A Study of Transverse Buckling Effect on the Characteristics of Nuclides Burnup Wave in a Fast Neutron Multiplying Media

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1 Introduction

Nuclear energy is abundant and is environment-friendly. There is practically no greenhouse gas emission from nuclear power plants and hence no contribution to global warming. Therefore, the nuclear option for electricity generation appears to be an ideal one for sustainable development. However, Three Mile Island accident in 1979 and Chernobyl accident in 1986 raised the matters/issues of nuclear safety. Fukushima accident in March 2011 has raised the matter/issues of management of long-term decay heat removal in the reactors and spent fuel storage pool. The consequences of these accidents led to the higher emphasis to be given on improvement in the technology that includes reliability of components/equipment, intrinsic safety, passive safety features in shutdown systems and decay heat removal systems design, proper management of care to be taken of long-term station blackout and measures against proliferation of fissile material. In view of the above, the new innovative concepts are being explored. One of the concepts is breed and burn concept. This implies soliton-type fuel burning waves in nuclear reactors as a working principle of reactor model [1–5]. These are termed traveling wave reactors (TWRs). In these reactors, the processes of frequent fuel loading and handling of spent fuel are eliminated by designing a reactor that burns like a CANDLE [6,7] for years with self-regulation. CANDLE is also the abbreviation of Constant Axial Neutron flux, nuclear densities, and power during life of energy producing reactors. Over the years, a larger number of studies have been carried out on this concept [5–23]. The studies do establish the necessary and sufficient conditions for the establishments of soliton-like burnup wave in thermal and fast multiplying media and high fuel burnup can be achieved. At the corporate level, Terra Power Company, Bellevue, WA is working on the design and development of TWRs [24–26] and has plans to construct the reactor. The studies have shown that higher fuel burnup can be achieved by using the new concept of TWR.

Studies have also been conducted to achieve higher fuel burnup by inventing better fuel reshuffling methodologies.

The nuclides burnup wave studies have also been found applicable in reactor control and in configuring the neutron poison in the control rods. This pertains to balancing of the burnup rate of fissile material (negative reactivity addition) with neutron poison burnup rate (positive reactivity addition) in the control rods and achieving flatter curve of reactivity versus burnup up and time. For instance, in heterogeneously placed control rods, if the rod thickness is more than the mean free path of neutrons in neutron poison of the rod, the rod would depict self-shielding effect; that is, neutron absorption would be larger near the surface of the rod and diminishes in the depth of absorber region. The reactivity worth of the rods versus time can be tailored by appropriately adjusting the poison region of the rod and the concentration of the poison in the absorber region. The propagation of burn up of poison in the poisoned region of the rods with self-shielding effect follows the wave propagation characteristics. This is very well demonstration first by Seifritz [9,11]. Later, the idea was extended to diffusive media by Hugo [11,12]. Some recent work on this topic by Van Dam is reported in Refs. [27] and [28].

At Indian Institute of Technology (IIT) Kanpur, our focus has been on the characterization of build-up and establishment of nuclides burnup wave in absorbing, diffusive and multiplying media [20–23]. In these studies, the emphasis has been on the investigation of the transient part of the development of the burnup waves. New parameters were defined to characterize the transient part of the waves as: (a) transient time (TT)—time elapsed in establishing the steady burnup wave; (b) transient length (TL)—distance covered in establishing the steady-state burn up. The TT and TL parameters would become more clear from the results presented in Sec. 4 and the results ofRefs. [22] and [23]. The characteristics of the steady-state part of the waves are expressed in terms of full width at half maxima (FWHM) and full width at 10% of maximum (FW10M). The studies [22,23] have discovered that the above parameters characterize well the transient and steady-state part of the nuclide burnup wave in neutron absorbing, diffusive and multiplying media.
This work is an extension over the studies reported earlier in Ref. [23]. In this work, the size of the system was taken large so that the neutron leakage radially is negligible. In this work, results are presented on the investigations of the effect of transverse neutron leakage expressed by transverse neutron flux buckling, on the characteristics of buildup and propagation of nuclear burnup wave in a multiplying medium. The results of the parametric study with respect to leakage are also presented.

