A minimalist model of terminal lakes: Qinghai Lake (China) and Lake Chad (N Africa)

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ABSTRACT

The ratio of the areas of a terminal lake to its total basin provides a geomorphological measure, which characterises the geobotanical state of water limited climates. A minimalist model of this lake area ratio is introduced combining the water balances of lake and land, the Budyko–Schreiber rainfall–runoff chain over land and, as a closure, the land and lake potential evaporation to be of similar magnitude. The following results are analysed. (i) Model diagnostics quantify a dryness threshold separating water from energy limited climate regimes, which coincides with an upper bound for the area ratio of terminal lakes to remain hydrologically closed. (ii) Model validations for Qinghai Lake and Lake Chad demonstrate model and observed water flux budgets to be in close agreement. (iii) Finally, a paleo-climate sensitivity study for Lake Chad demonstrates that the minimalist model appears to be a viable tool for future and paleo-climate estimates based on lake area ratio changes only and a single reference water flux. Furthermore, a stochastic interpretation of the rainfall–runoff chain allows estimates of water flux variability.

Key words | climate change, closed lakes, paleo climate, paleo environment, rainfall–runoff chain, thresholds

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INTRODUCTION

Lakes are a remarkable pattern of the Earth's surface which, jointly with river flow and vegetation, characterise regional climates. In particular, terminal (closed or endorheic) lakes which, embedded in a closed basin, describe the hydrological relation between geomorphological structure of the land–lake topography, the geobotanical properties of the land–vegetation–climate system and the large-scale atmospheric conditions. In this sense, lake-basin area ratio (as a geomorphological measure) and vegetation classes (a geobotanic quantity) represent climate variables corroborating response and feedback to the atmosphere's forcing of the global circulation (in terms of the net incoming radiation and precipitation) and to the Earth's climate system. A first attempt introducing vegetation as a climate variable was proposed by Koeppen (1936) leading to a quantitative classification of the land surface climate state based on vegetation information. Early proposals to derive the regional surface water and energy flux climates from terminal lake area ratio data have been applied by Broecker & Orr (1958) and Snyder & Langbein (1962). Employing lake area-dependent equilibrium water balance models they obtained quantitative paleo-climate indicators (see Broecker & Orr (1958), Snyder & Langbein (1962), and references therein). For example, paleo-climate precipitation has been estimated (Kutzbach 1980; Hastenrath & Kutzbach 1985) combining parameterisations of land and lake evaporation, a diagnostic rainfall–runoff relation (Schreiber 1904), and the observed ratios of lake and basin areas. This approach has been applied in many other studies.

Here we introduce a minimalist model also combining geomorphological (lake area ratio) and geobotanic (dryness ratio) indicators to estimate the environmental climate state in terms of its climate mean water and energy cycle (section below). To substantiate its applicability (see Fraedrich & Sielmann 2011), the minimalist approach is validated (the

doi: 10.2166/nh.2013.015

following section, Lakes revisited) for a large terminal lake in China (Qinghai, Rhode *et al.* (2010)) and in Africa (Lake Chad, Kutzbach (1980)). A conclusion provides a critical appraisal of the results.

TERMINAL LAKES: CLIMATOLOGICAL EMBEDDING AND MODEL

This section introduces the minimalist terminal lake model, commencing with the climatological embedding, followed by the derivation of the model and the assumptions leading to it. An outline of the model analysis follows, which includes an assessment of the mean fluxes, their sensitivity and variability.

Hydrological setting

The land surface hydrology is controlled by long-time means of the rainfall–runoff chain commencing with atmospheric water supply P and ending with soil water runoff Ro:

$$P = Ro + E \tag{1}$$

The partitioning of the water supply between runoff, Ro and evaporation, E, requires a suitable parameterisation of the land surface water fluxes. Here we use the approach introduced by Schreiber (1904) which, a century later, is referred to as Budyko's hypothesis (Budyko 1974; Kutzbach 1980; Dooge 1992; Wang & Takahashi 1998; Koster & Suarez 1999; Arora 2002; Zhang *et al.* 2004; Yang *et al.* 2006; Sharif *et al.* 2007):

