Joint estimation of hydraulic conductivities of two sand samples in a W-tube system with a bi-exponential response

Mariusz Kaczmarek, Filip Wolny and Marek Marciniak

ABSTRACT

The motivation for this paper is the practical problem of interpreting bi-exponential rate-of-rise curves that are observed during many field piezometric tests. In the authors' previous study, a laboratory W-tube system of water flow through two samples of sand was introduced with an adequate mathematical model. The desired bi-exponential character of water flow was obtained by placing two different samples of sand in separate but connected columns. In the present paper, a so-called inverse problem is solved. The optimization procedure is applied in order to jointly estimate a pair of hydraulic conductivity values based on experimentally recorded bi-exponential rate-of-rise curves. The obtained values of hydraulic conductivities are presented and compared to the values determined from independent constant-head permeability tests conducted for the analyzed sands. The results of this identification procedure varied in accuracy. The mean percent errors between the hydraulic conductivity values measured jointly and independently for the analyzed experimental series were in the range of 8.8 to 37.2%. The discussion presents the restrictions of this interpretational method and suggests further modeling plans.

Key words | dual-permeability, flow modeling, heterogeneity, hydraulic conductivity, numerical optimization techniques, permeability test

Mariusz Kaczmarek (corresponding author)
Institute of Mechanics and Applied Computer Science,
Kazimierz Wielki University,
Chodkiewicza Street 30,
Bydgoszcz 85-064,
Poland
E-mail: *mkk@ukw.edu.pl*

Filip Wolny
Marek Marciniak
Institute of Physical Geography and Environmental Planning,
Adam Mickiewicz University,
Dziegielowa Street 27,
Poznan 61-680,
Poland

Filip Wolny
Institute of Groundwater Ecology,
Helmholtz Zentrum München-German Research Center for Environmental Health,
Ingolstaedter Landstrasse 1,
Neuherberg 85764,
Germany

INTRODUCTION

Piezometric permeability tests are usually interpreted by assuming homogeneity of water-bearing formations, however many of these formations are characterized by the presence of different systems of void space (Shapiro & Hsieh 1998; Wu *et al.* 2005; Altunkaynak 2007; Alexander *et al.* 2011). In such cases, qualitative deviations in the rise of the water level can be observed (Wang *et al.* 2004; Marciniak *et al.* 2010, 2013). The research described here is motivated by some pressure-induced permeability tests that result in recording bi-exponential rate-of-rise curves which clearly differ from the usual exponential rise of the water level. The bi-exponential curves may indicate the existence of two distinct pathways for water flow within a water-bearing formation (Szymkiewicz 2008; Balogun *et al.* 2009).

In the authors' earlier paper (Marciniak *et al.* 2013), a laboratory model of an inhomogeneous formation, called W-tube system, was considered. Numerous experiments were conducted in this system for pairs of sand samples representing a wide range of permeability values, which resulted in recording bi-exponential curves that were successively interpreted by applying an adequate mathematical model.

This paper aims to present the procedure of a joint estimation of hydraulic conductivities for two sand samples in the W-tube system, whenever a bi-exponential curve is observed. This so-called inverse problem is solved using optimization techniques along with the results of tests and the model for W-tube response. The obtained values of hydraulic conductivity are presented and compared to the

doi: 10.2166/nh.2015.320

values determined from constant-head permeability tests conducted for the analyzed sands. The discussion presents possible sources of errors, restrictions of the considered interpretation method, and suggestions for further modeling.

MOTIVATION

The research presented in this paper is motivated by the unusual results of some PARAMEX tests first observed in the Belchatow lignite mine in Poland. The PARAMEX test, described in the following section, is a pressure-induced permeability test which requires initiating and recording water level movements inside a piezometer (Marciniak 1999, 2001), similarly as in a slug or boil test (Cooper *et al.* 1965, 1967; Bouwer & Rice 1976; Bredehoeft & Papadopulos 1980; Cardiff *et al.* 2011; Chen *et al.* 2012). Nearly 12% of these tests resulted in obtaining bi-exponential rate-of-rise curves (Marciniak *et al.* 2013). Any attempts to find an exponent that would characterize the curves have proved to be unsuccessful (Figure 1).

Some piezometers, where bi-exponential movement was observed, are screened in Late Cretaceous limestone rubble. The filter zone of others is located within layered formations consisting of different Quaternary sands. These types of lithology suggest the possible presence of dual-permeability flow conditions within the screened formations that may be responsible for the deviations of the exponential rise of the water level.

The authors of this paper introduced a triple-columned system (Marciniak *et al.* 2013) that allows observing bi-exponential water level movement in laboratory conditions along with an adequate mathematical model of water flow. Owing to its shape, the system was called a 'W-tube' and, to some degree, is considered to re-enact the hydraulic properties of field piezometric tests in dual-permeability formations. The desired character of water flow was obtained by placing two different samples of sand in separate but connected columns (Wolny 2012; Marciniak *et al.* 2013). The laboratory nature of the presented research was dictated by the possibility of selecting numerous sets of hydraulic conductivity values for testing. Moreover, laboratory tests also allowed verifying the values of conductivities derived from the bi-exponential rate-of-rise curve with the values obtained for the analyzed sand samples using the constant-head method. The verification of research results would be severely restricted in the field.

The successful solution of the inverse problem, i.e., determining the hydraulic conductivities of two sands in the W-tube based on the experimentally observed bi-exponential rate-of-rise curve, can serve as an idea to elaborate interpretation algorithms based on permeability tests such as the PARAMEX, slug or boil test. Such algorithms could be implemented in field conditions, whenever a bi-exponential water level rise is observed. This would increase the reliability of piezometric permeability tests in heterogeneous formations.

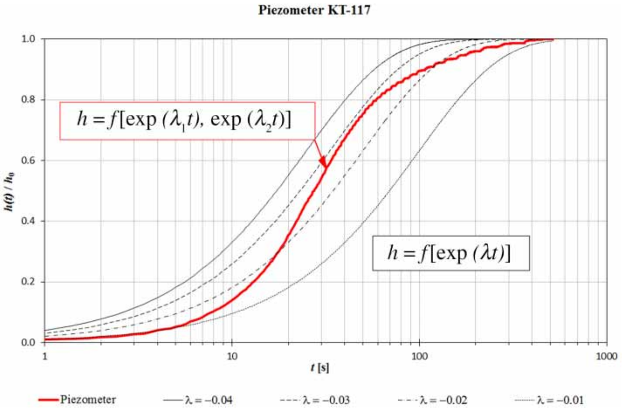

Figure 1 | Experimentally recorded bi-exponential water level rise in a piezometer (bold line) and attempts to fit theoretical exponential curves (thin lines) to the field data (logarithmic time scale, only the rise of the water table is shown).

