Simple methods for quick determination of aquifer parameters using slug tests

A. Ufuk Şahin

ABSTRACT

The slug test is still one of the simplest and cost-effective methods to interpret the hydraulic parameters for aquifer analysis. This study introduces two new estimation approaches for the slug test, the time shift method (TSM) and arc-length matching method (AMM), to identify aquifer parameters in a reliable and accurate manner, which was established on the idea that any change in the normalized drawdown or arc-length measurements of the data curve at the predefined drawdown levels is linked with the variation of storativity. These approaches remove the need for superimposition of the type curves and the field data. The proposed methods are straightforward to apply and automatize the parameter estimation process. TSM and AMM were tested with a number of numerical experiments including synthetically generated data augmented with random noise, hypothetical slug tests conducted in a heterogeneous rock-fracture system, and well-known real field data. The skin effect was also implemented to evaluate its impact on the estimation performance of the suggested approaches. The results verified that both proposed methods are able to produce estimates of hydraulic parameters more accurately than existing methods. The proposed methods could serve as a viable supplementary interpretation tool for slug test analysis.

Key words | arc-length matching method, rock-fracture systems, skin effect, slug test, time shift method

A. Ufuk Şahin
Department of Civil Engineering,
Hacettepe University,
06800 Ankara,
Turkey
E-mail: *ufuksahin@hacettepe.edu.tr*

INTRODUCTION

In financial and logistical aspects, slug tests are traditionally utilized for rapid evaluation of near-well characteristics of an aquifer by injection or extraction of an amount of water volume instantaneously in a well. The application procedure for slug tests is straightforward and easy due to the requirements for minimum test equipment compared to other information collection methods such as core samples, geological logs, and pumping tests that have been used as a complementary part of the site characterization (Butler *et al.* 1996). Freeze & Cherry (1979) recommended the use of slug tests, since they are able to provide adequate information about site character and are more reliable than pumping tests.

The first mathematical analysis on the interpretation of slug tests was conducted by Hvorslev (1951). A number of researchers (Cooper *et al.* 1967; Bouwer & Rice 1976; Bredehoeft & Papadopulos 1980; Barker & Black 1983; Moench & Hsieh 1985; Karasaki *et al.* 1988; Bouwer 1989; Hyder *et al.* 1994; Butler 1998; McElwee & Zenner 1998) subsequently developed the mathematical models to formulate the temporal responses of slug tests. Ramey *et al.* (1975) analyzed the effect of skin, which is defined as head loss induced by the well face, and treated it by using an infinitesimal skin thickness assumption. Peres *et al.* (1989) proved that the time derivative of constant discharge solution leads to slug test solution with the infinitesimal skin by means of Duhamel's theory; and this transformation is also valid for any well/aquifer system. Faust & Mercer (1984) proposed an analytical solution and developed a numerical solution to evaluate the finite-thickness skin on slug test analysis.

doi: 10.2166/nh.2016.232

Faust & Mercer (1984) also demonstrated that a positive skin is responsible for the shifting of the slug response horizontally with respect to no-skin condition, which induces an unreliable estimation performance whereas a negative skin makes no significant contribution to the behavior of the slug response. Moench & Hsieh (1985) introduced a semi-analytical solution in the Laplace domain to understand the influence of finite-thickness skin using pressurized slug tests. Yang & Gates (1997) employed a finite element model for a slug test conducted in a confined aquifer with a finite-thickness skin, treating the well as a line source. Yeh & Yang (2006) proposed an analytical solution for the slug test response in a confined aquifer with finite-thickness skin using the Bromwich integral technique to convert the Laplace domain solution to the time domain. Yeh *et al.* (2008) also provided a semi-analytical slug test solution for partially penetrating a well in a confined aquifer considering the presence of finite-skin thickness.

The above discussion summarizes some examples of forward problem approaches to be used in slug test analysis. Each model has definite advantages in application but may fail to estimate the aquifer parameters if the model assumptions are violated, as is frequently encountered in practice, as discussed in Osborne (1993). Butler *et al.* (1996) suggested a guideline, which includes several modifications such as using different initial head displacements, selecting proper analysis methods, installing an observation well near the slugged well, performing pre- and post-analysis for the available data, utilizing data acquisition equipment to improve the quality of the collected data, etc., in order to eliminate the discrepancy between the parameter estimations of the slug test and those obtained from other site characterization techniques such as pumping tests. McElwee *et al.* (1995a, 1995b) presented sequential researches on the sensitivity analysis of slug tests conducted in the slugged and observation well. McElwee *et al.* (1995a, 1995b) also revealed that the high correlation between transmissivity (T) and storativity (S) makes the determination of both parameters difficult in a reliable sense using the slugged well data. McElwee *et al.* (1995a, 1995b) concluded that the temporal and spatial dependence of the sensitivities for the aquifer parameters would be reduced using the slug response collected in the observation well. Drilling an observation well solely for use in slug tests, however, seems impractical in terms of financial considerations (Butler *et al.* 1996).

Determination of aquifer parameters regarded as an inverse problem is important in site characterization. The curve matching process is one of the most frequently employed techniques to estimate aquifer parameters. Sageev (1986) characterized the response of the slug impulse as early, intermediate, and late times and developed the type curves for each stage to analyze the effect of wellbore storage and skin on each segment. Sageev (1986) also investigated the effective radius as well as parameter estimation; this model is however strictly valid for confined aquifers. Dax (1987) linearized the slug test well function, given by Cooper *et al.* (1967), to estimate aquifer parameters easily by formulating a new coefficient termed the decay coefficient; however, the hydraulic conductivity estimation requires the storage coefficient as *a priori* information. Peres *et al.* (1989) proposed a deconvolution method to remove the wellbore storage effect from the slugged well data with the infinitesimal skin effect, and established a new type curve to identify the aquifer parameters, converting slug test data to the equivalent constant head response as if it was obtained in the extraction well. This technique can be readily used if the storage coefficient is known as *a priori* information. Chakrabarty & Enachescu (1997) then improved the work of Peres *et al.* (1989), removing the requirement for a known S to predict the slug test parameters by a curve match process. Singh (2007) developed unimodal type curves to predict aquifer parameters for the Cooper *et al.* (1967) slug test model. Together with the wellbore storage and skin effect, identification of heterogenic characteristics of the field being investigated is another challenging task to be considered in aquifer analysis assessment. Recently, Paradis *et al.* (2015) developed a hydraulic tomography based model to resolve the heterogeneity in T, S, and anisotropy using the slug test data.

The graphical curve matching methods for slug test analysis, however, might not yield a unique match when aquifer storativity is so small. In addition, the interpretation of the type curve matching process is subjective, which induces bias in the estimations. Several approaches developed for the different aquifer types have been devised to eliminate the potential risks of graphical matching methods. For instance, recent research proposed by Sahin & Ciftci

(2016) introduces an area matching process to eradicate the requirement of the curve matching process to obtain aquifer parameters using constant-head pumping-test data.

This study is intended to identify the aquifer parameters in lieu of the type curve matching approach for the interpretation of slug test data by establishing two new estimation procedures, referred to as the time shift method (TSM) and arc-length matching method (AMM), respectively. One of the merits of this study is that there is no need to superimpose the type and data curves in these approaches, which provides greater flexibility in the estimates of aquifer parameters. The introduced methods are also as simple as the classical methods, require no complex calculations, and allow the practitioner to obtain more accurate and reliable estimations by avoiding the curve matching process.