$$Ro/P = \exp(-PE/P) = \exp(-D) \tag{2}$$

That is, the runoff ratio, $C = Ro/P$, is parameterised in terms of the dryness ratio, $D = PE/P$, which represents water demand (corresponding to the water equivalent of energy available for evaporation) or potential evaporation, PE, versus water supply or rainfall, P. This parameterisation can also be formulated in terms of the evaporation ratio $F = E/P = 1 - C$. A theoretical derivation of Schreiber's parameterisation (Fraedrich 2010) is based on a biased coin-flip Ansatz to describe the land's hydrological cycle in terms of a rainfall–runoff chain. It commences with exponentially distributed daily random rainfall, which provides the water input to the biosphere representing the fast water reservoir (interception by vegetation and wetted ground) covering the land surface; only rainfall exceeding the potential evaporation supplies the soil which, as the integrating slow system, provides the discharge. Furthermore, it allows an estimate of the natural variability of the land surface climate regime including variances, covariances and extremes in drought and wetness. For a comprehensive application, see Fraedrich & Sielmann (2011).

Dryness ratio

The dryness ratio D enters the Schreiber parameterisation as a climate indicator and fundamental physical measure, which characterises the Earth's land surface (Budyko 1974). It is defined by the flux ratio of water demand over water supply:

$$D = PE/P \tag{3}$$

This ratio is used for geobotanic diagnostics combining physical flux properties with vegetation (Budyko 1974). Two basic geobotanic climate regimes are distinguished: the energy limited regime, $0 < D < 1$, is separated from the water limited regime $D > 1$ at the threshold $D = 1$. Furthermore, a dryness ratio spectrum describes the land surface biosphere in terms of vegetation types ranging from tundra, $D < 1/3$, and forests, $1/3 < D < 1$, via steppe and savanna, $1 < D < 2.0$, semi-desert $2.0 < D < 3.0$, to deserts $3.0 < D$. These bounds are common in the hydrology community (and far from being outdated; see, for example, Shen & Chen (2010)). The two regimes separate the vegetation spectrum at $D = 1$. Energy limitation occurs, if available energy supporting the water demand PE is low; thus runoff exceeds evaporation for given precipitation, $E \sim PE$. Water limitation occurs, if available energy is so high that water supply by precipitation evaporates, which then exceeds runoff, $E \sim P$. Thus, from dryness ratio and precipitation the hydrological cycle can be deduced, that is, potential evaporation PE, evaporation E and runoff Ro (using the respective flux ratios).

Bowen ratio

The Bowen ratio, which describes the latent-versus-sensible heat flux ratio connecting water and energy flux balances, is introduced to complete the non-dimensional flux ratios introduced so far. Replacing potential evaporation by net radiation (Budyko 1974) $N = PE$, leads to the Bowen ratio, $B = H/E$, which can also be parameterised in terms of Schreiber's formula using the steady state energy flux balance $N = E(1 + B)$:

$$B^* = \{D/(1 - C)\} - 1 = D/\{1 - \exp(-D)\} - 1$$

The star superscript is used to identify the approximation induced by the Schreiber–Budyko relation. For example, at $D = 1$, the Bowen ratio attains the value $B^* \sim 0.4$ related to observed climates (Budyko 1974).

Lake area ratio

An additional regional climate indicator is the lake area ratio of terminal (or closed) lakes. The terminal lake area ratio combines the areas of lake, a_{lake} and watershed, a_{land}, to obtain

$$A = a_{\text{lake}}/(a_{\text{lake}} + a_{\text{land}}) \tag{4}$$

Lake overflow occurs at $A = 1$, when $a_{\text{land}} = 0$. An equilibrium model of terminal lakes can be derived from the lake area averaged water fluxes (see, for example, Mason *et al.* (1994)):

$$P_{\text{lake}} = Ro_{\text{lake}} + E_{\text{lake}} \tag{5}$$

Lake properties, like precipitation and runoff (=lake inflow) are indicated by subscripts. A lake area ratio A, which depends on basin and on lake evaporation, E and E_{lake}, and on the common precipitation, P, can be derived. Combining Equations (4) and (5) with the land surface water balance (Equation (1)) after weighting by the land and lake areas, a_{lake} and a_{land}, respectively, one obtains

$$A = (P - E)/(E_{\text{lake}} - E) \tag{6a}$$

or $A = Ro/(Ro + PE–P)$. Additional assumptions have been made. (i) The catchment runoff provides the water inflow to the lake, $Ro\ a_{\text{land}} = –Ro_{\text{lake}}\ a_{\text{lake}}$. (ii) Exchanges between lake and aquifer are null unless implicitly included in the runoff term. (iii) Precipitation over lake and catchment are the same, $P = P_{\text{lake}}$. (iv) Systematic biases are neglected in estimating lake properties; these are, for example, the 'vapour blanket effect' of lakes in hot and dry environments (postulated by Fraedrich (1972b), and measured by Burba *et al.* (1999); see also Fraedrich *et al.* (1977)) and the 'Lake Victoria effect' of dynamically enhanced rainfall over lakes in tropical environments, which plays a role when climate induced land–lake differences generate a rainfall surplus over the lake induced by the nocturnal land–sea breeze convergence (Flohn & Fraedrich 1966; Fraedrich 1972a; Hastenrath & Kutzbach 1985; and textbooks). Here we introduce a minimalist version of the lake area model, to provide estimates of paleoprecipitation (and thus the water flux balances) from lake area sensitivities.