THE PRESSURE-INDUCED PERMEABILITY FIELD TEST

The pressure-induced permeability test is conducted in the following manner. First, the upper part of a piezometer is sealed off with a sealing device. Simultaneously, a probe is placed inside the piezometer to measure water level fluctuations. During the test, air is pumped into the sealed piezometer to decrease the water level. Next, pumping is stopped and the water level stabilizes. Afterwards, the air valve of the sealing device is opened and the air inside the piezometer decompresses, leading to the free rise of the water level to its initial state. The rate at which water returns to its initial level determines the hydraulic conductivity of the analyzed aquifer.

The mathematical model of water level rise within the piezometer characterizes two possible cases: damped oscillations or aperiodic movement (Krauss 1974, 1977). By using the model in the identification algorithms of the PARAMEX test, it is possible to calculate the hydraulic conductivity of an aquifer (Marciniak 2012).

So far, the pressure-induced permeability tests have been conducted in over 1,300 piezometers in various regions of Poland. In a vast majority of cases, an aperiodic movement of the water level is observed. The water level's rate-of-rise as a function of time, $h(t)$, can be described by the following exponential function:

$$h(t) = h_0[1 - \exp(\lambda \cdot t)] \qquad (1)$$

where h_0 is the initial water level (cm), $\lambda < 0$ is an exponent dependent on the hydraulic conductivity (1/s).

Tests that resulted in observing bi-exponential water movement were examined thoroughly. Initially, piezometer damage, i.e., the presence of faulty sand packs around the piezometers and leaky isolating clay seals, was taken into consideration. Such types of damage have been observed during pressure-induced permeability tests. However, in such cases, the first stage of water level rise is rapid and short, and the next stage is exponential. In order to find hydraulic connections resulting in the bi-exponential behavior of the water level, several laboratory systems of water flow have been investigated by Marciniak *et al.* (2013). Only the W-tube model of water flow presented in this paper allowed such behavior to be observed. This system of water flow indicates that a weak hydraulic connection between the two subdomains of a dual-permeability medium is required to observe curves deviating from the exponential rise.

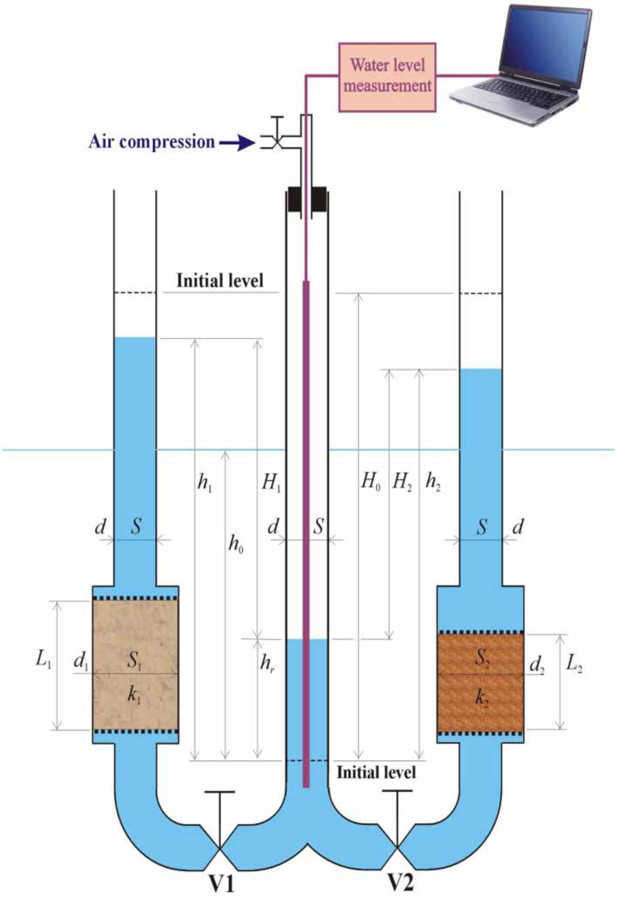

Figure 2 | The W-tube system used in the permeability tests.

THE LABORATORY W-TUBE MODEL OF A PIEZOMETRIC TEST

The W-tube system consists of three hydraulically connected columns (Figure 2). The two outer columns are equipped with cylindrical chambers where different sand samples are placed. The central column, representing the piezometer, contains a probe that measures the water level in this column as a function of time.

The preparation of a test requires filling all columns with water to the same level. Afterwards, the water level in the central column is displaced down and the water in the outer columns is displaced up by pumping air into the central column using an air compressor. The water level stabilizes over time. A test is initialized at the same moment when the air valve located at the top of the central column is opened, causing water to return to the initial level due to its filtration through the samples in the outer columns. Since the W-tube is equipped with two valves, V1 and V2, it is also possible to perform experiments for singles samples of sand. Closing one of the valves lead to a U-tube system with a possibility of generating exponential curves typical for a homogeneous water-bearing medium.

The mathematical model of the W-tube test is based on a balance of forces for water flowing through the sand samples. It was assumed that the inner cross-sections of all columns, denoted as S, are equal. S_1 and S_2 are the cross-sections of the samples of sand, L_1 and L_2 denote the heights of these samples, k_1 and k_2 are their hydraulic conductivities. Initially, the level of water in the central column is lower than in the adjacent columns and serves as the reference point for the measurements of the water level h_r in the central column and the h_1 and h_2 levels in the outer columns. The proposed mathematical model neglects the inertial forces related to the motion of water in the columns and in the pore space of the samples. The interaction forces between the fluid and the water and the porous material are approximated according to the linear Darcy's law. Taking into consideration the hydrostatic forces caused by the water columns as well as the gravitational forces and the interaction forces between the water and the porous material, the following balance equations for the pore water can be written (Rehbinder 1992; Marciniak *et al.* 2013):

$$\begin{cases} \rho g(h_1 - h_r)S_1 + \dfrac{\rho g}{k_1} L_1 \dfrac{dh_1}{dt} S = 0 \\ \rho g(h_2 - h_r)S_2 + \dfrac{\rho g}{k_2} L_2 \dfrac{dh_2}{dt} S = 0 \end{cases} \tag{2}$$

where ρ is the density of water, g is the acceleration due to gravity, and the derivative dh_i/dt denotes the rate at which the water level changes. This change is equal to the filtration velocities in the appropriate samples.

