

1 Uniform thermal expansion reactivity evaluation based on reactivity coefficient for
2 sodium-cooled fast reactor

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24 **ABSTRACT**

25 *This paper presents a method for calculating reactivity changes caused by uniform thermal expansion in*
26 *sodium-cooled fast reactors using reactivity coefficients. The reactivity changes due to both Z-axis and radial*
27 *expansion are separated into two effects: the shape change effect and the macroscopic cross-section change*
28 *effect. These effects are expressed using the reactivity coefficients of materials in the reactor core. The*
29 *reactivity change due to Z-axis expansion considers the expansion of the fuel, cladding, and wrapper tube,*
30 *while the radial expansion considers the fuel pin, structural materials, and reactor core support structures.*
31 *To confirm the accuracy of this approach, calculations were performed for simplified reactor geometry with*
32 *different height-to-diameter ratios. The results show that reactivity changes behave almost linearly for*
33 *expansions of up to 2%. Additionally, the difference between the expansion reactivity obtained by using*
34 *reactivity coefficients and those by direct calculations considering core expansion is within a few percent.*

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35 **1 Introduction**

36 Various types of sodium-cooled fast reactors have been studied, and it is known
37 that the nuclear characteristics, such as sodium void reactivity, differ significantly
38 depending on the reactor design [1-5]. Therefore, in the design of fast reactors, transient
39 analysis is generally performed to ensure core safety during accidents such as
40 unprotected transient overpower [6,7]. In transient analysis of severe accidents for a fast
41 reactor, it is important to evaluate the reactivity introduced by the expansion of the core
42 and changes in fuel and coolant temperatures. In the neutron transport calculation, the
43 reactivity feedback introduced by changes in fuel and coolant temperature can be
44 accounted for by using cross sections corresponding to the transient conditions. However,
45 the geometric change caused by the expansion of the core requires a separate evaluation
46 from neutron transport calculations. A direct approach to account for geometric changes
47 is the coupling of neutron transport calculation with structural analysis codes. Directly
48 accounting for geometric changes in neutron transport calculations requires performing
49 neutron transport calculations for continuously changing geometry. As a result, the
50 computational time increases significantly. Therefore, it is effective to evaluate expansion
51 reactivity based on reactivity coefficients in core design.

52 A method that uses reactivity coefficients to account for the assembly bowing due
53 to changes in the temperature distribution within the fuel assemblies has been proposed
54 [8,9]. This method is based on the perturbation theory, and the verification was
55 conducted using Monte Carlo calculations [9]. In addition to the assembly bowing, there
56 are expansions, such as the vertical expansion of the reactor core due to increased fuel

57 temperature, and the expansion of reactor core support plates due to the increase in inlet
58 coolant temperature, which are significant in the analysis of the fast reactor Monju [7].
59 For sodium-cooled fast reactors using oxide fuels, formulas for calculating these
60 expansion reactivities are provided [10]. However, the assumptions used in deriving the
61 reactivity evaluation formula based on reactivity coefficients, and the approximation
62 errors have not been sufficiently discussed.

63 In our previous study [11], as part of verification of calculations using reactivity
64 coefficients, we evaluated an error due to reactivity component separation using the
65 Monte Carlo code MVP [12]. However, the results include statistical errors, and the
66 calculation did not utilize reactivity coefficients. Therefore, in this paper, the aim is to
67 evaluate the accuracy of calculations using reactivity coefficients by using the diffusion
68 theory code CITATION [13]. In this study, to conduct an fundamental investigation of
69 expansion reactivity based on reactivity coefficients, we assume that the core expands
70 uniformly in the Z-axis or radial direction, and non-uniform expansions, such as bowing,
71 are left for future work. Section 2 presents the derivation of Z-axis expansion reactivity
72 using reactivity coefficients. Section 3 presents the derivation of radial expansion
73 reactivity. Section 4 shows the differences from formulas derived in previous study [10].
74 Section 5 discusses the errors in calculations using reactivity coefficients. Section 6
75 presents the conclusions.

76

77 **2 Derivation of Z-axis expansion reactivity**

78 In this study, the shape change effect is defined as the reactivity introduced due
79 to a change in the reactor core height while keeping the macroscopic cross-section
80 constant. The macroscopic cross-section change effect is defined as the reactivity
81 introduced due to a change in the macroscopic cross-section while keeping the reactor
82 core height constant. The total reactivity inserted when the core expands along Z-axis can
83 be approximately separated into the shape change effect and the macroscopic cross-
84 section change effect as

$$\rho_z = \frac{\{k(H', \Sigma') - k(H, \Sigma)\}}{k(H', \Sigma')k(H, \Sigma)} \approx \rho_{z,s} + \rho_{z,\Sigma}, \quad (1)$$

85 where, H and H' represent the core height before and after expansion, respectively, and
86 Σ and Σ' are the macroscopic cross-sections before and after expansion. The term $\rho_{z,s} =$
87 $\frac{k(H', \Sigma) - k(H, \Sigma)}{k(H', \Sigma)k(H, \Sigma)}$ represents the geometric change effect, while $\rho_{z,\Sigma} = \frac{k(H, \Sigma') - k(H, \Sigma)}{k(H, \Sigma')k(H, \Sigma)}$
88 represents the macroscopic cross-section change effect.