METHODOLOGY

Cooper *et al.* (1967) described an analytical model to understand the transient behavior of slug test in a confined aquifer as follows:

$$\nabla \cdot (T \cdot \nabla h) = S\frac{\partial h}{\partial t} \quad \text{for} \quad r \geq r_w \tag{1}$$

with the following boundary conditions and initial condition as:

$$\begin{aligned} &\int_0^{2\pi} T\frac{\partial h}{\partial r} r d\theta = \pi r_c^2 \frac{dH_{wb}}{dt} \quad &&\text{at} \quad r = r_w \\ &h(r_w, t) = H_{wb}(t) && \\ &H_{wb} = H_{wb}(0) &&\text{at} \quad t = t_0, r = r_w \end{aligned} \tag{2}$$

where T and S are the aquifer parameters transmissivity and storativity, respectively, h denotes the hydraulic head, H_{wb} is the head in the wellbore, $H_{wb}(0)$ is the initial excess head, r_c is the casing radius, r_w is the well radius, and t, r, and θ indicate time, radial, and angular directions, respectively (Batu 1998). The solution of Equation (1) is given with two non-dimensionless parameters as follows:

$$F(\alpha, \beta) = \frac{8\alpha}{\pi^2}\int_0^\infty \frac{\exp(-\beta x^2/\alpha)}{x\psi(x)}dx \tag{3}$$

where

$$\psi(x) = (xJ_0(x) - 2\alpha J_1(x))^2 + (xY_0(x) - 2\alpha Y_1(x))^2 \tag{4}$$

$$\alpha = \left(\frac{r_w}{r_c}\right)^2 S \tag{5}$$

and

$$\beta = \frac{Tt}{r_w^2} \tag{6}$$

where J_0 and J_1 are the zero and first order Bessel functions of the first kind, Y_0 and Y_1 are the zero and first order Bessel functions of the second kind.

Slug well function in Equation (3), developed by Cooper *et al.* (1967), can be generated for several α values using a number of alternative numerical integration schemes. Slug test parameters are typically obtained from a curve matching process: the type curves and the field data are superimposed on semi-logarithmic scale on the time axis and selecting the convenient α value which satisfies, in principle, the perfect match. Once α is properly identified, any point on the normalized drawdown level, which is identical for both curves, will have a corresponding β value, which in turn allows prediction of the transmissivity as given in Equation (6).

In the graphical curve matching method, the type curve itself is used as an identification parameter to assess the α value. The matching process has, indeed, inherently potential drawbacks due to the smaller α values, at which the type and site curves are difficult to match visually since type curves are so close to each other. The determination of the α value in an accurate and reliable manner is so significant in the estimation process because this directly affects the predictions of T and S. Therefore, two different 'identifier parameters' termed arc-length and time shift, were proposed in order to remove the subjectivity of the classical curve matching, which may cause unreliable estimates of slug test parameters, to acquire the aquifer parameters in an easy and automatic fashion. The variation of the identifier parameters over α values was elaborated for each method. The radial basis function collation method (RBFCM) was employed to avoid the curve matching and to interpolate any missing value in the available data.

Time shift method process

The motivation of TSM is to elaborate the elapsed time for the selected normalized drawdown shown as F_1 in Figure 1 to be halved (or any desired value shown as F_n in Figure 1), which corresponds to a unique time shift (Δ) for each α value. Dimensionless time can be expressed as:

$$F(\alpha, \beta) = \eta \rightarrow \beta = G(\alpha, \eta) \quad (7)$$

where η is the normalized drawdown level for a particular time and G is the transfer function. This inversion given in Equation (7) could be performed by employing the Newton–Raphson iteration scheme as follows:

$$\beta^k = \beta^{k-1} + \frac{\eta - F}{\frac{\partial F}{\partial \beta}} \quad (8)$$

in which the superscript k denotes the iteration number and F is the slug well function appearing in Equation (3). The dimensionless time derivative term in Equation (8) can be easily found by applying the Leibniz rule as:

$$\frac{\partial F}{\partial \beta} = -\frac{8}{\pi^2} \int_0^\infty \frac{x \exp(-\beta x^2/\alpha)}{\psi(x)} dx \quad (9)$$

This iteration scheme in Equation (8) gives β values in a few iterations when using $\beta^0 = 0.001$ as an initial guess for all ranges of α values implemented in this particular study. The theoretical time shift (Δ) is therefore shown as:

$$\Delta(\alpha, \eta_1, \eta_2) = \log_{10}(\beta_{\eta_2}/\beta_{\eta_1}) \quad (10)$$

The variation of the time shift Δ over the α values for several normalized drawdown differences is illustrated in Figure 2(a). If the perfect match between theoretical type curves and the field data is satisfied, the time shift between two particular normalized drawdown values for the field

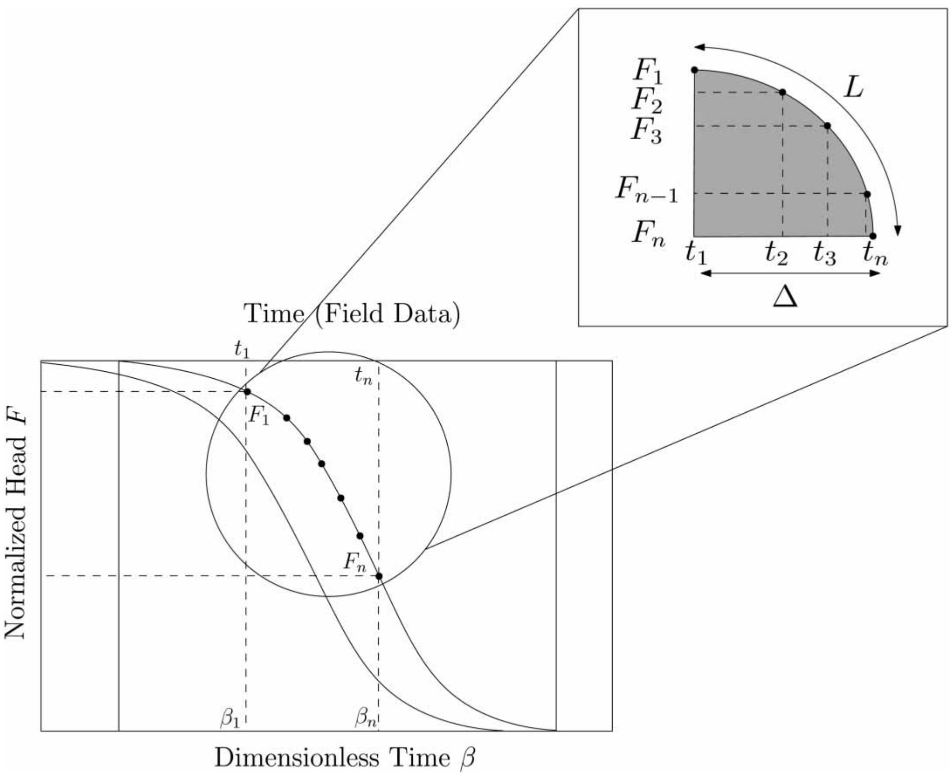

Figure 1 | Schematic illustration of identifier parameters.

data, as shown in Figure 1, is:

$$\Delta_f = \log_{10}\left(\frac{t_2}{t_1}\right) \tag{11}$$

Once Δ_f is computed, the α value could be easily approximated either using Figure 2(a) or RBFCM. The following application steps summarize the entire process as:

- Step 1. Calculate the time shift in the field data and estimate the corresponding α value.
- Step 2. Read β value for $F = 0.5$ from the theoretical type curve drawn from the estimated α value using Equation (3).
- Step 3. Calculate T using Equation (6) and S using Equation (5).