2 Mathematical Model

The modeling is similar to the one discussed in Refs. [19] and [23] where basically reactor is represented by the one-dimensional semi-infinite slab. The slab consists of two zones: ignition zone where Pu\(^{239}\) is fused to produce neutron current on the left boundary of the slab and breeding zone where \(U^{238}\) breeds Pu\(^{239}\). Here, transverse buckling is zero and hence there is no leakage of neutrons.

In this work, the reactor is represented by a cylinder of finite size; i.e., finite axial length and radius and with azimuthal symmetry (Fig. 1). The left and right boundaries of the cylindrical geometry are represented by \(z_i\) and \(z_s\), respectively. The plutonium fused region (the ignition zone) on the left boundary extends to distance \(d\), (see Fig. 1) while the rest on the right boundary is breeding zone. As mentioned above, Pu\(^{239}\) is fused at the left boundary and \(J\) is the neutron current density entering into the breeding zone through the left boundary. In this case, transverse buckling is finite and hence there is radial neutron leakage.

The neutron balance equation for the two-dimensional cylindrical system can be written as

\[
\frac{\partial N}{\partial t} = vD \frac{\partial^2 N}{\partial z^2} + vD \frac{\partial^2 N}{\partial r^2} + vD \frac{1}{r} \frac{\partial N}{\partial r} - vN\Sigma_t \sigma_t N_i + (V_f - 1)v\sigma_{\text{Pu}} NN_{\text{Pu}}
\]  

(1)

\[
\frac{\partial N_k}{\partial t} = -v\sigma_{sb} NN_k
\]  

(2)

\[
\frac{\partial N_{\text{Pu}}}{\partial t} = -N_k - v\sigma_{sb} NN_k
\]  

(3)

\[
\frac{\partial N_{\text{Pu}}}{\partial t} = \frac{1}{\tau_{\beta}} N_k - v(\sigma_{\text{Pu}} + \sigma_{\text{Pu}})NN_{\text{Pu}}
\]  

(4)

In the above equations, \(z\) and \(r\), respectively, represent the axial and radial directions; \(v\) and \(D\), respectively, represent neutron velocity and diffusion coefficient of the medium; \(\sigma_t\) and \(\sigma_f\), respectively, represent microscopic capture and fission cross-sections; \(V_f\) is the number of neutrons produced per fission and \(N\) is the neutron density. \(N_k\) is the nuclide density of \(U^{238}\) and \(N_k\) is the nuclide density of \(U^{239}\) and is the nuclide density of Pu\(^{239}\) and \(\tau_{\beta}\) is the mean lifetime for the \(\beta\)-decay process; the \(i\) in summation \((\Sigma)\) stands for \(U^{238}\) and \(U^{239}\).

Here, the second and third terms in Eq. (1) represent radial neutron leakage. It may be mentioned that, as done in our earlier study [23] and reported in other references [19,23], only the density of four components (neutron, \(U^{238}\), \(U^{239}\), and Pu\(^{239}\)) is used in studying the propagation of burnup wave in a multiplying medium and for \(U\)-Pu fuel cycle. The half-life of Pu\(^{239}\) being small (23.5 min), and the fission cross section of Pu\(^{239}\) being large compared to its capture cross section, the amount of formation of Pu\(^{239}\) is negligible. \(U^{238}\) formation is negligible and even is stated in transmutation reaction of \(U^{238}\) in formation of Pu\(^{239}\) [19,29]. The absorption of neutrons by Pu\(^{239}\) and Pu\(^{239}\) results in beta decay of both the nuclides. So, the \(\beta\) decay half-life of Pu\(^{239}\) is considered. Thus, the major effect of Pu\(^{239}\) is accounted in the analysis.