89 The reactivity change due to the variation in core height can be expressed as:

$$\rho_{z,s} = K_H \frac{\Delta H}{H}, \quad (2)$$

90 where ΔH is the change in core height and K_H is the reactivity coefficient of core height.
91 By representing the core height with the height of the fuel section, the expression
92 becomes

$$\rho_{z,s} = K_H \frac{\Delta H_f}{H_f}, \quad (3)$$

93 where subscript f represents the fuel. Using the linear thermal expansion coefficient, the
94 relative change in the core height direction is given by

$$\frac{\Delta H_f}{H_f} = \alpha_f \Delta T_f, \quad (4)$$

95 where α_f is the linear thermal expansion coefficient of the fuel, and ΔT_f is the fuel
 96 temperature change. Then, the shape change effect can be described as:

$$\rho_{z,s} = K_H \alpha_f \Delta T_f. \quad (5)$$

97 Moving on to $\rho_{z,\Sigma}$, since changes in the macroscopic cross-section can be represented by
 98 density changes, the reactivity introduced by density changes is decomposed as

$$\rho_{z,\Sigma} = \sum_i \frac{\partial keff}{\partial d_i/d_i} \frac{\partial d_i/d_i}{\partial H_i/H_i} \frac{\Delta H_i}{H_i}, \quad (6)$$

99 where d_i and H_i represent the density and the height of material i (fuel, cladding, or
 100 wrapper tube). Then we introduce the density reactivity coefficient described as

$$K_{d,i} = \frac{\partial keff}{\partial d_i/d_i}. \quad (7)$$

101 In addition, we use an approximation of:

$$\frac{\partial d_i/d_i}{\partial H_i/H_i} \approx -1. \quad (8)$$

102 Using Eqs. (7) and (8), Eq. (6) can be rewritten as

$$\rho_{z,\Sigma} = - \sum_i K_{d,i} \alpha_i \Delta T_i. \quad (9)$$

103 y summing $\rho_{z,s}$ and $\rho_{z,\Sigma}$, the Z-axis expansion reactivity is described as:

$$\rho_z = K_H \alpha_f \Delta T_f - \sum_i K_{d,i} \alpha_i \Delta T_i = (K_H - K_{d,f}) \alpha_f \Delta T_f - K_{d,cl} \alpha_{cl} \Delta T_{cl} - K_{d,wr} \alpha_{wr} \Delta T_{wr}, \quad (10)$$

104 where, subscript cl and wr represents the cladding and wrapper tube, respectively. In Eq.

105 (10), the reactivity associated with the thermal expansion of the wrapper tube is also

106 included, but in events where the fuel temperature changes significantly, this reactivity
107 becomes relatively small.

108

109 **3 Derivation of radial expansion reactivity**

110 The radial expansion considered in the present study includes the expansion of
111 the reactor core support plates at the bottom of the core, which leads to an increase in
112 the coolant flow paths, and the radial expansion of fuel pins which involves the fuel-
113 cladding interaction. While the interaction depends on the fuel material and its burnup,
114 the fuel pellet and cladding are collectively considered as one region in the derivation.
115 Similar to z-axis expansion reactivity, the radial expansion reactivity is approximately
116 separated into the shape change effect and the macroscopic cross-section change effect
117 as

$$\rho_r = \frac{\{keff(R', \Sigma') - keff(R, \Sigma)\}}{keff(R', \Sigma')keff(R, \Sigma)} \approx \rho_{r,s} + \rho_{r,\Sigma}, \quad (11)$$

118 Here, R and R' represent the core radius before and after expansion, respectively. The

119 term $\rho_{r,s} = \frac{keff(R', \Sigma) - keff(R, \Sigma)}{keff(R', \Sigma)keff(R, \Sigma)}$ represents the geometric change effect, while $\rho_{r,\Sigma} =$

120 $\frac{keff(R, \Sigma') - keff(R, \Sigma)}{keff(R, \Sigma')keff(R, \Sigma)}$ represents the macroscopic cross-section change effect.

121 Let us consider a situation that the lower core support plate expands, leading to
122 the radial expansion of the reactor core. The reactivity $\rho_{r,s}$ can be described by
123 introducing the core radial reactivity coefficient as

$$\rho_{r,s} = K_R \frac{\Delta A_{support}}{A_{support}} = 2K_R \left((1 + \alpha_{support} \Delta T_{support})^2 - 1 \right) \quad (12)$$

$$\approx 2K_R \alpha_{support} \Delta T_{support},$$

124 where A is the area, K_R is the reactivity coefficient of core radial, and subscript *support*
 125 represents the core support plate. Since the expansion ratio is generally less than a few
 126 percent, the term $(\alpha_{support} \Delta T_{support})^2$ becomes negligible and the approximation is
 127 justified.