Minimalist model

Water balance (Equation (1)) and Schreiber–Budyko formula (Equation (2)) determine the lake–land equilibrium climate state. This approach has been suggested by Kutzbach (1980) and succesfully employed to paleo-lakes since. Instead of parameterising land and lake radiative fluxes and the related Bowen ratios, which requires a large set of parameters, we assume the potential evaporation of lake and catchment to be of the same magnitude:

$$E_{\text{lake}} = f\,PE$$

Thus only land information is required to determine the lake's structural behaviour (here the f-factor serves as a tracer, discussed later). The minimalist lake area ratio can now be re-formulated (from Equation (6a)) reducing the two surface flux ratios D and C, to a single one employing the means of the stochastic (biased coinflip) rainfall–runoff chain. That is, introducing Schreiber's parameterisation (Equation (2); Schreiber 1904; Fraedrich 2010) to the lake area ratio (Equation (6)) turns the model (Equation (6a)) to a minimalist one which depends only on the dryness ratio $D = PE/P$, as an indicator of the geobotanic state of the environment:

$$A^* = (1-F)/(D-F) = C/(D-1+C)$$
$$= \exp(-D)/(f\,D - 1 + \exp(-D)) \tag{6b}$$

The star superscript is used to identify the approximation induced by the Schreiber–Budyko relation. Given the relative lake area A^*, the dryness ratio D can be determined as all other flux ratios. With E_{lake} being parameterised by the potential evaporation PE of the catchment basin and setting $f = 1$, the maximum possible terminal lake area ratio is attained at $A^* = 1$, when the dryness ratio yields the threshold value at $D = 1$, which separates water from energy limited climates. This structure change occurs under the topographically idealised condition of a constant height basin boundary. Then the terminal lake reaches its natural overflow threshold at its largest possible extent or $a_{\text{land}} = 0$. That is, closed or terminal lakes, $0 < A < 1$, exist only under water limited land conditions, $1 < D < \infty$ and the lake area ratio decreases for increasing dryness ratio D. In contrast, overflow occurs under energy limited regimes (and ideal topography) for $D < 1$ (Figure 1). Of course, natural basins are not limited by such idealised conditions, because outlets occur at lower heights with water flowing into the lake from the remaining parts of the catchment, $a_{\text{land}} > 0$.

Pan evaporation over land has been suggested as an estimator for lake surface evaporation with an f-factor, $f \sim 0.7$

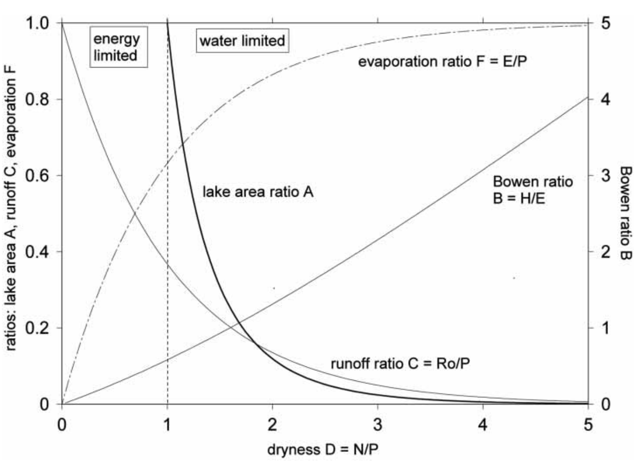

Figure 1 | Minimalist model of terminal lakes in a climate state diagram: dryness ratio dependent climate variables are the lake area ratio *A*, the Bowen ratio *B*, the runoff ratio *C* and evaporation ratio *F*.

