A Neuro-Mimetic Dynamic Scheduling Algorithm for Control: Analysis and Applications

Harpreet S. Kwatra
Francis J. Doyle III
School of Chemical Engineering, Purdue University,
West Lafayette, IN 47907-1283 USA

Ilya A. Rybak
James S. Schwaber
Neural Computation Program, E.I. duPont de Nemours & Co.,
Wilmington, DE 19880-0328 USA

A simple neuronal network model of the baroreceptor reflex is analyzed. From a control perspective, the analysis suggests a dynamic scheduled control mechanism by which the baroreflex may perform regulation of the blood pressure. The main objectives of this work are to investigate the static and dynamic response characteristics of the single neurons and the network, to analyze the neuromimetic dynamic scheduled control function of the model, and to apply the algorithm to nonlinear process control problems. The dynamic scheduling activity of the network is exploited in two control architectures. Control structure I is drawn directly from the present model of the baroreceptor reflex. An application of this structure for level control in a conical tank is described. Control structure II employs an explicit set point to determine the feedback error. The performance of this control structure is illustrated on a nonlinear continuous stirred tank reactor with van de Vusse kinetics. The two case studies validate the dynamic scheduled control approach for nonlinear process control applications.

1 Introduction

A neuronal network architecture for scheduled control of nonlinear process systems was presented recently by Doyle III et al. (1994). The network was originally employed in a simple model of the mammalian baroreceptor reflex (Doyle III et al. 1997). This neuronal network was observed to exhibit a scheduled control activity. Therefore, it was proposed to apply the network in a dynamic scheduled control framework for applications in nonlinear process control. This article presents new results in two main categories: (1) qualitative and quantitative analyses of dynamics of both the single
neuron model and the network and (2) case studies of control of a conical tank and a nonlinear continuous stirred tank reactor (CSTR) with van de Vusse kinetics.

The baroreceptor vagal reflex is responsible for the regulation of blood pressure and displays a rich range of controlled dynamic behavior. The components of the reflex include pressure sensors in the major blood vessels, a neuronal processor, and several classes of motor actuators of the heart. The pressure signal is encoded by first-order neurons. This information is projected onto second-order neurons in the nucleus tractus solitarius (NTS) (Schwaber 1987), which process this information and modulate actuator signals to the heart to change cardiac rate, contractile timing and pattern, and contractile strength. The closed-loop behavior is fairly intuitive: as pressure rises above “normal,” a control signal is generated in the NTS, which lowers the heart rate.

The first-order sensory neurons are well described as highly sensitive, rapidly adapting neurons that transduce and encode each pulse of pressure with a train of spikes on the rising phase of pressure. Because of the distribution of pressure thresholds, the dynamic first-order neurons have a distributed sensitivity to the measured variable (blood pressure) and its rate of change. Data in the literature (Donoghue et al. 1982; Czachurski et al. 1988; Bradd et al. 1989; Rogers et al. 1993) suggest that these neurons map the spatial pressure distribution onto the inputs to the second-order neurons and “schedule” their activities during dynamic changes of the pressure. This observation and general principles of sensory system organization suggest that second-order neurons have a “competitive” response to the input from the first-order neurons. It is plausible that a lateral inhibition organization provides both the competition between the second-order neurons and their scheduled response. This competition may be simulated by a simple neuronal network model in which each second-order neuron responds and provides a control signal in a definite static (dependent on the thresholds of the corresponding first-order neurons) and dynamic (dependent on the velocity of pressure increase) range of pressure changes.

Such behavior can be exploited in the formulation of scheduling algorithms for use in controller design. Just as competition between second-order neurons leads to a selective dynamic response, this attribute can be utilized to apply the appropriate level of nonlinear feedback compensation for a control system. One particularly attractive feature of this biological algorithm is the graceful transition exhibited between operating regimes: there does not appear to be a discrete set of operating levels but rather a continuum. In addition, the transition is dynamic; it is dependent not only on the magnitude of the blood pressure but on its rate of change as well. These two characteristics of the algorithm are in direct contrast to traditional scheduling approaches, which utilize lookup tables for a discrete set of operating levels (Aström & Wittenmark 1989).
Figure 1: Simplified architecture of the baroreceptor vagal reflex. The lateral inhibitory function proposed for the inhibitory interneurons is implemented by the first-order neurons themselves for the sake of simplicity. Solid lines represent synaptic excitation; dashed ones signify inhibition. A $9 \times 6$ network of nine first-order and six second-order neurons is shown.

2 A Model of the Baroreceptor Reflex

An analysis of the experimental results (Rogers et al. 1993; Doyle III et al. 1997) reveals the following dynamic properties of the baroreceptor reflex control structure:

1. The second-order NTS neurons respond to blood pressure changes with bursts of activity that have a characteristic frequency much lower than the frequency of the cardiac cycle.

2. The responses suggest that NTS neurons are inhibited immediately after and perhaps immediately before the bursts.

3. It is reasonable to assume that this bursting activity is the source of regulatory signals that cause the compensatory changes in pressure.