Owing to the conservation of mass in the system, the changes of the water levels in the columns for any time increment dt can be given by the following equation:

$$dh_1 + dh_2 + dh_r = 0 \tag{3}$$

After conducting several mathematical calculations described by the authors in a previous paper (Marciniak *et al.* 2013), the following formula was obtained:

$$h_r = \frac{H_0}{3}[2 + (B-1)\exp(\lambda_1 t) - (B+1)\exp(\lambda_2 t)] \tag{4}$$

where the constant B is derived from the following equation:

$$B = \frac{A_2 + A_1}{2\sqrt{A_1^2 + A_2^2 - A_1 A_2}} \tag{5}$$

and $\lambda_{1,2}$ equals:

$$\lambda_{1,2} = -A_1 - A_2 \pm \sqrt{A_1^2 + A_2^2 - A_1 A_2} \tag{6}$$

Furthermore, the constants A_1 and A_2 are related to the geometric parameters of the analyzed samples of sand: S_1, S_2, L_1, L_2, and to their hydraulic conductivities: k_1 and k_2:

$$A_1 = \frac{k_1 S_1}{L_1 S}, A_2 = \frac{k_2 S_2}{L_2 S}. \tag{7}$$

By solving Equation (4), it is possible to determine the bi-exponential response of the water level in the central column as a function of time. The same approach can also be used to model water flow in a U-tube system, when one of the valves is closed. All of the other assumptions are identical. It should be underlined that the W-tube model re-enacts the results of field piezometric tests conducted in heterogeneous formations, i.e., the rate-of-rise curves, rather than the exact character of water flow occurring during such tests. However, there are many similarities between the field and laboratory setups, which will allow modifying the W-tube model in the future to fully describe the field situation.

MATERIALS AND METHODS

Four sand types, marked as I, II, III, and IV, were chosen for laboratory testing. These sands were classified as very coarse, gravelly, fine, and silty sand, respectively, and they cover a broad range of hydraulic conductivities typical for most water-bearing formations, i.e., from 10^{-5} to 10^{-3} m/s. Their conductivity values were determined empirically from the grain size distributions and experimentally in a permeameter using the constant-head method (Table 1). Moreover, minor adjustments to the W-tube apparatus made it possible to perform additional constant-head permeability measurements for all samples placed in the small cylindrical chambers. Since testing required filling the W-tube with the same type of sand more than once, the hydraulic conductivity of every analyzed sample was known. The results of hydraulic conductivity measurements performed in the W-tube are shown in Figure 3.

The cylindrical chambers of the apparatus were filled with samples of sand either to the height of 25 or 50 cm (in order to double the amount of curves obtained when the chambers are filled to one height). To analyze the dual-permeability of all sand combinations, six experimental

Table 1 | Hydraulic conductivity values of the sand samples *k* [m/s]

		Hydraulic conductivity *k* [m/s]			
No.	**Sand type**	**From the Hazen equation**	**From the USBSC equation**	**Based on the constant-head test in a permeameter**	**Based on the constant-head test in the U-tube**
I	Very coarse sand	7.81×10^{-3}	3.77×10^{-3}	1.71×10^{-3}	$(2.24 \pm 0.25) \times 10^{-3}$
II	Gravelly sand	3.38×10^{-4}	4.90×10^{-4}	3.04×10^{-4}	$(1.66 \pm 0.23) \times 10^{-4}$
III	Fine sand	5.10×10^{-5}	2.74×10^{-5}	5.35×10^{-5}	$(4.99 \pm 0.96) \times 10^{-5}$
IV	Silty sand	n/a*	1.08×10^{-5}	1.81×10^{-5}	$(1.42 \pm 1.21) \times 10^{-5}$

*The following condition is not satisfied: $0.1 < d_{10} < 3$.

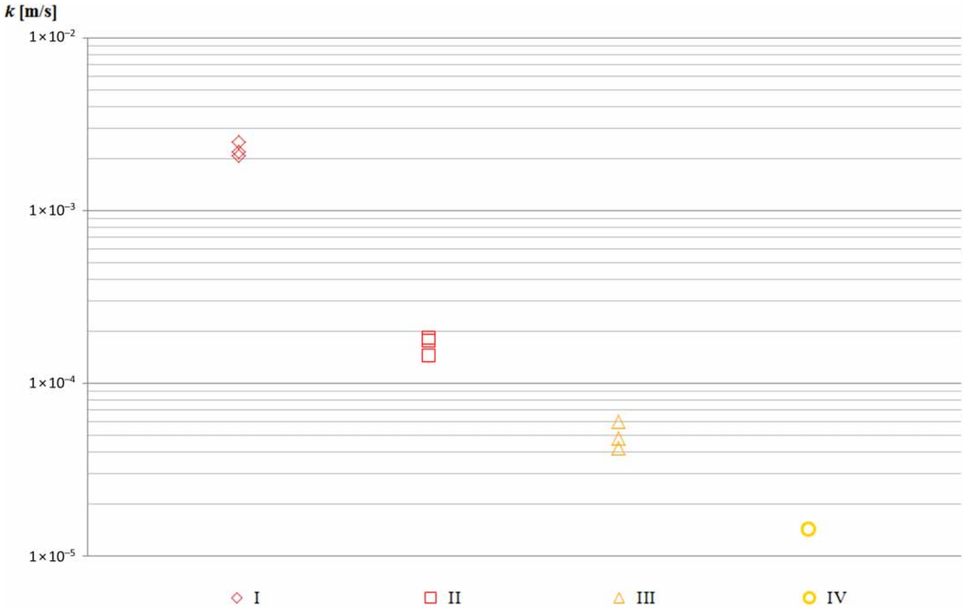

Figure 3 | Hydraulic conductivity measurements performed in the W-tube for all tested sand samples.

series were carried out, with two sand samples tested in each series: I_II, I_III, I_IV, II_III, II_IV, and III_IV.

A total of 24 rate-of-rise curves were recorded for single samples of sand during U-tube tests (with one valve of the apparatus closed), and another 24 curves were recorded for various combinations of sample type and height during the W-tube tests.

