Effect of bed roughness on 1-D entropy velocity distribution in open channel flow

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ABSTRACT

A theoretic-analytical formulation, based on entropy velocity profile law and classical relationships for uniform flow and friction factor, is proposed enlightening the general logarithmic relationship existing between the parameter $\Phi(M)$, defined as mean cross section velocity over maximum velocity, and the ratio water depth/bed roughness (D/d). The relationship $\Phi(M)$–D/d has been applied to a relevant set of experimental velocity measurement data collected both in laboratory and in field, showing different behaviour between small scale and large-intermediate roughness flows. In particular, the roughness influence becomes remarkable whenever shallow water flow conditions occur, that is when the ratio between the flow depth and the roughness height is less than 4, while $\Phi(M)$ tends to be constant as the value of D/d increases.

Key words | entropy velocity profile, friction factor, roughness, submergence, water discharge

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INTRODUCTION

Water flow measurements are basic data to be used in developing reliable surface-water supplies, providing crucial information on the availability of water discharge and its variability in time and space.

The knowledge of discharge, even to predict, is a necessity for water resource management, hydraulic design, hydrologic analysis, drought and flood forecasting, as well as water quality monitoring.

Measurement of discharge at gaging stations requires information about the mean velocity in a number of subsections across the river/channel and knowledge of cross-section geometry at measuring locations. Multiple depth and velocity measurements are taken by the current-meter across the channel in order to calculate the total stream discharge at a given moment. Such discharge values are used to define the cross-section rating curve, which represents an essential operative tool for river flow management even during flood events.

In natural rivers, velocity distribution is affected by channel geometry, vegetation and roughness. In very wide open channel flows, velocity increases monotonically from 0 at the channel bed to the maximum value at the water surface, and the distribution may be considered as one-dimensional (1-D). In the case of channels which are not considerably wide, besides the influence of the boundary, the velocity varies even along the transverse direction and a two-dimension distribution might be taken into account. Thus, the maximum velocity occurs at or below the water surface inducing dip-phenomenon and the position of maximum velocity is also influenced by the aspect ratio (B/D with B channel width and D water depth) (Ferro 2003; Yang *et al.* 2004). Several classical laws have been developed to describe the velocity distribution, such as log and power velocity distribution. Furthermore, in order to determine the mean velocity and the discharge in rivers and streams, numerous methods are available including the use of empirical formulas.

Originally derived for uniform flows, Manning's equation is also a well-known empirical formula, but its application under unsteady non-uniform flow conditions is extremely difficult. This is due to the variation of both the energy slope and Manning's roughness coefficient as well as in time and from cross section to cross section along the flow direction. Considering the limitations of classical

methods, Chiu (1987) derived the velocity distribution using the concept of informational entropy introduced by Shannon (1948) and evolved by Tsallis (1988), through which time-averaged velocity is assumed as a random variable. This velocity distribution has widely been employed in many different flow cases and enriched by relevant meaningful contributions, both theoretical and applied, derived from robust experimental results (Chiu 1989, 1991; Chiu & Said 1995; Xia 1997; De Araújo & Chaudhry 1998; Greco 1999, 2000; Greco & Mirauda 2004; Moramarco et al. 2004; Chiu & Hsu 2006; Marini et al. 2011; Mirauda et al. 2011). The main aspect of such a model is related to the need for only one parameter M known as entropy parameter. Such a parameter depends on the ratio between the cross section average and maximum velocities, $\Phi(M)$.

The way to evaluate $\Phi(M)$ represents a relevant issue in order to apply the entropy velocity distribution profile, causing a heated debate among researchers mainly addressed to the reasonable invariance of the velocity ratio for sections along the same river (Xia 1997; Ardiclioglu et al. 2007; Moramarco & Singh 2010). Therefore M should be assumed as a peculiar characteristic not only of the monitored site but also of the river reach where sites are located. This result might be very interesting to derive the rating curve to be employed on river monitoring and control at all stages. In fact, the knowledge of M, and hence of the entropic relationship, can be crucial to overcome the problem that velocity measurements during high floods can be made only in the upper portion of the flow area where the maximum velocity occurs.

On the other hand, the same experimental results, which ensure to assume the entropy parameter constant when high stages or floods occur, allow us to achieve a possible uncertainty concerning the variability of the ratio between the mean and maximum velocities during shallow flows, whereas the role played by bed roughness on the flow asset still remains relevant.

