

Parameter estimation of an improved nonlinear Muskingum model using a new hybrid method

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ABSTRACT

Although various techniques have been proposed to estimate the parameters of different versions of the Muskingum model, more rigorous techniques and models are still required to improve the computational precision of the calibration process. In this research, a new hybrid technique was proposed for Muskingum parameter estimation. Based on the conducted comprehensive literature review on the Muskingum flood routing models, a new improved Muskingum model with nine constant parameters was presented. Since the inflow-weighted parameter in the proposed model is a function of inflow hydrograph, it varies during the flood period and consequently can also be considered as a variable-parameter Muskingum model. The new hybrid technique was successfully applied for parameter estimation of the new version of Muskingum model for two case studies selected from the literature. Results were compared with those of other methods using several common performance evaluation criteria. The new Muskingum model significantly reduces the sum of the square of the deviations between the observed and routed outflows (SSQ) value for the double-peak case study. Finally, the obtained results indicate that not only the hybrid modified honey bee mating optimization-generalized reduced gradient algorithm somehow overcomes the shortcomings of both zero and first-order optimization techniques, but also the new Muskingum model appears to be the most reliable Muskingum version compared with the other methods considered in this study.

Key words | flood routing, hybrid method, nonlinear Muskingum model, parameter estimation

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INTRODUCTION

In general, river flood routing methods can be categorized into three main distinct approaches: (1) the lumped models (e.g., linear and nonlinear Muskingum), (2) the distributed models (e.g., dynamic wave, diffusion wave, and kinematic wave models), and (3) the semi-distributed models (e.g., Muskingum–Cunge family models). The first approach is exclusively capable of routing the flood hydrograph of a river regardless of considering reach characteristics. The continuity equation accompanied with a specific storage function is utilized in this category. The true meaning of flood routing using these models is nothing but to compute the Muskingum parameters exclusively by using historical

records. The distributed models utilize reach properties to route floods in any section of the reach by considering both mass conservation and transport momentum equations. Finally, the third category of models can be described as a tradeoff between the two first approaches since they predict floods along a river by using a simplified form of the Saint-Venant equations based on both physical concepts and river characteristics (Barati 2014).

The governing equations of the linear Muskingum model are the continuity equation and a linear storage relation which attributes the storage magnitude of the reach to inflow and outflow magnitudes using

Muskingum parameters:

$$\frac{dS_t}{dt} = I_t - O_t \quad (1)$$

$$S_t = K[xI_t + (1 - x)O_t] \quad (2)$$

where S_t = total storage at time t ; I_t and O_t = inflow and outflow magnitudes at time t , respectively; K = storage parameter; and x = a weighting parameter. The Muskingum parameters in the storage function (Equation (2)) are required to be calibrated using the historical flood events in the reach under consideration and these parameters are expected to capture the characteristics of that reach. These parameters can be utilized for flood routing for future events. Hence, parameter estimation is considered to be one of the major challenges of the Muskingum flood routing model.

Since the Muskingum model is one of the most widely used methods for flood routing, many studies have been conducted on improving both accuracy and efficiency of this model, particularly during the last two decades. However, based on the comprehensive literature review presented in the next section, more investigations seeking new improved Muskingum models are required in favor of achieving more rigorous flood routing results.

In this paper, a new Muskingum model was proposed which has nine parameters. Although all parameters of the proposed Muskingum model are constant during the flood period, this new model can be considered as a variable Muskingum model since the weighting parameter of this model varies with the inflow of the hydrograph. A new hybrid technique was also presented and applied for parameter estimation of the new Muskingum model. The new hybrid model is a combination of modified honey bee mating optimization (MHBMO) algorithm with generalized reduced gradient (GRG) algorithm. This hybrid method works in two consecutive steps in which the MHBMO algorithm performs in the first phase and the GRG algorithm acts in the second one. The parameters of the new Muskingum model were calibrated using the new hybrid model for two flood sets of data: (1) smooth and (2) multi-peak flood hydrographs. The objective function was the sum of the square of the deviations between the observed and routed outflows (SSQ). The obtained results show a reduction of 4.52%

and 99.7%, respectively, with regard to the best reported results in the previous literature.

