convective equilibrium (when a large-scale moisture supply is present). The criticism is essentially about the closure of the scheme (using moisture convergence) rather than about the rest of its design. Other weaknesses, such as not accounting for downdrafts or momentum transport, can also be missing features of other convection schemes. Similarly, a number of mass-flux schemes have used a moisture convergence closure (Bougeault 1985; Tiedtke 1989) even though CAPE closures are becoming more popular. The use of large-scale moisture tendencies in the closure makes this scheme “CISK-able” (Emanuel 1991). It could be of interest to examine in a linearized NWP model the relevance of this mechanism since the definition of the coupling between the large scale and convection can be altered. Further details are given in the conclusions.

### b. Evaluation in stand-alone mode

A number of atmospheric profiles taken from a 12-h forecast of a T42 319L60 (horizontal grid mesh about 60 km and 60 vertical levels) version of the ECMWF model (CY23R4; information on the features of ECMWF model cycles can be found online at http://www.ecmwf.int/products/data/technical/model_id/index.html) starting from the operational analysis at 1200 UTC 15 January 2002 have been used to evaluate the behavior of the Kuo symmetric scheme and to compare it to the ECMWF mass-flux scheme from Tiedtke (1989) and Gregory et al. (2000). An oceanic area has been chosen along the ITCZ ($0^\circ$–$10^\circ$S, $70^\circ$–$80^\circ$E) containing 297 profiles where deep convection is active in the Tiedtke’s scheme. Each scheme is driven by atmospheric profiles of $T$ and $q$ as well as by tendencies from the dynamics, radiation, and vertical diffusion. These profiles correspond to equilibrium values associated with the mass-flux scheme, but can provide useful information on the behavior of the Kuo symmetric scheme.

The mean profiles of large-scale tendencies show (Fig. 1a) a thermal destabilization favorable to the onset of moist convection with a large warming in the boundary layer up to 10 K day$^{-1}$ and an important cooling aloft reaching a minimum of $-12$ K day$^{-1}$ around level 40 (around 500 hPa). The humidity tendency is positive over the whole profile, with the largest values in the boundary layer (combined effects of surface evaporation and low-level moisture convergence).

The mean heating and moistening rates produced by the Kuo symmetric and the mass-flux schemes are compared in Figs. 1b and 1c. The profiles produced by the two schemes have similar vertical structures in the cloud layer even though values can differ by several
Kelvin per day at specific levels. Above and below the convective cloud, the Kuo symmetric does not produce any heating or cooling. The detrainment of cloud liquid water in the environment produces a significant cooling at cloud top with the mass-flux scheme. Similarly, large cooling and drying take place in the boundary layer. Figure 1d, in which the effect of downdrafts has been switched off in the mass-flux scheme, indicates that they contribute significantly to the convective tendencies in the boundary layer. The strong imbalance noticed in the subcloud layer between adiabatic and Kuo symmetric tendencies is a consequence of a stand-alone evaluation that uses input profiles produced by a model having a different convection scheme. The use of a moist static energy–conserving moist adiabat could mimic the effects of downdrafts below cloud base (Raymond and F...
Emanuel 1993). This improvement will be considered in the future.

As outlined by Emanuel (1991), the agreement in terms of convective tendencies with observations is not necessarily a pertinent criteria to evaluate convection schemes since the T and q profiles will change following the sum of adiabatic and convective tendencies that have roughly the same order of magnitude but with opposite signs. Therefore, apparently small errors in convective tendencies can have dramatic consequences on total tendencies. However, one must keep in mind that the main focus of this study is on the behavior of linearized schemes. Given the much greater complexity of Tiedtke’s mass-flux scheme, results from the Kuo symmetric scheme look realistic enough for applications in TL/AD models.

