Estimating models for reference evapotranspiration with core meteorological parameters via path analysis
Ye Liu, Miao Yu, Xiaoyi Ma and Xuguang Xing

ABSTRACT

Accurate estimation and reliable universal performance of reference evapotranspiration (ET₀) obtained from a few meteorological parameters are important for the rational planning of agricultural water resources and the effective management of water in irrigated regions. Meteorological data in southern China were used to calculate ET₀ using the standard Penman–Monteith formula and determined the core decision variable (hours of sunshine, N) and the limited decision variable (relative humidity, RH) using path analysis. Estimation models using an artificial neural network and wavelet neural network were established for the Wuhan and Guangzhou meteorological stations. The statistical indices were positively correlated with the decision contribution rates to ET₀. The ET₀ values for other stations in southern China were all estimated by these models, which were trained for the Guangzhou station, and then made a total comparison with Hargreaves–Samani (HS) and Priestley–Taylor (PT) empirical ET₀ models. Error analysis indicated that the root mean square error and the mean absolute per cent error were around 0.32 mm and 5.5%, respectively, with a high coefficient of determination and Nash–Sutcliffe efficiency over 0.9, indicating that these estimating models could be applied in more regions for universal analysis with high accuracy.

Key words | decision variables, estimation accuracy, neural network, path analysis, reference evapotranspiration, universal analysis

INTRODUCTION

Reference evapotranspiration (ET₀) is a critical parameter for calculating evapotranspiration and estimating actual crop water requirements and is widely used for choosing an irrigation system, optimizing crop planting structure, and enriching field water balance theory. Determining the characteristics of water consumption for a crop simultaneously could provide a reliable basis for the deployment of farmland water management and the allocation of agricultural water resources (Chen 1995; Temesgen et al. 2005; Trajkovic & Kolakovic 2009b; Chang et al. 2010; Kisi et al. 2012). The estimation and prediction of ET₀ are thus very important for developing a reasonable system of field irrigation and for improving agricultural water management.

The Penman–Monteith (PM) equation is recommended by the United Nations Food and Agriculture Organization (FAO) and has been accepted universally for calculating ET₀. The FAO defines ET₀ for a hypothetical crop with an assumed height of 0.12 m, a surface resistance of 70 s m⁻¹, and an albedo of 0.23, closely resembling the reference crop canopy evapotranspiration of an extensive surface of actively growing and adequately watered green grass of uniform height (Allan et al. 1998). The PM method had higher accuracy and wider applicability than the Hargreaves–Samani and Priestley–Taylor methods, but its application was limited by the difficulty of PM calculation, the required meteorological parameters, and the incompleteness of meteorological data collected by some small weather stations (Tabari & Talaei 2011; Ngongondo et al. 2013).

New estimation models have recently been proposed with the emergence of artificial neural network (ANN)
technology (Kisi & Çimen 2009; Adeloye et al. 2012; Baba et al. 2013; Shiri et al. 2015a). The back propagation (BP) neural network model, currently the most mature and popular neural network, provides a powerful fault-tolerant and nonlinear approximation capability for calculation, simulation and estimation. This model has been widely used for calculating and estimating \( ET_0 \) (Kumar et al. 2002; Cui et al. 2005; Khoob 2008). The meteorological parameters that are most influential during the establishment of a network model should be chosen as the inputs of the network; recent studies suggest about four parameters (Landeras et al. 2008; Dai et al. 2009; Traore et al. 2010; Shiri et al. 2011; Huo et al. 2012). Model concision and universal application, however, cannot be fully implemented using these many parameters, so fewer (one or two) decisive meteorological parameters should be used to estimate \( ET_0 \) for providing a reliable theoretical basis for real-time estimation and application, especially in developing countries and regions which suffer from lack of instruments and sensors (Shiri et al. 2014, 2015b).

Hence, the development of more efficient \( ET_0 \) estimating models is now of great importance when only few climatic data are available (Shiri et al. 2015). Many mathematical methods have been used to select the determining parameters, such as regression, correlation, sensitivity and trend analyses (Beven 1979; Huo et al. 2004; Verstraeten et al. 2005; Cao et al. 2007; Nova et al. 2007; Dinpashoh et al. 2011; Espadafor et al. 2011; Zhang et al. 2012; Li et al. 2013; Talaei et al. 2014; Tan et al. 2015). Many meteorological parameters, however, are strongly correlated with \( ET_0 \) and are not completely independent of each other. Regression equations or empirical formulae with fewer variables can thus easily be ineffective when analysing data using the least squares method and so are less reliable and convincing. Path analysis can identify the direct and indirect effects from independent and dependent variables and identify the most highly interacting influences from among all parameters than can a simple correlation analysis (Tao et al. 2013; Sun et al. 2014; Ambachew et al. 2015; Zhang et al. 2016).

On the other hand, the wavelet analysis provides a useful method to decompose the observed available data, in terms of both time and frequency (Daubechies 1990). Partal (2009) utilized wavelet transform to decompose and reconstruct the climate data for \( ET_0 \) estimation. Wavelet analysis, however, focuses on determining the required components of each selected climate factor instead of choosing the core parameters from all meteorological factors, which is different from path theory. Furthermore, the established models based on wavelet analysis are difficult to popularize, largely attributed to heavy workload and difficult consistent-component decision.