METHODS

The Muskingum model

Studies on the Muskingum flood routing method can be categorized into two general groups. Those in the first category are intended to recommend different optimization techniques in favor of achieving more rigorous parameter estimation results. On the other hand, the second type attempts to somehow improve the Muskingum model or flood routing procedure. In the first category, numerous optimization algorithms have already been applied for Muskingum parameter estimation so far. These techniques generally can be categorized into four branches: (1) zero-order optimization algorithms, e.g., genetic algorithm (GA) (Mohan 1997), particle swarm optimization (PSO) (Chu & Chang 2009), and MHBMO (Niazkar & Afzali 2015a); (2) first-order optimization algorithms, e.g., Nelder–Mead simplex (NMS) (Barati 2011) and GRG (Barati 2013); (3) second-order optimization algorithms, e.g., Broyden–Fletcher–Goldfarb–Shanno (BFGS) (Geem 2006); and (4) hybrid optimization algorithms, e.g., shuffled frog leaping algorithm and Nelder–Mead simplex (SFLA-NMS) (Haddad *et al.* 2015a, 2015b), GA-GRG (Barati 2012).

The zero-order techniques exclusively utilize some algorithm parameters. For instance, the MHBMO algorithm has five independent control parameters: (1) size of the initial population (N_{ipop}), (2) the number of workers (N_{worker}), (3) the speed of the queen at the start of the mating flight (S_{max}), (4) the speed of the queen at the end of the mating flight (S_{min}), and (5) the speed reduction factor (χ) (Niazkar & Afzali 2015a). This category initiates the parameter estimation process only when these controlling parameters are being evaluated. On the contrary, the first- and second-order models make the user guess one and two initial sets of values, respectively, for Muskingum parameters at the beginning of the calibration procedure, when the exact values are unknown and meant to be calibrated. The algorithm parameter evaluation in the zero-order methods and these initial-value vectors in the

first- and second-order methods significantly affect the speed, and more importantly, the accuracy of Muskingum parameter estimation. The need for an initial guess for Muskingum parameters in advance of the calibration process is considered to be one of the challenges of the first- and second-order optimization algorithms. This challenge is almost addressed in the hybrid models. In these techniques, two algorithms are combined and worked one after the other in a two-phase process. In the first step, a zero-order algorithm commonly commences the parameter estimation process by only assuming delicate values for its controlling parameters. As this algorithm conducted the calibration process either after a specified number of iterations or until a stopping criterion is satisfied, the obtained results for Muskingum parameters are utilized as initial values for the corresponding parameters in a first- or second-order algorithm in the second step. Hence, not only do the hybrid techniques overcome the shortcomings of the first- and second-order algorithms, they appear to improve the accuracy of Muskingum parameter estimation in comparison with the zero-order methods as well.

The second-type efforts have tried to search for sufficient ways on how to obtain more precise flood routing results specifically as shown in the two sub-sections below.

Appraisal of new modifications for the Muskingum model

These modifications were basically conducted by: (1) introducing new constant-parameter Muskingum models, e.g., four-parameter Muskingum model (Easa 2013b) and seven-parameter Muskingum model (Haddad *et al.* 2015b); and (2) utilizing variable-parameter Muskingum models, e.g., m -variable model (Easa 2013a) and VPNLMM-L (Karahana 2014).

Table 1 shows chronological improvement of the Muskingum flood routing model. In this table, α is a flow parameter, β is an exponent parameter, r is a dimensionless parameter for dividing flood period into sub-periods, and θ is a weighted parameter in weighted-average inflow modification.

In Table 1, the simplest version of the flood routing Muskingum model was proposed by McCarthy (1938), which has two constant parameters, i.e., K and x . In this model, storage variation with weighted inflow and outflows is presumed to be linear. Since the corresponding relation is

practically found to be nonlinear most times, Chow (1959), Gill (1978), and Gavilan & Houck (1985) separately introduced three different nonlinear Muskingum models. Among these nonlinear models, Gill's nonlinear Muskingum model is considered to be the most widely used variant of Muskingum models and much research has been specifically devoted to improve the accuracy of this model in the literature. In all the aforementioned models, it is assumed that no lateral flow is conveyed throughout the reach under investigation.