In such cases, the implementation of a numerical model might supply a correction of the friction factor in order to restore the energy fluxes, recalibrating the value of the location where the velocity profile predicts zero velocity, y_0, (Moramarco & Singh 2010) or rather assuming that the entropy parameter depends on the roughness or better on the relevance of roughness height, d, on water depth, D.

This paper deals with the analytic-theoretical derivation of the relationship between $\Phi(M)$ and the relative submergence, here defined as ratio D/d, using classical open channel flow equations. The $[\Phi(M)–(D/d)]$ relationship has been applied to a set of experimental velocity data collected both in laboratory and in field, showing a good response of the theoretical model but selecting different behaviour depending on the roughness scale. In fact, $\Phi(M)$ is strongly dependent on the ratio depth/roughness for values of D/d less than 4 when large or intermediate roughness scale occurs (Bathurst et al. 1981; Bathurst 1985), while it might be assumed constant to 0.66 for small roughness scale ($D/d > 4$) according to those observed during the high stages and floods (Moramarco & Singh 2010).

This result is very interesting not only from a scientific point of view but, indeed, it improves the suitability of the entropy velocity distribution not only at high stage but in shallow water flows as well, enforcing the model as a strongly operative tool.

ENTROPY VELOCITY PROFILE AND RELATIVE SUBMERGENCE IN OPEN CHANNEL FLOW

Shannon (1948) formulated the concept of entropy as a measure of information or uncertainty associated with the random variable or its probability distribution. Moreover, the principle of maximum entropy (POME) introduced the least-biased probability distribution of the random variable subject to given information in terms of constraints as well as the theorem of concentration for hypothesis testing, introducing what we normally refer to as the entropy theory. A quantitative measure of uncertainty associated with a probability distribution of a continuous random variable in terms of entropy, H, called Shannon entropy or informational entropy, is defined as

$$H = -\int_{-\infty}^{+\infty} p(x) \log p(x)\,\mathrm{d}x \tag{1}$$

where $p(x)$ is the continuous probability density function of random variable x.

Using POME, entropy can be maximized through the method of Lagrange multiplier as follows:

$$L = -\frac{1}{m-1}\int_{-\infty}^{+\infty} p(x)\left\{1 - [p(x)]^{m-1}\right\}dx + \sum_{i=1}^{N}\lambda_i\, g_i(x) \tag{2}$$

where m is a real number greater than 0, $g_i(x)$ is the ith constraint function, and λ_i is the Lagrange multiplier for each constraint, reflecting its weight in the maximization of entropy.

Chiu (1987, 1991) applied such a concept of entropy to open-channel flows, including the modelling of velocity distribution, shear stress and sediment concentration. Analysis of velocity distribution in the probability domain has an advantage in determining the cross-sectional mean velocity and the momentum and energy coefficients without dealing with the geometrical shape of cross sections, which tend to be extremely complex in natural channels (Chiu 1991).

To relate the entropy-based probability distribution to the spatial distribution, an assumption on the probability distribution in the space domain is needed.

Considering u to be the time-averaged and, therefore, time invariant velocity on an iso-velocity, which is assigned a value ξ. The value of u is almost 0 at ξ_0 which corresponds to the channel boundary, and u reaches $U_{\max}$ at $\xi_{\max}$, which may occur at or below the water surface. Under such circumstances, u monotonically increases from ξ_0 to $\xi_{\max}$. Then, at any value of the spatial coordinate less than ξ, the velocity is less than u, which can be written in the cumulative distribution function as

$$F(u) = \frac{\xi - \xi_0}{\xi_{\max} - \xi_0} \tag{3}$$

Thus, the Shannon entropy of velocity distribution can be written as:

$$H = -\int_0^{U_{\max}} p(u)\log p(u)\,du \tag{4}$$

Through a similar procedure, the probability density function of the velocity distribution is obtained by maximizing the Shannon entropy equation

$$L = \int_0^{U_{\max}} \frac{f(u)}{m-1}\left\{1 - [f(u)]^{m-1}\right\}du + \lambda_0\left[\int_0^{U_{\max}} f(u)\,du - 1\right] + \lambda_1\left[\int_0^{U_{\max}} uf(u)\,du - \bar{u}\right] \tag{5}$$

in which λ_0 and λ_1 are the Lagrange multipliers and the following constraint equations