4. It is plausible that each NTS neuron provides this regulation in a defined static and dynamic range of pressure.

These observations, combined with other physiological data and general principles of sensory system organization, support the plausibility of a simple baroreflex model. A diagram of the proposed network model for the closed-loop baroreflex is shown in Figure 1 (note that the focus of this model is restricted to the vagal baroreflex, which affects the cardiac output; the effects of the sympathetic system on the heart and the peripheral resistance in the vascular beds have been ignored). The first-order neurons, which are arranged in increasing order of pressure threshold, receive an excitatory input signal that is proportional to the blood pressure. The second-order neurons
receive synaptic excitation from the first-order neurons as depicted in Figure 1 by the solid lines. While the synaptic inhibition of the second-order neurons probably occurs via inhibitory neurons, a simplified model that does not use interneurons is employed, and their function is implemented using first-order neurons only. This is achieved by direct synaptic inhibition from the neighboring first-order neurons (i.e., the periphery of the receptive field; Hubel & Wiesel 1962), as depicted in Figure 1 by the dashed lines. A more realistic mechanism could employ inhibitory interneurons and reciprocal inhibition between the second-order neurons.

2.1 Model of a Single Neuron. Following the Hodgkin-Huxley formalism, the nonlinear dynamics of the membrane potential of a neuron can be described by the following differential equation (Doyle III et al. 1997):

$$c \dot{V} = g_0(V_r - V) + \sum_i g_i(E_i - V) + I \quad (2.1)$$

where $c$ is the membrane capacitance, $V$ is the membrane potential, $V_r$ is the resting membrane potential, $g_0$ is the generalized conductance, $g_i$ is the deviation in the conductance of the $i$th ionic channel, $E_i$ is the reversal potential of the $i$th ionic channel, and $I$ is the input current. Three types of conductances ($g_i$) are used in the current model: conductances for excitatory and inhibitory synaptic channels that are opened by action potentials (AP) coming from other neurons and a conductance for the potassium channel opened by AP generation in the neuron itself. The potassium channel conductance, threshold dynamics, and neuronal output are modeled by additional nonlinear differential equations. A detailed description, along with the parameter values, is available in Doyle III et al. (1997).

Although the model represents a considerable simplification of the more extensive multiple channel neuron kinetic models (Schwaber et al. 1993), it describes the behavior of the baroreflex neurons with sufficient accuracy for the purpose of network modeling.

Analysis of Dynamics of Single Neurons. The neuron model above is composed of several complex nonlinear differential equations. It is extremely difficult to interpret the input-output behavior of the model directly from these differential equations. This motivates an analysis of the dynamics of the neuron model in terms of its steady-state locus and dynamic response. Such an analysis validates the model against known experimental results, and also facilitates a system-level interpretation of the model. This interpretation is useful when the neuron model and its network are employed in process control architectures.

The response of a single first-order neuron model to several types of input was examined by computer simulation. The first simulation, shown in Figure 2a, demonstrates the well-known nonlinear steady-state behavior of the baroreceptor neurons. Below the threshold blood pressure of 105 mmHg,
Figure 2: (a) Nonlinear steady-state behavior of a first-order neuron (baroreceptor). Spiking frequency is calculated as the inverse of the interspike interval. (b) Dynamic response of a first-order neuron to positive and negative step changes in blood pressure. The upper graph is the trace of the spiking frequency calculated as the inverse of the interspike interval. The lower graph shows the blood pressure signal. Continued.
there is no response; the neuron starts spiking once the blood pressure exceeds the threshold value. Note that physiologically, there are about a hundred baroreceptors per nerve with pressure thresholds distributed from well below (35 mmHg) to well above (170 mmHg) the range of resting mean arterial pressure (Kirchheim 1976; Kumada et al. 1990). It is this distributed sensitivity to pressure that lends itself to the scheduling response of the neuronal network described in section 2.2.

Figure 2b shows the response of the neuron to positive and negative step changes in blood pressure. The dynamic nature of the neuron is evident here as the sudden rise in blood pressure results in a large increase in the spiking frequency, which then decays down to a lower value (adaptation). Correspondingly, a drop in blood pressure causes the neuron to stop spiking altogether, and then recover to the steady-state spiking frequency (postexcitatory depression). These characteristics validate the neuron model against published experimental data (Brown et al. 1976; Kirchheim 1976). A typical electrophysiological experiment for a rat using a step in blood pressure of 10 mmHg was observed to raise the baroreceptor discharge from around 35 Hz to 75 Hz, which then adapted to around 50 Hz (Brown 1980). Similarly, the negative step in pressure caused a marked depression in discharge with subsequent recovery to the original steady-state spiking frequency.
Figure 2c depicts the response of the neuron to an input signal that is similar to the physiological blood pressure trace. The pressure signal has a sharp rise, a slower fall, and a time period of around 150 ms (in the range of the cardiac period of a rat; Doyle III et al. 1997). The figure shows the neuron to fire action potentials mostly on the rising phase of the pressure pulse, which is consistent with observed behavior (Kirchheim 1976; Karemaker 1987). This sensitivity to the rate of change of pressure (or dP/dt) critically contributes to the dynamic nature of the scheduling observed for the neuronal network described in the next section.