128 Next, we consider the reactivity $\rho_{r,\Sigma}$ due to changes in the macroscopic cross-
 129 section. The changes in macroscopic cross-section can be represented by changes in
 130 density, thus we consider the densities of the fuel pin, structural materials such as
 131 wrapper tubes, and coolant. Since the total mass of the fuel pin and structural materials
 132 does not change after the expansion, we obtain

$$\frac{\Delta d_{p,homo}}{d_{p,homo}} = \frac{\Delta d_{s,homo}}{d_{s,homo}} = \frac{V_t}{V_t'} - 1 \approx -\frac{\Delta V_t}{V_t}. \quad (13)$$

133 where $d_{p,homo}$ and $d_{s,homo}$ are the homogenized densities of the fuel pin and structural
 134 materials, respectively, and V_t and V_t' are the total volumes before and after expansion.
 135 The change in homogenized coolant density of the core can be expressed as

$$\begin{aligned} \Delta d_{c,homo} &= \frac{d_{c,homo} V_t + d_c \Delta V_c}{V_t'} - d_{c,homo} \\ &= \frac{d_{c,homo} V_t + d_c (\Delta V_t - \Delta V_p - \Delta V_s)}{V_t'} - d_{c,homo} \\ &= \frac{-d_{c,homo} \Delta V_t + d_c (\Delta V_t - \Delta V_p - \Delta V_s)}{V_t'}. \end{aligned} \quad (14)$$

136 where subscript c represents the coolant. In Eq. (14), we use a relationship of

$$\Delta V_t = \Delta V_p + \Delta V_s + \Delta V_c. \quad (15)$$

137 Next, the following relationship is utilized:

$$d_c = d_{c,homo} \frac{V_t}{V_c}. \quad (16)$$

138 Using Eq. (16), Eq. (14) becomes

$$\begin{aligned} \Delta d_{c,homo} &= d_{c,homo} \frac{\Delta V_t}{V_t} \left(-1 + \frac{V_t}{V_c} \right) - d_{c,homo} \frac{\Delta V_p + \Delta V_s}{V_c} \\ &= d_{c,homo} \frac{\Delta V_t}{V_t} \left(\frac{V_p + V_s}{V_c} \right) - d_{c,homo} \frac{\Delta V_p + \Delta V_s}{V_c}. \end{aligned} \quad (17)$$

139 Hence, the relative change in $d_{c,homo}$ is described as

$$\frac{\Delta d_{c,homo}}{d_{c,homo}} = \frac{\Delta V_t}{V_t} \left(\frac{V_p + V_s}{V_c} \right) - \frac{\Delta V_p + \Delta V_s}{V_c}. \quad (18)$$

140 The reactivity introduced by relative density changes is obtained by multiplying by the
 141 density reactivity coefficients as

$$\begin{aligned} \rho_{r,\Sigma} &= K_{d,p} \frac{\Delta d_{p,homo}}{d_{p,homo}} + K_{d,s} \frac{\Delta d_{s,homo}}{d_{s,homo}} + K_{d,c} \frac{\Delta d_{c,homo}}{d_{c,homo}} \\ &= -K_{d,p} \frac{\Delta V_t}{V_t} - K_{d,s} \frac{\Delta V_t}{V_t} + K_{d,c} \frac{\Delta V_t}{V_t} \left(\frac{V_p + V_s}{V_c} \right) - K_{d,c} \frac{\Delta V_p}{V_c} - K_{d,c} \frac{\Delta V_s}{V_c}, \end{aligned} \quad (19)$$

142 where, $K_{d,p}$, $K_{d,s}$, and $K_{d,c}$ the density reactivity coefficients of fuel pin, structural
 143 materials, and coolant. Using the following relationships:

$$\frac{\Delta V_t}{V_t} = 2\alpha_{support} \Delta T_{support} \quad (20)$$

$$\frac{\Delta V_p}{V_p} = 2\alpha_p \Delta T_p \quad (21)$$

$$\frac{\Delta V_s}{V_s} = 2\alpha_s \Delta T_s \quad (22)$$

144 we can rewrite $\rho_{r,\Sigma}$ as

$$\rho_{r,\Sigma} = 2\alpha_{support} \Delta T_{support} \left(K_{d,c} \frac{V_p + V_s}{V_c} - (K_{d,p} + K_{d,s}) \right) - 2\alpha_p \Delta T_p K_{d,c} \frac{V_p}{V_c} \quad (23)$$

$$-2\alpha_s \Delta T_s K_{d,c} \frac{V_s}{V_c}$$

145 By summing $\rho_{r,s}$ and $\rho_{r,\Sigma}$, the radial expansion reactivity is described as

$$\rho_r = 2\alpha_{support} \Delta T_{support} \left(K_R + K_{d,c} \frac{V_p + V_s}{V_c} - (K_{d,p} + K_{d,s}) \right) - 2\alpha_p \Delta T_p K_{d,c} \frac{V_p}{V_c} - 2\alpha_s \Delta T_s K_{d,c} \frac{V_s}{V_c} \quad (24)$$

146 When the wrapper tube is the only structural material expanding outside the fuel pins,

147 Eq. (24) becomes:

$$\rho_r = 2\alpha_{support} \Delta T_{support} \left(K_R + K_{d,c} \frac{V_p + V_s}{V_c} - (K_{d,p} + K_{d,s}) \right) - 2\alpha_p \Delta T_p K_{d,c} \frac{V_p}{V_c} - 2\alpha_{wr} \Delta T_{wr} K_{d,c} \frac{V_{wr}}{V_c} \quad (25)$$

148

149 4 Differences from conventional formulas

150 The expansion reactivity calculation formulas for the fast reactor using oxide fuel
 151 are shown in a technical report [10]. Here, the differences between the calculation
 152 formulas shown in the report and that derived in the present study are discussed.

