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Evaluating uncertain public projects with rival and non-rival benefits

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1. Introduction

Governments must often make decisions about the allocation of public funds among competing projects of uncertain value. With the benefits and costs of public projects evaluated in terms of consumers' willingness to pay, consistent accounting requires that the social cost of a project's risk be assessed according to the same metric. In an influential paper, Arrow and Lind (1970) argue that, when this approach to evaluating public projects is adopted, the social cost of project risk in a large economy is negligible, and projects should be evaluated solely on the basis of their expected values. The argument is that, as the number of consumers becomes large, the private risk borne by any one consumer becomes small and, more decisively, the aggregate (social) cost of risk becomes small as well – a result known as the Arrow-Lind Theorem.

A serious limitation of the Theorem, however, pointed out by Fisher (1973) and by Foldes and Rees (1977), is that the result does not apply to public projects that produce goods whose benefits are consumed collectively. For these Samuelson (1954) public goods, the cost of risk borne by an individual consumer associated with project benefits is unaffected by the size of the population, and hence the social cost of risk for these projects is positive. By contrast, for the Arrow-Lind Theorem to hold, the public project must, in the limit, have no effect on any consumer; that is, not only must the taxes paid to finance the project approach zero for each consumer as the population grows, but the benefit enjoyed by each consumer must also approach zero. Therefore, the project must have not only a fixed cost, but also a fixed aggregate expected benefit. Hence, a public project for which the Arrow-Lind Theorem is applicable must produce benefits by providing the services of a private good, and then the project is simply a private investment undertaken by government on behalf of taxpayers.

Thus, at one extreme public projects provide the services of a private good and consumers' risk preferences have no bearing on project evaluation in a large economy, while at the other extreme projects provide the services of a public good and consumers' risk preferences are paramount in project evaluation, whatever the size of the economy. Often, however, public expenditures provide both private (rival) and public (non-rival) consumption benefits. Public expenditures on education offer a classic example, enhancing the production of human capital by individuals – a private good – while also cultivating an ethic of civic responsibility and respect for the rule of law – a public good. Likewise, national defense expenditures protect private property as well as the economy's infrastructure, and the construction and maintenance of dams provide both hydroelectric power and pollution abatement when used as a substitute for fossil fuels.

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In this paper we focus on public projects that provide a mixture of rival and non-rival consumption benefits. We adopt the basic elements of the Arrow-Lind model, but elaborate by assuming that a proportion α of the project's benefits are rival in consumption while the proportion $1 - \alpha$ of the project's benefits are non-rival in consumption. When $\alpha = 1$ the Arrow-Lind Theorem applies and the social cost of the project's risk is negligible in a large economy; when $\alpha = 0$ the Theorem does not apply and the social cost of the project's risk is positive regardless of the size of the economy. It follows that, the social cost of risk must decline in an overall sense as the degree of rivalry increases from $\alpha = 0$ to $\alpha = 1$, and as the size of the population increases, at least when benefits are perfectly rival.

Our analysis shows that, while expected project benefit for each consumer declines as α increases, the private and social costs of project risk also decline uniformly as α increases if consumers are risk averse and exhibit constant or increasing absolute risk aversion (CARA or IARA). Although the social cost of risk bearing increases with greater rivalry if decreasing absolute risk aversion (DARA) is sufficiently strong, the social cost of risk declines with greater rivalry if relative risk aversion is less than one and relative prudence is less than two. Regardless of its effects on the cost of risk, greater rivalry increases the social value of a public project, suggesting that projects with largely rival consumption benefits should prevail over projects with similar cost, but largely collective consumption benefits.

We also show that the private cost of risk bearing is lower with a larger population if all benefits are non-rival and consumers exhibit DARA, or if benefits are at least partially rival, consumers exhibit CARA or DARA, and the social cost of risk declines with greater rivalry. We conclude that intuitive predictions regarding the influence of rivalry and population size on the cost of bearing the risk associated with public projects of uncertain value are confirmed when relative risk aversion is less than one and relative prudence is less than two.

2. A model of public projects with collective and private benefits

The economy is comprised of $n > 1$ identical consumers who derive utility from consumption of a single good. Each consumer is endowed with a sure income A plus a random, zero-mean component ε. The public project produces an output with expected benefit $\bar{\theta}$ at a total cost of c. Each consumer pays a lump-sum tax of c/n to finance the project and enjoys the consumption benefits of both its private good and public good services. We assume that these benefits are random. For example, public expenditures on national defense are more valuable in a state of war than in peace, but which state shall obtain is uncertain; public expenditures on education have an uncertain effectiveness in generating both rival and non-rival benefits; hydroelectric power generation depends on river flow, which in turn depends on uncertain rainfall.