2.2 A Network of First-Order and Second-Order Neurons. The general architecture of a neuronal network with excitation and inhibition was discussed in the beginning of section 2. Specifically, a $4 \times 1$ neuronal structure, in which four first-order neurons converged onto one second-order neuron, was employed as a basic block for the network. Allowing for overlap between the first-order neurons, six of these blocks were combined to yield the neuronal network shown in Figure 1. Note that in this network, the second-order neurons “share” the inputs from the first-order neurons, so as to provide a narrower pressure sensitivity. As will be seen below, such a network architecture gives rise to a scheduled response from the second-order neurons, in specific narrow ranges of the input signal.

Analysis of Dynamics of the Network. Just as for the individual neuron models, the response of the network is nonlinear and complex. In fact, the inhibition-excitation architecture of the network makes its response even more complicated and difficult to predict. Once again, this motivates an analysis of the dynamics of the network before it is employed in a simulation of the baroreceptor reflex.

Static Response of the Network. First, the static response of the network is analyzed. By static, it is meant that the rate of change of the input signal is small. To implement this analysis, the input (blood pressure) is ramped up very slowly (at the rate of 0.6 mmHg/sec), and the action potentials for each of the second-order neurons are recorded. Note that the input signal drives the nine first-order neurons of Figure 1, and the output signals from these first-order neurons in turn drive the six second-order neurons. Figure 3a shows that the static response of the neurons is distributed in pressure space. The second-order neurons of the network fire in specific ranges (receptive fields) of the blood pressure input signal. This aspect of the network enables it to “schedule” the network input signal to the output of the network and may be exploited in algorithms for control. In fact, the shapes of the response graphs resemble one-dimensional radial basis functions (Hunt et al. 1992) and could be employed in a similar manner for modeling and control.

Dynamic Response of the Network. By virtue of the fact that the first-order neurons in the network are intrinsically dynamic (see section 2.1), the outputs of the network exhibit significant dynamic behavior as well. This is demonstrated in Figure 3b, in which the responses of the second-order neu-
Figure 3: (a) Static receptive fields of the six second-order neurons of Figure 1. (b) Dynamic receptive fields of the six second-order neurons of Figure 1, at 10 Hz.

...urons are plotted for the case when the network is driven by a sinusoid of frequency 10 Hz (close to the heart rate of a rat), added to a slowly ramping input signal (at the rate of 0.6 mmHg/sec). This input excites the dynamics of the network at the frequency of the sinusoid. The dynamic responses...
appear sparser than the corresponding static ones seen in Figure 3a, indicating decreased overall network activity for inputs with high-frequency content. More interestingly, the figure shows that the response from each of the second-order neurons spreads beyond the static response range. For instance, the static receptive field for the lower-most neuron ranges from 3.0 to 6.0 units of input level, while the dynamic receptive field is spread from 3.0 to 7.2 units. Due to the overlap in the dynamic receptive fields observed in Figure 3b, more than one second-order neuron may be active at any time depending on the frequency content of the input signal. If the input level is rising, a second-order neuron may fire even when the input value is below its static range of response. This may happen because the frequency content in the input signal might be sufficient to excite its dynamics. Note that while the scheduling functionality of the network is due to its excitation-inhibition architecture (see Figure 1), the sensitivity to the dynamic component of the input is attributable to the dP/dt sensitivity of the first-order neurons.

From the point of view of control, this dynamic nature of the network is especially relevant. The scheduling function of the network will have a dynamic component as well. In traditional control theory, gain scheduling is restricted to slowly time-varying scheduling variables, as changes in these are not explicitly taken into account in the control design. The dynamic network above would be expected to have interesting dynamic scheduling properties that could be exploited to optimize control performance in process system applications.

2.3 A Simple Model of the Cardiovascular System. The previous sections described the models of the first- and second-order neurons and the neuronal network. Two more blocks have to be specified to complete the model of the baroreceptor reflex.

First, the sum of the outputs from the second-order neurons forms the input for an intermediate subsystem (see Figure 1), which is modeled as a simple linear filter. This dynamic system captures the effect of the neural circuitry that lies between the second-order neurons and the heart. This includes the interneurons in the NTS (Spyer 1994) and the motor neurons in the nucleus ambiguus (Loewy & Spyer 1990).

Next, a first-order approximation of the cardiovascular dynamics is described. The blood pressure decays exponentially from a current level to the level $P_0$ with the time constant $\tau_P$. At selected time points, the pressure responds with a jump to the level $P_m$ in response to the pumping action of the heart. One of the driving forces for this subsystem is the disturbance $P_i$, which represents the effects of an external agent (e.g., a drug infusion). The second input is the feedback signal from the intermediate subsystem. A more detailed description is available in Doyle III et al. (1997).