In fact, many studies have analysed the selection of meteorological parameters and the estimation accuracy during the establishment of neural network models for \( ET_0 \) estimation, but most were suitable only for a limited area and were unable to obtain effective estimates when the study area expanded. The universal analysis of \( ET_0 \) estimation based on an ANN model is extremely necessary. Moreover, wavelet neural network (WNN) is a new kind of network that combines the classic ANN and the wavelet analysis, which hybridize the flexibility and learning abilities of the neural network (Zhang & Benveniste 1992; Hsieh et al. 2011; Ong & Zainuddin 2016; Sharma et al. 2016). In several recent studies, the application of the wavelet analysis coupled with neural network can improve the efficiency of the traditional ANN model (Chauhan et al. 2009; Falamarzi et al. 2014; Wang et al. 2015). It is valuable to establish the WNN model and compare it with the ANN model for \( ET_0 \) estimation and promotion.

When the daily meteorological data from various capital cities in southern China for 1969–2010 had been prepared as the origin data, the study first calculated the reference \( ET_0 \) values using the PM equations, applied path analysis for the \( ET_0 \) values and various meteorological parameters, analysed the strength of the interactions between the meteorological parameters, and finally selected a few core decision parameters. Meteorological stations which had the same selected parameters were merged into a group, and a base station which had the most influential corresponding decision parameters was chosen from each group. Then, the ANN and WNN models could be established by investigating the estimation accuracy and reliability with the selected meteorological parameters, the data from each base station were used to train these neural network models, and the data from the other stations were used for the estimation and universal analysis. The estimation accuracy and reliability of these models were
analysed and compared with those of some empirical equations in order to provide solid technical support for applying the model.

MATERIALS AND METHODS

Study area and data sources

China is divided into northern and southern regions by the Qinling Mountains and the Huaihe River. The southern region more urgently needs to improve land use and farming productivity due to its higher population density and smaller area of arable land. Meteorological data from various municipalities and capital cities in the south were selected for the study to raise the level of agricultural development in this region (Figure 1). Summer (June–August) meteorological data for 1969–2010 were obtained from the website of the China Meteorological Data Sharing Service System (http://cdc.nmic.cn/home.do). The data included daily hours of sunshine (N, h), average temperature (T\text{mean}, °C), maximum temperature (T\text{max}, °C), minimum temperature (T\text{min}, °C), relative humidity (RH, %) and wind speed (U\text{2}, m s\textsuperscript{-1}).

Reference evapotranspiration

The PM equations are based on the energy balance and water vapour diffusion theories and so provide a better theoretical basis and much higher accuracy than other methods for calculating $ET_0$ and take into account both the physiological characteristics of crops and the changes in aerodynamic parameters (Allan et al. 1998). The PM FAO-56 formula, recommended after years of research and improvement, is:

$$ET_0 = \frac{0.408\Delta(R_n - G) + \gamma \frac{900}{T + \frac{273}{u_2}(e_s - e_a)}}{\Delta + \gamma(1 + 0.34u_2)}$$

where $ET_0$ is the reference evapotranspiration calculated by the PM method (mm·d\textsuperscript{-1}), $R_n$ is the net radiation at the crop surface (MJ·m\textsuperscript{-2}·d\textsuperscript{-1}), $G$ is the soil heat-flux density (MJ·m\textsuperscript{-2}·d\textsuperscript{-1}), $T$ is the mean daily air temperature at a height of 2 m (°C), $U_2$ is the wind speed at 2 m (m·s\textsuperscript{-1}), $e_s$ is the saturation vapour pressure (kPa), $e_a$ is the actual vapour pressure (kPa), $\Delta$ is the slope of the vapour-pressure curve (kPa·°C\textsuperscript{-1}), and $\gamma$ is the psychometric constant (kPa·°C\textsuperscript{-1}).

Figure 1 | Distribution of meteorological stations.
In addition, some well-known empirical models were considered to compare the performance of $ET_0$ calculated by the PM method. The Priestley–Taylor method (Priestley & Taylor 1972) simplified and adjusted the PM method which calculated $ET_0$ without $RH$ and $U_2$. Moreover, the Hargreaves–Samani method (Hargreaves & Samani 1985) could compute $ET_0$ only by the temperature data. These two equations are listed as follows:

**Priestley–Taylor:**

$$ET_{0 \text{PT}} = \frac{1.26}{\lambda} \frac{\Delta}{\Delta + \gamma} (R_n - G) \tag{2}$$

**Hargreaves–Samani:**

$$ET_{0 \text{HS}} = 0.0023 \frac{R_n}{\lambda} (T_{\text{mean}} + 17.8) \sqrt{T_{\text{max}} - T_{\text{min}}} \tag{3}$$

where $ET_{0 \text{PT}}$ is the reference evapotranspiration estimated by the Priestley–Taylor method (mm·d$^{-1}$), $ET_{0 \text{HS}}$ is the reference evapotranspiration estimated by the Hargreaves–Samani method (mm·d$^{-1}$), $R_n$ is the daily extraterrestrial radiation (MJ·m$^{-2}$·d$^{-1}$), and $\lambda$ is the latent heat of evaporation (MJ·kg$^{-1}$).

**Path analysis theory**

Path analysis was first proposed in 1921 as a mathematical and statistical method by the geneticist Sewell Wright. Nowadays, the method is broadly used in agriculture and energy demands, revealing direct or indirect relationships between some morphological characters (Mokhtassi Bidgoli et al. 2006; Yu et al. 2012; Zhang et al. 2016). However, little information is available on the use of this technique to evaluate the affecting factors of $ET_0$. Given the fact that all the meteorological variables are strongly correlated and ultimately lead to multi-collinearity, traditional trend and correlation analyses cannot quantify the interactions among the meteorological factors when filtering the suitable parameters. Path analysis is a standardized partial regression statistical technique of partitioning the correlation coefficients into direct and indirect effects, thus the direct effect and contribution of each factor to $ET_0$ could be calculated.