(see Li *et al.* 2007), which suggests a smaller lake than pan evaporation. This accounts for the vapour-blanket, fetch and many other effects, which influence lake evaporation and leads to a larger lake area ratio (Equation (6b)) at similar dryness conditions. For $f > 1$, the 'Lake Victoria' effect may be implemented due to enhanced surface temperatures over the lake. However, as the physically plausible threshold, which limits the lake area ratio at $A = 1$, coincides with the dryness ratio threshold $D = 1$, we do not employ the f-factor as a correction and set $f = 1$ in the following.

Note that the dryness ratio is employed here as a climate state geobotanic indicator to validate terminal lakes and to estimate their sensitivity for future scenarios or paleoclimate studies; the runoff ratio can also be used (as in Rhode *et al.* (2010), see their Figures 5 and 6) or the Bowen ratio (see Kutzbach 1980).

LAKES REVISITED: QINGHAI AND CHAD

In the following, closed lakes (Qinghai and Chad) are revisited to validate the minimalist model with the present day climate (or reference state) combining geomorphological, geographic and geobotanic indicators. That is, basin size and lake area define the lake area ratio, A, as geomorphological indicator which, by the Schreiber–Budyko Ansatz, determines the basin's climate in terms of a geobotanic indicator; that is the ratios of dryness D, runoff C, evaporation F or the Bowen ratio B. Furthermore, given also the reference precipitation P, the full hydrological cycle can be estimated by the minimalist model (Figure 1); that is, evaporation over the lake PE, over land E, land runoff Ro (or lake inflow) and, with the Bowen ratio, also the sensible heat flux H over land.

The subsequent validation is a model diagnostic which, given the observed geographic–geomorphological input (lake area ratio A), compares the model output with parameters (flux ratios) of the observed climate reference state to characterising the related vegetation type and water cycle intensity (dryness ratio D and the other related). In addition, given the observed rainfall, estimates of the water fluxes like runoff and evaporation are also obtained and compared with observations. Note that it is our aim to merely demonstrate the minimalist Ansatz and to provide an educated first guess for, say, paleo-climate estimates, and not to replace comprehensive model approaches.

Qinghai Lake: A validation analysis

Qinghai Lake (Koko Nor) is situated in the cold and arid climate of the north-eastern Qinghai-Tibetan plateau (37°N, 3,200 m). It is China's largest closed-basin lake, which is affected by the major Asian circulation systems: the East and South-West Asian monsoon in summer and the dry westerlies of northern Eurasia. The lake is considered to be sensitive to climate change with its area almost doubling between the extremes from present day to maximum extent during earlier climate states in the Holocene (10 to 5 kyrs bp), $a_{\mathrm{lake}} \sim 4{,}300$ to $7{,}655\ \mathrm{km}^2$, which is embedded in a basin area of $a_{\mathrm{lake}} + a_{\mathrm{land}} \sim 29 \times 10^3\ \mathrm{km}^2$. This corresponds to a lake area ratio range, $A \sim 0.145$ to $A_{\max} \sim 0.258$, or interval $\delta A \sim 0.113$. This and further information on the present day hydrology of Qinghai Lake is taken from Rhode *et al.* (2010, Section 3.3) (see also Walker (1993), Boehner (1994), Lehmkuhl & Haselein (2000), An *et al.* (2006), Herzschuh (2006), Colman *et al.* (2007) and Li *et al.* (2007)).

First, land water fluxes are analysed. The observed annual mean fluxes (1959–2000, see Rhode *et al.* (2010) and Li *et al.* (2007)) are rainfall with standard deviation, $P \sim 0.36 \pm 0.065$ m/yr (and range 0.25 to 0.40 m/yr), which is balanced by evaporation $E \sim 0.30$ m/yr (from shore to mountains, 1.00 to 0.30 m/yr) plus surface runoff $Ro \sim 0.054$ to 0.059 m/yr per unit catchment area of $25.4 \times 10^3\ \mathrm{km}^2$ (or stream flow of $\sim 1.5\ \mathrm{km}^3$/yr without groundwater discharge $0.6 \times 10^9\ \mathrm{m}^3$/yr or 0.023 m/yr). This provides the observed runoff or evaporation ratios, $C = Ro/P \sim 0.15$ or $F = E/P \sim 0.84$. Using the Schreiber–Budyko formula (Equation (2)), the dryness ratio $D \sim 1.8$ to 1.9 can now be derived which, employing the minimalist model (Equation (6b)), gives the lake area ratio $A \sim 0.145$. This value is almost identical to the observed (see above). Note also that the long-term mean of the measured annual surface runoff ratios, $\langle C \rangle \sim 0.14$ (Rhode *et al.* 2010) is also surprisingly close to the runoff ratio, $C(A) \sim 0.15$, derived from the long-term mean lake area ratio using the minimalist model. Including groundwater drainage, $Ro \sim 1.5$ plus $0.6\ \mathrm{km}^3$/yr, the runoff ratio increases by about 15% but this value may be reduced choosing a larger evaporation value for the basin evaporation estimate ranging from shore to mountain (1.00 to 0.30 m/yr).