On the afferent side, the input signal to the first-order neurons depends on the pressure $P$ via the amount of stretch in the blood vessels, modeled here by a simple linear relationship, $I = k_P P$, where $k_P$ is a “tuning” coefficient.
2.4 Computer Simulation of the Baroreceptor Reflex. The analyses of the dynamics of the single neuron and the network provide a reasonable starting point for a simulation of the simple model of the baroreceptor reflex. For the model described, a network with four first-order neurons and one second-order neuron is employed. In the general case of multiple second-order neurons, the sum of the outputs from the second-order neurons may be used.

Figure 4 shows the responses of the four first-order neurons (the top two rows) and one second-order neuron (the lower left trace) to a fluctuating pressure signal (the bottom right trace). The cardiovascular system experiences a positive disturbance at time 500 ms, which causes the mean pressure to rise. Due to the barotopical distribution of thresholds, the first-order neurons respond sequentially to increasing mean blood pressure. Hence, the first-order neuron with the lowest threshold (number I) displays the greatest amount of activity. The middle pair of first-order neurons (numbers II and III) excites the second-order neuron, while the other two first-order neurons (numbers I and IV) inhibit it.

When the feedback loop is open (not shown here), the persistent disturbance drives the mean pressure to levels much higher than the ones seen in Figure 4. When the feedback loop is closed, the second-order neuron provides the pressure control. As the pressure enters the sensitive range of the second-order neuron, a signal burst is sent to the intermediate block. This block drives the heart with a negative feedback signal, leading to a tem-
porary decrease in the pressure level. The persistent external signal drives the pressure up again, and the trend is repeated. The low-frequency bursts exhibited by the second-order neuron are similar to the electrophysiological recordings for NTS neurons (Rogers et al. 1993).

While the network behavior of the proposed baroreflex model is a reasonable first approximation of the experimentally recorded neuronal behavior (Doyle III et al. 1997), there is ample scope for refinement. However, the simulation successfully illustrates the dynamic scheduled response of a network of complex neurons and how the neurons in this specific network can be employed to model and predict the behavior of the baroreceptor reflex. This architecture of the network can be exploited in process control algorithms, the subject of discussion in the second half of the article.

3 Applications to Nonlinear Process Control

From a control perspective, an interesting feature of the network model of section 2.2 is that individual second-order neurons are active in a narrow static and dynamic range of pressure changes. While the scheduling activity is a result of the network architecture, its dynamic nature is due to the intrinsic behavior of the neurons themselves. These static and dynamic components of the scheduling activity were analyzed in section 2.2.

The scheduling functionality of the neuronal network described by the receptive fields in Figures 3a and 3b is of a discrete nature. While the overlapping of the dynamic receptive fields may provide smoother transitions among the scheduled outputs or channels, the range in which a channel is active is nevertheless discrete (e.g., the lower-most neuron in Figure 3b is active from 3.0 to 7.2 units of input). This discrete nature can be traced back to the first-order neurons (baroreceptors), which are active only above their pressure thresholds. Motivated by this insight, a discrete gain scheduling approach is adopted in the current control design. Note, however, that the baroreceptor reflex employs a large number of baroreceptors and NTS neurons such that the discrete scheduling would approach a continuous nonlinear scheduling function. Based on the last observation, a dynamic gain scheduling controller of the continuous nonlinear form is being researched (Kwatra & Doyle III 1995).

Gain scheduling, which has been in use in the chemical process industry for more than a decade, is a powerful method for compensating for process nonlinearities when some measurable indicator of changing plant dynamics is available (Äström & Wittenmark 1989). It may then be possible to eliminate the influence of varying dynamics by changing the parameters of the regulator as a function of the measured scheduling variable. Typical applications include distillation column control (Shinskey 1977), pH control (Gulaian & Lane 1990), and batch process control (Luyben 1990). Experience has shown gain scheduling to be both effective and easy to use for process
systems (Åström & Hägglund 1990). Given its success, any improvements that build on its strengths would be clearly motivated.

In classical gain scheduling, the gains (or other controller parameters) are scheduled according to the instantaneous values of the scheduling variable, and no convolution over time is involved. Even if the scheduling variable measures dynamic aspects of the system, variations in it are not explicitly dealt with during control design. For instance, although aircraft velocity (Mach number) is often used as the exogenous scheduling variable for the control of aircraft acceleration (Reichert 1992), the converse is not true; that is, acceleration is not normally employed as a scheduling variable for the control of aircraft velocity. Typically each controller is obtained for a fixed value of the scheduling variable, independent of past values and time. This feature of the traditional gain scheduling design methodology is directly responsible for its limitation to slow variations in the scheduling parameter (Shamma & Athans 1992). By contrast, the scheduling activity of the network of neurons of the earlier sections has a distinct dynamic component. The dynamic scheduling activity of the network holds promise for exploitation in scheduling algorithms for control of nonlinear process systems, as shown below.