$$C_1 = \int_0^{U_{\max}} f(u)\,du = 1 \tag{6}$$

$$C_2 = \int_0^{U_{\max}} uf(u)\,du = \bar{u} \tag{7}$$

$$f(u) = \exp(\lambda_0 - 1 + \lambda_1 u) \tag{8}$$

Thus, Chiu's velocity distribution results as

$$u = \frac{U_{\max}}{M}\ln\left[1 + (e^M - 1)F(u)\right] = \frac{U_{\max}}{M}\ln\left[1 + (e^M - 1)\frac{\xi - \xi_0}{\xi_{\max} - \xi_0}\right] \tag{9}$$

where M is the dimensionless entropy parameter introduced in the entropy-based derivation (Chiu 1988; Chiu & Said 1995; Chiu & Tung 2002; Luo & Singh 2011; Cui & Singh 2013). Hence, M can be used as a measure of uniformity of probability and velocity distributions. The value of M can be determined by the mean, U_m, and maximum velocity values derived from the following equation:

$$\Phi(M) = \frac{U_m}{U_{\max}} = \left(\frac{e^M}{e^M - 1} - \frac{1}{M}\right) \tag{10}$$

This parameter has proved to be useful for characterizing and comparing various patterns of velocity distributions and the status of the open-channel flow system, which can be expressed by the location of mean and maximum velocity and their relationships. The mean velocity value, the location of the mean velocity (Chiu & Said 1995; Chiu & Tung 2002) and the energy coefficient (Chiu & Murray 1992) can be obtained from M. The use of the entropy parameter predetermined for

a channel section can greatly ease discharge estimation, especially in unsteady flow (Chiu & Murray 1992).

The mean velocity, in fact, is another main characteristic of channel flow. With the known mean velocity value, the flow discharge, sediment transport and pollutant transport can be obtained. A linear relation between mean and maximum velocities was discovered by collecting the velocity data in some cross-sections of the Mississippi River (Xia 1997).

Equation (10), indeed, represents the fundamental relationship, from an applied point of view, of the entropy velocity distribution and the assessment of the entropy parameter pass through the knowledge of the ratio between mean and maximum velocities, $\Phi(M)$.

Therefore, moving inside the domain of classical hydraulic and open channel flow equations (Rouse 1950; Streeter 1961; Roberson 1968), it should be possible to take into account a conspicuous budget of relationships among mean velocity, maximum velocity, dynamic and geometric characteristics of the flow. Let us assume the Chezy equation to derive the mean velocity as follows:

$$U_m = C\sqrt{gR_iS_0} = Cu_* \tag{11}$$

in which $u_* = \sqrt{g\,R_i\,S_0} =$ shear velocity, $R_i =$ hydraulic radius, $S_0 =$ energy slope, $g =$ gravity acceleration and $C =$ dimensionless resistance coefficient. C can be expressed through the generalised Colebrook equation, or similar, extended to the open channel flow in fully developed turbulent flow as follows:

$$C = \frac{1}{k}\ln\left(\frac{13.3\,\varphi\,R_i}{\varepsilon}\right) = \frac{1}{k}\ln\frac{D}{d} + \frac{1}{k}\ln C_0 \tag{12}$$

where $\varepsilon =$ equivalent bottom roughness, $\varphi =$ shape coefficient, $D =$ water depth, $d =$ characteristic bottom roughness height (i.e. d_{50} or d_{84}) and $C_0 =$ dimensionless coefficient introduced in order to express C as a function of D/d. Thus Equation (11) becomes

$$\frac{U_m}{u_*} = \frac{1}{k}\ln\frac{D}{d} + \frac{1}{k}\ln C_0 \tag{13}$$

The maximum velocity conveys an important factor concerning channel flow as it defines the range of velocity distribution. The location of maximum velocity is of interest, as the maximum velocity does not always occur at the water surface, but some distance below it, which is called the dip phenomenon. It is stated that the dip phenomenon is caused by the secondary currents (Nezu & Nakagawa 1993), which is the circulation in a transverse channel cross section as the longitudinal flow component is called primary flow. Because the secondary motion will transport the low momentum fluid from the near bank to the middle of the flow, the high-momentum fluid moves from the free surface toward the bed.