153 In the technical report [10], the formula for calculating z-axis expansion reactivity
 154 is expressed as

$$\rho_z = (K_H - K_{d,f}) \alpha_f \Delta T_f - K_{d,s} \alpha_s \frac{V_{cl}}{V_s} \Delta T_{cl} - K_{d,s} \alpha_s \frac{V_{wr}}{V_s} \Delta T_{wr} \quad (26)$$

155 In Eq. (26), the expansion reactivity for both cladding and wrapper tubes is calculated
 156 using the density reactivity coefficient of structural materials. The contribution of
 157 cladding and wrapper tube are taken by using a proportion with the total volume. The

158 thermal expansion coefficients of the cladding and wrapper tubes are assumed to be the
159 same as the structural materials. While Eq. (26) is suitable when cladding and wrapper
160 tubes are made from the same material, Eq. (10) is more appropriate when their materials
161 are different.

162 In the technical report [10], the formula for calculating the radial expansion
163 reactivity can be expressed as

$$\rho_r = 2\alpha_{support}\Delta T_{support} \left(K_R + K_{d,c} \frac{V_p + V_s}{V_c} - (K_{d,p} + K_{d,s}) \right) - 2\alpha_s \Delta T_{cl} K_{d,c} \frac{V_p}{V_c} - 2\alpha_s \Delta T_{wr} K_{d,c} \frac{V_{wr}}{V_c}, \quad (27)$$

164 here, a coefficient 2 of the term containing K_R is not shown in the technical report [10],
165 but is modified in the paper. In the second and third terms on the right side of Eq. (27),
166 the thermal expansion coefficient used for considering the expansion of the fuel pin is the
167 coefficient of the structural materials. It can be confirmed that previous study has
168 represented the linear expansion coefficient of the pin using the coefficient of structural
169 materials. On the other hand, in the present formula shown in Eq. (25), the radial
170 expansion of the fuel pin is treated using a linear expansion coefficient α_p , which accounts
171 for the expansion of both the fuel and the cladding.

172

173 **5 Accuracy of Uniform Expansion Reactivity Evaluation Based on Reactivity Coefficients**

174 This chapter discusses the accuracy of the evaluation results of expansion
175 reactivity based on reactivity coefficients. The calculation condition and results are
176 respectively shown in Section 5.1 and 5.2.

177

178 **5.1 Calculation Conditions**

179 **5.1.1 Expansion Ratio in Sodium-Cooled Fast Reactors**

180 In this paper, we consider the expansion of the fuel and the lower core support
181 structure. In a technical report for a sodium-cooled fast reactor with MOX fuel, formulas
182 for calculating thermal expansion coefficients are presented [10]. Figures 1(a) and 1(b)
183 show the thermal expansion coefficients for the structural material of the lower support
184 plate (SUS316) and the MOX fuel obtained from the calculation formulas presented in this
185 technical report. The boiling point of sodium is approximately 900°C, thus, Fig. 1(a) shows
186 that the thermal expansion coefficient of the lower support plate is less than $2.2 \times 10^{-5}/K$.
187 Therefore, even if the sodium temperature increases by 100°C, the expansion ratio
188 remains below 2.2E-3, which is less than 1%. Additionally, Fig. 1(b) indicates that the
189 thermal expansion coefficient is below $2.2 \times 10^{-5}/K$ even at around 2400°C, which is near
190 the melting point of MOX fuel. As for metallic fuels, the average thermal expansion
191 coefficient for U-Pu-Zr alloys is reported to be $1.7 \times 10^{-5}/K$ [14,15]. Therefore, for the MOX
192 fuel or U-Pu-Zr alloys used as metallic fuels, even if the fuel temperature increases by
193 500°C, the expansion ratio would be around 1% or less. In this paper, assuming large
194 temperature changes, the accuracy of expansion reactivity for expansion ratios of up to
195 2% is discussed.

196

197 **5.1.2 Core Geometry and Calculation Method**

198 Two simplified core geometries (Core A and B) with varying core height to diameter
199 ratio were used (Fig. 2). The fuel region is based on the inner fuel specifications of the
200 Monju [7]. To obtain the 70-group cross-sections based on JENDL-4.0 [16], a pin-cell
201 calculation is performed using SLAROM-UF [17]. Using the 70-group macroscopic cross-
202 sections for the MOX fuel, SUS316, and sodium obtained from the pin-cell calculation, the
203 homogenized 70-group cross-sections for the fuel assembly are calculated. The
204 calculation condition for the 70-group macroscopic cross-sections is the same as that used
205 in the transient calculation for Monju [7]. The homogenized cross section was calculated
206 using volume weighting. For more precise calculations, it would be appropriate to use the
207 flux-volume-weighted homogenized cross section, which accounts for the neutron flux
208 distribution within the assembly. Since this study is not aimed at precisely determining
209 core characteristics but rather focuses on discussing the approximation error of the
210 expansion reactivity, the volume-weighted cross section was used for the calculation.