Next, the lake water fluxes are analysed validating the minimalist lake approach (see Figure 2). The observed rainfall $P \sim 0.36$ m/yr (as over land) plus inflow, $Ro_{\mathrm{lake}} = (1\text{–}1/A)\ Ro_{\mathrm{land}} \sim 0.32$ to 0.35 m/yr (without the groundwater discharge 0.14 m/yr) support the lake's evaporation $E_{\mathrm{lake}} \sim 0.68$ to 0.71 m/yr, estimated as residual of the lake's water balance. Now employing the minimalist lake model, the observed lake area ratio A determines (see Figure 1) the climate state's dryness ratio $D = 1.89$. Thus the lake evaporation $E_{\mathrm{lake}} = PE = D\ P \sim 0.68$ m/yr, given the observed rainfall mean, is well simulated. That is, the minimalist model provides the full water flux budget once the geomorphological indicator (the lake area ratio) is known and one of the long-term mean water fluxes (precipitation). In a similar manner, evaporation and runoff ratios, 0.8 and 0.15, and the respective water fluxes, 0.31 and 0.054 m/yr, compare well with the observed ones (see above).

Finally, a rainfall dependent water flux diagram (Figure 2) corroborates the hydrological flux balances of both land and lake, which are related to the dryness ratio only.

Water flux diagram

In summary, all land and terminal lake water fluxes (y-axis) depending on rainfall (x-axis) can be presented in a diagram

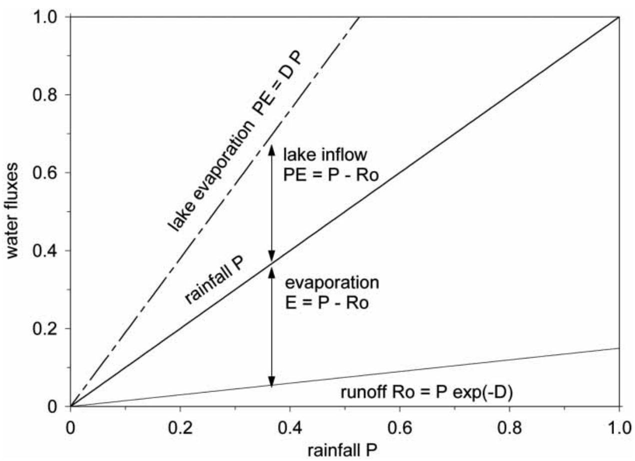

Figure 2 | Water fluxes derived from the minimalist model of terminal Lake Qinghai with lake area ratio $A \sim 0.145$ dependent dryness ratio, $D \sim 1.9$. (i) Land water fluxes comprise evaporation E and runoff Ro adding up to precipitation P (main diagonal). (ii) Lake evaporation $PE = E_{\mathrm{lake}} = D\ P$ for $D \geq 1$ exceeds the main diagonal (1:1) because it is balancing the sum of precipitation P (main diagonal) and inflow (Ri) from the catchment, $Ro_{\mathrm{lake}} = P\text{–}PE = (1\text{–}D)\ P < 0$.

calibrated by the dryness ratio D of the drainage area, which is determined by the lake area ratio A (Equation (6b)). The main diagonal or (1:1) line separates water fluxes on land (below) and lake (above the diagonal). Given the Qinghai Lake area ratio $A \sim 0.145$ (and thus the related dryness or runoff ratios, $D \sim 1.89$ or $C \sim 0.15$) and the rainfall $P \sim 0.36$ m/yr, one obtains the following results:

1. Land evaporation (below the main diagonal) $E = P\ (1\text{–}C) \sim 0.31$ m/yr and runoff $Ro = P\ C \sim 0.05$ m/yr add up to the rainfall $P \sim 0.36$ m/yr (main diagonal).
2. Lake evaporation (above the main diagonal), $E_{\mathrm{lake}} = PE = D\ P \sim 0.68$ m/yr (steepest line) is balanced by rainfall $P = P_{\mathrm{lake}}$ (main diagonal) plus inflow from the catchment, $Ro_{\mathrm{lake}} \sim 0.32$ m/yr. The latter agrees with the measured runoff from the drainage area, $Ro_{\mathrm{land}}(1\text{–}1/A) = 0.32$ m/yr, because land runoff and lake inflow balance, $a_{\mathrm{land}}\ Ro_{\mathrm{land}} + a_{\mathrm{lake}}\ Ro_{\mathrm{lake}} = 0$.