Yang et al. (2004) investigated the mechanism of dip phenomenon as dependent on the secondary currents in open-channel flow. In their study, a dip-modified log law for the velocity distribution in open channel was developed and its result was good, thus

$$\frac{u(y)}{u_*} = \frac{1}{k}\ln\frac{y}{y_0} + \frac{\alpha}{k}\ln\left(1 - \frac{y}{D}\right) \tag{14}$$

in which $y =$ distance from the bottom, $\alpha =$ dip-velocity derived as follows:

$$y_{\max} = \frac{D}{1+\alpha} \tag{15}$$

with $y_{\max} =$ distance from the bottom at which the maximum velocity, $U_{\max}$ occurs.

Thus, combining Equations (14) and (15) results

$$\frac{U_{\max}}{u_*} = \frac{1}{k}\ln\left(\frac{D}{y_0(1+\alpha)}\right) + \frac{\alpha}{k}\ln\left(\frac{\alpha}{1+\alpha}\right) \tag{16}$$

with $y_0 =$ location where the log velocity profile predicts the zero value, assumed to be proportional to the characteristic bottom roughness height, d, as suggested by Rouse (1965) and Ferro (2003)

$$y_0 = C_\xi d \tag{17}$$

in which $C_\xi =$ experimental parameter, therefore:

$$\frac{U_{\max}}{u_*} = \frac{1}{k}\ln\left(\frac{D}{d}\right) + \frac{1}{k}\ln\left(\frac{\alpha^\alpha}{C_\xi(1+\alpha)^{1+\alpha}}\right) \tag{18}$$

Finally, the ratio between Equations (13) and (18) explicitly proposes $\Phi(M)$ as function of relative submergence D/d

$$\Phi(M) = \frac{U_m}{U_{\max}} = \frac{\ln\left(\dfrac{C_0 D}{d}\right)}{\ln\left[\dfrac{D}{d} \dfrac{\alpha^{\alpha}}{C_{\xi}(1+\alpha)^{1+\alpha}}\right]} = A_{\Phi}\ln\frac{D}{d} + B_{\Phi} \quad (19)$$

where A_{Φ} and B_{Φ} numerical coefficients and the ratio D/d represents the relative submergence.

Equation (19) gives reason for a possible effect of bed roughness on the entropy velocity distribution in open channel flow dependent on the roughness scale whether large, intermediate or small (Bathurst 1985).

LABORATORY AND FIELD DATA ANALYSIS

The above mentioned dependence between the entropy parameter, M, through the ratio $\Phi(M)$, and the relative submergence, D/d, has been studied referring to a wide volume of data, collected both in the laboratory and field. Such a database has resulted particularly significant covering a relevant interval of relative submergence, ranging from 1.9 up to 17, water discharge, from few litres up to some cubic metres per second, and slope. In fact, giving a quick overview of the data set, it should be possible to outline the following.

(1) *Field data* collected on several monitoring cross sections along different rivers in Southern Italy (Follone and Amato rivers in Calabria and Basento, Sinni, Agri and Cavone in Basilicata) sorted in two main classes:

(a) *inbank flow*: referred to natural rivers with a full set of velocity measurements and slope in the range 0.2–0.8%, water discharge in the range 0.15–9 mc/sec, mean sediment diameter, d_{50}, in the range 3–8.6 cm, assumed as the roughness height, and relative submergence (here assumed $D/d = D/d_{50}$), in the range 4–17;

(b) *low stage flow*: referred to suitable velocity measurements carried on natural streams with low water depth and slope in the range 0.1–1%, water discharge in the range 0.017–1.9 mc/sec, mean sediment diameter, d_{50}, in the range 3–6 cm, assumed as the roughness height, and relative submergence (here assumed $D/d = D/d_{50}$), in the range 1.2–4.

(2) *Laboratory data* sampled on a rectangular flume with a regular bed roughness, d, slope in the range 0.05–1%, water discharge in the range 7–72 L/sec and relative submergence (D/d), in the range 2–7 (Mirauda *et al.* 2011). The laboratory activities were developed in order to simulate low stage flows assumed to be significantly representative of intermediate and high roughness open channel flows. The experimental tests were carried out in the Hydraulics Laboratory of Basilicata University, on a free surface flume of 9 m in length and with a cross section of 0.5×0.5 m, whose slope can vary from 0 up to 1%. A set of wood spheres of 0.035 m in diameter (d) was placed on the bed reproducing homogenous roughness. These elements were located in order to obtain a roughness concentration, λ, expressed as the ratio between the total projected area of the spheres and the reference area, equal to 0.15, corresponding to the maximum flow resistance (Rouse 1965).