In contrast, O'Donnell (1985) ceased no-lateral-flow assumption and presented a new three-parameter Muskingum model (Table 1). In the recommended model, lateral flow is hypothesized to be not only a linear multiplication of inflow at each time step but also constantly distributed along the reach. After a distinct gap in the time table of Muskingum model improvement, a variable-exponent Muskingum model was presented by Easa (2013a). This model has the same formation as Gill's model while the exponent parameter in this model, i.e., m , can possess the desirable number of discrete values in the flood period. In this variable-exponent model, the flood period is divided into a user-defined number of sub-periods using a new non-dimensional parameter, i.e., r . The exponent parameter can have a different value in each sub-period. In other words, unlike previous models in which all Muskingum parameters are kept constant during the flood period, the exponent parameter is a discontinuously variable in this model. The lack of a standard approach for selecting an appropriate number of sub-periods is one of its critical shortcomings (Karahana 2014). The next modified version of the Muskingum flood routing model was presented by combining Chow's and Gill's nonlinear Muskingum models, which has four constant parameters (Easa 2013b). Vatankhah (2014a) proposed a new five-parameter Muskingum model using Gill's and Galvin and Houck's nonlinear models. The idea of considering a linear lateral flow with respect to inflow in each time interval in the linear Muskingum model of O'Donnell was extended for Gill's nonlinear model (Karahana *et al.* 2015). This new model, which has five parameters, was also proposed with a modification for flood routing scheme, which will be covered later in this section. Additionally, Karahana (2014) divided the flood period in respect to peak time inflow hydrograph into two

Table 1 | Chronological review of Muskingum flood routing models

No.	Researchers	Number of parameters	Storage equation
1	McCarthy (1938)	2	$S_t = K[xI_t + (1-x)O_t]$
2	Chow (1959)	3	$S_t = K[xI_t^\alpha + (1-x)O_t^\alpha]$
3	Gill (1978)	3	$S_t = K[xI_t + (1-x)O_t]^m$ where m is the constant exponent parameter
4	Gavilan & Houck (1985)	4	$S_t = K[xI_t^{\alpha_1} + (1-x)O_t^{\alpha_2}]$
5	O'Donnel (1985)	3	$S_t = K[x(1+\beta)I_t + (1-x)O_t]$ where βI is lateral flow and β is the lateral inflow factor
6	Easa (2013a)	$2L + 1$	$S_t = K[xI_t + (1-x)O_t]^{m(u_i)}$ where $m(u_i)$ is the variable exponent parameter and $u_i = rI_{max}$ and L is a user-defined parameter
7	Easa (2013b)	4	$S_t = K[xI_t^\alpha + (1-x)O_t^\alpha]^m$
8	Vatankhah (2014a)	5	$S_t = K[xI_t^{\alpha_1} + (1-x)O_t^{\alpha_2}]^m$
9	Karahan (2014)	10	$S_t = K[x(1+\beta)I_t + (1-x)O_t]^m$ where βI is lateral flow and $I_{ave} = [\theta I_t + (1-\theta)I_{t-1}]$ and $L = 2$ (rising and falling inflow limbs)
10	Easa <i>et al.</i> (2014)	6	$S_t = [xK_1I_t^{\alpha_1} + (1-x)K_2O_t^{\alpha_2}]^m$
11	Easa <i>et al.</i> (2014)	$2L + 2$	$S_t = K[xI_t^\alpha + (1-x)O_t^\alpha]^{m(u_i)}$ where $m(u_i)$ is the variable exponent parameter and $u_i = rI_{max}$ and L is a user-defined parameter
12	Easa (2014)	$4L + 1$	$S_t = K(u_i)[x(u_i)I_t^{\alpha(u_i)} + (1-x(u_i))O_t^{\alpha(u_i)}]^{m(u_i)}$ where $m(u_i)$ is the variable exponent parameter and $u_i = rI_{max}$ and L is a user-defined parameter
13	Easa (2014)	$2L + 3$	$S_t = K[x(u_i)[(1+\beta)I_t]^\alpha + (1-x(u_i))O_t]^{m(u_i)}$ where $x(u_i)$ is the variable exponent parameter and $u_i = rI_{max}$ and L is a user-defined parameter
14	Karahan <i>et al.</i> (2015)	5	$S_t = K[x(1+\beta)I_t + (1-x)O_t]^m$ where βI is lateral flow and $I_{ave} = [\theta I_t + (1-\theta)I_{t-1}]$
15	Easa (2015)	$2L + 3$	$S_t = K[xI_t + (1-x)O_t]^{m(u_i)}$ where $u_i = I_t/I_{max}$ and $m(u_i) = a + b \ln(1 + cu_i)$ or $m(u_i) = a + b \exp(-cu_i)$ and L is a user-defined parameter
16	Haddad <i>et al.</i> (2015b)	7	$S_t = [x(C_1I_t^{\alpha_1}) + (1-x)(C_2O_t^{\alpha_2})]^m$
17	Zhang <i>et al.</i> (2016)	$2L + 2$	$S_t = K[x(1+\beta)I_t + (1-x)O_t]^{m(u_i)}$ where $m(u_i)$ is the variable exponent parameter and $u_i = rI_{max}$ and L is a user-defined parameter