Arc-length matching method

The arc length of the curve pair between predefined normalized drawdown levels, shown as L in Figure 1, is selected as a matching identity in order to estimate the α value. The arc length of the theoretical type curve for two arbitrarily chosen β values can be calculated as:

$$L = \int_{\mu_1}^{\mu_2} \sqrt{1 + \left(\frac{\partial F}{\partial \mu}\right)^2}\, d\mu \tag{12}$$

where $\mu = \ln\beta$ and

$$\frac{\partial F}{\partial \mu} = -\frac{8}{\pi^2}\int_0^\infty \frac{\beta x \exp(-\beta x^2/\alpha)}{\psi(x)} dx \tag{13}$$

The axes of the slug well function can be interchangeably used to calculate arc length between any two normalized drawdown levels as follows:

$$L = \int_{F_1}^{F_2} \sqrt{1 + \left(\frac{\partial \mu}{\partial F}\right)^2}\, dF \tag{14}$$

If the type curve and field data are assumed to be perfectly matched, the arc length appearing in Equation (14) can be approximated from the field data as:

$$L_f \approx \sum_{i=1}^{n-1} \sqrt{1 + \left(\frac{\Delta(\ln t_i)}{\Delta F_i}\right)^2}\, \Delta F_i \tag{15}$$

where n is the total data points in the interest interval.

The variation of arc length, L, over the α values for several normalized drawdown levels is also shown in Figure 2(b). Similar to the TSM procedure, α value could be easily estimated either using Figure 2(b) or RBFCM. Steps 2 and 3 in the TSM are then applied to obtain the aquifer parameters.

RESULTS AND DISCUSSION

The estimation performance of the proposed methods was tested with a number of numerical experiments. As a first step, these approaches were investigated using synthetically generated slug test data which were augmented with substantial noise to mimic the non-smooth test data. TSM and AMM were also examined by a real field data set and compared with the existing techniques in the literature. To show the performance of the implemented methods, several error metrics such as the coefficient of determination (R^2), the root mean squared error (RMSE), the scattered index (SI), and the mean absolute error (MAE) were employed as:

$$R^2 = 1 - \frac{\sum_{i=1}^{N}(\hat{y}_i - y_i)^2}{\sum_{i=1}^{N}\left(y_i - \frac{1}{N}\sum_{i=1}^{N} y_i\right)^2} \qquad RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\hat{y}_i - y_i)^2}$$

$$SI = \frac{RMSE}{\frac{1}{N}\sum_{i=1}^{N} y_i} \qquad MAE = \frac{1}{N}\sum_{i=1}^{N}|\hat{y}_i - y_i| \tag{16}$$

where N is the total number of data, $\hat{y}_i$ is the estimated data value, and y_i is the actual data value.

Homogeneous aquifer with random noise

A number of numerical experiments were conducted in confined aquifer settings by generating log T values in a range of

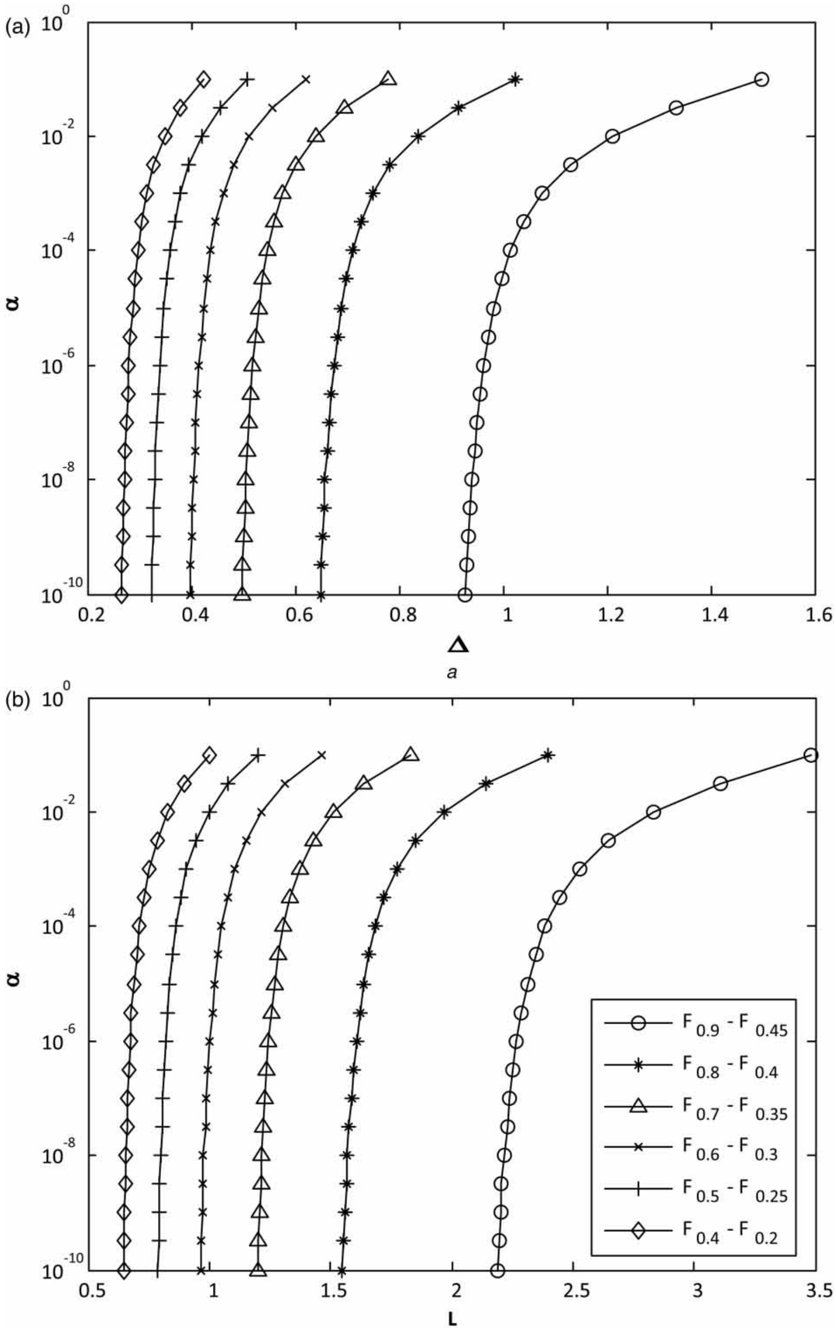

Figure 2 | The variation of α value for several normalized drawdown levels with (a) time shift and (b) arc length.

-3 to -1 with an increment of 0.05 m^2/min (a total of 41 distinct T values). The aquifer storativity, S, was assumed as 1×10^{-4} for each experiment. Each slug test in the hypothetical aquifer commenced from 0.01 min and continued to 100 min with uniform logarithmic time steps of 0.1 min in order to observe transient responses in a finite-radius of the well, r_w, of 0.05 m. The casing radius, r_c, was assumed to be equal to that of the slug well. One percent of observed temporal slug response was added to each generated data set as a random noise in order to mimic the possible error sources (i.e., measurement errors) during the test period.