Solving the inverse problem for the laboratory model of a piezometric permeability test required implementing numerical optimization techniques incorporated in the MATLAB computational environment. This procedure was performed in accordance with the mathematical model of water flow in the W-tube presented in the previous section. Using the numerical techniques, the best fitting hydraulic conductivities were calculated for every experimentally registered rate-of-rise curve. For U-tube experiments, when an exponential curve was recorded, a single value of hydraulic conductivity was calculated. For W-tube experiments, when the superposition of flow parameters for two sand samples was investigated and bi-exponential water level movement was observed, a pair of corresponding hydraulic conductivities was obtained. The numerically determined values of hydraulic conductivity k_W were compared with the values derived from constant-head measurements (in accordance with Darcy's law) k_D performed in the W-tube apparatus for each sand sample by calculating the absolute and percent errors (Equations (8) and (9), respectively):

$$\Delta k = |k_W - k_D| \tag{8}$$

$$\delta_{k\%} = \delta_k \cdot 100\% = \frac{|k_W - k_D|}{k_D} \cdot 100\% \tag{9}$$

Furthermore, theoretical rate-of-rise curves were generated for the numerically obtained k_W values. To assess their agreement with the experimental curves, a root mean square error (RMSE) was calculated for every experimentally recorded curve and the corresponding results of mathematical modelling:

$$RMSE = \frac{1}{n}\sqrt{\sum_{1}^{n}(s_W - s_D)^2} \tag{10}$$

where s_D is the water level recorded during an experiment, s_W is the water level calculated from the mathematical model based on numerically obtained hydraulic conductivities k_W, n is the number of measurements.

Table 2 | Results of hydraulic conductivity identification for experimental series I_II

Experiment	Sand sample	Darcy test k_D [m/s]	W-tube test k_W [m/s]	Absolute error Δk [m/s]	Percent error $\delta_{k\%}$ [%]
I25	I	3.31×10^{-3}	2.87×10^{-3}	4.40×10^{-4}	13.3
I50	I	3.31×10^{-3}	2.96×10^{-3}	3.50×10^{-4}	10.6
II25	II	2.88×10^{-4}	3.00×10^{-4}	1.20×10^{-5}	4.2
II50	II	2.88×10^{-4}	2.33×10^{-4}	5.50×10^{-5}	19.1
I25_II25	I	3.31×10^{-3}	3.42×10^{-3}	1.10×10^{-4}	3.3
	II	2.88×10^{-4}	2.77×10^{-4}	1.10×10^{-5}	3.8
I25_II50	I	3.31×10^{-3}	3.60×10^{-3}	2.90×10^{-4}	8.8
	II	2.88×10^{-4}	2.43×10^{-4}	4.50×10^{-5}	15.6
I50_II25	I	3.31×10^{-3}	3.42×10^{-3}	1.10×10^{-4}	3.3
	II	2.88×10^{-4}	3.03×10^{-4}	1.50×10^{-5}	5.2
I50_II50	I	3.31×10^{-3}	3.52×10^{-3}	2.10×10^{-4}	6.3
	II	2.88×10^{-4}	2.52×10^{-4}	3.60×10^{-5}	12.5
			average	$\mathbf{1.40\times10^{-4}}$	**8.8**

Table 3 | Results of hydraulic conductivity identification for experimental series I_III

Experiment	Sand sample	Darcy test k_D [m/s]	W-tube test k_W [m/s]	Absolute error Δk [m/s]	Percent error $\delta_{k\%}$ [%]
I25	I	3.48×10^{-3}	3.11×10^{-3}	3.70×10^{-4}	10.6
I50	I	3.48×10^{-3}	3.46×10^{-3}	2.00×10^{-5}	0.6
III25	III	7.71×10^{-5}	1.11×10^{-4}	3.39×10^{-5}	44.0
III50	III	7.71×10^{-5}	8.60×10^{-5}	8.90×10^{-6}	11.5
I25_III25	I	3.48×10^{-3}	3.59×10^{-3}	1.10×10^{-4}	3.2
	III	7.71×10^{-5}	1.12×10^{-4}	3.49×10^{-5}	45.3
I25_III50	I	3.48×10^{-3}	3.74×10^{-3}	2.60×10^{-4}	7.5
	III	7.71×10^{-5}	7.91×10^{-5}	2.00×10^{-6}	2.6
I50_III25	I	3.48×10^{-3}	3.85×10^{-3}	3.70×10^{-4}	10.6
	III	7.71×10^{-5}	1.15×10^{-4}	3.79×10^{-5}	49.2
I50_III50	I	3.48×10^{-3}	3.97×10^{-3}	4.90×10^{-4}	14.1
	III	7.71×10^{-5}	8.63×10^{-5}	9.20×10^{-6}	11.9
			average	$\mathbf{1.46\times10^{-4}}$	**17.6**

It should be noted that the agreement of the experimental curves and the ones generated using the mathematical model for k_D values from constant-head tests was discussed in the authors' previous paper (Marciniak *et al.* 2013).

RESULTS

The hydraulic conductivities of each sand sample determined from constant-head measurements k_D and the values calculated based on the mathematical model using numerical optimization techniques k_W are presented in Tables 2–7. The values of absolute and percent errors between k_D and k_W are given in the above-mentioned tables. Moreover, examples of rate-of-rise curves are depicted in Figures 4–9 for every experimental series. In each figure three curves are presented: (1) the experimentally recorded rate-of-rise curve for an experiment with two sand types fully filling the chambers of the apparatus; (2) the corresponding curve calculated using the mathematical model for hydraulic conductivities derived from numerical optimization (k_W values); and (3) the corresponding curve calculated using the mathematical model for hydraulic

Table 4 | Results of hydraulic conductivity identification for experimental series I_IV