sub-periods, and applied this model with different parameter values for the rising and falling limbs. Since he also utilized the modification for flood routing scheme proposed in Karahan *et al.* (2015), the model can be considered as a ten-parameter model in which Muskingum parameters can discontinuously change after inflow peak-time. A new constant six-parameter Muskingum model was suggested by Easa *et al.* (2014). The exponent-variable model (Easa 2013a) was combined with the four-parameter model (Easa 2013b) and a new exponent-variable model was introduced (Easa *et al.* 2014). Moreover, the obtained model was modified by Easa (2014) by extending the idea of variable-exponent model. The latter has the same formation as the four-parameter Muskingum model (Easa 2013b), in which each of the four parameters can discontinuously vary in the flood

period. The technique utilized for dividing the flood period into several sub-periods is the same as the one suggested in Easa (2013a). Easa (2014) proposed a new variable-weighting parameter Muskingum model, which conceptually considers lateral flow the same as the one in Karahan *et al.*'s model (2015). The variable exponent-parameter model in Table 1 was modified, where the exponent parameter can continuously vary by logarithmic or exponential functions during flood period (Easa 2015). Each of these functions has three new parameters and the number of parameters for this model consequently is equal to five. Additionally, a new seven-parameter Muskingum model was proposed by Haddad *et al.* (2015b). More recently, Zhang *et al.* (2016) proposed a new Muskingum model which has a discontinuous variable exponent. This exponent varies in different

sub-periods of the flood routing period in the same way as the one suggested by Easa (2013a). Although the recommended model considers the linear lateral flow, it has the same shortcomings as Easa's model since it is a modified form of Easa's model. Finally, it should be noted that Muskingum models with a greater number of parameters achieve better results since they efficiently provide more superior fitting capacity in comparison with Gill's nonlinear model (Table 1).

Improving the flood routing scheme

The flood routing scheme is mainly a step-by-step procedure for calculating routed outflow hydrograph from the continuity equation and the storage function. Although the same process is more or less followed in all studies in the literature, some modifications were sporadically utilized. For instance, if one rearranged the storage function of Gill's Muskingum model in Table 1 for the outflow, the following equation will be obtained:

$$O_t = \left(\frac{1}{1-x} \right) \left(\frac{S_t}{K} \right)^{\frac{1}{m}} - \left(\frac{x}{1-x} \right) I_t \quad (3)$$

In favor of achieving better routing results, some researchers utilized I_t in this equation while others attempted to use I_{t-1} instead. Moreover, an arithmetic average, $0.5(I_t + I_{t-1})$ (Niazkar & Afzali 2015a) and weighted average, i.e., $\theta I_t + (1-\theta)I_{t-1}$ (Karahana *et al.* 2015) were suggested for substitution of I_t in Equation (3). Although the latter modification brings about a new parameter, i.e., θ , to the parameter calibration list, its assessment clearly shows that this modification performs sufficiently well for all utilized data sets.