In this study, only one dependent $y$ is $ET_0$ and some independents $x_i$ are the meteorological variables. Assuming that $r_i$ is the simple correlation coefficient of $x_i$ and $y$, $r_{ij}$ is the simple correlation coefficient between $x_i$ and $x_j$, then the canonical equations of path analysis (Stafford & Seiler 1986; Sarawgi et al. 1997) can be proposed as follows:

$$\begin{bmatrix}
    1 & r_{12} & \cdots & r_{1n} \\
    r_{21} & 1 & \cdots & r_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    r_{n1} & r_{n2} & \cdots & 1
\end{bmatrix}
\begin{bmatrix}
    P_1 \\
    P_2 \\
    \vdots \\
    P_n
\end{bmatrix}
= \begin{bmatrix}
    r_1 \\
    r_2 \\
    \vdots \\
    r_n
\end{bmatrix}$$

where $P_i$ is the direct path coefficient, and represents the direct effects between $x_i$ and $y$; $r_{ij}P_i$ is the indirect path coefficient, and shows the indirect effects of $x_i$ on $y$ through $x_j$. Another important indicator is the decision contribution rate ($R_{dc}^i = r_iP_i$), which expresses the direct contributions of $x_i$ on $y$. Hence, the key decisive parameters with the largest influence on $ET_0$ can be accurately obtained via the above indicators $P$ and $R_{dc}$.

**Artificial neural network**

ANN technology provides an alternative for estimating nonlinear systems including $ET_0$, and the feed forward BP neural network is becoming the most mature and popular ANN. Many studies have proved that the one-hidden-layer ANN model can approximate any arbitrary precision continuous function, which is generally used for estimating $ET_0$ in practical applications (Kumar et al. 2002; Landeras et al. 2008; Dai et al. 2009; Trajkovic & Kolakovic 2009a; Traore et al. 2010; Huo et al. 2012). The classical architecture of ANN applied to estimate $ET_0$ is composed of three layers (Figure 2): the input layer where the meteorological data chosen by the above path analysis are introduced into the model; the output layer where the reference $ET_0$ values calculated by the PM formulae are obtained; and the hidden layer where the network is learned and processed. The mathematical explanation of this model is given by the equations below.

$$y_k = f_1 \left( \sum_{i=1}^{n} W_{ik} x_i + \theta_k \right) = \left[ 1 + \exp \left( -\sum_{i=1}^{n} W_{ik} x_i - \theta_k \right) \right]^{-1} \tag{5}$$
\[ P = f_2 \left( \sum_{k=1}^{m} V_k y_k + \lambda \right) = \sum_{k=1}^{m} V_k y_k + \lambda \]  

(6)

where \( i(1–n) \) is the \( i \)th input layer neuron and \( n \) is the number of input layer neurons; \( x_i \) is the different meteorological factors of the input layer; \( k(1–m) \) is the \( k \)th hidden layer neuron and \( m \) is the number of hidden layer neurons; \( y_k \) is the input vector of the hidden layer. In addition, \( P \) is the calculation output as \( ET_{0i} \) and the number of output layer neurons is only 1. \( f \) is the transfer functions between the adjacent layers, including the sigmoid function \( f_1(x) \) and the purelin function \( f_2(x) \). The upper layer nodes and the lower layer nodes are connected by the weights \( W_{ik} \) and \( V_k \), and the thresholds \( \theta_k \) and \( \lambda \).

Overall, the two steps of forward propagation of a signal and the BP of the error are executed alternately in the BP neural network model using the iterative gradient-descent technique to gradually minimize the quadratic error function defined in Equation (7). Once the target error and training parameters are established, the network model can approximate a nonlinear continuous function with any arbitrary precision by continuously correcting the network weights and thresholds.

\[ E = \frac{1}{2} \sum_{n=1}^{N} (P_n - O_n)^2, \]  

(7)

in which \( P_n \) is the computed output by this ANN model, and \( O_n \) is the observed value of \( ET_{0i} \) calculated by the PM method, \( N \) is the number of training data sets.

**Wavelet neural network**

WNN is an ANN model constructed based on wavelet analysis. The typical architecture of WNN is shown in Figure 3. Compared with the classical ANNs, the neurons in the hidden layer are named wavelons and the activation function is a wavelet family instead of the conventional sigmoid function (Alexandridis & Zapranis 2013). A family of wavelets can be constructed from a cluster of ‘mother wavelets’ \( \varphi(a,b)(x) \) (described in Equation (8)) formed by dilation \( a \) and translation \( b \) (Chauhan et al. 2009). In this study, the Gaussian wavelet function defined by Equation (9) was chosen as the activation function, then the mathematical representation of the WNN model is given by Equation (10).

\[ \varphi^{a,b}(x) = \frac{1}{\sqrt{a}} \cdot \varphi \left( \frac{x-b}{a} \right) \]  

(8)

\[ \varphi(t) = -t \cdot \exp \left( -\frac{t^2}{2} \right) \]  

(9)

\[ P = \sum_{k=1}^{m} V_k \varphi \left( \sum_{i=1}^{N} W_{ik} x_i - \theta_k \right) \]  

(10)

The training parameters of WNN models consist of the dilation factors \( a_k \), the translation factors \( b_k \), and the weight coefficients between the wavelet neurons and the input/output layer \( w_{ik}, v_k \). These parameters were computed and adjusted during the training process, and the optimized
values could be obtained by the same quadratic error function represented in Equation (7).