The possibility of solitary wave propagation depends on the ratio, \(\eta = n_{eq}/n_{cr}\). Since \(\eta\) is composed of a different combination of constants, the three variants of \(\eta\), i.e., \(\eta > 1\), \(\eta = 1\) and \(\eta < 1\) are possible. The necessary condition for the existence of a solitary wave is \(\eta > 1\), i.e., \(n_{eq} > n_{cr}\), [13].

3 Numerical Calculations

To facilitate the numerical calculations, the distance and time are normalized to diffusion length and mean neutron lifetime,
respectively. The mean neutron lifetime ($\tau_n$) and neutron diffusion length ($L$) of the medium are expressed as

$$\tau_n = \frac{1}{\Sigma_{i} N_i \sigma_{ui}}$$

$$L = \sqrt{\frac{\sigma t}{\Sigma_{i} N_i \sigma_{ui}}}$$

Further, it is assumed that Pu$^{240}$ and U$^{238}$, the nuclides produced by neutron capture of U$^{239}$ and Pu$^{239}$, are equivalent to U$^{238}$ since the available U$^{238}$ only gets converted to U$^{239}$ and then further to Pu$^{239}$. This helps to close the set of equations without changing the nature of the studied process. To make the calculations simple, the neutron capture cross sections of U$^{239}$ and Pu$^{239}$ were taken same as of U$^{238}$ ($\sigma_{\alpha} = \sigma_{\beta} = \sigma_{\beta}$) [19]. Thus, Eqs. (2) and (3), can be written as

$$\frac{dN_{8k}}{dt} = -\nu \sigma_{\beta} N (N_8 - N_{8k} - N_9)$$

$$\frac{dN_9}{dt} = -\frac{1}{\tau_9} N_9 + \nu \sigma_{\beta} N (N_8 - N_9)$$

The nuclide concentration and neutron density are normalized as follows:

$$n_i = \frac{N_i}{N_{tot}}$$

$$n = N \tau_v \nu \sigma_8$$

The distances are normalized in terms of diffusion length ($l$) and time by mean lifetime for $\beta$ decay ($\tau_\beta$). So, one can write the diffusion and burnup equations in a dimensionless format as

$$\frac{\beta}{\tau_\beta} \frac{dn(z,t)}{dt} = \frac{\partial^2 n(z,t)}{\partial z^2} + n(z,t) \left\{ \frac{n_{pu}(z,t)}{n_{nu}} - \frac{2.405}{R} \right\} - q(z,t)$$

$$\frac{dN_8}{dt} = -n(z,t) \left\{ n_8(z,t) - n_9(z,t) - n_{pu}(z,t) \right\}$$

$$\frac{dN_9}{dt} = n(z,t) \left\{ n_9(z,t) - n_0(z,t) \right\} - n_0(z,t)$$

$$\frac{dN_{pu}(z,t)}{dt} = n_0(z,t) \left\{ 1 - \frac{n(z,t)}{n_{eq}} n_{pu}(z,t) \right\}$$

The constant $\beta = \tau_\beta/\tau_\beta \approx 0.6 \times 10^{-12} \approx 0$). Here, $q(z,t)$ represents the external source of neutrons. The neutron source is required for burnup wave ignition. The external neutron production can be modeled in two ways. One in an above-mentioned way and in the second model, a neutron current is allowed to fall on the boundary at $z = 0$. The neutron current $j_0$ should be included in the boundary condition. It is possible to vary the solitary wave velocity by changing source strength.