Two control architectures are described. First, a control scheme inspired by the baroreceptor reflex model is used to implement disturbance rejection in a conical tank. The shortcomings of this control scheme are addressed in a second control architecture that uses error feedback and a set point. This control scheme is demonstrated on a nonlinear CSTR with van de Vusse kinetics.

3.1 Control Structure I. The first control structure (see Figure 5) is based on the network control scheme used in the simulation of the baroreceptor reflex (see section 2.4). The cardiovascular system of Figure 1 is replaced

Figure 5: Schematic of control structure I.
by the nonlinear process being controlled, with the output scaled to match the pressure variable that is fed to the network. The neuronal network is identical to that used in the baroreflex simulation except that the outputs of the second-order neurons are passed through proportional-integral (PI) controllers before being summed. The PI controllers facilitate additional tuning specific to the nonlinear process being controlled. The intermediate subsystem is the same as before: a linear first-order filter. This control design was tested on a simple nonlinear process system. The system under consideration is a first-order process with a gain that varies nonlinearly as a function of the process output. A physical example of such a system is a level-control problem in a tank with a cross-sectional area that varies with height (e.g., a conical or spherical holding tank).

**Case Study: Level Control in a Conical Tank.** Holding tanks that have variation in cross-sectional area as a function of height are common in the chemical process industry. A conical tank model demonstrates control structure I. The differential equation of a conical tank can be written as

\[ \tau \frac{dy}{dt} = -y^2 + k y^2 u + d, \]  

where \( y \) (controlled variable) is the level of the fluid in the tank, \( u \) (manipulated variable) is the volumetric feed rate to the tank, \( d \) is a disturbance input, and \( \tau \) and \( k \) are constants. This model was obtained from a material balance on the tank, assuming that the discharge coefficient is proportional to \( \sqrt{y} \). Note that the first two terms on the right-hand side of equation 3.1 are nonlinear in \( y \).

Regulator control of the conical tank was implemented with control structure I. The gains of the PI controllers were tuned so as to yield identical local responses around several operating points, as is done in input-output linearization techniques (Isidori 1989). The integral action of all the controllers was identical and chosen such that the disturbance was rejected within 60 sec. The simulation of the control system for a large negative disturbance is shown in Figure 6. The control for the same disturbance using a linear controller obtained by internal model control (IMC) design techniques (Morari & Zafiriou 1989) for the steady-state operating point is also shown. The graph suggests that the dynamic network controller adjusts for the rising fluid level in the tank and prevents the overshoot that occurs for the linear control. This dynamic aspect of the scheduling activity is of interest in scheduling control algorithms.

Control structure I suffers from several shortcomings. First, although it demonstrated reasonable regulator control (disturbance rejection) in this example, it has no facility to implement servo control (set point tracking), a necessary component of most chemical process control applications because grade changeovers and set point adjustments are common. Second, the individual controllers in the feedback loop are driven by the outputs of
Figure 6: Regulator response of a conical tank controlled by a bank of PI controllers, which was dynamically scheduled by a $6 \times 3$ network.

Figure 7 shows the block diagram of control structure II. The scheduling network is grouped together in a block labeled “scheduler,” and the control elements are placed in the “regulator.” For instance, in the level control problem, the regulator block would simply contain the PI controllers, and the scheduler would have the network of

A second control structure was developed to overcome these problems. While control structure I was consistent with the neurobiological bases in the baroreceptor reflex model, control structure II was modified to meet the requirements of typical process control problems.

### 3.2 Control Structure II

Figure 7 shows the block diagram of control structure II. The scheduling network is grouped together in a block labeled “scheduler,” and the control elements are placed in the “regulator.” For instance, in the level control problem, the regulator block would simply contain the PI controllers, and the scheduler would have the network of

the second-order neurons, as opposed to the usual error feedback signal. The implication is that zero offset is not guaranteed in this control design.
neurons. For the general control problem, the regulator block would contain an array of an appropriate type of controllers, such as PID controllers, and IMC controllers, and the scheduler block would have a suitably sized network of first-order and second-order neurons.

A second, and perhaps debatable, modification is the introduction of a set point signal in the control loop. This is necessary from the process control point of view, because almost all process control problems involve setting and adjusting the set point to achieve the desired output from the process. From the point of view of neurophysiology, this is less justifiable because there is no known explicit set point for the baroreceptor reflex. However, since it is known that the central nervous system does have signals converging onto the crNTS (cardiorespiratory NTS) and the medulla (Spyer 1994), it may be reasonable to speculate that some type of implicit set point signal is conveyed to the network of second-order and higher-order neurons of the reflex.