Experiment	Sand sample	Darcy test k_D [m/s]	W-tube test k_W [m/s]	Absolute error Δk [m/s]	Percent error $\delta_{k\%}$ [%]
I25	I	3.30×10^{-3}	3.46×10^{-3}	1.60×10^{-4}	4.8
I50	I	3.30×10^{-3}	3.45×10^{-3}	1.5×10^{-4}	4.5
IV25	IV	1.90×10^{-5}	1.50×10^{-5}	4.00×10^{-6}	21.1
IV50	IV	1.90×10^{-5}	1.85×10^{-5}	5.00×10^{-7}	2.6
I25_IV25	I	3.30×10^{-3}	4.75×10^{-3}	1.45×10^{-3}	43.9
	IV	1.90×10^{-5}	2.11×10^{-5}	2.10×10^{-6}	11.1
I25_IV50	I	3.30×10^{-3}	4.52×10^{-3}	1.22×10^{-3}	37.0
	IV	1.90×10^{-5}	2.29×10^{-5}	3.90×10^{-6}	20.5
I50_IV25	I	3.30×10^{-3}	4.00×10^{-3}	7.00×10^{-4}	21.2
	IV	1.90×10^{-5}	2.00×10^{-5}	1.00×10^{-6}	5.3
I50_IV50	I	3.30×10^{-3}	4.53×10^{-3}	1.23×10^{-3}	37.3
	IV	1.90×10^{-5}	2.04×10^{-5}	1.40×10^{-6}	7.4
			average	$\mathbf{4.11\times10^{-4}}$	**18.1**

Table 5 | Results of hydraulic conductivity identification for experimental series II_III

Experiment	Sand sample	Darcy test k_D [m/s]	W-tube test k_W [m/s]	Absolute error Δk [m/s]	Percent error $\delta_{k\%}$ [%]
II25	II	2.12×10^{-4}	2.16×10^{-4}	4.00×10^{-6}	1.9
II50	II	2.12×10^{-4}	2.03×10^{-4}	9.00×10^{-6}	4.2
III25	III	6.21×10^{-5}	1.11×10^{-4}	4.89×10^{-5}	78.7
III50	III	6.21×10^{-5}	7.97×10^{-5}	1.76×10^{-5}	28.3
II25_III25	II	2.12×10^{-4}	2.77×10^{-4}	6.50×10^{-5}	30.7
	III	6.21×10^{-5}	1.15×10^{-4}	5.29×10^{-5}	85.2
II25_III50	II	2.12×10^{-4}	2.28×10^{-4}	1.60×10^{-5}	7.5
	III	6.21×10^{-5}	7.97×10^{-5}	1.76×10^{-5}	28.3
II50_III25	II	2.12×10^{-4}	1.59×10^{-4}	5.30×10^{-5}	25.0
	III	6.21×10^{-5}	1.37×10^{-4}	7.49×10^{-5}	120.6
II50_III50	II	2.12×10^{-4}	2.02×10^{-4}	1.00×10^{-5}	4.7
	III	6.21×10^{-5}	8.17×10^{-5}	1.96×10^{-5}	31.6
			average	$\mathbf{3.24\times10^{-5}}$	**37.2**

Table 6 | Results of hydraulic conductivity identification for experimental series II_IV

Experiment	Sand sample	Darcy test k_D [m/s]	W-tube test k_W [m/s]	Absolute error Δk [m/s]	Percent error $\delta_{k\%}$ [%]
II25	II	2.38×10^{-4}	1.95×10^{-4}	4.30×10^{-5}	18.1
II50	II	2.38×10^{-4}	1.91×10^{-4}	4.70×10^{-5}	19.7
IV25	IV	1.90×10^{-5}	1.50×10^{-5}	4.00×10^{-6}	21.1
IV50	IV	1.90×10^{-5}	1.85×10^{-5}	5.00×10^{-7}	2.6
II25_IV25	II	2.38×10^{-4}	2.22×10^{-4}	1.60×10^{-5}	6.7
	IV	1.90×10^{-5}	1.80×10^{-5}	1.00×10^{-6}	5.3
II25_IV50	II	2.38×10^{-4}	2.48×10^{-4}	1.00×10^{-5}	4.2
	IV	1.90×10^{-5}	2.37×10^{-5}	4.70×10^{-6}	24.7
II50_IV25	II	2.38×10^{-4}	2.09×10^{-4}	2.90×10^{-5}	12.2
	IV	1.90×10^{-5}	1.69×10^{-5}	2.10×10^{-6}	11.1
II50_IV50	II	2.38×10^{-4}	2.14×10^{-4}	2.40×10^{-5}	10.1
	IV	1.90×10^{-5}	2.52×10^{-5}	6.20×10^{-6}	32.6
			average	$\mathbf{1.56\times10^{-5}}$	**14.0**

Table 7 | Results of hydraulic conductivity identification for experimental series III_IV

Experiment	Sand sample	Darcy test k_D [m/s]	W-tube test k_W [m/s]	Absolute error Δk [m/s]	Percent error $\delta_{k\%}$ [%]
III25	III	8.09×10^{-5}	1.05×10^{-4}	2.41×10^{-5}	29.8
III50	III	8.09×10^{-5}	8.41×10^{-5}	3.20×10^{-6}	4.0
IV25	IV	1.90×10^{-5}	1.50×10^{-5}	4.00×10^{-6}	21.1
IV50	IV	1.90×10^{-5}	1.85×10^{-5}	5.00×10^{-7}	2.6
III25_IV25	III	8.09×10^{-5}	1.14×10^{-4}	3.31×10^{-5}	40.9
	IV	1.90×10^{-5}	1.78×10^{-5}	1.20×10^{-6}	6.3
III25_IV50	III	8.09×10^{-5}	8.66×10^{-5}	5.70×10^{-6}	7.0
	IV	1.90×10^{-5}	2.42×10^{-5}	5.20×10^{-6}	27.4
III50_IV25	III	8.09×10^{-5}	4.41×10^{-5}	3.68×10^{-5}	45.5
	IV	1.90×10^{-5}	3.69×10^{-5}	1.79×10^{-5}	94.2
III50_IV50	III	8.09×10^{-5}	9.00×10^{-5}	9.10×10^{-6}	11.2
	IV	1.90×10^{-5}	2.74×10^{-5}	8.40×10^{-6}	44.2
			average	$\mathbf{1.24\times10^{-5}}$	**27.9**

conductivities obtained from constant-head tests in the apparatus (k_D values). A logarithmic timescale was used for each figure. Every set of curves $h = h(t)$ was normalized using the following equation (Equation (11)):

$$\frac{h_r(t)}{h_0} = f(\log t) \tag{11}$$

Furthermore, a breakdown of hydraulic conductivity identification results (k_D and k_W values) for all experimental series is shown in Figure 10.

The calculated RMSE values for the experimental curves and the ones derived from the mathematical model based on numerically obtained hydraulic conductivities k_W are shown in Table 8.