To study the time evolution of solitary wave, Eqs. (15)–(18) should be solved; the boundary conditions are given below [19]:

(i) At the right boundary, the neutron current coming into the cylinder is taken as zero

$$\frac{dN_{8k}(z,t)}{dz} + \frac{1}{2} n_{8k}(z,t) = 0$$

The constant $\gamma = L/\nu \tau a$)

(ii) On the left boundary, a neutron current density of $J$ is applied to initiate the burnup wave. Under the normalized condition, $J$ becomes $j_0 = \tau_\beta J/Nl$. That is

$$\frac{dN_8}{dt} = -n(z,t) \left\{ n_8(z,t) - n_9(z,t) - n_{pu}(z,t) \right\}$$

$$\frac{dN_9}{dt} = n(z,t) \left\{ n_9(z,t) - n_0(z,t) \right\} - n_0(z,t)$$

$$\frac{dN_{pu}(z,t)}{dt} = n_0(z,t) \left\{ 1 - \frac{n(z,t)}{n_{eq}} n_{pu}(z,t) \right\}$$

The initial medium consists of fertile U$^{238}$ with a plutonium fuse up to a distance $d$, which is given by the formula in Eq. (21) [19]

$$n_{pu}(z,t) = a_0 n_{nu} \left\{ \cos \left( \frac{\pi z}{d} \right) + 1 \right\}$$

Crank–Nicolson method [30,31] is used for numerically solving diffusion Eq. (15). This method is used for parabolic partial differential equations. For increasing the accuracy, the average of spatial derivatives of consecutive time steps is taken (instead of taking spatial derivative at a single time-step). The burnup equations (Eqs. (16)–(18)) and boundary conditions (Eqs. (19) and (20)) are discretized by the finite difference scheme. MATLAB programming environment is used for carrying out the simulation in a similar way as in Ref. [23]. To check the validity of the step size chosen for spatial and temporal steps and to validate the code for numerical simulation, the results in Ref. [23] are also reproduced and then the radial leakage term was added in the code for further calculations.

4 Results

4.1 Validation of Calculations. The code is validated by reproducing the results of Ref. [23]. The inputs are same as in Refs. [19] and [23]: $j_0 = 0.001$, $\gamma = 6 \times 10^{-2}$, $n_{eq} = 0.1$, $\eta = 2.0$, $a_0 = 1.65$, and $d = 8$. Normalized length $= 15$, $\beta = 0.6 \times 10^{-12} = 0$, diffusion length $= 10$, mean neutron lifetime ($\tau_\beta$) = 3.41 days. The simulation results are presented in Figs. 2–6. The spatial profiles of neutron density and nuclide concentrations are shown in the figures. Figure 2 shows spatial profiles of neutron density, each curve depicting a time interval of one unit. It can be observed that over a distance of 5 units (=50 cm) and time of about 22 units (~76 days), the neutron density builds up. Beyond 50 cm and 76 days, the spatial shape of neutron profile remains constant, indicating that solitary wave is in steady-state and transient part is over before 50 cm and 76 days. Thus, the transient length (TL) and transient time (TT) for this case are 50 cm and 76 days, respectively. Figure 3 shows the spatial profile of neutron density at the transient time of 76 days. In Fig. 3, the TL, FWHM, and FW10M are also depicted.

Figure 4 shows the U$^{239}$ spatial profile. Initially, the U$^{239}$ concentration is zero but as time progresses, the U$^{239}$ builds up. On further advancement of the time, these U$^{239}$ nuclides get consumed in the fission reaction. In the middle portion, it can be seen clearly that the plutonium concentration reaches a steady-state value.

Figure 6 shows the U$^{238}$ concentration. Uranium gets converted into Pu$^{239}$ as it absorbs the neutron so the concentration of the U$^{238}$ decreases with the time.

The transient parameters for 99% criteria of asymptotic state and for $\eta = 2$ are calculated to be: TT = 76 days, TL = 50 cm, and velocity of wave propagation at equilibrium = 7 m/year. In Ref. [23], the transient parameters for 99% criteria of asymptotic
state and for $\eta = 2$ were calculated to be as: $TT = 76$ days, $TL = 49$ cm, velocity of wave propagation at equilibrium $= 6.6$ m/year. This result validates the step size chosen for spatial and temporal steps and also validates the code for numerical simulation. The minute variation in the result is due to considering the constant $\beta = 0$ for decreasing the computational time.