Furthermore, the data considered in the present study have been compared to those reported by Moramarco & Singh (2010) referring to two cross sections, Santa Lucia and Ponte Nuovo, on the Tiber river.

The field (U_m; $U_{\max}$) is actually affected by the scaling effect due to the different physical domains existing between flume and rivers, as well as between low and inbank flows, as shown in Figure 1. In the same figure, the linear regressions differentiated among the two main data set, field and laboratory data, and in between the bulk of field data are reported, showing quite similar values of $\Phi(M)$.

Moreover, while pairs (U_m; $U_{\max}$) referred to inbank flows and Moramarco & Singh (2010) data seem to be distributed around the linear regression showing a uniform trend of spreading, and with slope close to 0.66, the low stage flow data and laboratory measurements allow us to observe a general tendency to overestimate the real ratio $\Phi(M)$ when the velocities decrease. In fact, low stage flow data set presents a linear high regression with a slope close to 0.63, while laboratory measurements present an average slope of about 0.68 but increasing as the velocity
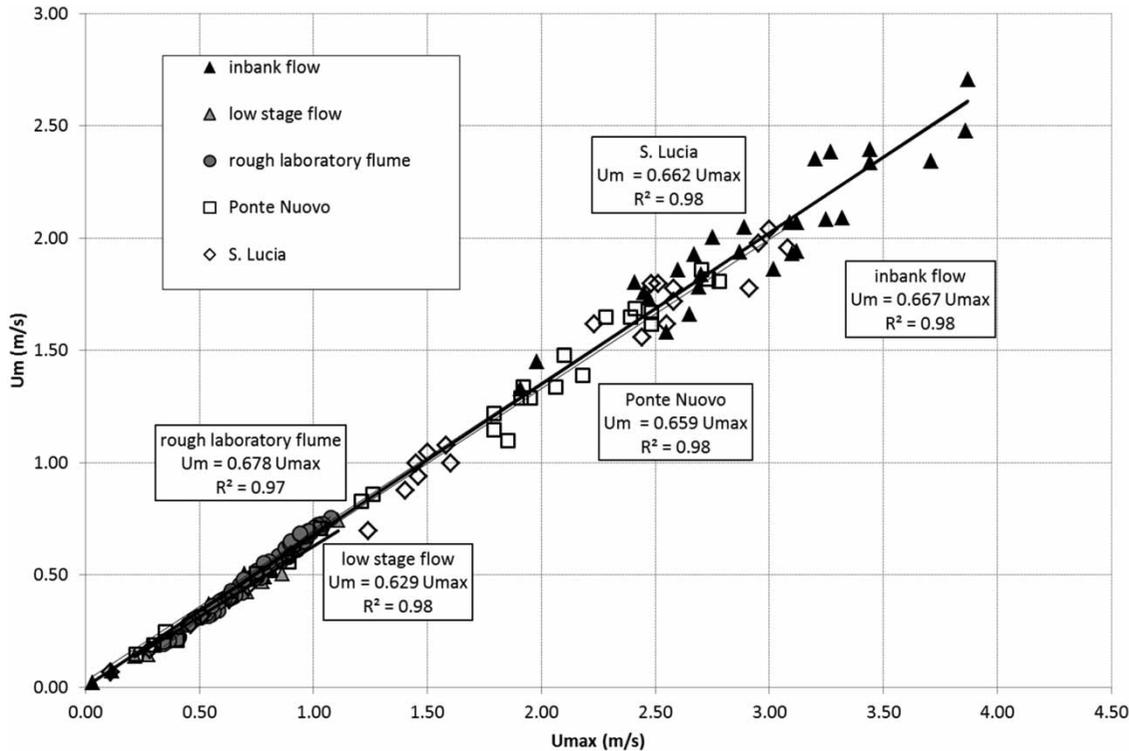
Figure 1 | Mean and maximum velocities observed in laboratory and field (S. Lucia and Ponte Nuovo derived by Moramarco & Singh 2010).

increases. That is, for low velocity, generally corresponding to high roughness flows, the values of the ratio between the mean and maximum velocities tend to decrease. This indicates a possible dependence of $\Phi(M)$ on local flow condition changes. Thus, assuming the relative submergence as an inducing factor of the variation in the value of $\Phi(M)$, as theoretically demonstrated above, and plotting this ratio versus the corresponding relative submergence D/d, it is possible to observe different behaviour occurring on the roughness scale (Figure 2).