Another modification in flood routing procedure was conducted in calculating the storage value. Traditionally, Euler's method was utilized for computation of new storage value (Tung 1985), while other explicit time integration schemes, e.g., improved Euler's method (Vatankhah 2014b), fourth-order Runge-Kutta method (Vatankhah 2014a, 2014b), Runge-Kutta-Fehlberg method (Vatankhah 2014b), were also applied for this purpose. Although the application of these higher order explicit schemes generally improves the accuracy of Muskingum flood routing results, they did not

perform better than conventional Euler's method for all the utilized data sets. According to the state-of-the-art of the Muskingum model, more investigations on Muskingum models and new techniques for more accurate parameter estimation are still required.

Proposed Muskingum model

The formation of the storage relation plays an essential role in both accuracy and efficiency of the Muskingum flood routing method since the parameters in this relation, i.e., Muskingum parameters, are supposed to capture reach characteristics for routing future flood events. In essence, the greater the number of Muskingum parameters, the better the flood routing model performs in fitting historical records. On the other hand, more parameters may cause the calibration process to become more complex. Hence, the tradeoff between the number of Muskingum parameters, model accuracy, and smooth calibration process should always be a factor in the selection of a proper storage relation. Additionally, as the variable-parameter Muskingum model performs much better in comparison with the constant-parameter models, it can be implied that Muskingum parameters vary during the flood period. Putting all these factors together, a new Muskingum model is proposed in order to improve the accuracy of the Muskingum model in routing floods. This new model utilizes nine constant parameters in order to relate reach storage to inflow and outflow values:

$$S_t = K[x_1 I_t^{\alpha_1} (1 + x_2 I_t^{\alpha_2}) + x_3 O_t^{\alpha_3}]^m \quad (4)$$

In this new model, the inflow value is multiplied by a variable function of inflow in the storage relation. Therefore, the proposed model can be rewritten as follows:

$$S_t = K[f(I_t) I_t^{\alpha_1} + x_3 O_t^{\alpha_3}]^m \quad (5)$$

where

$$f(I_t) = x_1 (1 + x_2 I_t^{\alpha_2}) \quad (6)$$

Hence, the new Muskingum model has a variable inflow-weighted parameter.

This new model, unlike the 6th, 12th, 13th, and 14th models in Table 1, considers variable parameter, i.e., inflow-weighted parameter, without dividing the flood period into sub-periods. These modifications divide flood period into a user defined number of sub-periods, i.e., L , while selection of an appropriate number of sub-divisions is not standardly addressed in application of these models. This definitely produces confusion for users in practice. Not only the way in which these models divide flood period bringing additional parameter(s) to the list of parameters required to be calibrated, but also the technique utilized for this division is quite questionable. Moreover, although these modifications used a greater number of parameters, they did not perform as efficiently as more complex constant-parameter models. To be more specific, the 6th model in Table 1, a variable-exponent Muskingum method, for $L = 10$ has 21 parameters. It achieves the SSQ value equal to 24.203 by the Premium Solver for Wilson's data (Easa 2013a), whereas the 7th model in Table 1, a four-constant-parameter Muskingum model, obtains 7.67 for SSQ for the corresponding data by any algorithm (Easa 2013b). It also may be implied that the variation of the exponent parameter in the Muskingum model does not have a significant effect on the accuracy of routing results. In contrast, the proposed model not only considers variable inflow-weighted parameter, which is a function of inflow, but it also predicts floods with no requirement for flood period division. The applicability and performance of the proposed model in the simulation process is presented in the following sections.

Simulation procedure of the proposed model

In the routing procedure for the nine-parameter Muskingum model, Euler's method and the weighted average technique proposed by Karahan *et al.* (2015) are employed. In order to estimate the parameters of the proposed Muskingum model, the routing procedure can be conducted as follows.

First, by rearranging Equation (4), the magnitude of the outflow O_t can be expressed as:

$$O_t = \left[\frac{1}{x_3} \left(\frac{S_t}{K} \right)^m - \frac{x_1}{x_3} I_{\text{mod}}^{\alpha_1} (1 + x_2 I_{\text{mod}}^{\alpha_2}) \right]^{\frac{1}{\alpha_3}} \quad (7)$$

where $I_{\text{mod}} = \theta I_t + (1 - \theta) I_{t-1}$ and I_{mod} is the modified inflow at time t .