Wagneur (1991) examined the behavior of the Kuo symmetric scheme for deep convective profiles from the Global Atmospheric Research Programme (GARP) Atlantic Tropical Experiment (GATE) and noticed that this scheme was superior to five other Kuo-type schemes.

c. Evaluation in a global model

A preliminary evaluation of the Kuo symmetric scheme within a global version of the GEM model has been performed by examining the nonlinear tendencies (i.e., trajectory) produced in the TL model. Indeed, the linearization of the TL model is performed around an evolving trajectory from the NL model. Model prognostic variables (temperature, wind components, specific humidity, and surface pressure) are stored at every time step of an NL integration. From these variables, all other quantities needed for linearizing the TL model are recomputed. In particular, at each time step of the TL integration, the NL convective tendencies are computed since they are required to compute the linearized convective tendencies and are also used as input to the TL large-scale precipitation scheme [fractional stepping approach as explained in Fillion and Mahfouf (2000)].

Given this particular role played by the NL convection scheme in the TL model, it is important to evaluate its behavior in that context.

A 24-h integration of the TL version of GEM (resolution 120 × 60 points on the horizontal with 28 vertical levels) using a simplified vertical diffusion scheme (Laroche et al. 2002), a simple large-scale condensation scheme (Haltiner and Williams 1980), an orographic drag scheme (Zadra et al. 2003), and the Kuo symmetric scheme has been performed. The trajectory (around which the linearization is done) has been produced from an NL integration of GEM at the same resolution but with a comprehensive package of physical parameterization schemes (including radiation, cloud, and land surface processes). In particular, the NL GEM model has a convection scheme, based on the original Kuo (1974) and Anthes (1977) versions, that is coupled to a cloud scheme from Sundqvist (1978) for the generation of stratiform clouds and precipitation. The semi-Lagrangian temporal integration scheme allows a time step of 1 h for the chosen horizontal resolution. Initial conditions have been taken from the CMC global analysis of 7 February 2001 at 1200 UTC. The initial conditions of the TL model that are not relevant for the current evaluation are explained in the next section.

Figure 2 shows a zonal mean of total precipitation (24-h accumulation) for the NL and TL schemes (as explained above, the precipitation in the TL model also corresponds to the trajectory; the perturbed precipitation is examined in the next section). The main large-scale features are present in both schemes: maxima above 7 mm day−1 near 10°S corresponding to the ITCZ, subtropical minima, secondary maxima over midlatitudes associated with the storm tracks, and negligible values near the poles. The criteria for triggering convection are very similar between the two schemes, which explains their consistency. The design of the TL model is such that both schemes use almost the same input atmospheric profiles at each time step; therefore differences are only the result of the internal behavior of the moist physics. Intensities are somewhat smaller with Kuo symmetric in the Tropics by about 1.5 mm day−1. Stratiform precipitation in the NL package (Sundqvist’s scheme) being rather different from the TL scheme, values along the storm tracks differ by a factor of 2 between the two schemes. This difference mostly comes from a larger extent of areas of low rainfall intensities with the Sundqvist’s scheme that does not require supersaturation to be reached for generating precipitation. When the NL model is run with the Kuo symmetric scheme, the zonal mean precipitation is very similar to the one obtained in the TL model, as is also shown in Fig. 2. It means that the large-scale forcing as well as the mechanisms generating rain are properly captured by the TL scheme despite the lack of radiative and surface processes in the trajectory and a much simpler diagnostic of surface precipitation. In a final test, the NL scheme within the TL model has been simplified by neglecting the detrainment term in the convective tendencies [i.e., imposing K = 0 in (1) and (2)]. The justification of this experiment will appear more clearly in the next section when defining the linearized version. The precipitation with this simplified version is slightly reduced compared to those obtained with the complete scheme, but the main large-scale fea-
tures are kept. Therefore, the drying and warming by induced subsidence is rather well simulated by canceling the large-scale tendencies, which are primarily due to vertical advection in most tropical situations. As previously noticed by Bougeault (1985), clouds tend to modify their environment by mixing processes with a much slower time scale than the induced subsidence. When computing the corresponding time scales associated with the values of K they range between 2 and 10 days, except in intense rainy areas (which explains why nonnegligible differences take place along the ITCZ).