The auxiliary or scheduling variable from the process is fed back to the scheduler block, which contains the dynamic scheduling network of neurons. In Figure 7, the process output itself serves as the scheduling variable. Within the scheduler block, this signal forms the input to the first-order neurons. The outputs of the second-order neurons are passed onto the regulator block. The values of these outputs or channels vary between zero and around unity—zero denoting that the channel is inactive and unity indicating that the channel is fully active. This scheduling activity of the network is dynamic, as illustrated in section 2.2. As a result, two neighboring channels may be partially active as the scheduling variable makes the transition from one level to another. In addition, there is sensitivity to the rate of change of the scheduling variable, which makes the scheduling functionality truly dynamic.

The feedback error signal is obtained by subtracting the process output from the set point and is then fed into the regulator block along with the scheduler channel outputs. Within the regulator block, each of the channel outputs is simply multiplied with the error signal and fed into the controller corresponding to that channel (see Figure 8). This activates the controllers corresponding to the active scheduler channels, in proportion to their level of activity. Hence, if only one channel is active, the error signal gets multiplied by unity and fed into the controller corresponding to that channel. If two neighboring channels are active with, say, outputs of 0.5 unit each, the error signal gets equally divided between the two corresponding controllers. The output from all the controllers is added to give the control input for the process.

This feedback control loop is applied to a nonlinear chemical reactor in the following section.

Case Study: Dynamic Scheduled Control of a Nonlinear CSTR. A Reaction with van de Vusse Kinetics. van de Vusse (1964) described a chemical reaction mechanism that has recently been used by several researchers as
Figure 8: Exploded view of the regulator block in control structure II.

a benchmark for nonlinear studies (Kantor 1986; Doyle III & Morari 1992; Engell & Klatt 1993). It involves both a consecutive and a parallel reaction that give rise to undesired by-products. The reaction equation can be written as:

$$A \xrightarrow{k_1} B \xrightarrow{k_2} C \quad 2A \xrightarrow{k_3} D.$$  

(3.2)

The control objective, then, is to maximize the rate of production of the desired product ($B$) and to minimize the rates of production of the by-products ($C$ and $D$) by way of manipulation of the catalyst and the reaction conditions (Kantor 1986).

Engell and Klatt (1993) describe a specific example of the above chemical reaction in which cyclopentenol ($B$) is obtained from cyclopentadiene ($A$) by acid-catalyzed electrophilic addition of water in dilute solution. The undesired by-products are dicyclopentadiene ($D$) and cyclopentanediol ($C$). The reaction kinetics employ rate expressions for $k_1$, $k_2$, and $k_3$ that obey the Arrhenius equation. The coolant dynamics can be neglected, leading to a third-order nonlinear dynamic system. This gives rise to a single input–single output (SISO) system in which the concentration of the desired product $B$ is controlled by manipulating the inflow of $A$ to the reactor. The differential equations describing the system and the particular parameters used in the simulations here are given in Engell and Klatt (1993).

**IMC Controller Design.** The IMC design methodology was chosen to obtain the bank of controllers due to its relative ease in derivation and tuning (Morari & Zafiriou 1989). The CSTR dynamics were linearized around six...
Table 1: Transfer Function Gains, Zeros, and Poles

<table>
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<tr>
<th>Transfer function</th>
<th>K</th>
<th>z1</th>
<th>z2</th>
<th>p1</th>
<th>p2</th>
<th>p3</th>
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<td>-2.871e-05</td>
<td>-0.0074</td>
<td>0.0235</td>
<td>-0.0357</td>
<td>-0.0136 + 0.0056i</td>
<td>-0.0136 - 0.0056i</td>
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<td>0.0282</td>
<td>-0.0333</td>
<td>-0.0122 + 0.0047i</td>
<td>-0.0122 - 0.0047i</td>
</tr>
<tr>
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<td>0.0328</td>
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</tr>
<tr>
<td>IV</td>
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<td>-0.0091 + 0.0026i</td>
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<tr>
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<td>-0.0165</td>
<td>-0.0053 + 0.0009i</td>
<td>-0.0053 - 0.0009i</td>
</tr>
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equally spaced points in the operating region of 0.8 to 1.0 mol/ℓ product concentration C_B (Engell & Klatt 1993). The resulting six transfer functions were of the type

\[ G(s) = K \frac{(s - z_1)(s - z_2)}{(s - p_1)(s - p_2)(s - p_3)}. \]  

(3.3)

The zero \( z_2 \) is in the right \( s \)-half plane, while the other zero and the poles are all in the left \( s \)-half plane (see Table 1). Integral square error (ISE) optimal IMC controllers were derived for the six transfer functions, and first-order filters were appended to them. These are equivalent to classical feedback controllers of the form

\[ C(s) = \left( -1/K \right) \frac{(s - p_1)(s - p_2)(s - p_3)}{s(s - z_1)(\lambda s + \lambda z_2 + 2)}. \]  

(3.4)

The six IMC-type controllers were placed in the “regulator” block of the feedback control loop of Figure 7. The IMC filter time constant \( \lambda \) was set to 20 seconds.