Note that, for a closed catchment-terminal lake system in equilibrium, the total rainfall balances the total evaporation at the surface and there is no net divergence or convergence of atmospheric or subsurface water.

Variability

A biased coinflip Ansatz provides the stochastic interpretation of the Schreiber–Budyko formula (Fraedrich 2010) as an equation of state for land surface climates. This leads to statistical measures of variability, for example, standard deviation of rainfall (and other water fluxes). Consider independently occurring rainfall episodes of, say, 10-day duration (that is, the common rain-bearing synoptic scale trough-ridge systems, including their associated half-period of dryness), with the only information of the rainfall mean, P_k, then, under maximum entropy conditions, the event rainfall is exponentially distributed (a result widely employed by hydrologists, see Eagleson (1978) and Rodriguez-Iturbe et al. (1999)):

$$\mathrm{prob}(p_k \geq X) = \int_X^{\infty} f(p_k)dp_k = \exp(-X/P_k) \qquad (7)$$

The sum of $k = 1, \ldots, K$ independent exponentially distributed events comprises a rainfall total, which is gamma (Erlang) distributed, $\Gamma(K,p_{k=1}) = (p/p_{K=1})^{K-1} \exp(-p/p_{K=1})/(p_{K=1}\ \Gamma(K))$, with scale and shape parameters, K and $p_{K=1}$, the mean $P = K\ P_k$, standard deviation, $\sigma = K^{1/2}\ P_k = K^{-1/2}\ P$, coefficient of variation $Cv = \sigma/P = K^{-1/2}$, and skewness $\gamma = 2\ K^{-1/2}$. Analysing the moments of the other fluxes is beyond the scope of this paper and will be presented later. Now, the coefficient of rainfall variability is $Cv = \sigma/P = K^{-1/2} \sim 0.165$, considering large-scale synoptic events of the period of about 10 days (or $K \sim 36$). Given the observed rainfall mean, $P \sim 0.36$, this corresponds to a standard deviation of the annual means, $\sigma \sim Cv\ P \sim 0.06$ m/yr which is close to the observed standard deviation estimate, $\sigma \sim \pm\ 0.065$ m/yr.

Lake Chad: A sensitivity analysis

The Lake Chad basin covers an area of about 2.5 million km^2, which is situated in Northern Africa (6 to 24°N, 7 to 24°E). Its climatological setting shows a large meridional gradient of rainfall with 1.6 m/yr and larger (0.15 m/yr and smaller) in the southern (northern) regions dominated by the West-African monsoonal circulation. The vegetation is characterised by desert and steppe in the north, savanna and woodland in the south (Odada et al. 2006). The complex structure of the Lake Chad basin allows the classification of present lake states (following Lemoalle et al. (2008), Odada et al. (2006) and Olivry et al. (1996)) and a paleo lake state, which can be related to the following set of topographical thresholds:

1. *Large Lake Chad* occupies a northern and southern water pool covering a maximum surface area of about 25,000 km^2 which, near 283.5 m asl, may lead to occasional northeastward outflows through the Bahr El Ghazal valley. It provides an upper bound of normal (or intermediate) Lake Chad whose single water body (surface area ~20,000 km^2 at 281 m asl) persisted until 1973 to 1975. That is, before the 1970s the lake area ratio $A_{\mathrm{normal}} \sim 0.008$ could increase up to $A_{\mathrm{large}} \sim 0.01$. Lake Chad, predominantly fed by the River Chari receives a total annual river inflow 43.6 km^3/yr or $Ro_{\mathrm{normal}} \sim 0.017$ m/yr which represents a pre-1970 mean (see Odada et al. (2006) and Lemoalle et al. (2008)). Since then, Lake Chad has experienced a drastic recession and functions as Small Lake Chad.