3. The linearized convection scheme

As explained in the introduction, the design of useful linearized parameterization schemes for variational data assimilation is not straightforward. The validity of the TL approximation needs to be examined for finite-size perturbations similar to the type of corrections applied to the a priori model state in order to get an improved (analysis) state. The breakdown of the TL approximation has two origins: strong nonlinearities and thresholds in the NL scheme that cannot be described in the TL scheme. Mahfouf (1999), Janisková et al. (1999), and Laroche et al. (2002) have demonstrated that for vertical diffusion the major problem is associated with strong nonlinearities of the eddy diffusivity coefficients around neutral stability (since stable regimes are associated with low values of these coefficients and unstable regimes with much larger values). They showed that by neglecting the linearization of these coefficients, a reasonable (and useful) behavior of the TL scheme can be obtained to describe the temporal evolution of analysis increments in the boundary layer. When dealing with moist processes, nonlinearities can be present but thresholds are also a source of mismatch between the NL and TL schemes. The main difference between thresholds and nonlinearities is that practical solutions can be found to deal with nonlinearities (by removing part of the linearization of such nonlinear processes or smoothing them) whereas for thresholds, solutions proposed by Xu (1996) and more recently by Mu and Wang (2003) have never been examined in NWP models. Thus, even with a linear convection scheme the validity of the TL approximation can be rather poor by the existence of a trigger function. In this study we compare errors produced by nonlinearities separately from errors produced by discontinuities to estimate if one source is more critical than the other when linearizing moist processes.


**a. Methodology**

The impact of a TL version of a physical parameterization scheme can be evaluated by including it in the TL model and comparing its response to the difference between two NL integrations (with full physics). If the inclusion of the physical scheme in the TL model reduces the error with respect to the NL model, it can then be considered as beneficial. The initial conditions $x'$ of the TL model should reflect the size and structure of typical errors that need to be corrected in a variational assimilation. That is why analysis increments or scaled singular vectors are often considered as initial conditions. In this study, $x'$ corresponds to analysis increments from the CMC analysis at 1200 UTC 7 February 2001. A first 24-h NL integration starting from the analysis $x$ is performed, producing a model state $\text{NL}(x)$; then a second NL integration starting from a perturbed state (background) $x + x'$ (where $x'$ is minus the analysis increment) produces a model state $\text{NL}(x + x')$. Then a TL integration starting from $x'$ linearized at each time step around $x$ (which needs to be stored) is done to obtain an evolved perturbed state $\text{TL}(x')$. Results are then quantified in terms of spatial mean absolute error defined by

$$
\varepsilon = [\text{NL}(x + x') - \text{NL}(x) - \text{TL}(x')] \tag{9}
$$

where various versions of TL are compared against the same “truth” defined as the difference of two NL integrations performed with a complete package of physical parameterizations. It is important to underline that in the context of a validation for an incremental 4DVAR, the NL runs could be performed at higher resolution than the TL runs in order to also account for such differences in the comparison (Trémolet 2003). The experiments presented hereafter use the same resolution for the TL and NL integrations, which means that the noticed differences could potentially be larger.

**b. Description and preliminary evaluation**

The linearized equations of the Kuo symmetric scheme are derived from (1) and (2):

$$
\left( \frac{\partial T}{\partial t} \right)_{\text{conv}} = \frac{1}{C_p} \left[ -\left( \frac{\partial s'}{\partial t} \right)_{\text{LS}} + K'(s_c - s) + K(s'_c - s') \right],
$$

$$
\left( \frac{\partial q'}{\partial t} \right)_{\text{conv}} = \left[ -\left( \frac{\partial q'}{\partial t} \right)_{\text{LS}} + K'(q_c - q) + K(q'_c - q') \right],
$$

These tendencies are applied to the perturbed variables $T'$ and $q'$ when the criteria to trigger convection from the trajectory variables are satisfied.