Randomly selected two data sets over 41 distinct slug test data were analyzed with graphical based matching methods such as the traditional curve matching process (TCM) and Singh's (2007) curve match method (SCM). Figure 3 illustrates the randomly selected two slug test data sets used in this example, Case 1 and Case 2, which were generated with 1% of random noise using $T = 0.001$ and $T = 0.1$ m^2/min, respectively. Figure 3(a) shows the estimated aquifer parameters by TCM for Case 1, which are $T = 0.0013$ m^2/min and $S = 1 \times 10^{-5}$. The same aquifer parameters obtained by SCM, as shown in Figure 3(b), were $T = 0.0012$ m^2/min and $S = 1 \times 10^{-5}$. The TCM for Case 2, shown in Figure 3(c), yields $S = 1 \times 10^{-5}$ whereas SCM, as given in Figure 3(d), predicts $S = 1 \times 10^{-4}$. When the Case 1 data were analyzed by the proposed TSM and AMM, the S estimates were 6.3981×10^{-5} and 6.3772×10^{-5}, respectively, with almost the same RMSE value of 0.0045 and the same estimations of $T = 0.001$ m^2/min. A similar estimation performance can be observed for Case 2 data, represented in Figure 3(c). These examples indicate that graphical based methods contain bias in the estimation of aquifer parameters due to the subjectivity of matching values.

Following the application procedures of TSM and AMM, the transmissivity and storativity values of each experiment were estimated selecting $\eta_1 = 0.5$ and $\eta_2 = 0.125$, as shown in Figure 4. These analyses were also implemented by employing a number of pre-defined normalized drawdown values which were considered to reflect the different characters of the slug test such as early, intermediate, and late time behavior. Table 1 summarizes the error analysis for several η_1/η_2 ratios utilized in this example. In

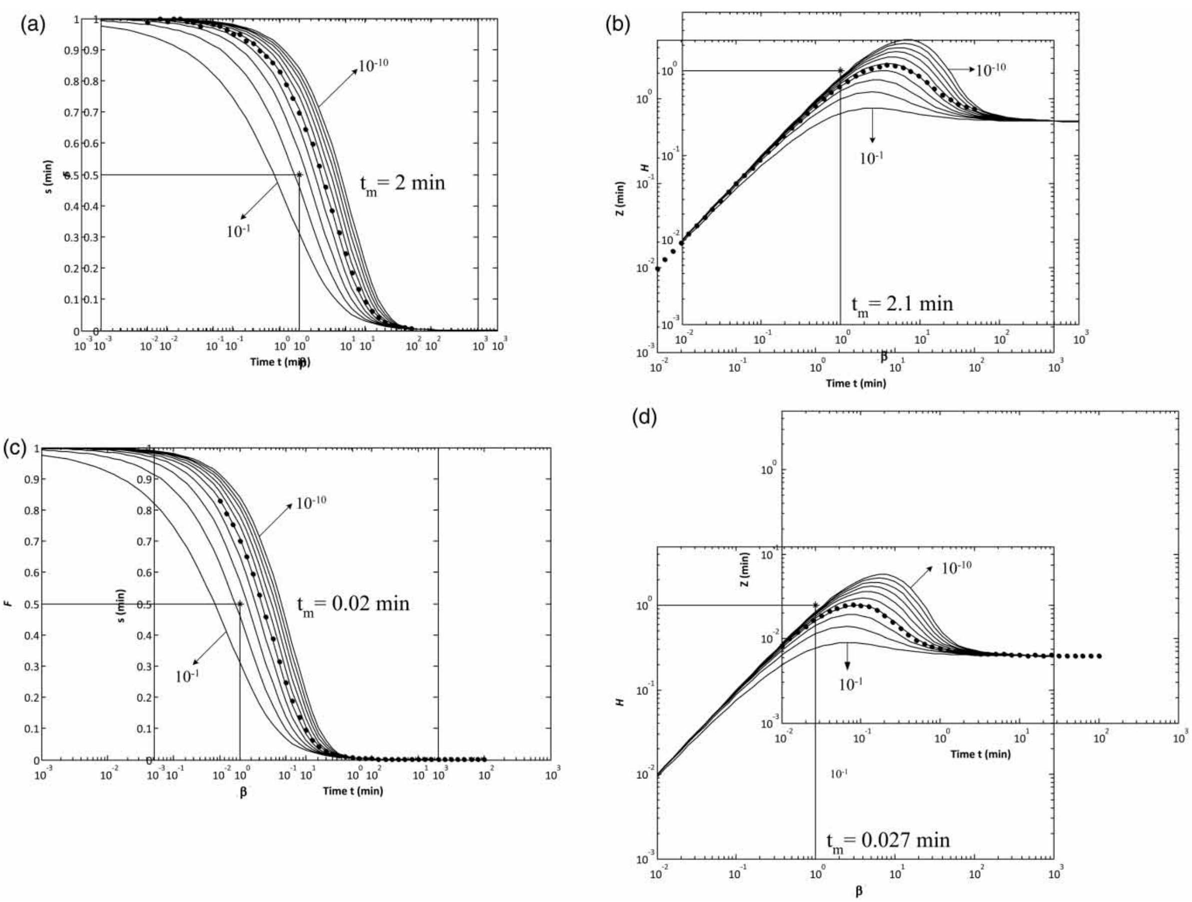

Figure 3 | The performance of the curve matching process: (a) TCM for Case 1, (b) SCM for Case 1, (c) TCM for Case 2, and (d) SCM for Case 2.

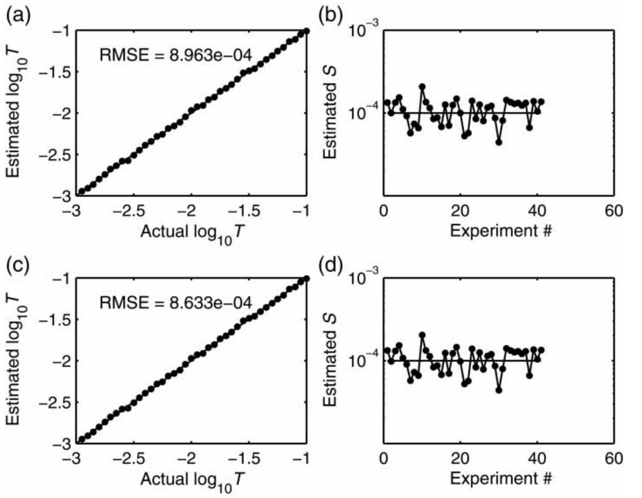

Figure 4 | The overall performance of: (a) T estimates by TSM, (b) S estimates by TSM, (c) T estimates by AMM, and (d) S estimates by AMM.

Table 1, the error metrics for drawdown analysis are given as the mean value of 41 distinct experiments. According to Table 1, the performance of the proposed methods increases if the late time behavior was preferred, which is not explicitly understood from the classical curve match based methods. For instance, choosing η_1 and η_2 values as 0.250 and 0.166, which is assumed to reflect late time behavior of the slug test, in both methods yields better estimation results in comparison with those employing 0.800 and 0.533. In general, when η_1/η_2 increases, the errors between the actual and estimated parameters reduce for both of the proposed methods for every time level used in this example.