**Statistical indices**

Five statistical indices, root mean square error (RMSE), mean absolute error (MAE), mean absolute per cent error (MAPE), Nash–Sutcliffe efficiency (NSE) and the coefficient of determination ($R^2$), were selected to evaluate the efficiency of the alternative ANN models and these empirical $ET_0$ equations:

$$\text{RMSE} = \sqrt{\frac{1}{M} \sum_{i=1}^{M} (E_i - C_i)^2}, \quad (11)$$

$$\text{MAE} = \frac{1}{M} \sum_{i=1}^{M} |E_i - C_i|, \quad (12)$$

$$\text{MAPE} = \frac{1}{M} \sum_{i=1}^{M} \frac{|E_i - C_i|}{C_i} \times 100\%, \quad (13)$$

$$\text{NSE} = 1 - \frac{\sum_{i=1}^{M} (C_i - E_i)^2}{\sum_{i=1}^{M} (C_i - \bar{C})^2}, \quad (14)$$

and

$$R^2 = \frac{\left[ \sum_{i=1}^{M} (C_i - \bar{C})(E_i - \bar{E}) \right]^2}{\sum_{i=1}^{M} (C_i - \bar{C})^2 \sum_{i=1}^{M} (E_i - \bar{E})^2}, \quad (15)$$

where $M$ is the number of observed data points, $E_i$ is $ET_0$ obtained by the estimated models, $C_i$ is $ET_0$ calculated with the PM method, $\bar{E}$ is the average of the data arrays of $E_i$, and $\bar{C}$ is the average of the data arrays of $C_i$.

In addition, the linear regression equation $y = ax + b$ was also introduced to calibrate the performance of these estimation models. Where $y$ is the $ET_0$ values calculated by the PM method, $x$ is the $ET_0$ values estimated by any other empirical method or ANN model; $a$ is the slope and $b$ is the intercept.

**RESULTS AND ANALYSES**

**Selection of decisive meteorological parameters**

The six meteorological factors, average temperature ($T_{\text{mean}}$), maximum temperature ($T_{\text{max}}$), minimum temperature ($T_{\text{min}}$), relative humidity ($RH$), wind speed ($U_2$) and daily hours of sunshine ($N$), were strongly coupled, determining the direct influence of each parameter on $ET_0$ was difficult. Thus, path analysis could be implemented to detect the critical parameters. The study first calculated the correlation coefficient ($r$) between $ET_0$ and each meteorological parameter, then determined the direct and indirect effects of each parameter on $ET_0$ using the canonical equations and mathematical formulae of path analysis, and finally obtained $P$ and $R_{dc}$. The analyses of the meteorological data from the Wuhan and Guangzhou stations are presented in Tables 1 and 2, respectively.

All meteorological parameters except $U_2$ were significantly correlated with $ET_0$ at the Wuhan and Guangzhou stations. The $P$ of $N$ was 0.7560 and 0.8216 and the $R_{dc}$ of $N$ was 0.7023 and 0.7985 for the Wuhan and Guangzhou stations, respectively, much higher than those for the other parameters. Thus, $N$ was selected as the crucial decisive parameter affecting $ET_0$. The $P$ and $R_{dc}$ of $RH$ were the highest (in absolute values) among the other five meteorological parameters, and only the $P$ of $RH$ was negative. Then, $RH$ would be chosen as the second decision parameter and as the limited decisive variable. These two parameters had the most influence on $ET_0$, which provided a theoretical basis for the subsequent few-parameter estimation model.

**Establishment of neural network models for $ET_0$ estimation**

The meteorological parameters $T_{\text{mean}}, T_{\text{max}}, T_{\text{min}}, RH, U_2$ and $N$ must all be used to calculate $ET_0$ by the PM equations, but $N$ and $RH$ were the most significant variables identified by path analysis. This study used these two decisive parameters and the calculated $ET_0$ values as input
and output factors, respectively, using the neural network toolbox in MATLAB to establish the ANN and WNN models.

The above two estimated models with single (N) or double (N and RH) parameters were established and compared using the summer meteorological data and calculated ET₀ values for the Wuhan and Guangzhou stations for 1969–2010. The number of nodes of the corresponding hidden layer was set at three by trial-and-error to simplify and extend the utility of these models with no loss of information or data regulation. The study used the data for 1969–2003 to train the corresponding parameters and the data for 2004–2010 to validate the above models with network structures of 1 × 3 × 1 and 2 × 3 × 1. The total amount of water consumed during a few days in actual agricultural production, but not the daily precise consumption, was required for irrigation. The daily ET₀ was thus replaced by the 5-day average ET₀. The estimated and calculated ET₀ were then obtained. The scatterplots are shown in Figure 4, and the statistical error indices for the estimates are listed in Table 3.

The ANN models with single (N) or double (N and RH) parameters produced reasonable and effective estimates for the Wuhan and Guangzhou stations (Figure 4 and Table 3). For the single-parameter model for the Wuhan and Guangzhou stations, the RMSE was 0.41 and 0.29 mm, the MAE was 0.32 and 0.24 mm, the MAPE was 6.9 and 5.4%, and the NSE was 0.8362 and 0.8834, respectively. The introduction of RH significantly reduced all error statistical indices, with RMSES of 0.35 and 0.16 mm, MAEs of 0.24 and 0.12 mm, MAPEs of 5.5 and 2.9%, and NSES of 0.9588 and 0.9660 for the Wuhan and Guangzhou stations, respectively. Simultaneously, the estimation situations obtained by the WNN models were very similar to the above ANN models, and the statistical indices from the WNN models were mostly superior compared with the ANN models. It could be concluded that using the Gaussian wavelet function as the activation function improved the neural