The mean absolute error $\varepsilon$ between the TL and NL models [defined by (9)] is presented in Table 1 for three forecast ranges (6, 12, and 24 h) for specific humidity. This variable is chosen since the influence of moist processes is the largest (Mahfouf 1999). A first TL integration (TL1) without moist processes has been performed. It can be considered as a baseline experiment against which the inclusion of moist physics should be compared. In a second experiment (TL2) the large-scale condensation has been activated in the TL run and a reduction of $\varepsilon$ can be noticed at all ranges, demonstrating the positive impact of including such a scheme in the TL model. The reduction of error is about 6%, which is consistent with previous studies. When the TL version of the Kuo symmetric is included, in a third experiment (TL3), a further gain is obtained in the short range (6 h) but is lost after 12 h. Results after 24 h are unacceptable since the error is much larger than without the linearized convection scheme. This is a clear demonstration of the possible detrimental impact of a linearized physical parameterization scheme. This global degradation is in fact the result of few model columns where convective tendencies are becoming unrealistically large during the course of the integration. A time series of perturbed convective precipitation $R'_c$ at a problematic point shows the growth with time of noise, where from one time step to the next $R'_c$ has opposite and increasing values (Fig. 3). The noise stops when convection from the trajectory run is switched off (after 19 h).

A number of numerical solutions have been tried unsuccessfully in the TL model (reduction of time step, implicit temporal scheme). Following a solution proposed by Mahfouf (1999) to solve a similar problem for the linearized vertical diffusion scheme, the linearization has been simplified by assuming no perturbations.
of cloud properties \((s'_c = 0, q'_c = 0)\) and also no perturbations of the detrainment coefficient \((K' = 0)\), leading to a set of simplified linearized equations for convection:

\[
\left(\frac{\partial T'}{\partial t}\right)_{\text{conv}} = -\left(\frac{\partial T'}{\partial t}\right)_{LS} - \kappa K'T',
\]

\[
\left(\frac{\partial q'}{\partial t}\right)_{\text{conv}} = -\left(\frac{\partial q'}{\partial t}\right)_{LS} - \kappa Kq'.
\]

The value of \(\kappa\) is set to one.

Indeed, the main problem concerns nonlinearities associated with \(K\) and \(x_c\) (where \(x\) stands for either \(s\) or \(q\)) since the linearization of

\[
\frac{\partial x}{\partial t} = K(x_c - x)
\]

leads to

\[
\frac{\partial x'}{\partial t} = \left[\frac{\partial K}{\partial x}(x_c - x) + K\left(\frac{\partial x}{\partial x} - 1\right)\right]x',
\]

where an exponential growth is possible whenever the term in brackets is positive. By setting

\[
\frac{\partial K}{\partial x} = 0 \quad \frac{\partial x}{\partial x} = 0
\]

what is left from the previous equation is only an exponential damping of the perturbation.

Similar difficulties in linearizing convection schemes have been reported by Errico and Raeder (1999) (where too-large Jacobian elements are set to zero). Mahfouf (1999) and M. Janisková (2001, personal communication) have set the perturbation of the mass flux to zero in order to avoid the development of noise in
The ECMWF and Météo-France TL models respectively. The previously linearized Kuo-type schemes did not show such problems (Zou et al. 1993; Vukčićević and Errico 1993) because the vertical structure of the heating profile is imposed from an analytical formula instead of being proportional to the difference between cloud and environmental properties.

The impact of such a simplified scheme is reported in Table 1 for experiment TL4 and also in Fig. 3 where large oscillations of \( R^c \), were noticed. It can be seen that this revised scheme has a positive impact on the evolution of analysis increments of specific humidity at all ranges. Results are improved with respect to the dry TL model and also with respect to a TL model with only large-scale condensation. The improvement with respect to the large-scale condensation is about 3% [similar to what Mahfouf (1999) has obtained with the ECMWF model]. Various intermediate solutions have been tried by just setting \( K^c = 0 \) or \( x^c = 0 \). Setting the perturbed cloud properties to zero had a larger impact than just imposing \( K^c = 0 \). This is because \( K^c \) is also modified when \( x^c \) is set to zero. As already stated in the evaluation of the NL scheme, the dominant contribution to the total convective tendencies is the compensating subsidence term (which is approximated in the Kuo symmetric by minus the large-scale tendencies) and convective precipitation was not too strongly modified when imposing \( K^c = 0 \). The linearization of this simplified NL scheme corresponds to (12) and (13) with \( \kappa = 0 \). The behavior of the TL convection scheme is almost identical by setting either \( \kappa = 1 \) or \( \kappa = 0 \). An interesting feature of the further simplification made by imposing \( \kappa = 0 \) is that the resulting tendencies do not depend upon any particular convection scheme. The only information required from the NL integration is the trigger function (the onset of convection and the corresponding vertical extent). Assuming that this information has been stored from a prior NL model integration, the above-linearized scheme is independent of the design of the NL scheme and in fact does not require the linearization of a convection scheme.