As a final remark, both proposed methods are able to produce aquifer parameters in good accordance with the generation values for homogenous aquifer settings even if the slug test data contain relatively small noise.

Layered aquifer with heterogeneous fracture

The estimation performance of the proposed methods was also tested in a hypothetical aquifer with a number of heterogeneous fracture layers. Shapiro & Hsieh (1998) conceptualized a model for the slug response in fractures intersecting a borehole under the assumptions: (i) the vertical flow across the fracture is negligible; and (ii) each fracture is isotropic and homogeneous. Slug response at the extraction well in the Laplace domain is given as:

$$\bar{s} = \frac{1}{p + 2\sum_{i=1}^{N} \sqrt{p\xi_i\alpha_i}\,\frac{K_1\left(\sqrt{p\xi_i/\alpha_i}\right)}{K_0\left(\sqrt{p\xi_i/\alpha_i}\right)}} \qquad (17)$$

with

$$\alpha_i = \left(\frac{r_w}{r_c}\right)^2 S_i, \quad \xi_i = \frac{T_i}{T_f}, \quad T_f = \sum_{i=1}^{N} T_i \qquad (18)$$

where $\bar{s}$ is normalized drawdown in the Laplace domain, N is the total number of layers, T_f denotes the formation transmissivity for the system of fractures, p is the Laplace variable, K_0 and K_1 are the Bessel functions of the second kind with zero and first order, respectively. Equation (17) can be converted in the time domain by employing Stehfest's (1970) algorithm as:

$$f[t] \approx \frac{\ln 2}{t} \sum_{n=1}^{M} C_n \bar{f}\left[\frac{n \ln 2}{t}\right] \qquad (19)$$

where

$$C_n = (-1)^{n+M/2} \sum_{k=\lfloor \frac{n+1}{2} \rfloor}^{\min(n,M/2)} \frac{k^{M/2}(2k)!}{(M/2-k)!k!(k-1)!(2k-n)!} \qquad (20)$$

with M being an even number and $\lfloor\rfloor$ being the integer part of $(n+1)/2$.

Two different slug test scenarios similar to the work of Shapiro & Hsieh (1998) were implemented using Equation (17). A 125-min long slug test was conducted in a fully penetrated extraction well which has equal casing and well radii of 0.075 m. Figure 5(a) exhibits the temporal head changes of both the slug test scenarios. In the first scenario, T values of 20-layered fractures were generated by a normal distribution with a mean of $\log_{10}(T) = -5$, and variance of $\log_{10}(T) = 2$, which provides a relatively large range of T values that varies in an order of 10^{-10} and 10^{-2} m^2/min, as seen in Figure 5(b). The total transmissivity of the fractures, T_f, was therefore realized as 45.93×10^{-4} m^2/min. S values were also generated to be log-normally

Table 1 | Estimation performance and error analysis of TSM and AMM for several η_1 and η_2 values

			Transmissivity analysis				Storativity analysis			Drawdown analysis			
Method	η_1	η_1/η_2	R^2	RMSE	SI	ME	Min AE	Max AE	MAE	R^2	RMSE	SI	ME
TSM	0.8	1.5	0.9661	0.0049	0.2185	-2.01×10^{-5}	3.75×10^{-6}	8.16×10^{-4}	1.39×10^{-4}	0.9999	0.0047	0.0128	-1.79×10^{-4}
		2	0.9906	0.0027	0.1234	4.41×10^{-4}	2.26×10^{-5}	4.66×10^{-4}	9.00×10^{-5}	0.9999	0.0040	0.0108	-1.70×10^{-4}
		3	0.9940	0.0021	0.0926	1.23×10^{-4}	2.67×10^{-5}	3.31×10^{-4}	6.56×10^{-5}	0.9999	0.0037	0.0099	-1.83×10^{-4}
		4	0.9952	0.0019	0.0838	2.00×10^{-4}	3.19×10^{-5}	3.08×10^{-4}	5.34×10^{-5}	0.9999	0.0036	0.0097	-1.61×10^{-4}
	0.5	1.5	0.9820	0.0035	0.1592	1.84×10^{-4}	1.21×10^{-5}	6.44×10^{-4}	1.05×10^{-4}	0.9999	0.0041	0.0111	-1.91×10^{-4}
		2	0.9904	0.0026	0.1172	5.17×10^{-5}	2.14×10^{-5}	2.92×10^{-4}	6.04×10^{-5}	0.9999	0.0036	0.0100	-1.96×10^{-4}
		3	0.9946	0.0020	0.0898	2.54×10^{-4}	3.75×10^{-5}	2.25×10^{-4}	4.92×10^{-5}	0.9999	0.0035	0.0096	-1.54×10^{-4}
		4	0.9981	0.0012	0.0536	1.07×10^{-4}	4.12×10^{-5}	1.86×10^{-4}	3.18×10^{-5}	0.9999	0.0034	0.0092	-1.50×10^{-4}
	0.25	1.5	0.9708	0.0046	0.2070	5.88×10^{-4}	1.92×10^{-5}	7.50×10^{-4}	1.48×10^{-4}	0.9999	0.0044	0.0124	-1.55×10^{-4}
		2	0.9903	0.0026	0.1169	-8.79×10^{-5}	2.92×10^{-5}	3.39×10^{-4}	4.31×10^{-5}	0.9999	0.0036	0.0099	-1.62×10^{-4}
		3	0.9968	0.0015	0.0690	-9.78×10^{-5}	3.49×10^{-5}	1.88×10^{-4}	3.15×10^{-5}	0.9999	0.0035	0.0095	-1.39×10^{-4}
		4	0.9988	0.0010	0.0450	2.04×10^{-4}	5.09×10^{-5}	1.74×10^{-4}	2.21×10^{-5}	0.9999	0.0035	0.0094	-1.42×10^{-4}
AMM	0.8	1.5	0.9658	0.0049	0.2193	2.74×10^{-5}	3.72×10^{-6}	8.33×10^{-4}	1.41×10^{-4}	0.9999	0.0047	0.0129	-1.76×10^{-4}
		2	0.9906	0.0028	0.1238	4.53×10^{-4}	2.16×10^{-5}	4.84×10^{-4}	9.07×10^{-5}	0.9999	0.0040	0.0108	-1.69×10^{-4}
		3	0.9939	0.0021	0.0939	1.39×10^{-4}	2.61×10^{-5}	3.33×10^{-4}	6.66×10^{-5}	0.9999	0.0037	0.0100	-1.83×10^{-4}
		4	0.9950	0.0019	0.0856	2.11×10^{-4}	3.10×10^{-5}	3.09×10^{-4}	5.47×10^{-5}	0.9999	0.0036	0.0097	-1.62×10^{-4}
	0.5	1.5	0.9820	0.0035	0.1594	1.50×10^{-4}	1.18×10^{-5}	6.36×10^{-4}	1.05×10^{-4}	0.9999	0.0041	0.0111	-1.91×10^{-4}
		2	0.9902	0.0026	0.1183	2.48×10^{-5}	2.12×10^{-5}	2.88×10^{-4}	6.03×10^{-5}	0.9999	0.0036	0.0100	-1.97×10^{-4}
		3	0.9950	0.0019	0.0862	2.07×10^{-4}	3.79×10^{-5}	2.19×10^{-4}	4.71×10^{-5}	0.9999	0.0035	0.0096	-1.52×10^{-4}
		4	0.9982	0.0012	0.0520	7.49×10^{-5}	4.09×10^{-5}	1.82×10^{-4}	3.10×10^{-5}	0.9999	0.0034	0.0092	-1.50×10^{-4}
	0.25	1.5	0.9722	0.0045	0.2012	5.17×10^{-4}	1.86×10^{-5}	6.95×10^{-4}	1.42×10^{-4}	0.9999	0.0044	0.0123	-1.54×10^{-4}
		2	0.9904	0.0026	0.1166	-1.11×10^{-4}	2.87×10^{-5}	3.38×10^{-4}	4.27×10^{-5}	0.9999	0.0036	0.0099	-1.62×10^{-4}
		3	0.9970	0.0015	0.0671	-1.02×10^{-4}	3.53×10^{-5}	1.84×10^{-4}	3.06×10^{-5}	0.9999	0.0035	0.0095	-1.39×10^{-4}
		4	0.9988	0.0010	0.0440	1.73×10^{-4}	5.04×10^{-5}	1.72×10^{-4}	2.15×10^{-5}	0.9999	0.0035	0.0094	-1.42×10^{-4}