| Table 1 | Path analysis between meteorological parameters and ET₀ in summer at the Wuhan station |
|---------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| Meteorological parameter | r | Direct effect P | Indirect effect | | | | | | | |
| | | T_mean | T_max | T_min | RH | U_2 | N | Total | R_dc |
| T_mean | 0.7585 | 0.1020 | – | 0.0220 | 0.0769 | 0.0914 | 0.0106 | 0.4556 | 0.6565 | 0.0773 |
| T_max | 0.8155 | 0.0231 | 0.0971 | – | 0.0667 | 0.0951 | 0.0106 | 0.5229 | 0.7924 | 0.0188 |
| T_min | 0.5391 | 0.0840 | 0.0934 | 0.0183 | – | 0.0657 | 0.0131 | 0.2646 | 0.4551 | 0.0453 |
| RH | – 0.7441 | – 0.1439 | – 0.0648 | – 0.0152 | – 0.0384 | – | – 0.0065 | – 0.4753 | – 0.6002 | 0.1071 |
| U_2 | 0.2438 | 0.1282 | 0.0084 | 0.0019 | 0.0086 | 0.0073 | – | 0.0894 | 0.1156 | 0.0313 |
| N | 0.9542 | 0.7360 | 0.0631 | 0.0164 | 0.0302 | 0.0929 | 0.0156 | – | 0.2182 | 0.7023 |

| Table 2 | Path analysis between meteorological parameters and ET₀ in summer at the Guangzhou station |
|---------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| Meteorological parameter | r | Direct effect P | Indirect effect | | | | | | | |
| | | T_mean | T_max | T_min | RH | U_2 | N | Total | R_dc |
| T_mean | 0.7707 | 0.0088 | – | 0.0508 | 0.0539 | 0.1146 | – 0.0065 | 0.5491 | 0.7618 | 0.0068 |
| T_max | 0.7899 | 0.0553 | 0.0081 | 0.0343 | 0.0432 | 0.1100 | – 0.0147 | 0.5880 | 0.7346 | 0.0437 |
| T_min | 0.4723 | 0.0660 | 0.0072 | 0.0361 | – | 0.0763 | 0.0019 | 0.2847 | 0.4062 | 0.0312 |
| RH | – 0.7039 | – 0.1504 | – 0.0067 | – 0.0404 | – 0.0335 | – | 0.0047 | – 0.4775 | – 0.5535 | 0.1058 |
| U_2 | 0.0178 | 0.0867 | – 0.0007 | – 0.0094 | 0.0014 | – 0.0081 | – | 0.0522 | – 0.0689 | 0.0015 |
| N | 0.9719 | 0.8216 | 0.0059 | 0.0396 | 0.0229 | 0.0874 | – 0.0055 | – | 0.1502 | 0.7985 |
network model accuracy better than utilizing the basic sigmoid function, when the networks trained and validated in the same local station.

In summary, the above results indicated that all models met the requirements of good estimates and acceptable accuracy in actual application, and the double-parameter
model also greatly improved the estimation accuracy and reliability compared to the single-parameter model. The error statistics were significantly reduced and the estimates were improved with the increase of \( P \) and \( R_{dc} \), indicating a positive correlation between them and suggesting that the meteorological parameters \( N \) and \( RH \) could be used as the inputs for these models. Both the single- and double-parameter neural network models, which produced estimation accuracies suitable for practical application, have significant potential for agricultural application, but the universal performance of these models should be studied further.

**Universal analysis of the estimation models**

Investigating the universality of the estimation models in multiple regions is important for improving the performance of the neural network structures and parameters. \( P \) and \( R_{dc} \) were calculated in the path analysis for all selected capital stations in the south (Tables 4 and 5, respectively). The hours of sunshine, \( N \), which had the largest \( P \), reaching 0.60–0.85, was selected as the core variable, and the relative humidity, \( RH \), which was the only negative parameter and had the second largest absolute value among all parameters, was selected as the limited variable. \( R_{dc} \), however, fluctuated between 0.53 and 0.81 when the single parameter \( N \) was selected as the input variable, so the error oscillation may be larger when applying the models on a larger scale. \( R_{dc} \) increased and maintained a range of 0.8–0.9 when \( RH \) was used as the second input variable, indicating that the double-parameter model was stable and highly credible when applied in the entire selected stations.

In summary, this study established the ANN and WNN models based on two parameters (\( N \) and \( RH \)), extracted the corresponding parameters of network structure, chose the city of Guangzhou as the benchmark station with the highest cumulative \( R_{dc} \) among all stations, and applied the data for 2004–2010 from other stations to verify and universally

### Table 3 | Comparison of \( ET_0 \) estimated errors using the 5-day average at the Wuhan and Guangzhou stations

<table>
<thead>
<tr>
<th>Station</th>
<th>Input parameter</th>
<th>Cumulative ( R_{dc} )</th>
<th>( RMSE ) (mm)</th>
<th>( MAE ) (mm)</th>
<th>( MAPE ) (%)</th>
<th>NSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wuhan</td>
<td>( N )</td>
<td>0.7023</td>
<td>0.4082 (0.3933)</td>
<td>0.3243 (0.3095)</td>
<td>6.9087 (6.6338)</td>
<td>0.8362 (0.8479)</td>
</tr>
<tr>
<td></td>
<td>( N, RH )</td>
<td>0.8094</td>
<td>0.3510 (0.2734)</td>
<td>0.2412 (0.2195)</td>
<td>5.4931 (5.0435)</td>
<td>0.9588 (0.9265)</td>
</tr>
<tr>
<td>Guangzhou</td>
<td>( N )</td>
<td>0.7985</td>
<td>0.2872 (0.2804)</td>
<td>0.2350 (0.2292)</td>
<td>5.4150 (5.2944)</td>
<td>0.8834 (0.8889)</td>
</tr>
<tr>
<td></td>
<td>( N, RH )</td>
<td>0.9043</td>
<td>0.1550 (0.1526)</td>
<td>0.1183 (0.1168)</td>
<td>2.8949 (2.8875)</td>
<td>0.9660 (0.9671)</td>
</tr>
</tbody>
</table>

*Note: The figures outside brackets are obtained based on ANN model; the figures inside brackets are all based on WNN model.*
analyse the models. The universal results from the ANN and WNN models are presented in Tables 6 and 7, respectively.