A further examination of the TL scheme has been performed within the ECMWF model having much higher horizontal and vertical resolutions (horizontal grid mesh about 120 km with 60 vertical levels). This was done for two reasons:

- to check if the numerical noise produced by the full TL convection scheme could be the result of feedbacks between the dynamical core and the physics, and
- to compare the simplified TL Kuo symmetric scheme with the operational ECMWF simplified TL mass-flux scheme.

The second item is of importance since the perturbed convective tendencies in the ECMWF TL model are written as

\[
\left( \frac{\partial T^c}{\partial t} \right)_{\text{conv}} = \frac{1}{C_p} \left[ \omega^c \frac{\partial \alpha^c}{\partial p} \right],
\]

where the mass-flux \( \omega^c \) from the trajectory is recomputed in TL/AD integrations representing a significant increase in computing cost as noticed by Barkmeijer et al. (2001) for tropical singular vectors. It was interesting to examine what results could be achieved in terms of TL approximation without the knowledge of the mass flux.

Table 2 provides improvement factors defined by

\[
\lambda_\chi = 100 \times \frac{e_{\text{mod}} - e_{\text{ref}}}{e_{\text{ref}}},
\]

where \( \lambda_\chi \) represents the reduction in error of a modified version (mod) against a reference one (ref) for a model variable \( \chi \). The ECMWF TL model (T159L60–CY26R1) has been integrated for 24 h with initial conditions corresponding to analysis increments produced at 1200 UTC 15 March 2001. The reference version is an adiabatic TL model and three versions of TL convection schemes have been considered: the ECMWF simplified mass-flux scheme (Mahfouf 1999), the simplified Kuo symmetric scheme, and the full Kuo symmetric scheme. Results in Table 2 reveal that the simplified Kuo symmetric scheme produces improvements comparable to those given by the mass-flux scheme. Results are slightly worse for humidity than for temperature. Given the simplicity of the Kuo symmetric scheme these results reveal the soundness of the linearization. The degradation with respect to the mass-flux scheme is mostly due to the approximation in the Kuo symmetric scheme to describe vertical transport by convection. The inconsistency with the NL scheme is probably of secondary importance. Indeed, when the Betts–Miller scheme was included in the ECMWF TL model, the improvement factor was even slightly higher (unpublished result).

The degradation produced by the full linearized version of Kuo symmetric is obvious in Table 2 with much

<table>
<thead>
<tr>
<th></th>
<th>ECMWF mass-flux scheme</th>
<th>Kuo symmetric simplified</th>
<th>Kuo symmetric full</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_\chi ) (%)</td>
<td>-13.7</td>
<td>-12.4</td>
<td>-8.5</td>
</tr>
<tr>
<td>( \lambda_\nu ) (%)</td>
<td>-19.9</td>
<td>-16.5</td>
<td>-13.1</td>
</tr>
</tbody>
</table>
smaller values of $\lambda_T$ and $\lambda_q$. As in the GEM model, this
global degradation in the ECMWF model corresponds
to a single vertical column where increments reach un-
realistic values (>100 K for temperature after 12 h).

c. Complementary evaluation

This section provides a complementary evaluation of
the behavior of the linearized moist physics, in terms of
perturbed precipitation fluxes. First, we want to illustra-
tion on time series at specific points the behavior of the
TL scheme versus the difference between two NL in-
tegrations. Then, we would like to assess the impor-
tance of pathological points in the degradation of the
TL approximation. Pathological points correspond to
points with a nonzero rainfall rate in one NL integra-
tion and a zero rainfall rate in the other. Is the TL
approximation much worse for pathological points? We
also want to examine if the TL approximation is better
for stratiform precipitation than for convective precipi-
tation. Finally, we are interested in a possible degrada-
tion coming from inconsistencies between NL and TL
physics.