211 The core calculations are performed using CITATION based on diffusion theory.
212 The calculations are carried out in an R-Z geometry with 100 divisions in both the radial
213 and Z-axis directions of the fuel region. The convergence criteria are set to 5×10^{-4} for the
214 neutron flux in each region and 1×10^{-6} for the multiplication factor. The boundary
215 condition is set to vacuum. Using CITATION, the reactivity coefficients presented in
216 Sections 2 and 3 are calculated. Then, the Z-axis and radial expansion reactivities are
217 obtained using the volumes of each material (e.g., coolant) in the Monju. In this
218 calculation, the density reactivity coefficient (e.g., $K_{d,f}$) and reactivity coefficient due to
219 shape changes (e.g., K_H) are obtained using 1% expansion in the axial or radial directions.

220 The expansion reactivity obtained from the reactivity coefficients is compared to the
221 expansion reactivity directly calculated by expanding the geometry in CITATION.

222

223 **5.2 Numerical Results**

224 **5.2.1 Z-axis Expansion Reactivity**

225 As mentioned in Section 2, the reactivity introduced by the thermal expansion of
226 the wrapper tube in the Z-axis direction is small. Therefore, in this section, we discuss the
227 reactivity obtained using the formulas derived in Section 2, considering two cases: when
228 only the fuel expands and when both the fuel and cladding expand. Figure 3(a) compares
229 reactivity for Core A with only fuel expansion, while Figure 3(b) includes both fuel and
230 cladding expansion. As shown in Fig. 3, the reactivity due to Z-axis expansion shows
231 approximately linear behavior within the expansion range of 0 to 2%. This indicates that
232 interpolation and extrapolation of reactivity using the reactivity coefficients are justified.
233 Additionally, the difference between the reactivity calculated using reactivity coefficients
234 and the direct calculation is within a few percent. Figure 4 shows a comparison of Z-axis
235 expansion reactivity for Core B. The magnitude of axial reactivity is smaller than Core A
236 since Core B is flatter than Core A. Similar to Core A, the reactivity in Core B also shows
237 linear behavior, and the difference between the reactivity calculated using reactivity
238 coefficients and the direct calculation closely matches, being within a few percent. These
239 results show that Z-axis expansion reactivity can be accurately estimated using the
240 derived formulas.

241 For Monju, since the cladding and wrapper tubes are made of the same material,
242 the z-axis expansion reactivity is identical between the present formulas and the
243 conventional formulas.

244

245 **5.2.2 Radial Expansion Reactivity**

246 In our previous study [11], it was confirmed that changes in the amount of sodium
247 within the core before and after expansion have a relatively small impact on expansion
248 reactivity. Therefore, for radial expansion, we focus on the reactivity introduced by the
249 expansion of the lower core support plate. Figures 5 and 6 show comparisons of radial
250 expansion reactivity for Core A and Core B, respectively. Compared to Core A, the
251 magnitude of radial reactivity is larger for Core B. For both core geometries, the reactivity
252 due to radial expansion shows approximately linear behavior within the expansion range
253 of 0 to 2%. Additionally, the difference between the reactivity calculated using reactivity
254 coefficients and the direct calculation is within a few percent. These results demonstrate
255 that the radial expansion reactivity can also be accurately estimated using the derived
256 formulas.

257 Next, let us discuss the reactivity difference between the present and
258 conventional formulas. One major difference is that the conventional formulas do not
259 consider the expansion of the fuel in the term related to fuel pin expansion. Table 1 shows
260 a comparison of the radial expansion reactivity obtained using the conventional formulas
261 and the present formulas when the radial expansion ratio is 1%. To simplify the discussion,
262 the interaction between the fuel and the cladding was ignored in the calculation. These

263 results confirm that even at a 1% expansion of the fuel, the difference between the
264 present and conventional formulas is about $3 \times 10^{-5} \Delta k/k'$ or less.

265

266 **6 Conclusion and future work**

267 In this study, to evaluate the uniform axial and radial expansion reactivity in
268 sodium-cooled fast reactors with low computational cost, we derived an evaluation
269 formula based on reactivity coefficients and confirmed the accuracy. For axial expansion,
270 the expansion of fuel, cladding, and wrapper tubes are considered. For radial expansion,
271 the expansion of the fuel pin, lower core support plate, and structural materials are
272 considered. In these derivations, the expansion reactivity is divided into the shape change
273 effect and the macroscopic cross-section change effect, and these effects are expressed
274 by the reactivity coefficients of materials in the core.