According to Bathurst et al. (1981) and Bathurst (1985), different flow resistance occurs depending on relative submergence, thus $\Phi(M)$ can be assumed strongly dependent on the ratio depth/roughness for values of D/d less than 4 when large and intermediate roughness scale occurs (Bathurst et al. 1981; Bathurst 1985), while ratio $\Phi(M)$ might be assumed constant almost uniform for small roughness scale ($D/d > 4$). This last issue agrees with that observed during high stages and floods by Moramarco & Singh (2010) which assume a constant ratio between mean and maximum velocities and close to 0.66.

Equation (19) can be applied to the present data, taking into account the constraint introduced by two main regimes of large-intermediate roughness ($D/d < 4$) and small roughness obtaining

$$\begin{cases} A_\Phi = 0.11 \\ B_\Phi = 0.51 \end{cases} \Rightarrow \frac{D}{d} < 4 \\ \qquad\qquad \text{and} \\ \begin{cases} A_\Phi = 0 \\ B_\Phi = 0.66 \end{cases} \Rightarrow \frac{D}{d} > 4 \tag{20}$$

Thus Figure 2 can be plotted as Figure 3, in which two sub-domains can be selected through the value of the relative submergence. The theoretical approach discussed in the previous paragraph finds a robust result in the analysis of real data, allowing us to consider the ratio between mean and maximum velocities as variable once shallow flow condition occurs.

The result obtained allows us to improve the use of the entropy velocity law during operative activities in field. In fact, once the roughness condition is defined, velocity measurements can be collected in the cross section at all
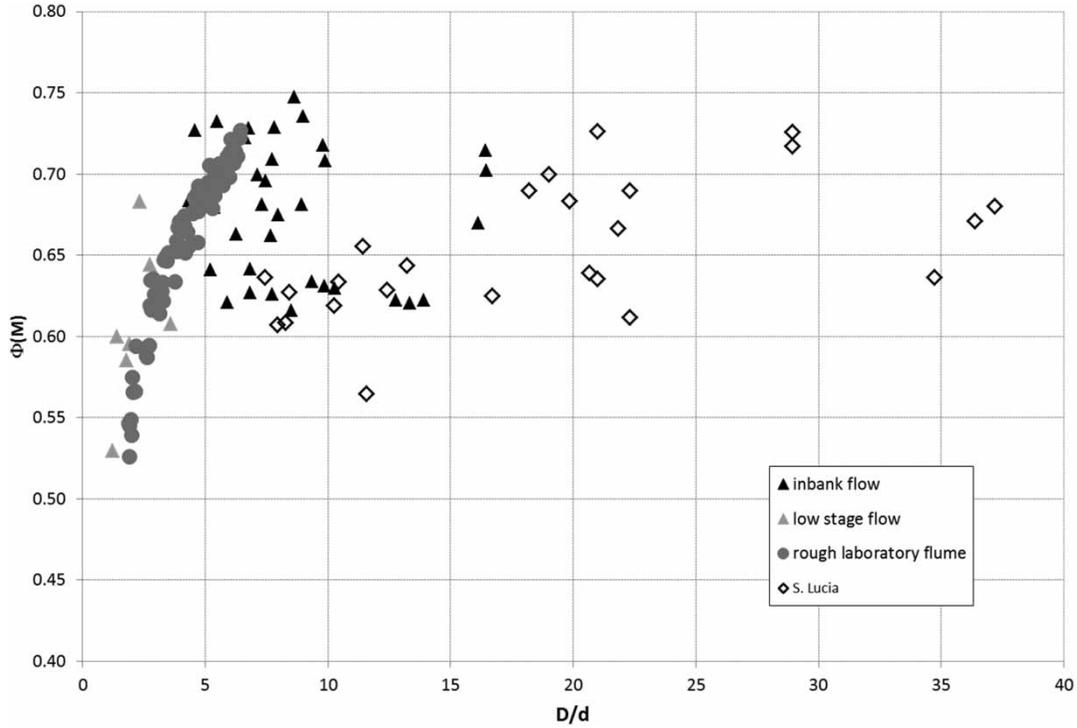
Figure 2 │ Mean and maximum velocities ratio, $\Phi(M)$, versus relative submergence, D/d, for laboratory and field data set (S. Lucia derived by Moramarco & Singh 2010).
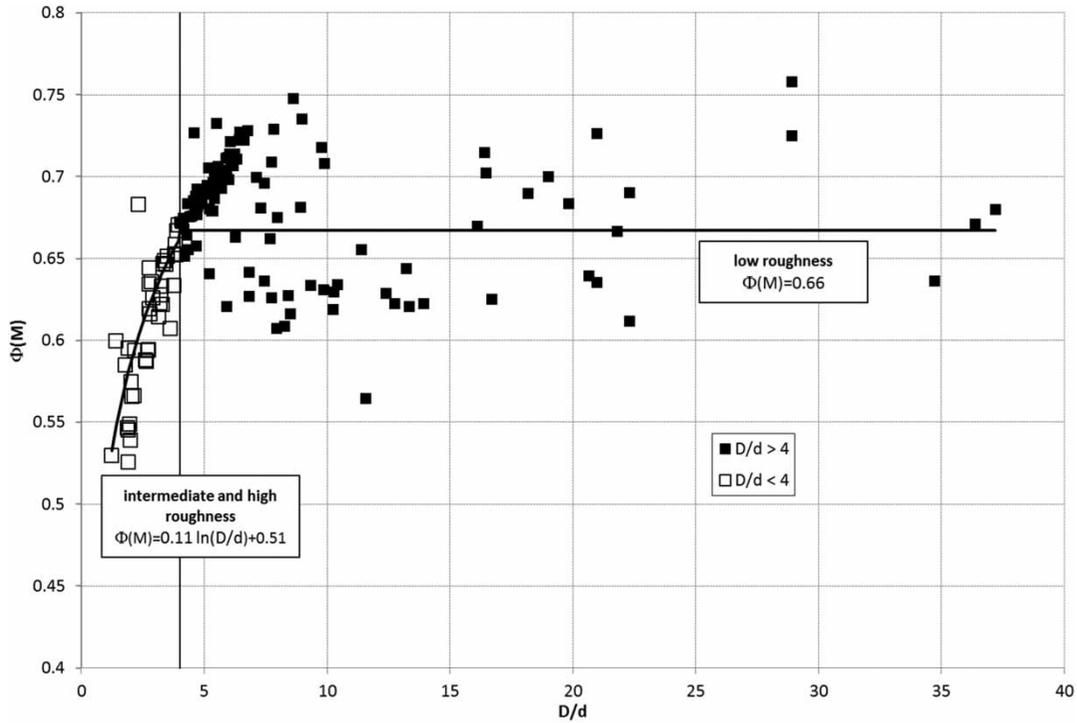