The calculated statistical data estimated by ANN model in Table 6 indicated that the cumulative $R_{dc}$ decreased smoothly from 0.90 to 0.80 at each station, $RMSE$ climbed gradually from 0.14 to 0.31 mm, $MAE$ grew modestly from 0.11 to 0.24 mm, $MAPE$ increased slightly from 2.5 to 5.4%. $R^2$ remained within the excellent range of 0.928–0.973, $NSE$ could be maintained at a level of above 0.80. In addition, the fitted equations and higher $R^2$ values for all stations indicated high estimation accuracy and consistent universal performance. These results suggested that the ANN model established for the Guangzhou station based on two meteorological parameters ($N$ and $RH$) had a strong regional universality in southern China.

Despite the fact that the WNN model has a higher performance of elasticity and plasticity, the universal estimation results listed in Table 7 are not as satisfactory as the traditional ANN model. Among all the stations, two-thirds of $RMSE$ exceeded 0.30 mm, half of $MAE$ and $MAPE$ were

### Table 5 | Decision contribution rates ($R_{dc}$) between each meteorological parameter and $ET_0$ at the capital stations in southern China

<table>
<thead>
<tr>
<th>Station</th>
<th>$R_{dc}$</th>
<th>$T_{mean}$</th>
<th>$T_{max}$</th>
<th>$T_{min}$</th>
<th>RH</th>
<th>$U_2$</th>
<th>$N$</th>
<th>$N - RH$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guangzhou</td>
<td>0.0068</td>
<td>0.0437</td>
<td>0.0312</td>
<td>0.1058</td>
<td>0.0015</td>
<td>0.7985</td>
<td>0.9044</td>
<td></td>
</tr>
<tr>
<td>Nanning</td>
<td>0.0275</td>
<td>0.0567</td>
<td>0.0154</td>
<td>0.0683</td>
<td>0.0080</td>
<td>0.8136</td>
<td>0.8820</td>
<td></td>
</tr>
<tr>
<td>Kunming</td>
<td>0.0262</td>
<td>0.0420</td>
<td>0.0082</td>
<td>0.1186</td>
<td>0.0303</td>
<td>0.7599</td>
<td>0.8785</td>
<td></td>
</tr>
<tr>
<td>Haikou</td>
<td>-0.0082</td>
<td>0.1037</td>
<td>0.0194</td>
<td>0.1010</td>
<td>0.0044</td>
<td>0.7703</td>
<td>0.8713</td>
<td></td>
</tr>
<tr>
<td>Guiyang</td>
<td>0.0024</td>
<td>0.0529</td>
<td>0.0367</td>
<td>0.1456</td>
<td>0.0373</td>
<td>0.7112</td>
<td>0.8567</td>
<td></td>
</tr>
<tr>
<td>Chongqing</td>
<td>0.0449</td>
<td>0.0037</td>
<td>0.0486</td>
<td>0.1381</td>
<td>0.0578</td>
<td>0.7109</td>
<td>0.8490</td>
<td></td>
</tr>
<tr>
<td>Fuzhou</td>
<td>-0.0296</td>
<td>0.0839</td>
<td>0.0587</td>
<td>0.1867</td>
<td>0.0302</td>
<td>0.6515</td>
<td>0.8382</td>
<td></td>
</tr>
<tr>
<td>Changsha</td>
<td>0.0463</td>
<td>0.0238</td>
<td>0.0596</td>
<td>0.1763</td>
<td>0.0276</td>
<td>0.6530</td>
<td>0.8293</td>
<td></td>
</tr>
<tr>
<td>Hangzhou</td>
<td>0.0273</td>
<td>0.0547</td>
<td>0.0520</td>
<td>0.1919</td>
<td>0.0193</td>
<td>0.6339</td>
<td>0.8258</td>
<td></td>
</tr>
<tr>
<td>Shanghai</td>
<td>-0.0646</td>
<td>0.1232</td>
<td>0.0818</td>
<td>0.2148</td>
<td>0.0265</td>
<td>0.5978</td>
<td>0.8125</td>
<td></td>
</tr>
<tr>
<td>Nanchang</td>
<td>0.0543</td>
<td>0.0540</td>
<td>0.0454</td>
<td>0.1398</td>
<td>0.0201</td>
<td>0.6707</td>
<td>0.8105</td>
<td></td>
</tr>
<tr>
<td>Wuhan</td>
<td>0.0773</td>
<td>0.0188</td>
<td>0.0453</td>
<td>0.1071</td>
<td>0.0313</td>
<td>0.7023</td>
<td>0.8094</td>
<td></td>
</tr>
</tbody>
</table>