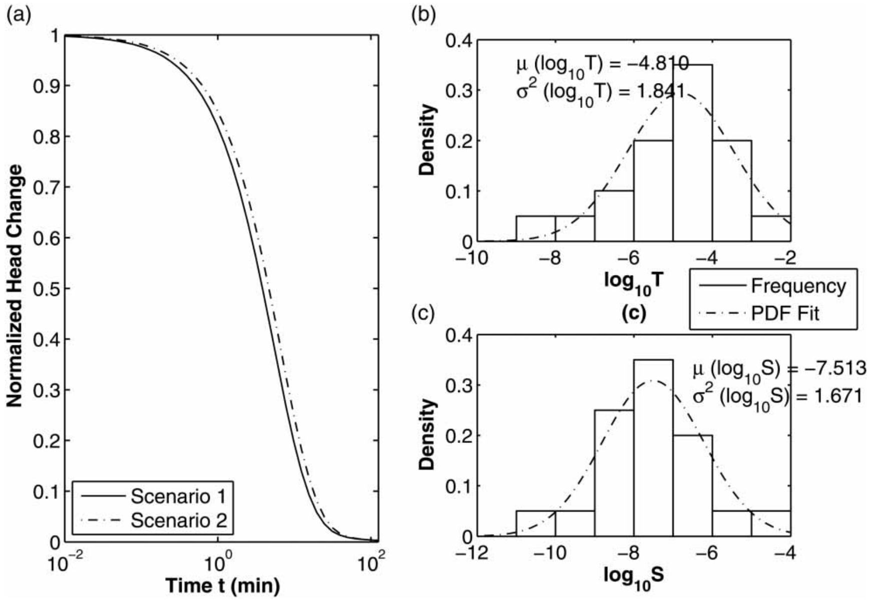

Figure 5 | Fracture model for slug test scenarios: (a) head responses, (b) generated *T* distribution, and (c) generated *S* distribution.

distributed with a geometric mean equal to 10^{-7} and a variance of $\log_{10}(S) = 2$, also shown in Figure 5(c). *T* and *S* values were assumed to be uncorrelated. In the second scenario, the generated *T* and *S* values were assumed to be negatively correlated, which means the fracture with the larger *T* value is linked with the lower *S* value. Shapiro & Hsieh (1998) stated that this situation can be observed when the fractures contain a filling material of lower permeability with a larger storativity than unfilled fractures.

For this particular example, identifying aquifer parameters via classical curve matching, it is difficult to obtain a good visual match since the α value is small. Table 2 summarizes the estimation performance of the proposed methods for both test scenarios. For instance, TSM predicted *T* and *S* values as 43.90×10^{-4} m^2/min and 1.2642×10^{-7}, respectively, for test Scenario 1 and those obtained by AMM were 44.09×10^{-4} m^2/min and 1.1999×10^{-7}, respectively. When *T* and *S* are uncorrelated as simulated in

Table 2 | Results and error analysis for fracture model

	Aquifer parameters		Drawdown comparison			
	T (m^2/min)	S	R^2	RMSE	SI	MAE
Scenario 1						
Proposed TSM ($\eta_1 = 0.5$, $\eta_2 = 0.25$)	4.39×10^{-3}	1.26×10^{-7}	1.0000	0.0002	0.0002	0.0001
Proposed AMM ($\eta_1 = 0.5$, $\eta_2 = 0.25$)	4.41×10^{-3}	1.20×10^{-7}	1.0000	0.0001	0.0002	0.0001
Cooper *et al.* (1967), curve match	1.90×10^{-3}	1.00×10^{-3}	0.9991	0.0156	0.0214	0.0115
Singh (2007)	4.70×10^{-3}	1.00×10^{-7}	0.9998	0.0066	0.0091	0.0037
Scenario 2						
Proposed TSM ($\eta_1 = 0.5$, $\eta_2 = 0.25$)	4.45×10^{-3}	3.08×10^{-9}	1.0000	0.0001	0.0002	0.0001
Proposed AMM ($\eta_1 = 0.5$, $\eta_2 = 0.25$)	4.46×10^{-3}	2.92×10^{-9}	1.0000	0.0002	0.0002	0.0001
Cooper *et al.* (1967), curve match	2.00×10^{-3}	1.00×10^{-4}	0.9995	0.0137	0.0183	0.0083
Singh (2007)	4.00×10^{-3}	1.00×10^{-8}	0.9999	0.0060	0.0081	0.0033

Scenario 1, the S estimates of proposed methods yield the geometric mean value of the fractures as tabulated in Table 2.

In Scenario 2, the performance of AMM was also checked with randomly selected drawdown levels, as shown in Figure 6. As seen in the figure, the different data pairs utilized in the AMM approach lead to almost equal T estimates. A similar result was obtained for TSM under the same estimation scheme.

As seen from these results, the proposed methods could be noted to produce reliable estimates of the aquifer parameters. The representative transmissivity T_f as explained in Shapiro & Hsieh (1998) was easily acquired via the proposed approaches.

The skin effect

The Yeh & Yang (2006) analytical solution for the slug response with the finite skin thickness was utilized to draw the limitation of suggested parameter estimation schemes. A number of 100-min-long slug tests were conducted in a hypothetical aquifer with several skin factors, such as 0.1, 1, 5, and 10. A composite aquifer system having different material properties near the wellbore and formation zones may be disturbed during the well drilling process, which eventually leads to changes in hydraulic conductivities. If the undisturbed zone has higher hydraulic conductivity than the wellbore zone, a positive skin ($\lambda > 1$) may be observed.