Figures 4 and 5 show the temporal evolution of in-
stantaneous and accumulated precipitation for both the
trajectory and the perturbation (estimated from finite
differences and from the TL model) at two specific
points characterized by the dominance of large-scale
precipitation in one case and by convective precipi-
tation in the other. These points have been selected as
representative of locations where the TL model pro-
vides a reasonable answer compared to the NL model.
Important remarks can be given from these figures.
First it appears that the 24-h perturbed accumulated
rainfall rate has a magnitude comparable to the refer-
ence rainfall rate: 4 mm against 11 mm for the strati-
form case and 10 mm against 17 mm for the convective
case after 24 h. The important sensitivity of moist pro-
cesses (coming from internal nonlinearities) means that
even for small initial perturbations (analysis incre-
ments) the response in terms of perturbed precipitation
can be very large (i.e., in a regime where the TL ap-
proximation may become invalid). The time evolution
of instantaneous precipitation reveals a much smoother
behavior in the TL model than in the NL model since it
is less affected by thresholds, but errors at a given time
can be extremely large (>1000%). The accumulated
erculated rainfall from the TL model is much more
comparable to the NL model for both the stratiform
and convective cases. From this example, it seems that
there is little hope in trying to assimilate instantaneous
rainfall rates within a variational framework. The
agreement on accumulated precipitation is encouraging
for using such observations in variational assimilation.

Since it is somewhat misleading to draw firm con-
clusions from just two locations, statistics have been per-
formed and are summarized in Table 3.

Results are sorted in three categories: instantaneous
versus accumulated rainfall rates, stratiform versus con-
vective rainfall rates, pathological versus all points. In-
stantaneous rainfall corresponds to values at 24 h, but
results are very similar at other times. The pathological
points cannot be described correctly by the TL model
(with the traditional linearization approach). The num-
ber of pathological points is around 50% for instanta-
aneous rain and drops down to about 30% when accu-
mulated. This confirms the interest for assimilating ac-
cumulated precipitation since the number of points
where the TL scheme is virtually useless is reduced by
a factor of 2. A second important remark is that strat-
iform precipitation has a larger fraction of pathological
points than convective precipitation. However, pa-
Theologies associated with convective precipitation are certainly more critical because of the dominance of convective rainfall over the globe (such a ratio is probably scheme dependent, but one could argue that for more elaborate cloud schemes part of the stratiform rain has a convective origin) and because of the largest corresponding intensities. This is revealed by the rms errors that are much larger for convective than stratiform rain. A comparison with the mean absolute value of the perturbed precipitation shows that these errors are very large (roughly twice the value of the perturbed field). Rms errors are significantly reduced on accumulated fields, but the relative error w.r.t. the perturbed field remains important. The smallest errors correspond to the accumulated stratiform precipitation, but they are also associated with smaller rates. An interesting feature is the fact that errors for nonpathological points are not smaller than errors for pathological points. Intuitively one could expect the behavior of the TL model to produce very large errors in situations where switches are modified in the two NL integrations, since such behavior cannot be described in the TL model. As illustrated by Fig. 6, areas associated with pathological points are located in the margins of intense rainy systems. They are associated with smaller rainfall rates (which explains why a small perturbation in the initial conditions can either trigger or switch off rain). Even though the relative error can be very large, the absolute error is smaller than for nonpathological points. On the contrary, nonpathological points are associated with intense rainfall rates where the behavior of the NL scheme has a tendency to saturate in situations where the rainfall should be increased. Such NL behavior cannot be handled properly by the TL scheme. Internal discontinuities can also appear in situations where the convection remains active in both NL runs. Fillion and Belair (2004) noticed that the change in cloud top between two NL runs can significantly degrade the validity of the TL approximation. However, such spatial discontinuities can be smoothed (Zupanski 1993), as opposed to temporal discontinuities.