### Table 6 | Comparison of $ET_0$ estimated errors based on ANN model at the capital stations in southern China

<table>
<thead>
<tr>
<th>Station</th>
<th>Cumulative $R_{dc}$</th>
<th>$RMSE$ (mm)</th>
<th>$MAE$ (mm)</th>
<th>$MAPE$ (%)</th>
<th>$NSE$</th>
<th>Linear regression equation</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guangzhou</td>
<td>0.9044</td>
<td>0.1415</td>
<td>0.1121</td>
<td>2.6955</td>
<td>0.9660</td>
<td>$y = 0.9854x - 0.0014$</td>
<td>0.9717</td>
</tr>
<tr>
<td>Nanning</td>
<td>0.8820</td>
<td>0.1383</td>
<td>0.1023</td>
<td>2.4493</td>
<td>0.9657</td>
<td>$y = 1.0201x - 0.0976$</td>
<td>0.9664</td>
</tr>
<tr>
<td>Kunming</td>
<td>0.8785</td>
<td>0.1361</td>
<td>0.1113</td>
<td>3.2502</td>
<td>0.8182</td>
<td>$y = 0.8632x + 0.0783$</td>
<td>0.9531</td>
</tr>
<tr>
<td>Haikou</td>
<td>0.8713</td>
<td>0.1590</td>
<td>0.1239</td>
<td>2.6204</td>
<td>0.9399</td>
<td>$y = 1.0861x - 0.2938$</td>
<td>0.9615</td>
</tr>
<tr>
<td>Guiyang</td>
<td>0.8567</td>
<td>0.1511</td>
<td>0.1192</td>
<td>3.5607</td>
<td>0.8316</td>
<td>$y = 0.9297x - 0.0161$</td>
<td>0.9627</td>
</tr>
<tr>
<td>Chongqing</td>
<td>0.8490</td>
<td>0.1922</td>
<td>0.1423</td>
<td>3.7743</td>
<td>0.9683</td>
<td>$y = 1.0624x - 0.2981$</td>
<td>0.9734</td>
</tr>
<tr>
<td>Fuzhou</td>
<td>0.8382</td>
<td>0.2454</td>
<td>0.1845</td>
<td>3.8932</td>
<td>0.8801</td>
<td>$y = 1.1862x - 0.5786$</td>
<td>0.9568</td>
</tr>
<tr>
<td>Changsha</td>
<td>0.8293</td>
<td>0.2382</td>
<td>0.1844</td>
<td>3.9798</td>
<td>0.9329</td>
<td>$y = 1.1785x - 0.7627$</td>
<td>0.9608</td>
</tr>
<tr>
<td>Hangzhou</td>
<td>0.8258</td>
<td>0.2767</td>
<td>0.2076</td>
<td>4.7596</td>
<td>0.9204</td>
<td>$y = 1.1532x - 0.5945$</td>
<td>0.9433</td>
</tr>
<tr>
<td>Shanghai</td>
<td>0.8125</td>
<td>0.3131</td>
<td>0.2393</td>
<td>5.3582</td>
<td>0.8629</td>
<td>$y = 1.1932x - 0.6010$</td>
<td>0.9281</td>
</tr>
<tr>
<td>Nanchang</td>
<td>0.8105</td>
<td>0.2568</td>
<td>0.2000</td>
<td>4.1421</td>
<td>0.9264</td>
<td>$y = 1.1557x - 0.7046$</td>
<td>0.9459</td>
</tr>
<tr>
<td>Wuhan</td>
<td>0.8094</td>
<td>0.2583</td>
<td>0.2057</td>
<td>4.8053</td>
<td>0.9175</td>
<td>$y = 1.0999x - 0.5577$</td>
<td>0.9344</td>
</tr>
</tbody>
</table>
over 0.24 mm and 5.5%. Especially in Fuzhou and Shanghai stations, the RMSE and MAE peaked at around 0.45 mm and 0.34 mm. Furthermore, for the Kunming and Guiyang stations, the MAPE soared to 8.9% and 8.5%, the NSE dropped to 0.70 and 0.82. Given that the fitting equation slope $a$ was less than 1 and the intercept $b$ was very close to 0, it could be inferred that the WNN model overestimated the $ET_0$ values partly attributed to the higher latitude values in these two cities. Fortunately, the fitted equations and $R^2$ values for all stations still performed well and other statistical indices ranged into an acceptable scope that could adapt the actual requirement in universal application.

**Comparison of the estimation models with the empirical equations**

All the validation data results from the selected 12 meteorological stations, which were calculated by the above neural network models, were mixed together and compared with some empirical models in order to further evaluate the generalization ability. The total performance of the ANN or WNN models and other empirical equations for the period of 2004–2010 are shown in Figure 5, and the statistical indices for the comparison are listed in Table 8.

Some famous empirical models for calculating $ET_0$ are generally developed under specific agricultural conditions or using limited climate data, so that their calculated performance cannot be more accurate than the results obtained by the PM method. The Hargreaves–Samani model with only temperature data as inputs, presented the poorest performance with the RMSE, MAE, MAPE, NSE and $R^2$ equal to 0.7927 mm, 0.6580 mm, 17.1346%, 0.5039 and 0.6004, respectively. All these statistical indices could be noticeably improved by adding the radiation item as the next input with using the Priestley–Taylor model, but the scatter plot in Figure 5 shows that this model overestimated most of the $ET_0$ values, thus making the calibration process vitally necessary. While the ANN model was introduced to estimate $ET_0$ values using only two meteorological parameters ($N$ and $RH$) selected by path analysis, the performances were tremendously improved by comparison of the empirical models, at the values of 0.3152 mm, 0.2367 mm, 5.5969%, 0.9215 and 0.9341 for RMSE, MAE, MAPE, NSE and $R^2$, respectively. For the WNN model, the MAPE and $R^2$ were slightly better than the ANN model but the other criteria were a little worse. Actually, when the estimated results are mixed together, no significant difference was found in these two models' accuracy as the per cent changes of those corresponding indices were less than 3.0%. Overall, the neural network estimated models with fewer inputs could exhibit much better accuracy in estimating $ET_0$ values than the empirical equations. Two meteorological parameters, $N$ and $RH$, detected by the theory of path analysis, were proved to be

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**Table 7** Comparison of $ET_0$ estimated errors based on WNN model at the capital stations in southern China