2. *Small Lake Chad* is located near the Chari River outflow. At maximum height of 280 m asl, it rises up to a 'great barrier' level separating southern and northern pools, and comprises island archipegalos, reed beds and open water. It covers a surface area of about 10,000 km^2 (corresponding to a lake area ratio $A_{\text{small}} \sim 0.004$), which is characterised by a large variability of half its magnitude and a presently open water pool of about 1,500 km^2. The mean water volume provided annually is 22.6 km^3/yr (or $Ro_{\text{small}} \sim 0.009$ m/yr) and represents the 1971–1990 mean (see Lemoalle *et al.* (2008) and Odada *et al.* (2006)).
3. *Lake Mega-Chad* has occupied an area of 0.35×10^6 km^2 in the Holocene (about 10 to 5 ky bp) which corresponds to a lake area ratio $A_{\text{mega}} \sim 0.14$. This lake size was limited by the Mayo Kebbi water outlet (at about 320 m asl) towards Benue River outside the Lake Chad basin. Originally noticed by Tilho (1925), it has been investigated since then (see Kutzbach (1980), Leblanc *et al.* (2006), Sepulchre *et al.* (2008) and Krinner *et al.* (2012), to name but a few).

Validation

The water budget of Normal Lake Chad (1954–1969) has been extensively analysed (see Odada *et al.* (2006) and Lemoalle *et al.* (2008), referring to Olivry *et al.* (1996)) and serves as reference for validating the minimalist model. The Normal Lake Chad area ratio, $A_{\text{normal}} \sim 0.008$ corresponds to the dryness and runoff ratios $(D,C)_{\text{normal}} \sim (3.6, 0.027)$. Given the mean annual pre-1970 runoff, $Ro \sim 0.017$ m/yr, we obtain minimalist model estimates of catchment mean precipitation ($P = Ro/C$), $P \sim 0.63$ m/yr, and lake evaporation ($E_{\text{lake}} = PE = D\ P$), $PE \sim 2.3$ m/yr. These water fluxes correspond surprisingly well with the observed data presented by Odada *et al.* (2006) and Lemoalle *et al.* (2008). Their estimates of the mean lake evaporation are 2.3 and 2.17 m/yr, respectively. The annual average zonal mean precipitation over Africa from 6° to 24°N (meridional extent of Lake Chad catchment) decreases from about 1.3 to 0.05 m/yr or less (see Krinner *et al.* (2012), Figure 1) leading to an estimate of an average of about 0.65 m/yr.

This validated dataset provides the reference state, A_0 and P_0, for the subsequent sensitivity analysis, to estimate the change of area ratio, δA, from present day to Mega-Chad conditions using a partially linear tangent approach of the minimalist equilibrium model.

Sensitivity

Under water limited conditions, $D > 1$, a sensitivity analysis of the reference climate state (subscript 0) is expressed in terms of dryness ratio variations:

$$\delta D/D_0 = \delta PE/PE_0 - \delta P/P_0 \tag{8a}$$

For dryness sensitivity to be negative (left), the increase in rainfall needs to be larger than the increase of potential evaporation. Under water limited or dry climate conditions, $D > 1$, a changing dryness ratio, $\delta D = (\delta PE - D_0\ \delta P)/P_0$, shows that only a small amount of rainfall increase is required to shift the dryness to a wetter state (and vice versa): $\delta PE \ll D_0\ \delta P$. Thus, in particular, the sensitivity of dry (steppe and savanna) and very dry (semi-desert and desert) climates is strongly affected by changes in rainfall, even if these changes are rather small; that is, $\delta D \sim -D_0\ \delta P/P_0$, or

$$D \sim D_0(1 - \delta P/P_0) \tag{8b}$$

which gives the dryness changing with rainfall or vice versa, $D \sim D_0\ (2 - P/P_0)$ or $P \sim P_0\ (2 - D/D_0)$.

Mega-Chad

Mega-Chad (subscript 'mega') is characterised by a substantially larger area ratio $A_{\text{mega}} \sim 0.14$ and wetter climate with dryness and runoff ratios $(D, C)_{\text{mega}} \sim (1.90, 0.15)$ deduced by the minimalist model. Our sensitivity analysis yields a rainfall estimate of $P_{\text{mega}} \sim P_0\ (2 - D_{\text{mega}}/D_0) \sim P_0\ (2 - 1.9/3.6) \sim 0.93$ m/yr and standard deviation bounds, 0.89 and 0.96 m/yr, determined by the variability Ansatz (Equation (5)). This estimate compares favourably with the Krinner *et al.* (2012) model simulations. In the Holocene, the annual average zonal mean precipitation over Africa between 6° and 24° N (meridional extent of Lake Chad catchment) has increased to about 1.6 to 0.2 m/yr (see Krinner *et al.* (2012), Figure 1) leading to an average of about 0.9 m/yr. They also note that 'the relative precipitation increase' (compared to the control run) is particularly strong, in excess of 50%, in the central and western Sahara, where the annual mean precipitation rates are about 0.25 m/yr.