275 We compared the derived formulas with conventional formulas. While the
276 formulas are generally consistent, there are some differences in some expressions. To
277 confirm the accuracy of the proposed formulas, numerical calculations were performed
278 using two different core geometries (Core A and Core B). Reactivities due to Z-axis and
279 radial expansions were calculated using both the reactivity coefficients and direct
280 calculation considering core expansion. As a result, for the expansion range of 0 to 2%,
281 the differences between the reactivity calculated using the reactivity coefficients and the
282 direct calculation were within a few percent. For both Z-axis and radial expansions, the
283 reactivity changes show linear behavior within the expansion range, demonstrating that
284 interpolation and extrapolation using the reactivity coefficients are feasible.

285 This study assumes uniform expansion in both radial and axial directions. However,
286 during transient conditions in fast reactors, non-uniform expansion, such as fuel assembly
287 bowing, can also occur. Future work is expected to develop methods that can be applied
288 to such non-uniform expansions.
289
290

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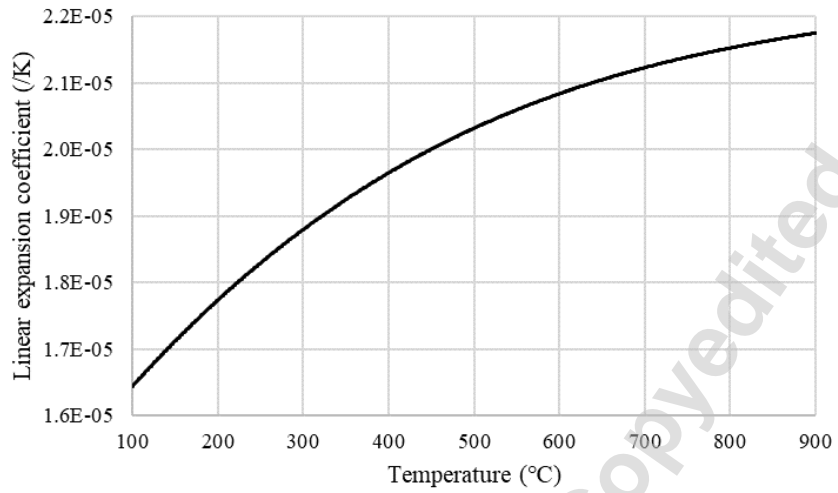
Figure Captions List

- Fig. 1 Linear expansion coefficient
- Fig. 2 Simple core geometry
- Fig. 3 Comparison of Z-axis expansion reactivity for Core A
- Fig. 4 Comparison of Z-axis expansion reactivity for Core B
- Fig. 5 Comparison of radial expansion reactivity for Core A
- Fig. 6 Comparison of radial expansion reactivity for Core B

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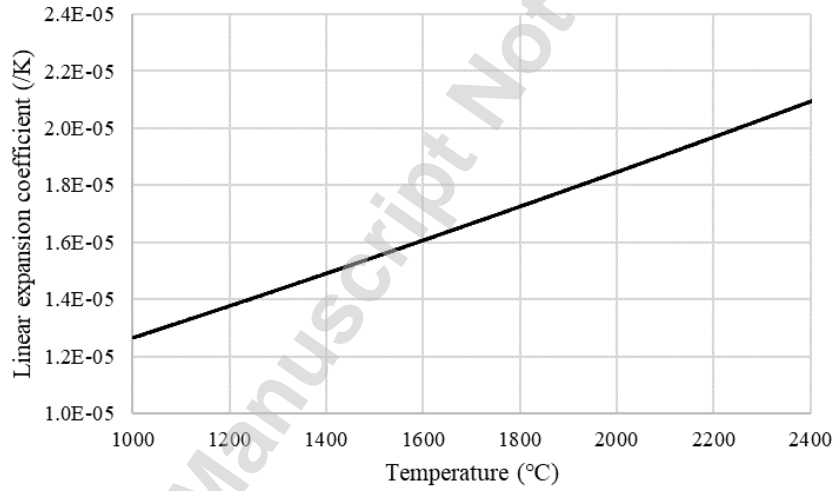
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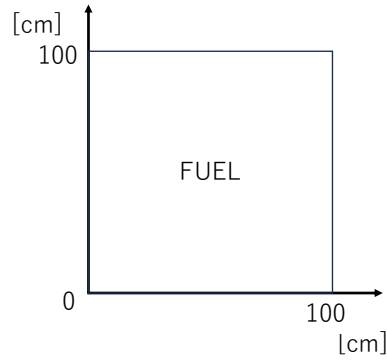
(a) SUS316