DISCUSSION AND CONCLUSIONS

The hydraulic conductivities of the analyzed sand samples identified in accordance with the mathematical model using numerical optimization techniques varied in

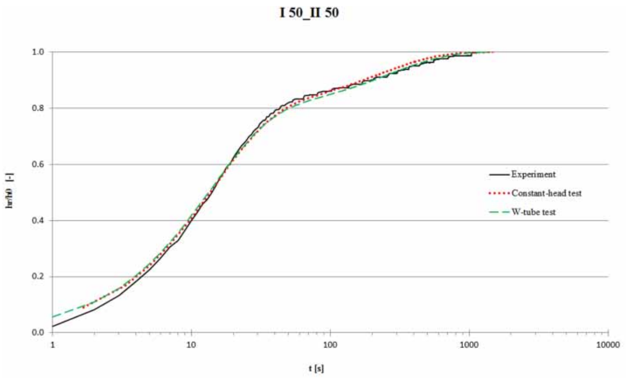

Figure 4 | Experimentally recorded rate-of-rise curve for I50_II50 test (bold line) compared to the results of simulations with k_D values from constant-head tests (dotted line) and k_W values from numerical optimization (dashed line).

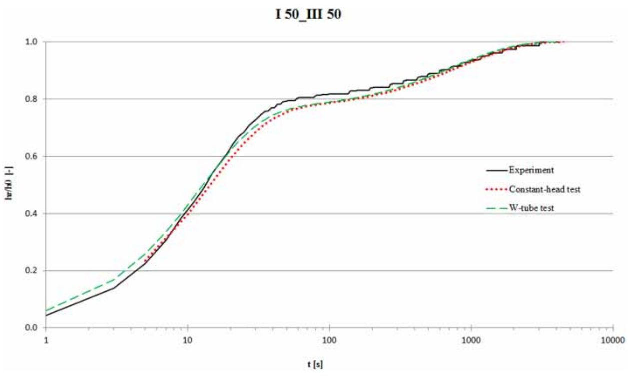

Figure 5 | Experimentally recorded rate-of-rise curve for I50_III50 test (bold line) compared to the results of simulations with k_D values from constant-head tests (dotted line) and k_W values from numerical optimization (dashed line).

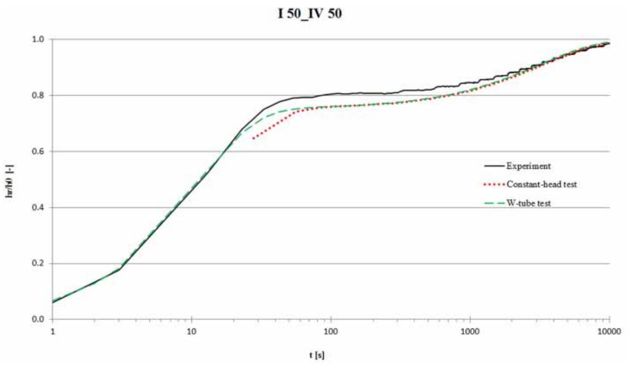

Figure 6 | Experimentally recorded rate-of-rise curve for I50_IV50 test (bold line) compared to the results of simulations with k_D values from constant-head tests (dotted line) and k_W values from numerical optimization (dashed line).

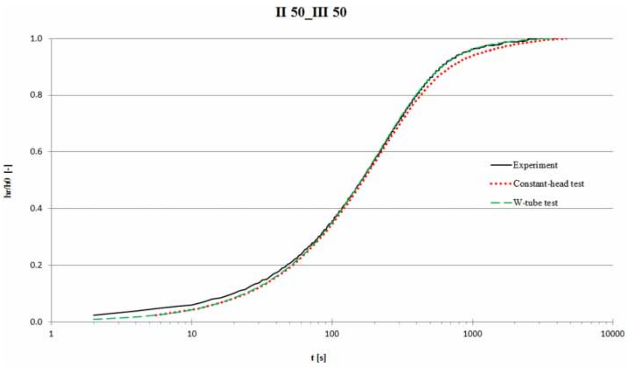

Figure 7 | Experimentally recorded rate-of-rise curve for II50_III50 test (bold line) compared to the results of simulations with k_D values from constant-head tests (dotted line) and k_W values from numerical optimization (dashed line).

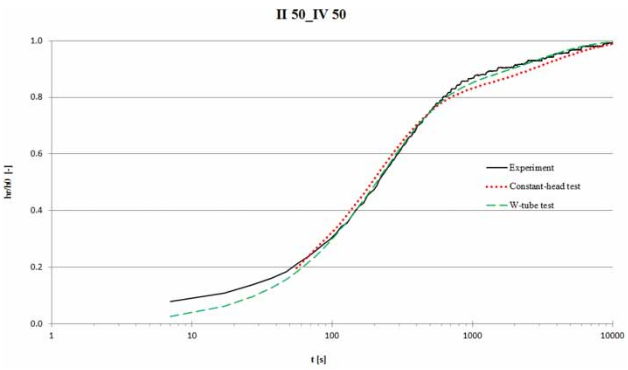

Figure 8 | Experimentally recorded rate-of-rise curve for II50_IV50 test (bold line) compared to the results of simulations with k_D values from constant-head tests (dotted line) and k_W values from numerical optimization (dashed line).

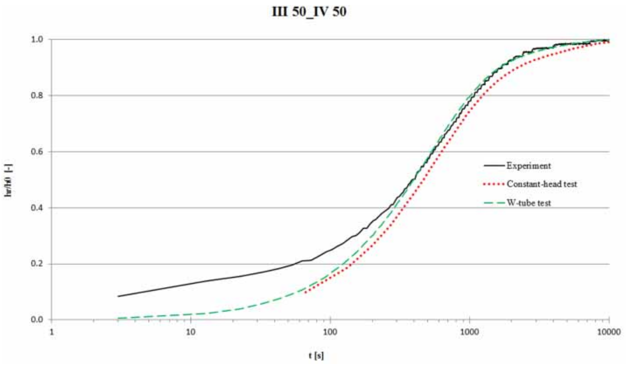

Figure 9 | Experimentally recorded rate-of-rise curve for III50_IV50 test (bold line) compared to the results of simulations with k_D values from constant-head tests (dotted line) and k_W values from numerical optimization (dashed line).

accuracy. The lowest errors were observed for series I_II. For this series, the mean percent error between the hydraulic conductivity values measured during constant-head tests and the values identified numerically was 8.8%. This indicates a good convergence of research results. Relatively small mean percent errors were obtained for series I_III,