**Simulation Results.** The IMC-type controllers in the regulator block are dynamically scheduled by a 9 \( \times \) 6 network of nine first-order and six second-order neurons placed in the scheduler block. The simulation is implemented using MATLAB software. Four simulation runs are presented here: three with set point changes in the concentration of \( B \) (servo problem) and one with a disturbance input in the inlet concentration of \( A \) (regulator problem).

The parameters for the CSTR model, set point changes, and disturbance input were chosen to be identical to those used by Engell and Klatt (1993) to facilitate a direct comparison. Their approach was based on traditional static gain scheduling. Their simulation results are reproduced here and are characterized by overshoots of up to 60% and response times of about 0.15 to 0.30 hour.
The first dynamic scheduled control simulation using the network of neurons and IMC-type controllers is for a step change in set point of $C_B$ from 0.85 to 0.95 mol/ℓ. Figure 9 shows that $C_B$ rises to 0.95 mol/ℓ with a response time of about 0.1 hour and with no significant overshoot. This is distinctly better than the static scheduling approach, which had an overshoot of about 40%.

The second simulation is for a set point change in $C_B$ from 0.85 to 0.86 mol/ℓ. The response time for this was around 0.1 hour, with no appreciable overshoot (see Figure 10). This compared favorably with the static scheduling approach, which had a response time of 0.3 hour and around 60% overshoot. In addition, the static scheduling control displayed excessive oscillations.

Figure 11 shows the case of set point change in $C_B$ from 0.95 to 0.85 mol/ℓ. It shows an inverse response, which is larger than that of the static scheduling approach; however, it is improved in the sense that it has no overshoot and a smaller response time (see Table 2).

Finally, Figure 12 shows the simulation of the regulator problem in which the control loop attempts to reject the step disturbance in $C_{AO}$ from 4.5 to 5.7 mol/ℓ. The result is better in the dynamic scheduled control case; there is no overshoot again, and the response times are comparable.

Discussion. A characteristic feature of the dynamic scheduled control simulations is that the controlled output trajectories are not smooth. This is a direct outcome of employing neurons in the scheduling network; the firing
Figure 10: Servo control for a step set point change in $C_B$ from 0.85 to 0.86 mol/ℓ; $C_{AO}$ is 5.7 mol/ℓ.

Figure 11: Servo control for a step set point change in $C_B$ from 0.95 to 0.85 mol/ℓ; $C_{AO}$ is 4.5 mol/ℓ.
Table 2: Summary of Results for the Nonlinear CSTR Case Study

<table>
<thead>
<tr>
<th>Magnitude of input</th>
<th>Settling time</th>
<th>Overshoot</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Static</td>
<td>Dynamic</td>
</tr>
<tr>
<td></td>
<td>scheduling (h)</td>
<td>scheduling (%)</td>
</tr>
<tr>
<td>0.10 set point</td>
<td>0.16</td>
<td>0.1</td>
</tr>
<tr>
<td>0.01 set point</td>
<td>0.30</td>
<td>0.1</td>
</tr>
<tr>
<td>−0.10 set point</td>
<td>0.15</td>
<td>0.1</td>
</tr>
<tr>
<td>0.12 disturbance</td>
<td>0.15</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Figure 12: Regulator control for a step disturbance in \( C_{A0} \) from 4.5 to 5.7 mol/ℓ; set point \( C_B \) is 0.9 mol/ℓ.

of the neurons is not smooth but is composed of spikes and intermediate durations of inactivity. Smooth process curves are desirable, and future work will incorporate a low-pass filter to accomplish this.

The individual linear controllers employed in the regulator block are themselves guaranteed stable functions due to the IMC methodology used. Around the steady states that each one of them is designed at, they provide stable closed-loop control, optimal in the integral square error sense. However, away from the steady states and during transitions, stability and performance would need to be inferred from extensive simulations or robustness analysis.
Overall, the dynamic scheduled control scheme performed better than the traditional static gain scheduled approach employed by Engell and Klatt (1993), especially in terms of reduced process overshoot. Conceptually, this is understandable, because the dynamic scheduler compensates for a controlled variable that is changing quickly and therefore prevents an overshoot. The simulations demonstrate the viability of the dynamic scheduled control scheme for process control problems. The next step would be to apply the algorithm to a complex nonlinear control problem of industrial relevance.

4 Conclusion and Future Work

This article consists of two main parts. The first part describes models of neurons and networks, and their application in simulation of the baroreceptor reflex. Dynamics of the single neuron model are analyzed by simulations, which revealed remarkable similarity of the model responses with the static and dynamic characteristics of the mammalian baroreceptors. A network of first-order and second-order neurons is simulated, and the static and dynamic receptive fields of the second-order neurons are traced. This analysis provided a basis for building a simple network model of the baroreceptor reflex. The proposed neuronal network model exhibits control of blood pressure disturbance in a fashion similar to that observed experimentally (Doyle III et al. 1997). These simulation and analysis results are the first of their kind in modeling efforts of the baroreceptor reflex.