In summary, the minimalist model of terminal lakes provides sensitivity estimates for past climates without requiring the large number of land and/or lake surface parameters (such as albedo, net radiation, Bowen ratio; see Kutzbach (1980), Hastenrath & Kutzbach (1985) and Kieniewicz & Smith (2009)), but only two, which characterise the reference climate (subscript '0'): rainfall P_0 and lake area ratio A_0 (or dryness D_0). Here it is the present day climate.

SUMMARY AND OUTLOOK

A minimalist model of terminal lakes in water limited climates is introduced, validated and subjected to a sensitivity analysis to estimate the lake area change and its related hydrology and climate by combining geomorphological and geobotanic information: lake area ratio A and dryness D. This aim is achieved without using a large number of land and lake surface parameters (such as albedos, net radiations, Bowen ratios, etc.) apart from two, which characterise the climate: the lake's area ratio A and a reference estimate of rainfall P. Two closures have been employed. (i) One refers to the water/energy balance with potential evaporation of land and lake to attain similar magnitude. The first result of this Ansatz – maximum lake area ratio ($A = 1$) occurs at the dryness ratio ($D = 1$) separating water from energy limited regimes – appears to be conceptually correct and is satisfied by the model structure. (ii) The other closure is related to sensitivity analyses inducing, from an observed lake area ratio change δA, the associated water limited climate regime (for example, vegetation) and thus the hydrology (that is the waterfluxes) by assimilating the present day's observed reference state. Model validation (section Lakes revisited: Qinghai and Chad) has demonstrated that the minimalist approach and its underlying assumptions (presented in the section Lake area ratio) (i) satisfy the present day hydrological and climate data and flux balances of both land and lake and (ii) are relatively close to other model estimates.

Thus, following the Hortonian approach (see Dooge 1992), it is suggested to introduce the terminal lake area ratio as another important dimensionless parameter characterising the state of the surface climate, besides the flux ratios of dryness, runoff, evaporation and Bowen. As the validation analysis shows a surprisingly good agreement between land surface climate indicators and the dependent lake area ratio, this suggests that the minimalist lake model also appears to be a useful tool for climate diagnostics. This includes climate variability estimates as, for example, drought and wetness, which, based on the natural measure of the underlying statistics, can also be estimated for climate change conditions under the sensitivity assumption. Note that this conceptual first order estimate of water and energy fluxes (and their statistics) should not replace but support a comprehensive climate modelling approach.

In summary, the minimalist terminal lake model has been suggested to provide a parsimonious tool for quantitative estimates of environmental climate indicators. The model has been validated for the Qinghai Lake and Lake Chad comparing model output and observations in terms of the basic geomorphological (lake area ratio A), geobotanic (dryness ratio D) and hydrological (runoff or evaporation ratio C or F) information. The results encourage us to subject the minimalist model for validation in other terminal lake systems and to improve on the present model. The model is working well on Quinghai Lake. However, it may require more refinement for the Lake Chad analysis; in particular, (i) the role of floodplains, (ii) the complex role of infiltration process under Lake Chad, considered as a 10% water loss at the present time and as a water influx in the past), and (iii) the very high spatial variability of the precipitation. Spatial and time variability of rainfall may be included when introducing and analysing higher moments (see Fraedrich & Sielmann (2011), Section 4); this also holds for the role of floodplains to be considered as randomised lakes (enhancing the actual evaporation over land). The issue of infiltration, however, requires the introduction of time dependence and the incorporation of external fluxes affecting the otherwise closed system. This will be the subject of future research, including sensitivity estimates based on a fluctuation-dissipation Ansatz for a new closure.

ACKNOWLEDGEMENTS

Previous exchanges with John Kutzbach and Stefan Hastenrath and, in particular, with David Rhode (more recently) are appreciated; Lai informed me about a preprint version of the Rhode *et al.* (2010) paper when visiting the Qinghai Institute of Salt Lakes in Xining. Comments by the referees and Christophe Cudennec are gratefully acknowledged. Discussions are appreciated at the Royal Meteorological Institute (Brussels, February 11–12, 2010) workshop on 'The Complexity Paradigm: Understanding the Dynamics of Weather and Climate' and at the Isaac Newton Institute (Cambridge, 2010) workshop on 'Uncertainty in Climate Prediction: Models, Methods and Decision Support'.

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First received 16 January 2012; accepted in revised form 20 May 2013. Available online 26 June 2013