Following Arrow and Lind, we assume that the random benefits of the public project are independent of the endowed income risk and abstract from temporal considerations. For simplicity, we assume that the random rival and non-rival benefits are perfectly positively correlated. Thus, in consumption, the project yields a random, positive flow of services θ, with a portion α of these services providing benefits that are rival in consumption. The remaining portion provides benefits that are non-rival in consumption. The net benefit of the project to each consumer is measured by net willingness to pay, which in state θ is equal to

$$
\begin{aligned}
B(\theta, \alpha, n) &\equiv \theta(1-\alpha) + \frac{\theta\alpha - c}{n} \\
&\equiv \omega\theta - \frac{c}{n},
\end{aligned} \tag{1}
$$

so that all consumers receive the project's non-rival benefit $\theta(1-\alpha)$ along with an equal share of the project's rival benefit $\theta\alpha$, and pay the lump-sum tax. The second line, serves to define the coefficient $\omega = 1 - \alpha(n-1)/n$, which depends on the degree of rivalry and the size of the population.

The expected utility of the representative consumer is given by

$$
E_\theta E_\varepsilon[U(A + \varepsilon + B(\theta, \alpha, n))] = E_\theta[V(A + B(\theta, \alpha, n))], \tag{2}
$$

where E_θ and E_ε denote the expectation operators for the independent random variables θ and ε, and U is the consumer's von Neumann-Morgenstern utility function, assumed to be strictly concave, reflecting risk aversion. From a consumer's perspective, the endowed income risk ε represents an independent background risk when evaluating the net benefits of the project, and therefore the relevant criterion is the derived utility function for the project's net benefits,

$$
V(A + B(\theta, \alpha, n)) \equiv E_\varepsilon[U(A + \varepsilon + B(\theta, \alpha, n))], \tag{3}
$$

which accounts indirectly for the endowed background risk ε.[1] The cost to each consumer of bearing the project's risk is measured by the risk premium for V, denoted $\pi(\omega, W)$ and defined implicitly by

$$
E_\theta[V(W + \omega\theta)] = V(W + \omega\bar{\theta} - \pi(\omega, W)), \tag{4}
$$

where $W = A - c/n$.

For the two polar cases we have

$$
E_\theta[V(W + (\theta - c)/n)] = V(W + [\bar{\theta} - c)/n] - \pi(1/n, W)) \tag{5}
$$

when all benefits are rival, and

$$
E_\theta[V(W + \theta - c/n)] = V(W + [\bar{\theta} - c/n] - \pi(1, W)) \tag{5'}
$$

when all benefits are non-rival. Although expected net benefits are greater when all benefits are non-rival, deviation about the mean is lower when all benefits are rival. With CARA, the difference in expected net benefit has no bearing on the magnitude of the risk premiums, and thus with less risk the project having only rival benefits has the smaller risk premium, and $\pi(1/n, W) < \pi(1, W)$.

3. Changes in the collective-private benefit mix

Many public projects provide both collective and private benefits. In these instances, α lies between the extremes of zero and one. From the two polar cases, we would expect the risk premium, and thus the social cost of risk, to be lower for projects providing relatively more private consumption benefits. However, we find that this intuitive prediction is not universally valid. Nonetheless, the social value of the project increases as α increases, *ceteris paribus*. We first investigate the effect of rivalry on the risk premium.

3.1. Risk-bearing cost and greater rivalry

Since an increase in α, representing an increase in rivalry, reduces the coefficient ω while leaving wealth unaffected, an increase in rivalry reduces the social cost of project risk if and only if π increases as ω increases. Differentiating Equation (4) with respect to ω yields $E_\theta[V'(W+\omega\theta)\cdot\theta] = V'(W+\omega\bar{\theta}-\pi(\omega,W))\cdot(\bar{\theta}-\pi_\omega)$, which implies

$$\pi_\omega = \bar{\theta} - E_\theta[V'(W+\omega\theta)\cdot\theta]/V'(W+\omega\bar{\theta}-\pi). \tag{6}$$

The social cost of risk declines as the degree of project rivalry increases if and only if

$$d\pi/d\alpha = \pi_\omega\omega_\alpha = -\frac{n-1}{n}\pi_\omega \tag{7}$$

is negative. The following is an immediate consequence of Equations (6) and (7).[2]