<table>
<thead>
<tr>
<th>Station</th>
<th>Cumulative $R_{dc}$</th>
<th>RMSE (mm)</th>
<th>MAE (mm)</th>
<th>MAPE (%)</th>
<th>NSE</th>
<th>Linear regression equation</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guangzhou</td>
<td>0.9044</td>
<td>0.1526</td>
<td>0.1168</td>
<td>2.8875</td>
<td>0.9671</td>
<td>$y = 0.9888x - 0.0177$</td>
<td>0.9731</td>
</tr>
<tr>
<td>Nanning</td>
<td>0.8820</td>
<td>0.1430</td>
<td>0.1060</td>
<td>2.5419</td>
<td>0.9641</td>
<td>$y = 1.0217x - 0.1087$</td>
<td>0.9652</td>
</tr>
<tr>
<td>Kunming</td>
<td>0.8785</td>
<td>0.3439</td>
<td>0.3042</td>
<td>8.9030</td>
<td>0.7007</td>
<td>$y = 0.8749x + 0.1606$</td>
<td>0.9539</td>
</tr>
<tr>
<td>Haikou</td>
<td>0.8713</td>
<td>0.1979</td>
<td>0.1426</td>
<td>2.9153</td>
<td>0.9404</td>
<td>$y = 1.0832x - 0.2850$</td>
<td>0.9604</td>
</tr>
<tr>
<td>Guiyang</td>
<td>0.8567</td>
<td>0.3245</td>
<td>0.2831</td>
<td>8.5391</td>
<td>0.8282</td>
<td>$y = 0.9359x - 0.0418$</td>
<td>0.9609</td>
</tr>
<tr>
<td>Chongqing</td>
<td>0.8490</td>
<td>0.2192</td>
<td>0.1629</td>
<td>4.1429</td>
<td>0.9654</td>
<td>$y = 1.0765x - 0.3550$</td>
<td>0.9720</td>
</tr>
<tr>
<td>Fuzhou</td>
<td>0.8382</td>
<td>0.4271</td>
<td>0.3309</td>
<td>6.2844</td>
<td>0.8691</td>
<td>$y = 1.2109x - 0.6815$</td>
<td>0.9543</td>
</tr>
<tr>
<td>Changsha</td>
<td>0.8293</td>
<td>0.3890</td>
<td>0.2933</td>
<td>5.7415</td>
<td>0.8954</td>
<td>$y = 1.2559x - 1.0575$</td>
<td>0.9509</td>
</tr>
<tr>
<td>Hangzhou</td>
<td>0.8258</td>
<td>0.3561</td>
<td>0.2692</td>
<td>5.7362</td>
<td>0.9061</td>
<td>$y = 1.1898x - 0.7432$</td>
<td>0.9380</td>
</tr>
<tr>
<td>Shanghai</td>
<td>0.8125</td>
<td>0.4500</td>
<td>0.3357</td>
<td>6.6770</td>
<td>0.8515</td>
<td>$y = 1.2088x - 0.6632$</td>
<td>0.9214</td>
</tr>
<tr>
<td>Nanchang</td>
<td>0.8105</td>
<td>0.3544</td>
<td>0.2663</td>
<td>5.2477</td>
<td>0.8971</td>
<td>$y = 1.2107x - 0.9211$</td>
<td>0.9330</td>
</tr>
<tr>
<td>Wuhan</td>
<td>0.8094</td>
<td>0.3114</td>
<td>0.2374</td>
<td>5.5929</td>
<td>0.9046</td>
<td>$y = 1.1399x - 0.7108$</td>
<td>0.9232</td>
</tr>
</tbody>
</table>
the most crucial factors for $ET_0$ estimation in southern China.

**DISCUSSION AND CONCLUSION**

This study calculated $ET_0$ using the PM equations based on the summer meteorological data for 1969–2010 from 12 capital stations in southern China, determined the decisive variables $N$ and $RH$ using path analysis, established ANN and WNN models for estimating $ET_0$ to evaluate the accuracy and reliability based on actual production needs, analysed the universal performance of the neural network models and made the comparison with some empirical equations among all stations in southern China. The following main conclusions were drawn:

1. The path analysis identified $N$ and $RH$ as the two core meteorological parameters with the largest influence on $ET_0$. $N$ had a positive influence on $ET_0$ and was selected as the core decisive variable, and $RH$ had a negative influence on $ET_0$ and was selected as the limited decisive variable.
2. The single-parameter (N) and the double-parameter (N and RH) neural network models based on the path theory estimated ET₀ accurately for the Wuhan and Guangzhou stations. The cumulative decision contribution rates to ET₀ were positively correlated with the error statistical indicators, demonstrating the robustness and reliability of these estimation models.

3. The double-parameter (N and RH) ANN and WNN models had the highest P and Rdc and the best estimation accuracy at the Guangzhou station. This local model also had higher accuracy and more consistent reliability than some empirical models when applied to other stations in southern China, confirming that this model had significant potential in agricultural applications.

In summary, the neural network estimation models with few parameters based on the principle of path analysis theory performed well, with high accuracy, consistent reliability, and robust universality. Path analysis theory thus provided a scientific basis that could feasibly be applied to choose the decisive parameters. Only two meteorological parameters (N and RH), however, could be directly applied to establish these models for estimating ET₀ for actual production, whether or not the meteorological data were fully available for some regions in southern China. Moreover, when some comparisons are made by path analysis at a large scale, it is helpful and useful to extract some stations which have the same decisive parameters into the same group in order to make further universal estimation. These concise neural network models with fewer variables have higher potential and promotional value for actual production than the empirical models, not only near the large capital cities, but also in smaller neighbouring areas.

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