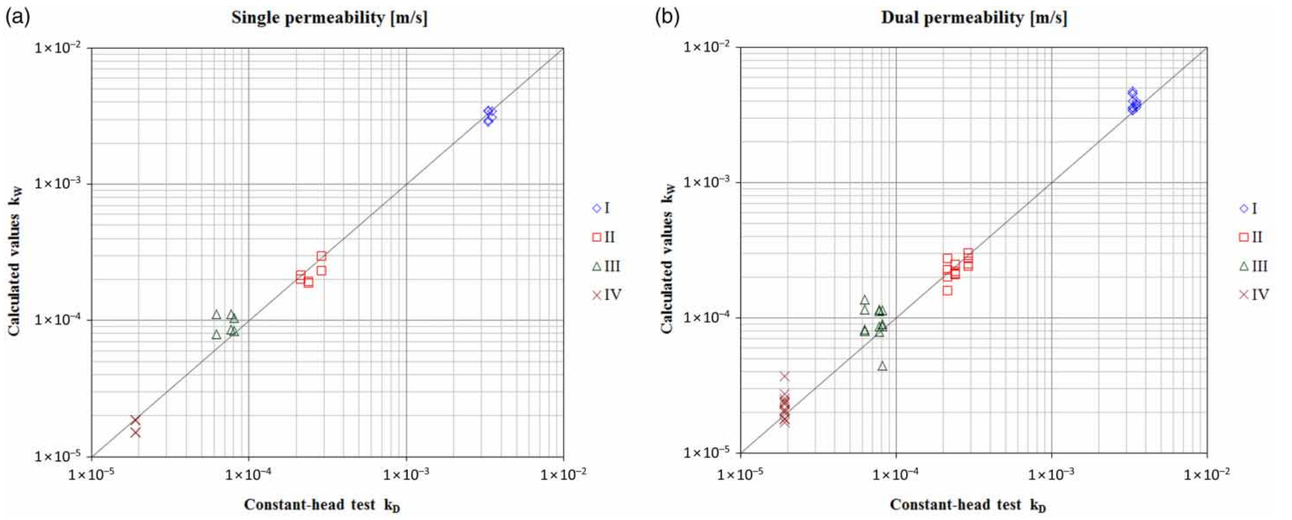

Figure 10 | Breakdown of hydraulic conductivity identification results for all experimental series: (a) experiments conducted in the U-tube; (b) experiments conducted in the W-tube.

Table 8 | RMSE values for the experimental curves and the curves calculated from the mathematical model for numerically obtained conductivities k_W

No.	Experiment	RMSE [cm]	No.	Experiment	RMSE [cm]	No.	Experiment	RMSE [cm]
Series I_II			Series I_IV			Series II_IV		
1.	I25	1.124	1.	I25	1.455	1.	II25	0.400
2.	I50	1.141	2.	I50	0.574	2.	II50	0.411
3.	II25	0.396	3.	IV25*	0.380	3.	IV25*	0.380
4.	II50	0.515	4.	IV50*	0.307	4.	IV50*	0.307
5.	I25_II25	0.691	5.	I25_IV25	0.781	5.	II25_IV25	0.467
6.	I25_II50	0.777	6.	I25_IV50	0.691	6.	II25_IV50	0.740
7.	I50_II25	0.460	7.	I50_IV25	0.588	7.	II50_IV25	0.272
8.	I50_II50	0.558	8.	I50_IV50	0.785	8.	II50_IV50	0.335
Series I_III			Series II_III			Series III_IV		
1.	I25	1.057	1.	II25	0.428	1.	III25	0.194
2.	I50	0.578	2.	II50	0.791	2.	III50	0.464
3.	III25	0.458	3.	III25	0.745	3.	IV25*	0.380
4.	III50	1.336	4.	III50	0.549	4.	IV50*	0.307
5.	I25_III25	0.722	5.	II25_III25	0.299	5.	III25_IV25	0.387
6.	I25_III50	0.635	6.	II25_III50	0.551	6.	III25_IV50	0.325
7.	I50_III25	0.577	7.	II50_III25	0.448	7.	III50_IV25	0.538
8.	I50_III50	0.730	8.	II50_III50	0.352	8.	III50_IV50	0.692

*The same IV25 and IV50 samples were used for series I_IV, II_IV, and III_IV.

I_IV, and II_IV – 17.6%, 18.1%, and 14.0%, respectively. The largest percent errors were obtained for series II_III, especially for the experiments with sand III filling the chamber of the W-tube to the height of 25 cm. For experiments III25, II25_III25, and II50_III25 the percent error for sand III was 78.7%, 85.2%, and 120.6%, respectively. The mean percent error for the whole experimental series was 37.2%.

The authors suggest that the changing consolidation of sand samples was one of the main causes of the observed errors. Since constant-head permeability measurements were always performed at the end of each experimental series, the analyzed samples of sand were probably more consolidated at this point in time than at the beginning of the series. Samples of sand III were the most affected by the changes in consolidation during the experimental series. Relatively high values of percent errors can be observed for the samples of this sand in every series.

In cases where there is only a small contrast of transmissivity between the two analyzed samples of sand, a high percent error was also observed. This is especially apparent in the identification results of experiment III50_IV25 – the percent error was 45.5% for sand III and 94.2% for sand IV.

Furthermore, regularity has also been observed for experiments conducted in the W-tube – percent errors for the sand with a higher value of hydraulic conductivity were usually smaller than for the sand with a lower value. This occurrence can be explained by the fact that the first part of such experiments (when most of the water level rise takes place) is determined by the flow parameters of the sand sample with a higher hydraulic conductivity. Therefore, the water flow through this sample has a greater influence on the recorded rate-of-rise curve and the accuracy of identifying the hydraulic conductivity of this sample must be greater.

All experimentally recorded rate-of-rise curves were compared to the curves calculated for the numerically obtained hydraulic conductivities using the presented mathematical model. RMSE values greater than 1 cm were observed only for three very rapid experiments with the highly permeable sand I in a U-tube configuration. In all other cases, the errors show minor discrepancies of the experimental and theoretical curves. This indicates that the hydraulic conductivity values obtained from calculations accurately reflect the hydraulic parameters of the sands used in laboratory studies.

The proposed method of determining the hydraulic conductivities of two sands in the W-tube can be used as an idea to elaborate interpretation algorithms based on permeability tests, including the pressure-induced PARAMEX test or other piezometric tests, such as the slug or boil test. The necessary algorithms could be implemented in field conditions, whenever a bi-exponential water level rise is observed, which would increase the reliability of piezometric permeability tests conducted in heterogeneous formations.