4.2 Effects of Neutron Leakage. As mentioned above, a two-dimensional cylindrical geometry is considered from $z_l = 0$ to $z_r = 15$. The boundary flux $j_0 = 0.001$ and the constants $\beta = 0$, $\gamma = 6 \times 10^{-2}$, $n_{eq} = 0.1$, $\eta = 2.0$, $\alpha_0 = 1.65$ and $d = 8$ [17]. The simulation results show ignition and propagation of the solitary wave. The spatial profiles of neutron density and nuclide concentrations are shown in Figs. 7–15 for different radius; that is, $R = 4.5$, 5.5, and 7.5 units (1 unit = 10 cm).

4.3.1 Radius = 4.5 Units (45 cm), Curve Plot Time Interval = 5 Units (17 Days). The spatial profiles of neutron density, uranium-238 density, and plutonium-239 density with respect to distance plotted at a time interval $[\Delta t = 5$ units ($= 17$ days)] of five units for the radius of the cylinder = 4.5 units are shown in Figs. 7, 8 and 9, respectively.

Figure 7 shows that the solitary burnup wave starts to build up but cannot propagate to a larger distance. The burnup wave dies out before propagating to the farthest end of the cylinder. Figure 8 shows that there is consumption of $U^{238}$ up to some distance but not to the full length of the cylinder, which is due to nonavailability of neutrons due to neutron leakage in the radial direction of the cylinder. Figure 9 shows that the generation of $Pu^{239}$ ceases out early before reaching the farthest end of the cylinder because of nonconversion of $U^{238}$ into $Pu^{239}$ due to radial leakage of the neutron.

4.3.2 Radius = 5.5 Units (55 cm), Curve Plot Time Interval = 5 Units (17 Days). The spatial profiles of neutron density, uranium-238 density and plutonium-239 density with respect to distance plotted at a time interval of five units for the radius of the cylinder equal to 5.5 units are shown in Figs. 10–12, respectively.
Figure 10 shows that the magnitude of the burnup wave increases up to a peak value and then magnitude decreases slowly. The negative slope of the peaks after reaching to the highest value is due to radial leakage of the neutrons. The neutron leakage introduces damping in the steady-state soliton burnup wave so the steady-state is not achieved. Figure 11 shows that the consumption of U^{238} is throughout the full length of the cylinder but the magnitude of the consumption decrease while progressing through...
the length of the cylinder, which is due to low-availability of neutron due to neutron leakage in the radial direction of the cylinder. Figure 12 shows that the Pu\textsuperscript{239} is generated throughout the full length of the cylinder. But the magnitude decreases while progressing to the right end of the cylinder because of nonconversion of U\textsuperscript{238} into Pu\textsuperscript{239} due to radial leakage of the neutron.

4.3.3 Radius = 7.5 Units (75 cm), Curve Plot Time Interval = 5 Units (17 Days). The spatial profiles of neutron density, uranium-238 density, and plutonium-239 density with respect to distance plotted at a time interval of five units for the radius of the cylinder equal to 7.5 units are shown in Figs. 13–15, respectively.

Fig. 8 Spatial profile of U\textsuperscript{238} for R = 4.5 units, \( \Delta t = 5 \) units (17 days)

Fig. 9 Spatial profile of Pu\textsuperscript{239} for R = 4.5 units, \( \Delta t = 5 \) units (17 days)

Fig. 10 Spatial profile of neutron density for R = 5.5 units
Figure 13 shows that the solitary burnup wave builds up and propagates to the full length of the cylinder. Figure 13 shows that the magnitude of the burnup wave increases up to the distance where it reaches steady-state and then magnitude remains almost constant. The slope of the peaks after the steady-state becomes almost horizontal this shows that the solitary wave propagation and its shape have very less effect of radial neutron leakage, i.e., the dependency of burnup solitary wave on neutron leakage has
been reduced to a greater extent. Figure 14 shows that the consumption of U\textsuperscript{238} is throughout the full length of the cylinder and the magnitude of the consumption is almost constant while progressing through the length of the cylinder. Figure 15 shows that the Pu\textsuperscript{239} is generated throughout the full length of the cylinder and with a constant magnitude while progressing to the right end of the cylinder.