Two complementary aspects were examined regarding possible simplifications to the moist physics. When the linearized convection scheme is not activated, the behavior of the linearized large-scale precipitation is significantly degraded: the rms error for 24-h accumulation increases from 0.66 to 1.37 mm day$^{-1}$. Even though errors on total precipitation could be found acceptable, the large differences in Jacobians between convective and stratiform precipitation schemes (Fillion and Mahfouf 2000) does not make such simplification suitable for variational applications. The linearized

![Fig. 5. (top) Perturbed accumulated and (bottom) accumulated convective precipitation over a 24-h period from the trajectory (NL ref), the difference between two NL integrations (NL pert), and the TL integration (TL pert).](image)

| Type of precipitation | Fraction of pathological points (%) | Rms error for nonpathological points (mm day$^{-1}$) | Rms error for all points (mm day$^{-1}$) | Mean $|R^2|$ (mm day$^{-1}$) |
|-----------------------|------------------------------------|---------------------------------------------|----------------------------------------|------------------|
| Instant CONV          | 52                                 | 8.45                                       | 7.70                                   | 4.41             |
| Instant STRAT         | 58                                 | 4.64                                       | 3.67                                   | 2.46             |
| Accum CONV            | 23                                 | 3.64                                       | 3.19                                   | 1.35             |
| Accum STRAT           | 35                                 | 0.74                                       | 0.66                                   | 0.38             |
moist physics is used to compute tendencies from the prognostic variables and to evaluate (as a diagnostic) a perturbed precipitation field (i.e., the observation operator in variational data assimilation). It is possible to use the linearized moist physics to produce a perturbed precipitation field but to ignore the tendencies from moist processes in the TL evolution of model variables. This simplification could be justified by the fact that the positive impact on prognostic variables is rather small, as shown in Table 1. However, it worsens considerably the estimation of perturbed rainfall rates. The result is more dramatic for stratiform precipitation that uses tendencies from convective processes. The rms error reaches 3.75 mm day\(^{-1}\). Finally, the use of a Kuo symmetric scheme in the NL model did not improve the behavior of the TL convection scheme. This result indicates that consistency between NL and TL schemes does not seem critical for improving the TL approximation (as previously noticed when the Kuo symmetric was included in the ECMWF model).

From the results presented above, it seems that the assimilation of accumulated precipitation (here 24 h) should be easier than the assimilation of instantaneous precipitation. In terms of pathologies and relative errors, stratiform precipitation does not behave much better than convective precipitation. Even though over specific regions (e.g., midlatitude winter) stratiform precipitation may dominate, for global applications the dominance of convective rain both in terms of occurrence and intensity puts the emphasis on improving linearized moist convection schemes. As shown by Fillion and Mahfouf (2003), even when stratiform precipitation is present it can be generated from convection and therefore the associated Jacobians reflect those of the convection scheme. Errors in terms of TL approximation are always very large and they are not dominated by pathologies. This is an important conclusion since it indicates that by having more linear moist physical processes some of the current errors could be reduced, as shown by Lopez and Moreau (2004). Two examples
have shown that for specific locations the behavior of the TL scheme can be very reasonable even for rather large perturbations but only in terms of accumulated rainfall. This is valid both for stratiform and convective rainfall rates.

4. Conclusions and perspectives

In this paper, a simple convection scheme proposed by C. Girard (Mailhot et al. 1998) based on an improved version of the classical Kuo scheme (in which the large-scale accession is defined in terms of moist enthalpy instead of moisture only) has been linearized for the TL/AD versions of the global version of the GEM model. It has been shown that the complete linearization of the scheme leads to important numerical noise similar to what was previously noticed for linearized vertical diffusion schemes (Mahfouf 1999; Janisiková et al. 1999; Laroche et al. 2002). When neglecting the perturbation of the detrainment coefficient and cloud properties the TL scheme becomes useful. In agreement with Mahfouf (1999) the inclusion of the convection scheme in the TL model improves the time propagation of humidity analysis increments (from 6 to 24 h) when compared to pairs of NL integrations. An evaluation has also been performed within the ECMWF TL model (having higher horizontal and vertical resolutions) where the noise of the complete linearized scheme has been recovered and the beneficial impact of the simplified linearization demonstrated. In particular, the simplified scheme has a positive impact similar to the ECMWF linearized mass-flux scheme. A detailed evaluation of the TL approximation in terms of perturbed precipitation (which is particularly relevant for variational assimilation) has shown that relative errors are not smaller for accumulated precipitation (over a 24-h period) than for instantaneous values, but the amount of pathological points (associated with discontinuities in the NL model) is significantly reduced when considering accumulated rain rates. Given the stochastic nature of precipitation such temporal smoothing is certainly compulsory for a deterministic data assimilation. It will be important to examine if the predictability of precipitating systems can be improved by assimilating accumulated instead of instantaneous rainfall rates. It is likely that the spatiotemporal details of precipitation observations cannot be captured in global analyses. However, improving the location and intensity of the large-scale diabatic forcing (especially in the Tropics) within the analysis through variational assimilation of precipitation should result in better medium-range forecasts over midlatitudes. This conjecture needs to be examined in future studies. The improved behavior of perturbed accumulated precipitation raises questions on the usefulness of derived rain rates from polar-orbiting satellites that can only measure instantaneous quantities. Multisensor strategies as envisaged by the future Global Precipitation Mission (GPM) could be more suitable for data assimilation purposes.