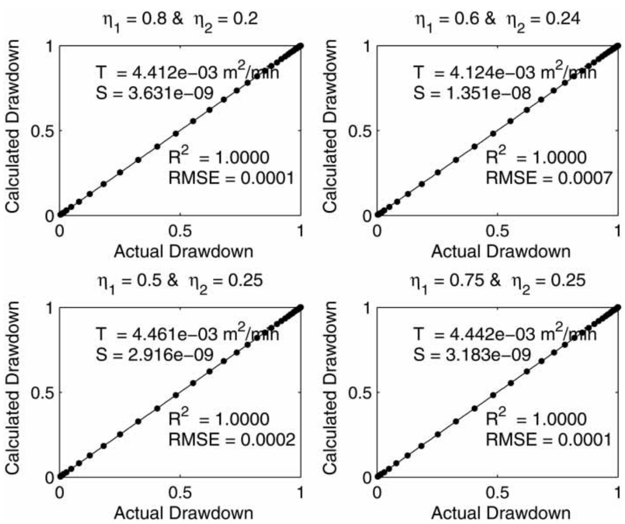

Figure 6 | AMM performance with different data pairs for Scenario 2.

In contrast, the hydraulic conductivity of the well proxy may increase as a result of a fracture in the well wall or extensive well development and become higher than that of the undisturbed zone, thus a negative skin ($\lambda < 1$) may occur (Yang & Yeh 2007; Yeh & Chen 2007). A total of 21 formation zone $\log_{10}(T)$ values ranging from -1 to $1\ m^2/min$ were employed to observe the temporal slug response in an extraction well with a radius of 1 m and a finite skin radius of 10 m. The S value for the skin and formation zone of each distinct test was assumed to be 1×10^{-4}. One percent of random noise was again added to mimic the unexpected disturbances (i.e., measurement error).

Figure 7 represents the estimates of the proposed TSM, which was applied to the available data set selecting η_1 and η_2 values as 0.50 and 0.25, respectively. AMM was also implemented, but since both methods showed almost the same level of accuracy, RMSE between the estimated formation T values and the generation ones was only given for TSM in Figure 7. As seen from Figure 7, TSM was able to predict the formation T values in good accordance with the generation values for negative skin condition ($\lambda = 0.1$). In general, the estimation error of the methods increases with the skin effect for the noisy data set. For a relatively large positive skin scenario ($\lambda = 10$), both methods systematically underestimated the formation zone T values. The primary reason behind this situation is that the skin effect and α value are both responsible for the right shift in the type curves, and the skin effect becomes dominant. Thus, the effect of skin is not identified individually for its larger values utilizing the proposed methods that were originally derived from no-skin condition. As discussed in Beckie & Harvey (2002), the estimated T values obtained from the slug test analysis are unbiased, which means near-well properties do not affect the estimates of T so much, whereas S is strongly dependent on T since the estimated S values are biased. Therefore, as verified with the results, the proposed methods could be safely employed to access the aquifer parameters unless skin factor is greater than 5, even if the available data contain relatively small substantial noise.

Real field data

The estimation performance of the proposed methods finally was analyzed with field data previously utilized by Cooper

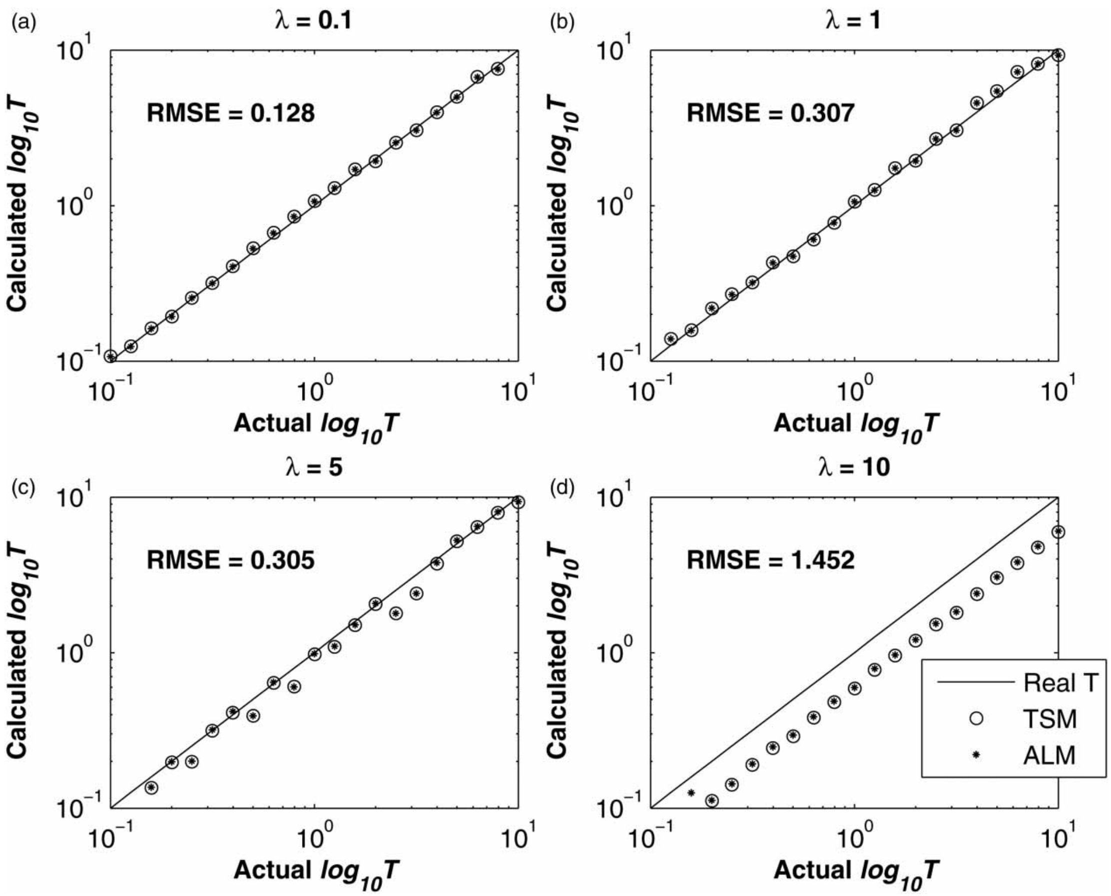

Figure 7 | TSM results under skin effect: (a) $\lambda = 0.1$, (b) $\lambda = 1$, (c) $\lambda = 5$, and (d) $\lambda = 10$.

et al. (1967). In these test data, 7.6 cm of well and casing radius was used to measure the slug responses of a 63-sec-long test. The initial head of the aquifer was reported as 0.336 m and the initial excess head in the well was 0.896 m. Applying TSM and AMM, *T* values were estimated as 4.909 and 4.738 cm^2/s, respectively. Similarly, *S* values for TSM and AMM were 1.174×10^{-3} and 1.473×10^{-3}, respectively. Table 3 summarizes the estimation results and the performance of the implemented methods used in the comparison. According to Table 3, the estimation capability of the proposed TSM and AMM is as high as the available techniques in the literature.