### 4.3 Discussion

The results in terms of parameters that characterize the soliton for different radius of the cylinder are summarized in Table 1. The steady-state criterion is used for calculation of the parameters where the slope of the maximum reaction rate is horizontal. When the slope of the maximum reaction rate is not horizontal, the steady-state percentage change in maximum reaction rate has been calculated for different times, at two days of interval. When the percentage change is approximately equal, it is assumed to be the beginning of the steady-state. For all the radii, the value of \( g \) is taken as 2.

![Spatial Profile of U\textsuperscript{238} Density at t = 5 units interval for R = 7.5 units](image1.png)

**Fig. 14** Spatial profile of U\textsuperscript{238} for \( R = 7.5 \) units

![Spatial Profile of Pu\textsuperscript{239} Density at t = 5 units interval for R = 7.5 units](image2.png)

**Fig. 15** Spatial profile of Pu\textsuperscript{239} for \( R = 7.5 \) units

<table>
<thead>
<tr>
<th>Table 1 Result with different radius</th>
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<tr>
<td>Normalized radius (dimensionless)</td>
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<td>Radius (cm)</td>
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<td>Normalized transient time (dimensionless)</td>
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<td>Transient time (days)</td>
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<td>Normalized transient length (dimensionless)</td>
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<td>Transient length (cm)</td>
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<td>Normalized velocity (dimensionless)</td>
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<td>Velocity (m/year)</td>
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<td>Normalized FWHM (dimensionless)</td>
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<td>FWHM (cm)</td>
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<td>Normalized FW10M (dimensionless)</td>
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<td>FW10M (cm)</td>
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shown in Fig. 16 is from 50 cm to 70 cm and then 200 cm to 1000 cm. From Fig. 16, it is clear that the change in TT is rapid for change in radius at a smaller value of radius but at the higher value of radius, TT tends to become flat.

Figure 17 shows the plot of TL in cm with respect to the radius in cm of the cylindrical medium. The range of the radii shown in Fig. 17 is from 50 cm to 70 cm and then 200 cm to 1000 cm. From the figure, it is clear that the change in TL is rapid for change in radius at smaller values of radius but at the higher value of radius, the variation in TL diminishes. The TL, just like TT values, starts saturating.

Conclusions

(1) The velocity of solitary wave propagation increases with the increase in the radius of the cylinder. In the present case, it saturates for R = 200 cm.
(2) When the neutron leakage is large, the wave propagation is slower.
(3) At a very small radius, the change in velocity with respect to the radius is sharp but at medium or large radius, the change is slower.

(4) FWHM and FW10M are relatively insensitive to radial neutron leakage.
(5) For the case considered, the TL is large in the beginning and saturates for R = 200 cm.
(6) For smaller radius, the TT is very high but it decreases with the increase in the radius. For R = 200 cm, the TT tends to become constant.
(7) At smaller radius, the TL decreases sharply with the increase in radius but at a larger radius, the TL decreases slowly with increase in radius.

In a nutshell, one can conclude that radial neutron leakage affects the burnup wave velocity relatively more and correspondingly the TT and TL. FWHM is relatively insensitive to neutron leakage. Larger the leakage, larger is the TT and TL and smaller are the velocity.