Despite smaller absolute errors, in relative terms, the TL version of the stratiform precipitation scheme does not behave much better than the TL version of the convection scheme. Emphasis should be put on improved linearized convection schemes because of the dominance of this type of precipitation (on the global scale) both in terms of occurrence and intensity. Moreover the fraction of pathological points is similar for the two types of precipitation. Most of the pathological points being located in marginal rainy areas leads to errors that are smaller than for nonpathological points. This result holds both for stratiform and convective precipitation. Therefore, even though switches cannot be handled in current TL/AD schemes it remains important to design more linear convection schemes, since the large errors noticed for nonpathological points could be significantly reduced. In our context, consistency between TL/AD and NL versions of parameterization schemes was not critical. Smoother schemes could also be helpful for triggering precipitation (e.g., if a natural transition could be found between shallow and deep convections), even though the assimilation of radiances sensitive to both cloud and rainwater contents can partially resolve this issue (Moreau et al. 2004).

This simple convection scheme will be used for applications such as 4DVAR, singular vectors, and key analysis errors. Regarding 4DVAR, it will be important in future studies to address the suitability of this simplified convection scheme beyond the incremental framework. Indeed, diabatic incremental and nonincremental 4DVAR assimilations have never been compared to evaluate the level of complexity of physical processes that is required for each approach. For singular vectors, preliminary results have shown that the full linearized convection scheme produced unrealistically large growth rates. In agreement with previous studies, Zadra et al. (2004) came to the conclusion that the simplified linearized moist convection scheme does not play a significant role in the enhancement of unstable modes over midlatitudes in winter. Such results are currently under study using the energetics of moist baroclinic instability. An important aspect to be examined concerns the modification of initial conditions in order to improve the model fit to observed precipitation in a variational framework. In that context we have further simplified the perturbed convective tendencies.
[by setting $\kappa = 0$ in (12) and (13)] of the TL scheme. With such simplification, it is possible to express the perturbed surface rainfall rate in two forms, either

$$R_1^p = \int_{p_b}^{p_t} \left( \frac{\partial q^*}{\partial t} \right) \frac{dp}{\bar{g}} \quad (19)$$

or

$$R_2^p = -\frac{C_p}{L} \int_{p_b}^{p_t} \left( \frac{\partial T^*}{\partial t} \right) \frac{dp}{\bar{g}} \quad (20)$$

When comparing these two formulations, they appear to behave similarly (in terms of mean and extremum values) and to have the same level of inaccuracy with respect to the NL behavior (see Table 4). However, the sensitivity of the scheme in terms of initial conditions should be rather different. Indeed, $R_1^p$ only depends on $q$ perturbations whereas $R_2^p$ only depends on $T$ perturbations. The first scheme can be considered as “CISK-able” since it will respond directly to moisture tendencies (i.e., moisture convergence) whereas the second scheme will only be sensitive to changes in temperature. The quasigeostrophic balance between mass and wind over midlatitudes for synoptic scales could allow more direct dynamical changes to the initial conditions with $R_2^p$ than with $R_1^p$. We plan to examine such aspects using the optimal perturbation framework proposed by Errico et al. (2003).

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