Proposition 1 π_ω is positive, and $d\pi/d\alpha$ is negative, if and only if

$$V'(W+\omega\bar{\theta}-\pi)\cdot\bar{\theta} > E_\theta[V'(W+\omega\theta)\cdot\theta]. \tag{8}$$

Applying the covariance rule to the random variables on the right-hand side of inequality Equation (8), while recognizing that V is risk averse, we obtain

$$0 > \mathrm{cov}(V'(W+\omega\theta),\theta) = E_\theta[V'\cdot\theta] - E_\theta[V']\cdot\bar{\theta}. \tag{9}$$

It follows that inequality Equation (8) holds if $V'(W+\omega\bar{\theta}-\pi) \geq E_\theta[V'(W+\omega\theta)]$, or equivalently, if

$$V'(W+\omega\bar{\theta}-\pi) \geq V'(W+\omega\bar{\theta}-\psi), \tag{10}$$

where ψ is the prudence premium for V introduced by Kimball (1990). Given risk aversion, inequality Equation (10) holds if and only if $\pi \geq \psi$, which requires that V exhibit constant or increasing absolute risk aversion (CARA or IARA).[3]

Corollary 1 π_ω is positive and $d\pi/d\alpha$ is negative if V exhibits CARA or IARA.

We next establish the existence of environments in which the social cost of project risk increases as the degree of rivalry increases. To this end, we introduce the random variable $\eta = \theta - \bar{\theta}$ and conclude that inequality Equation (8) fails to hold if and only if

$$\begin{aligned} E_\theta[V'(W+\omega\theta)\eta] &\geq [V'(W+\omega\bar{\theta}-\pi) - E_\theta V'(W+\omega\theta)]\bar{\theta} \\ &= [V'(W+\omega\theta-\pi) - V'(W+\omega\theta-\psi)]\bar{\theta}\,. \end{aligned} \tag{11}$$

The left-hand side of inequality Equation (11) is negative and the inequality holds if ψ exceeds π by a sufficiently wide margin. Since $\psi - \pi$ is directly related to the strength of DARA, we have the following result.

Proposition 2 π_ω is negative, and $d\pi/d\alpha$ is positive, if DARA is sufficiently strong.

Nonetheless, we can establish that greater rivalry reduces the social cost of risk for an important class of environments with DARA by introducing the function

$$H(\theta) \equiv V'(W + \omega\theta) \cdot \theta. \tag{12}$$

Since $V'(W + \omega\bar{\theta} - \pi) \cdot \bar{\theta} > V'(W + \omega\bar{\theta}) \cdot \bar{\theta}$, the following is a sufficient condition for inequality Equation (8),

$$H(\bar{\theta}) \geq E_\theta[H(\theta)]. \tag{13}$$

This sufficient condition holds if and only if H is concave. For the derivatives of H, we obtain $H' = V''\omega\theta + V'$ and

$$\begin{aligned} H'' &= [V'''\omega\theta + 2V'']\omega \\ &= V''\omega[2 - \hat{P}], \end{aligned} \tag{14}$$

where $\hat{P} = -\omega\theta V'''(W + \omega\theta)/V''(W + \omega\theta)$ is the index of partial relative prudence. For $\hat{P} = -bV''(a + b)$ to be less than or equal to two for all $b \geq 0$ and $a + b > 0$, it is necessary and sufficient that the index of relative prudence $P = -bV'''(b)/V''(b)$ be less than or equal to two for all $b > 0$. Hence, H is concave if $P \leq 2$.

Corollary 2 π_ω is positive, and $d\pi/d\alpha$ is negative, if $P \leq 2$.

Eeckhoudt, Etner, and Schroyen (2009) show that preference for 'harm disaggregation', or combining bad with good, is necessary and sufficient for $P \geq 2$, while their proof also shows that the opposite preference is necessary and sufficient for $P \leq 2$. The latter is consistent with DARA provided the index of relative risk aversion is less than P. Analyzing the portfolio problem, Choi, Kim, and Snow (2001) show that investment in the risky asset increases with first-order stochastic dominance improvement in the random rate of return if and only if relative risk aversion is less than or equal to one, and that investment declines with an increase in risk if and only if relative prudence is less than or equal to two. Thus, under conditions necessary and sufficient for the validity of intuitive predictions regarding investor responses to changes in the random rate of return, the social cost of risk declines with greater rivalry and preferences exhibiting DARA.