One of the important outcomes from the studies is that for a practical case of useful fuel burn-up wave development in TWRs, the radial neutron leakage is to be optimized carefully by adjusting the radius and length of the cylindrical reactor. The new parameters, i.e., transient length and transient time, in addition to wave velocity and FWHM, would serve the role of significant design parameters in the case of traveling wave reactors.

Nomenclature

- $B^2$ = buckling, 1/m$^2$
- $B_r^2$ = radial buckling, 1/m$^2$
- $B_z^2$ = axial buckling, 1/m$^2$
- $d$ = distance in medium to which plutonium is fused, m
- $D$ = diffusion coefficient of the medium, m
- $J$ = neutron current density entering the left boundary of cylindrical geometry, neutron/m$^2$ s
- $j_0$ = normalized neutron current density, dimensionless
- $l$ = diffusion length, m
- $n$ = normalized neutron density, dimensionless
- $N$ = neutron density, 1/m$^3$
- $n_{crt}$ = plutonium critical concentration under steady-state, $(N_{Pu}/N_{8})_{cr}$, dimensionless
- $n_{eq}$ = equilibrium concentration of plutonium under steady-state, $(N_{Pu}/N_{8})_{eq}$, dimensionless
- $N_{Pu}$ = normalized nuclide density of Pu$^{239}$, dimensionless
- $N_{8}$ = normalized nuclide density of U$^{238}$, dimensionless
- $n_{9}$ = normalized nuclide density of Np$^{239}$, dimensionless
- $N_{Pu}$ = nuclide density of Pu$^{239}$, 1/m$^3$
- $N_{tot}$ = sum of nuclide densities of all the nuclides, 1/m$^3$
- $N_8$ = nuclide density of U$^{238}$, 1/m$^3$
- $N_9$ = nuclide density of U$^{239}$, 1/m$^3$
- $N_{Pu}$ = nuclide density of Pu$^{239}$, 1/m$^3$
- $q$ = external source of neutrons, dimensionless
- $r$ = represents the normalized distance in radial direction, dimensionless
- $R$ = radius of cylinder, m
- $t$ = time, s
- $U$ = uranium, symbol
- $U_8$ = uranium-238, symbol
- $U_9$ = uranium-239, symbol
- $U_9$ = uranium-240, symbol
- $v$ = neutron velocity, m/s
- $V_f$ = number of neutrons produced per fission, dimensionless
- $z$ = represents the normalized distance in axial direction, dimensionless
- $z_l$ = normalized left boundary of the cylindrical geometry, dimensionless
\[ z_r = \text{normalized right boundary of the cylindrical geometry, dimensionless} \]

**Greek Symbols**

\[ \beta = \text{ratio of mean neutron lifetime to mean lifetime for the } \beta\text{-decay process, } \tau_d/\tau_\beta, \text{ dimensionless} \]

\[ \gamma = \text{constant, } 1/\tau_\gamma, \text{ dimensionless} \]

\[ \Delta t = \text{time interval, s} \]

\[ \eta = \text{ratio of equilibrium concentration of the plutonium to plutonium critical concentration, } n_{eq}/n_{ue}, \text{ dimensionless} \]

\[ \sigma_c = \text{microscopic capture cross-section, barn} \]

\[ \sigma_f = \text{microscopic fission cross-section, barn} \]

\[ \tau_d = \text{mean neutron lifetime, s} \]

\[ \tau_\beta = \text{mean lifetime for the } \beta\text{-decay process, s} \]

\[ \phi = \text{neutron flux, } 1/m^2/s \]

**Subscripts and Superscripts**

\[ c = \text{critical} \]

\[ eq = \text{equilibrium} \]

**Acronyms and Abbreviations**

CANDLE = constant axial shape of neutron flux, nuclear densities and power during life of energy producing reactors

FWHM = full width at half maximum

FW10M = full width at 10% of maximum

IIT = Indian Institute of Technology

TL = transient length

TT = transient time

TWR = travelling wave reactor

**References**