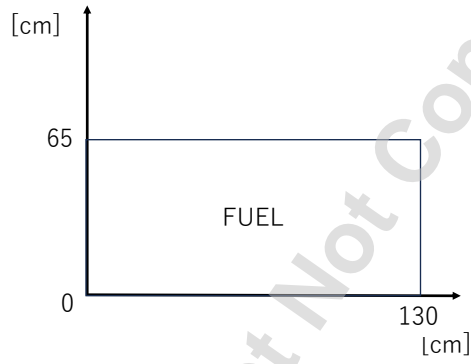
(b) MOX fuel

Fig. 1 Thermal expansion coefficient

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(a) Core A



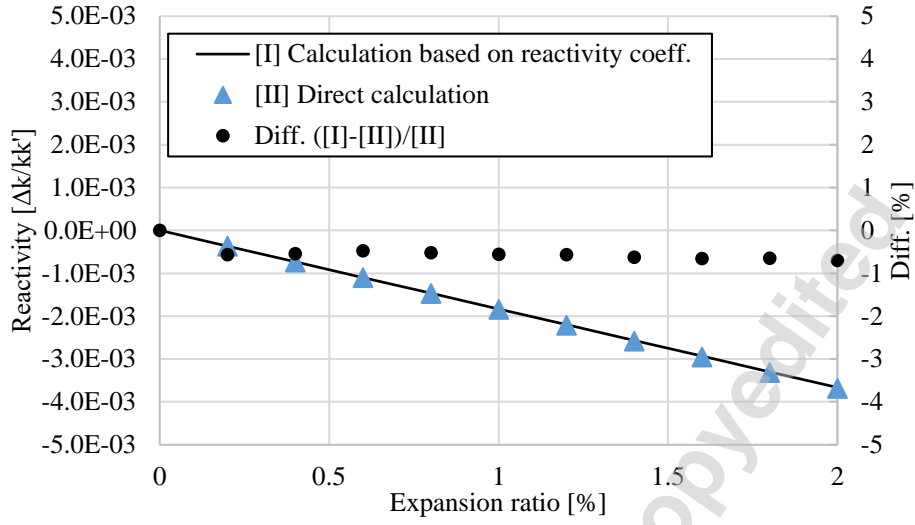
(b) Core B

Fig. 2 Simple core geometry

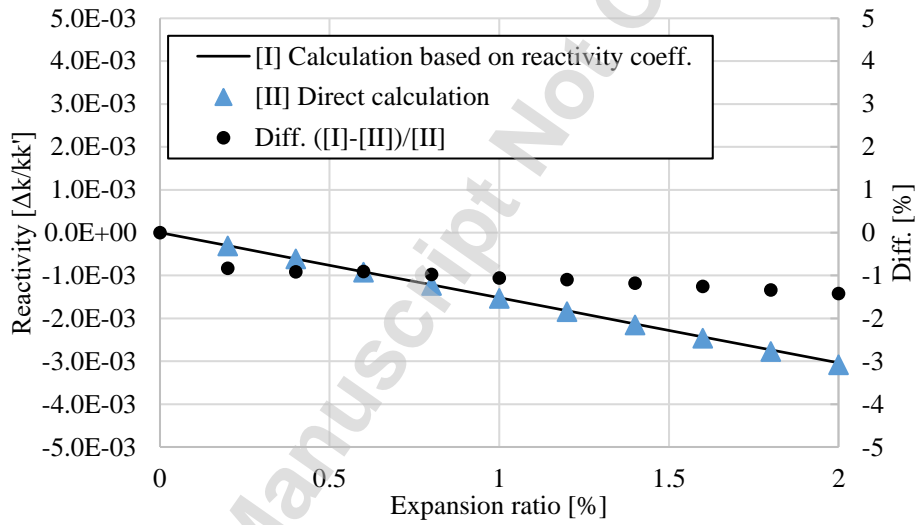
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(a) Fuel expansion only