Figure 3 │ General relationship between mean and maximum velocities ratio and relative submergence.
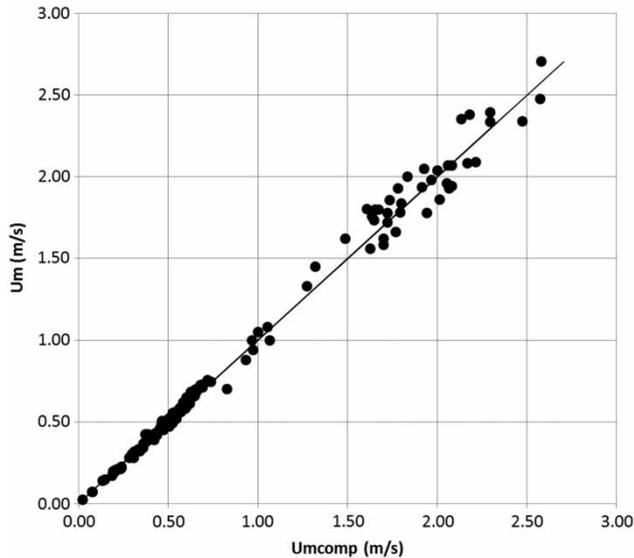
Figure 4 │ Observed mean velocity versus computed mean velocity for all data sets.

stage levels, low, medium, high and flood, obtaining the maximum one (assumed as the maximum value among all measures) to employ in Equation (10) together with value

of $\Phi(M)$ corresponding to the observed relative submergence. Of course, for $D/d > 4$, $\Phi(M)$ can be assumed constantly equal to 0.66.

Figure 4 reports the observed U_m for the investigated data set and the corresponding values U_{mcomp}, obtained through Equation (10) using $\Phi(M)$ derived by Equation (20). The comparison is very good even if the field data presents a wider spreading around the bisector than the laboratory data ones, that can be justified through the different levels of accuracy during the two measurement settings. Laboratory data collection can be assumed much more precise than the river data.