ACKNOWLEDGEMENTS

This work was partially supported by the Polish Ministry of Science and Higher Education (NCN Grant No. 0697/B/P01/2010/39, statutory activity – M.K.).

REFERENCES

Alexander, M., Berg, S. J. & Illman, W. A. 2011 Field study of hydrogeologic characterization methods in a heterogeneous aquifer. *Groundwater* **49** (3), 365–382. Doi:10.1111/j.1745-6584.2010.00729.x.

Altunkaynak, A. 2007 Steady-state groundwater flow model with variable hydraulic conductivity. *Hydrol. Sci. J.* **52** (1), 221–229. Doi:10.1623/hysj.52.1.221.

Balogun, A., Kazemi, H., Ozkan, E., Al-Kobaisi, E. & Ramirez, B. 2009 Verification and proper use of water–oil transfer function for dual-porosity and dual-permeability reservoirs. *SPE Res. Eval. Eng.* **12** (2), 189–199. Doi:10.2118/104580-PA.

Bouwer, H. & Rice, R. C. 1976 A slug test for determining hydraulic conductivity of unconfined aquifers with completely or partially penetrating wells. *Water Resour. Res.* **12** (3), 423–428. Doi:10.1029/WR012i003p00423.

Bredehoeft, J. D. & Papadopulos, S. S. 1980 A method for determining the hydraulic properties of tight formations. *Water Resour. Res.* **16** (1), 233–238. Doi:10.1029/WR016i001p00233.

Cardiff, M., Barrash, W., Thoma, M. & Malama, B. 2011 Information content of slug tests for estimating hydraulic properties in realistic, high-conductivity aquifer scenarios. *J. Hydrol.* **403** (1–2), 66–82. Doi:10.1016/j.jhydrol.2011.03.044.

Chen, C. S., Sie, Y. C. & Lin, Y. T. 2012 A review of the multilevel slug test for characterizing aquifer heterogeneity. *Terr. Atmos. Ocean. Sci.* **23**, 131–143. Doi:10.3319/TAO.2011.10.03.01(Hy).

Cooper Jr, H. H., Bredehoeft, J. D., Papadopulos, I. S. & Bennett, R. R. 1965 The response of well-aquifer systems to seismic waves. *J. Geophys. Res.* **70** (16), 3915–3926. Doi:10.1029/JZ070i016p03915.

Cooper Jr, H. H., Bredehoeft, J. D. & Papadopulos, I. S. 1967 Response of a finite diameter well to an instantaneous charge of water. *Water Resour. Res.* **3** (1), 263–269. Doi:10.1029/WR003i001p00263.

Krauss, I. 1974 Die Bestimmung der Transmissivitat von Grundwasserleitern aus dem Einschwingverhalten des Brunnen-Grundwasserleitersystems [In English: Determining the transmissivity of water-bearing media from an oscillation test conducted in a aquifer-well system]. *J. Geophys.* **40**, 381–400.

Krauss, I. 1977 Das Einschwingverfaren – Transmissivitatsbestimmung ohne Pumpversuch [In English: The oscillation method – determining transmissivity without pumping tests]. *GWF-Wasser/Abwasser.* **118**, 407–410.

Marciniak, M. 1999 In situ identification of the hydrogeological parameters of the aquifers and evaluation of the technical state of piezometers using sudden change in the hydraulic potential (the PARAMEX method) [In Polish: Identyfikacja parametrów hydrogeologicznych na podstawie skokowej zmiany potencjału hydraulicznego Metoda PARAMEX]. Wyd. Nauk. UAM, Poznan, Poland.

Marciniak, M. 2001 Identification of hydraulic conductivity coefficient of aquifers using the PARAMEX test. In: *New Approaches Characterizing Groundwater Flow* (K. P. Seiler & S. Wohnlich, eds.). Vol. 2, XXXI Congress IAH, Munich, Germany.

Marciniak, M. 2012 The PARAMEX test – a method of determining hydrogeological properties of groundwater recharge zone. In: *Proceedings of the 39th International Association of Hydrogeologists Congress 'Confronting Global Change'*, Niagara Falls, Canada.

Marciniak, M., Kaczmarek, M. & Wolny, F. 2010 Physical and mathematical model of a dual-permeability medium [In Polish: model fizyczny i matematyczny osrodka wodonosnego o podwójnej przepuszczalnosci]. *Biuletyn Panstwowego Instytutu Geologicznego* **442**, 129–136.

Marciniak, M., Kaczmarek, M. & Wolny, F. 2013 W-tube system with bi-exponential response – a model for permeability tests in heterogeneous aquifers. *J. Hydrol.* **501**, 175–182. Doi:10.1016/j.jhydrol.2013.08.012.

Rehbinder, G. 1992 Measurement of the relaxation time in the Darcy flow. *Transp. Porous Media* **8** (3), 263–275. Doi:10.1007/BF00618545.

Shapiro, A. M. & Hsieh, P. A. 1998 How good are estimates of transmissivity from slug tests in fractured rock? *Groundwater* **36** (1), 37–48. Doi:10.1111/j.1745-6584.1998.tb01063.x.

Szymkiewicz, A. 2008 Modelowanie przeplywu wody w utworach o podwojnej porowatosci [In English: Modelling of water flow in double-porosity media]. *Biuletyn Panstwowego Instytutu Geologicznego* **431**, 251–258.

Wang, X. S., Chen, C. X. & Jiao, J. J. 2004 Modified Theis equation by considering the bending effect of the confining unit. *Adv. Water Res.* **27**, 981–990. Doi:10.1016/j.advwatres.2004.08.

Wolny, F. 2012 Determining Dual-Permeability Parameters of Water-Bearing Formations [In Polish: Identyfikacja parametrow podwojnej przepuszczalnosci hydraulicznej utworow wodonosnych]. PhD thesis, Faculty of Geographical and Geological Sciences, Adam Mickiewicz University, Poznan, Poland.

Wu, C.-M., Yeh, T.-C. J., Zhu, J., Lee, T. H., Hsu, N.-S., Chen, C.-H. & Sancho, A. F. 2005 Traditional analysis of aquifer tests: comparing apples to oranges? *Water Resour. Res.* **41**, W09402. Doi:10.1029/2004WR003717.

First received 7 June 2015; accepted in revised form 3 November 2015. Available online 12 December 2015