3.2. Social value and greater rivalry

Differentiating the certainty equivalent wealth argument on the right-hand side of Equation (4) with respect to α yields

$$\partial[W + \omega\bar{\theta} - \pi(\omega, W)]/\partial\alpha = -\frac{n-1}{n}(\bar{\theta} - \pi_\omega) \tag{15}$$

as the effect of greater rivalry on individual net benefits. Substituting for π_ω from Equation (6), we obtain

$$-\frac{n-1}{n}(\bar{\theta}-\pi_\omega)=\frac{n-1}{n}\cdot\frac{E_\theta[V'\cdot\theta]}{\bar{V}'}, \tag{16}$$

which is positive since θ is positive. Multiplying by n yields the change in the project's social value as a result of greater rivalry.

Proposition 3 The social value of a public project of uncertain value increases as the degree of rivalry increases.

4. Changes in population

With a larger population, the coefficient ω is smaller, but the value of W is greater, resulting in potentially opposing effects on the private cost of risk bearing. Differentiating the individual risk premium with respect to n yields

$$d\pi/dn=[-\alpha\pi_\omega+c\pi_W]/n^2 \tag{17}$$

since $\omega_n=-\alpha/n^2$ and $W_n=c/n^2$. Differentiating Equation (4), which defines the risk premium, with respect to W yields $E_\theta[V'(W+\omega\theta)\cdot\theta]=V'(W+\omega\bar{\theta}-\pi)\cdot(1-\pi_W)$, which implies

$$\pi_W=1-E_\theta[V'(W+\omega\theta)\cdot\theta]/V'(W+\omega\bar{\theta}-\pi). \tag{18}$$

Substituting from Equations (6) and (18) into Equation (17) yields

$$n^2 d\pi/dn=-\alpha\left[\bar{\theta}-\frac{E_\theta[V'\theta]}{\bar{V}'}\right]+c\left[1-\frac{E_\theta[V']}{\bar{V}'}\right], \tag{19}$$

where $\bar{V}'=V'(W+\omega\bar{\theta}-\pi)$. It follows that

$$c\left[1-\frac{E_\theta[V']}{\bar{V}'}\right]<\alpha\left[\bar{\theta}-\frac{E_\theta[V'\theta]}{\bar{V}'}\right]=\alpha\bar{\theta}\left[1-\frac{E_\theta[V'\theta]}{\bar{V}'\bar{\theta}}\right] \tag{20}$$

is necessary and sufficient for the private cost of risk bearing to decline as the population increases.

Equation (17) reveals that, when all project benefits are non-rival and α equals zero, the sign of $d\pi/dn$ is the same as the sign of π_W, which is negative if and only if consumers exhibit decreasing absolute risk aversion. Inequality Equation (8), which is necessary and sufficient for $d\pi/d\alpha$ to be negative, implies that the final term multiplying $\alpha\bar{\theta}$ in Equation (20) is positive, while the term multiplying c is non-positive with CARA or DARA. In that event, $d\pi/dn$ is again negative.

Proposition 4 $d\pi/dn$ is negative, and the private cost of risk is lower with a larger population, if all benefits are non-rival and consumers exhibit DARA, or if some portion of benefits is rival, preferences exhibit CARA or DARA, and the cost of risk bearing declines with greater rivalry.

5. Conclusions

In the spirit of Arrow and Lind's analysis, we have reexamined the private and social costs of bearing the risks associated with public expenditure on projects of uncertain value. Beginning with the observation that the Arrow-Lind Theorem applies to projects whose benefits are provided by services that are wholly rival in consumption, we analyze the cost of risk for projects that provide a mix of rival and non-rival consumption benefits. Intuition suggests that both private and social costs of risk bearing decline as the degree of rivalry increases and as the size of the population increases, since these costs both vanish in the limit case with perfect rivalry and an infinite population. We confirm these intuitive predictions for selected economic environments, but show that if DARA is sufficiently strong these predictions fail to hold.

Despite these conflicting results, we show that the social cost of risk declines with greater rivalry when relative risk aversion is less than one and relative prudence is less than two, conditions that are necessary and sufficient for intuitive predictions regarding the effects of changes in risk in the portfolio problem. Additionally, we find that the social value of a public project increases for any finite population as the degree of rivalry increases.

Notes

1. Gollier and Pratt (1996) show that, in the presence of a zero-mean, independent background risk ε, V is more risk averse than U if U exhibits decreasing absolute risk aversion and decreasing absolute prudence.
2. Throughout, primes on univariate functions denote derivatives, and subscripts on multivariate functions denote partial derivatives.
3. See, for example, Gollier (2001, p. 25). Note that V inherits decreasing absolute risk aversion from U, but may not inherit CARA or IARA. See Gollier (2001, pp. 114ff).

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