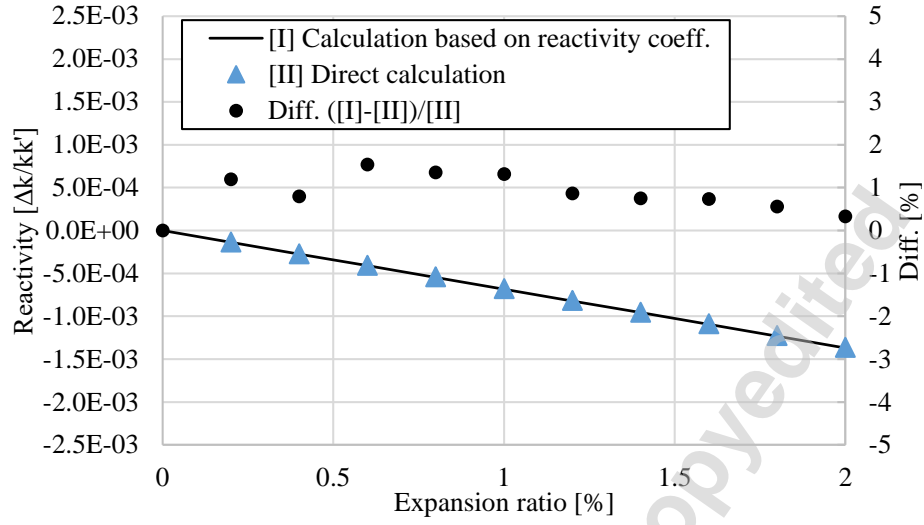


(b) Fuel and cladding Expansion

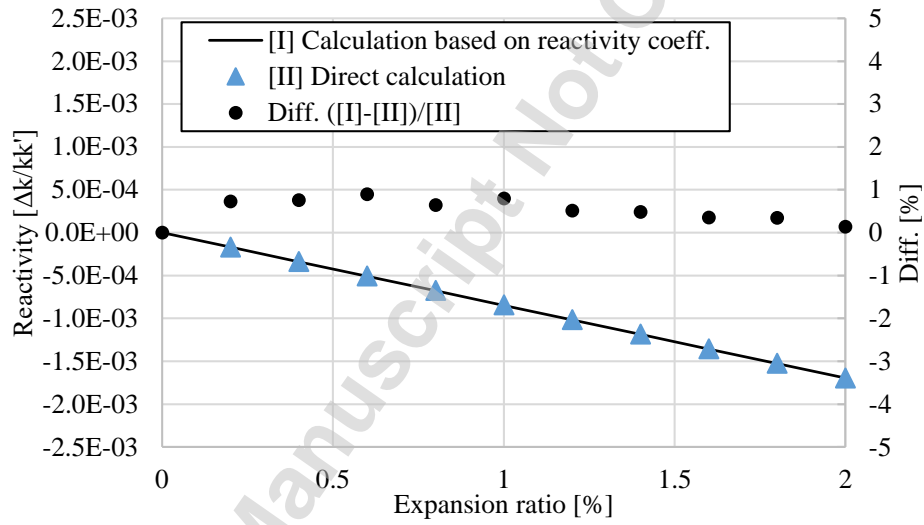
Fig. 3 Comparison of Z-axis expansion reactivity for Core A

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(a) Fuel expansion only



(b) Fuel and cladding Expansion

Fig. 4 Comparison of Z-axis expansion reactivity for Core B

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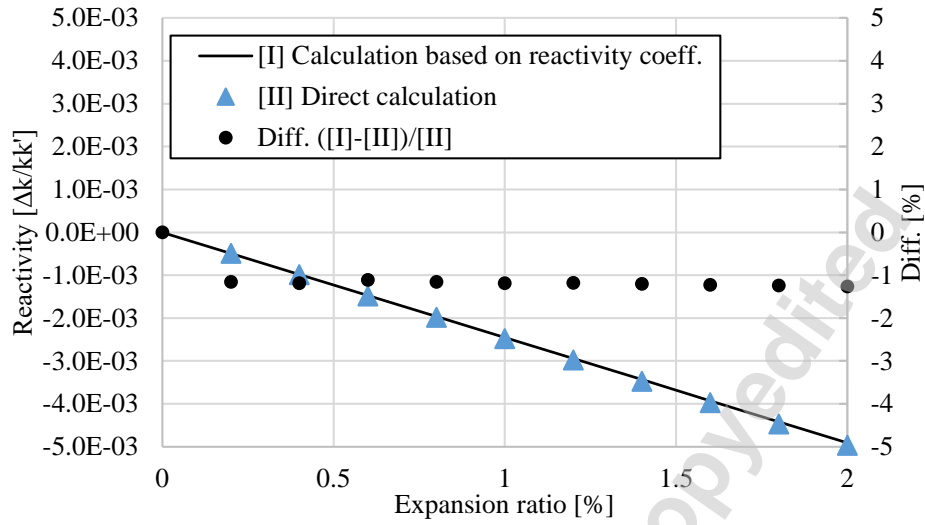


Fig. 5 Comparison of radial expansion reactivity for Core A

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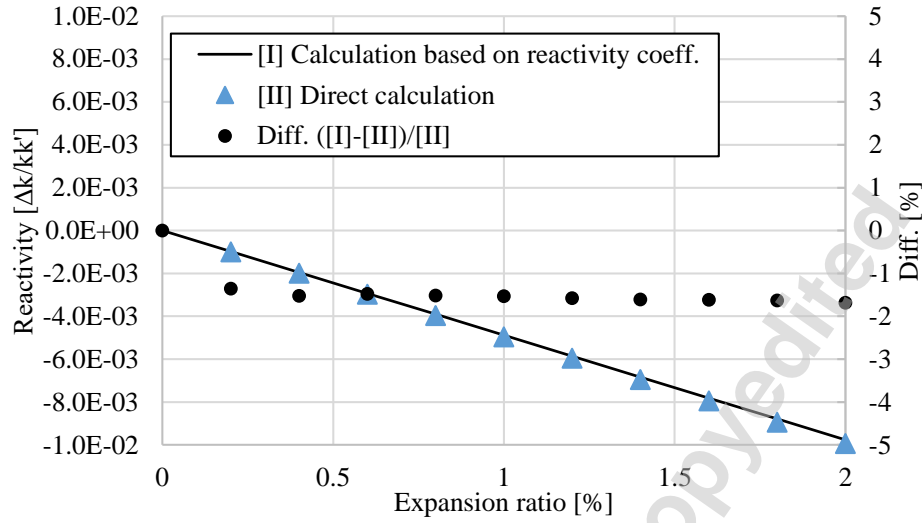


Fig. 6 Comparison of radial expansion reactivity for Core B

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Table Caption List

Table 1 Comparison of radial expansion reactivity.

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Table 1 Comparison of radial expansion reactivity.

Core geometry	(i) Present formulas $[\Delta k/k']$	(ii) Conventional formulas $[\Delta k/k']$	(i) -(ii) $[\Delta k/k']$
Core A	-2.3×10^{-4}	-2.4×10^{-4}	0.1×10^{-4}
Core B	-5.4×10^{-4}	-5.1×10^{-4}	-0.3×10^{-4}

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