The second part of the article reports on application of the network model of the baroreflex to nonlinear process control problems. The dynamic scheduling activity of the network is exploited in two control architectures. Control structure I is directly abstracted from the baroreceptor reflex model and is demonstrated on a nonlinear tank level control problem. Control structure II is developed to provide tools that are necessary for meaningful application in process control. This includes introduction of a set point and use of the feedback error signal. The control structure is illustrated on a nonlinear CSTR with van de Vusse kinetics. The simulation results show better control than traditional gain scheduling, especially in terms of reduced process overshoot. The two case studies validate the usefulness of the dynamic scheduled control approach.

A few comments on the issue of closed-loop stability for the dynamic scheduled control algorithm are relevant at this point. As is the case with classical gain scheduling, only local stability can be easily guaranteed, and global stability is inferred by extensive simulations. In general, global stability can be guaranteed under the conditions of sufficiently slow variations in the scheduling variable (Shamma & Athans 1992), but these bounds are difficult to obtain and usually very conservative. In light of the fact that the dynamic scheduled control approach implicitly accounts for fast changes in the scheduling variable, it is plausible that the related bounds on the
scheduling variable rate of change would be less conservative. Also note that for nonlocal operation of the controllers, their net contribution is a weighted (between 0 and 1) sum of the individual control actions. Thus, the weights themselves will not lead to unbounded action and the associated instability. A rigorous proof of stability during these transitions shall be obtained with potentially conservative conic sector bounding (Doyle III et al. 1989) in future work. The formulation of such bounds will be facilitated by the structured form of the dynamic scheduling controller; equal to a weighted contribution of several linear controllers. The nonlinearity arises from the dynamic nature of the weighting function. Since each weighting function $w_i$ is bounded (between 0 and 1), it can be cone bounded using a time-varying gain $\Delta_i$ as:

$$
\dot{x}_i = A_0 x_i + \Delta_i A_1 x_i + b_i u_i \\
y_i = c_i x_i.
$$ (4.1)

If the process being controlled can be similarly conic sector bounded using a time-varying gain, say, $\Delta_p$, then it would be possible to combine all the time-varying gains in the closed-loop in the M-$\Delta$ standard structured uncertainty form (Doyle 1982). Using extensions for time-varying and nonlinear uncertainty (Doyle III et al. 1989), conservative stability conditions could be derived.

The success of the dynamic gain scheduling algorithm is primarily due to the fact that while the traditional scheduled control approaches are built on the slowly varying scheduling variable assumption, the current approach employs an explicitly dynamic scheduling mapping. In the neuronal network implementation of the dynamic scheduled control algorithm, the dynamic scheduling is realized by the dynamic nature of the neurons, especially the $\text{d}P/\text{d}t$ sensitivity of the first-order neurons. Note that although neural net–based scheduling schemes (such as a radial basis function neural net) would lead to a simpler controller by avoiding the complex neuron dynamics, these would typically implement a static scheduling controller. Although dynamics could be introduced in the net by using feedback, the nodes are nevertheless static operators. The neuronal network used in this article employs dynamic neuronal blocks or nodes and is therefore inherently dynamic. On the other hand, all of the complex neuron dynamics may not contribute to the performance of the network controller. For the actual control of nonlinear process systems, the use of such neuronal networks may be unrealistic. Therefore, the underlying principles of the dynamic scheduled control exhibited by the above neuronal network need to be extracted and implemented in a nonlinear control framework (Kwatra & Doyle III 1995).

Through continued collaborative research activity, future work will seek to (1) increase the level of complexity in the neuron models to include multiple channel kinetics, (2) refine the system level models such as the model of the cardiovascular system, (3) analyze robustness properties of the pro-
posed dynamic scheduled control architectures, and (4) extract the underly-
ing principles of the dynamic scheduled control exhibited by the neuronal
network and apply them to nonlinear control theory.

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References

Åström, K. J., & Hägglund, T. (1990). Practical experiences of adaptive tech-
Addison-Wesley.
Bradd, J., Dubin, J., Due, B., Miselis, R. R., Monitor, S., Rogers, W. T., Spyer, K. M.,
& Schwaber, J. S. (1989). Mapping of carotid sinus inputs and vagal cardiac
Circulation Research, 1(46), 1–10.
discharge in normotensive and spontaneously hypertensive rats. Circulation
Research, 39, 488–496.
Morphology of electrophysiologically identified baroreceptor afferents and
stem projections of aortic baroreceptor afferent neurons in nodose ganglia of
proaches for chemical reactor control. DYCORD+ 92, IFAC, pp. 207–212.
(1997). Neuronal modeling of the baroreceptor reflex with applications in
process modeling and control. In D. Elliott (Ed.), Neural networks for control.
New York: Springer-Verlag.
motivated dynamic nonlinear scheduling algorithm for control. Proc. of the
American Control Conference, Baltimore (pp. 92–96).
Proc. of the American Control Conference, San Francisco (pp. 2941–2945).

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