However, the quality of applicative model response, obtained coupling Equations (20) and (10), can be better evaluated through the assessment of the resulting discharge. In fact, Figure 5 reports the percentage errors between the observed water discharge and the calculated one, outlining a very good response with relatively low values of uncertainty.

Thus, Equation (20) might be operatively employed in the monitoring and forecasting procedures for water discharge assessment in natural open channel at all stages
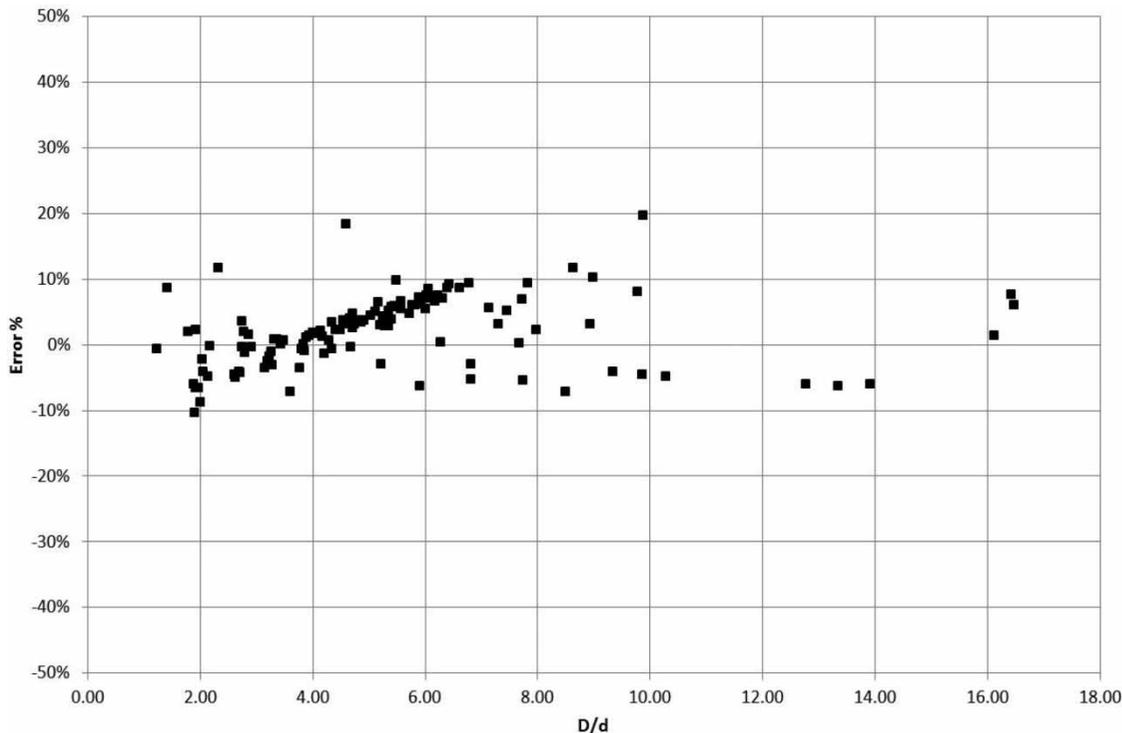

Figure 5 │ Observed percentage error in water discharge assessment.

low, medium and high flow or, rather, at different roughness regimes high, intermediate and low.

CONCLUSIONS

A simple but effective theoretic-analytical formulation, based on entropy velocity profile law and classical relationships for uniform flow and friction factor, is developed to propose a general logarithmic relationship existing between parameter $\Phi(M)$, defined as mean cross section velocity over maximum velocity, and the ratio water depth/bed roughness (D/d).

The $[\Phi(M)–(D/d)]$ relationship has been applied to a set of experimental velocity data collected both in the laboratory and field, showing a good response of the theoretical model but selecting different behaviours depending on the roughness scale. Ratio $\Phi(M)$ results are strongly dependent on the ratio depth/roughness for values of D/d less than 4, when large and intermediate roughness scales occur, while it might be assumed constant to 0.66 for small roughness scale $(D/d > 4)$ and flooding stages according to literature.

A comparison between the measured and the estimated discharges indicates a good performance of the present model improving the suitability of the entropy velocity distribution not only at a high stage but in shallow water flows as well, enforcing the model as a strongly operative tool.

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