Table 3 | Real field data analysis

	Aquifer parameters		Drawdown comparision			
Method	**T (cm^2/s)**	**S**	**R^2**	**RMSE**	**SI**	**MAE**
Proposed TSM ($\eta_1 = 0.5$, $\eta_2 = 0.25$)	4.9090	1.1740×10^{-3}	0.9990	0.0091	0.0266	0.0071
Proposed AMM ($\eta_1 = 0.5$, $\eta_2 = 0.25$)	4.7380	1.4730×10^{-3}	0.9990	0.0095	0.0279	0.0077
Cooper *et al.* (1967), curve match	5.3000	1.0000×10^{-3}	0.9983	0.0116	0.0355	0.0098
Batu (1998), curve match	5.2500	1.0000×10^{-3}	0.9984	0.0101	0.0305	0.0082
Singh (2007)	7.9890	1.0000×10^{-5}	0.9991	0.0190	0.0561	0.0136
Peres *et al.* (1989), semilog	4.8000					

CONCLUSION

In this study, two new parameter estimation methods, referred to as TSM and AMM, were introduced to analyze the slug test responses. These approaches contain simple application steps to access aquifer parameters. The performance of TSM and AMM was investigated with a number of data including homogeneous synthetic data augmented with random noise, layered fracture data, and real field data as well. The following key outcomes were drawn through the test case examples:

- The TSM and AMM are straightforward to implement, and avoid the curve matching process that might be difficult to apply for the test data having smaller α value.
- For homogeneous aquifer systems, these methods are able to provide the aquifer parameters better than the available curve match based methods.
- The representative T_f value for the fracture-rock matrix system was easily accessed by the proposed methods.
- The formation zone T values for a certain level of skin factor can be estimated accurately via the proposed methods.

As a general remark, the proposed TSM and AMM are evaluated as viable approaches to estimate the aquifer parameters. The established methods can form a basis for assessing the hydraulic parameters in lieu of the curve matching process employed for several aquifer types.

REFERENCES

Barker, J. A. & Black, J. H. 1983 Slug tests in fissured aquifers. *Water Resour. Res.* **19** (6), 1558–1564.

Batu, V. 1998 *Aquifer Hydraulics: A Comprehensive Guide to Hydrogeologic Data Analysis*. Wiley, New York, USA.

Beckie, R. & Harvey, C. F. 2002 What does a slug test measure: an investigation of instrument response and the effects of heterogeneity. *Water Resour. Res.* **38** (12), 26-1–26-14.

Bouwer, H. 1989 The Bouwer and Rice slug test–an update. *Ground Water* **27** (3), 304–309.

Bouwer, H. & Rice, R. C. 1976 A slug test for determining hydraulic conductivity of unconfined aquifers with completely or partially penetrating wells. *Water Resour. Res.* **12** (3), 423–428.

Bredehoeft, J. D. & Papadopulos, I. S. 1980 A method for determining the hydraulic properties of tight formations. *Water Resour. Res.* **16** (1), 233–238.

Butler, J. J. 1998 *The Design, Performance, and Analysis of Slug Tests*. Lewis, London, UK.

Butler, J. J., McElvee, C. D. & Liu, W. Z. 1996 Improving the reliability of parameter estimates obtained from slug tests. *Ground Water* **34** (3), 480–490.

Chakrabarty, C. & Enachescu, C. 1997 Using the deconvolution approach for slug test analysis: theory and application. *Ground Water* **35**, 797–806.

Cooper, H. H., Bredehoeft, J. D. & Papadopulos, I. S. 1967 Response of a finite diameter well to an instantaneous charge of water. *Water Resour. Res.* **3** (1), 263–269.

Dax, A. 1987 A note on the analysis of slug tests. *J. Hydrol.* **91** (1–2), 153–177.

Faust, C. R. & Mercer, J. W. 1984 Evaluation of slug tests in wells containing a finite–thickness skin. *Water Resour. Res.* **20** (4), 504–506.

Freeze, R. A. & Cherry, J. A. 1979 *Groundwater*. Prentice-Hall, Englewood Cliffs, NJ, USA.

Hvorslev, M. J. 1951 *Time Lag and Soil Permeability in Ground-Water Observations*. Bull. No. 36, Waterways Experiment Station, Corps of Engineers, US Army, p. 50.

Hyder, Z., Butler, J. J., McElwee, C. D. & Liu, W. 1994 Slug tests in partially penetrating wells. *Water Resour. Res.* **30** (11), 2945–2957.

Karasaki, K., Long, J. C. S. & Witherspoon, P. A. 1988 Analytical models of slug tests. *Water Resour. Res.* **24** (1), 115–126.

McElwee, C. D. & Zenner, M. 1998 A nonlinear model for analysis of slug-test data. *Water Resour. Res.* **34** (1), 55–66.

McElwee, C. D., Bohling, G. C. & Butler, J. J. 1995a Sensitivity analysis of slug tests, part 1. The slugged well. *J. Hydrol.* **164** (1–4), 53–67.

McElwee, C. D., Bohling, G. C., Butler, J. J. & Liu, W. 1995b Sensitivity analysis of slug tests, part 2. Observation wells. *J. Hydrol.* **164** (1–4), 69–87.

Moench, A. F. & Hsieh, P. A. 1985 Analysis of slug test data in a well with finite thickness skin. In: *The I.A.H. 17th International Congress on Hydrology of Rocks of Low Permeability*. International Association of Hydrogeology, Tucson, AZ, USA, January 1985. 17 (2), pp. 7–12.

Osborne, P. S. 1993 *Suggested Operating Procedures for Aquifer Pumping Tests*. US EPA Office, Office of Research and Development. EPA/540/S-93/503, Washington, DC, p. 23.

Paradis, D., Gloaguen, E., Lefebvre, R. & Giroux, B. 2015 Resolution analysis of tomographic slug test head data: two-dimensional radial case. *Water Resour. Res.* **51** (4), 2356–2376.

Peres, A. M. M., Onur, M. & Reynolds, A. C. 1989 A new analysis procedure for determining aquifer properties from slug tests data. *Water Resour. Res.* **25** (7), 1591–1602.

Ramey, H. J., Agarwal, R. G. & Martin, I. 1975 Analysis of slug test or DST flow period data. *J. Can. Pet. Technol.* **14** (3), 37–47.

Sageev, A. 1986 Slug test analysis. *Water Resour. Res.* **22** (8), 1323–1333.

Sahin, A. U. & Ciftci, E. 2016 An area matching process to estimate the hydraulic parameters using transient constant-head test data. *Hydrol. Res.* **47** (5), 919–931.

Shapiro, A. M. & Hsieh, P. A. 1998 How good are estimates of transmissivity from slug tests in fractured rock? *Ground Water* **36** (1), 37–48.

Singh, S. K. 2007 New methods for aquifer parameters from slug test data. *J. Irrig. Drain. Eng.* **133** (3), 272–275.

Stehfest, H. 1970 Algorithm 368: numerical inversion of Laplace transforms. *Commun. ACM* **13** (1), 47–49.

Yang, Y. J. & Gates, T. M. 1997 Wellbore skin effect in slug-test data analysis for low-permeability geologic materials. *Ground Water* **35** (6), 931–937.

Yang, S. Y. & Yeh, H. D. 2007 On the solutions of modeling slug test performed in a two-zone confined aquifer. *Hydrogeol. J.* **15** (2), 297–305.

Yeh, H. D. & Chen, Y. J. 2007 Determination of skin and aquifer parameters for a slug test with wellbore-skin effect. *J. Hydrol.* **342** (3–4), 283–294.

Yeh, H. D. & Yang, S. Y. 2006 A novel analytical solution for a slug test conducted in a well with a finite-thickness skin. *Adv. Water Resour.* **29** (10), 1479–1489.

Yeh, H. D., Chen, Y. J. & Yang, S. Y. 2008 Semi-analytical solution for a slug test in partially penetrating wells including the effect of finite-thickness skin. *Hydrol. Process.* **22** (18), 3741–3748.

First received 20 November 2015; accepted in revised form 24 March 2016. Available online 11 May 2016