Second, by combining the obtained relation (Equation (7)) and continuity equation (Equation (1)), the time rate of change of the storage volume can be declared as:

$$\frac{\Delta S_t}{\Delta t} = I - \left[\frac{1}{x_3} \left(\frac{S_t}{K} \right)^m - \frac{x_1}{x_3} I_{\text{mod}}^{\alpha_1} (1 + x_2 I_{\text{mod}}^{\alpha_2}) \right]^{\frac{1}{\alpha_3}} \quad (8)$$

where Δt is time interval.

Finally, the simulation procedure particularly includes the following steps.

Step 1: Determine the nine hydrologic parameters (K , x_1 , α_1 , x_2 , α_2 , x_3 , α_3 , m , and θ) by implementing the new hybrid optimization algorithm. The selection of values for these parameters can be obtained differently in the hybrid techniques as follows:

- In the first phase: The MHBMO algorithm randomly chooses initial values for the parameters in the defined bounds.
- In the second phase: The GRG algorithm utilizes the initial values of these parameters from the obtained result in the first phase.

Step 2: Calculate the initial storage volume by Equation (8), in which the initial outflow is assumed to be the same as the initial inflow. Therefore, the initial storage volume can be obtained by:

$$S_2 = K [x_1 I_1^{\alpha_1} (1 + x_2 I_1^{\alpha_2}) + x_3 O_1^{\alpha_3}]^m \quad (9)$$

Step 3: Check the applicability of the selected values for the parameters of the new proposed Muskingum model, i.e., (K , x_1 , α_1 , x_2 , α_2 , x_3 , α_3 , m , and θ). This check is particularly conducted in order to avoid producing complex values for the outflow using Equation (7). According to this check, the following inequality should be satisfied to obtain acceptable and feasible outflow values.

$$m < \frac{\log\left(\frac{S_t}{K}\right)}{\log\left(x_1 I_t^{\alpha_1} (1 + x_2 I_t^{\alpha_2})\right)} \quad (10)$$

Step 4: Compute the time rate of change of the channel storage using Equation (8).

Step 5: Estimate the next accumulated storage using the Euler's method by

$$S_{t+1} = S_t + \Delta S_t \quad (11)$$

It should be noted that negative values for reach storage is not acceptable and should be penalized in the routing procedure.

Step 6: Calculate the magnitude of the outflow at the next time using Equation (7). The third step to the sixth step should be repeated for all time intervals.

The new hybrid technique

The HBM algorithm was inspired from the mating process of honey bees, which makes this algorithm a phenomenon-mimicking algorithm. Not only is this algorithm a well-documented optimization technique, but it has also already been applied for several optimization problems in water resources engineering and especially Muskingum parameter estimation (Esmi Jahromi & Afzali 2014; Niazkar & Afzali 2015a, 2015b, 2016b; Afzali *et al.* 2016). The GRG algorithm, as a mathematical technique embedded in Excel spreadsheets, was also utilized for calibration of Gill's nonlinear Muskingum parameters (Barati 2013, Niazkar & Afzali 2016a). The new proposed hybrid model in this paper utilizes the combination of these two algorithms as a more powerful tool for efficient Muskingum parameter estimation. This new hybrid method not only overcomes the shortcomings of the zero-order algorithm, i.e., the MHBMO algorithm, but also provides a useful initial guess for the first-order method, i.e., the GRG algorithm. Therefore, the combination of these methods, i.e., the new hybrid model, performs more promisingly in finding optimum Muskingum parameter values.

This new hybrid approach works in two successive steps: (1) the MHBMO algorithm initiates the calibration process by assuming random values for Muskingum parameters and precedes the simulation process for several iterations; and (2) the GRG algorithm continues the routing procedure using the final values of Muskingum parameters obtained in the first phase.

In the first step of the new hybrid technique, a written code of the MHBMO algorithm in the MATLAB program

was utilized. In this regard, the controlling parameters of the MHBMO algorithm were evaluated exactly as the ones utilized in Niazkar & Afzali's study (2015a). The utilized range for Muskingum parameters is shown in Table 2. Finally, the obtained results for Muskingum parameters in the first step were used as the initial vector for the second phase. In the second phase, the routing procedure was simulated in Excel spreadsheets. The built-in optimization tool in Excel, i.e., the GRG algorithm, was utilized for the second phase of calibration of Muskingum parameters. Since the GRG algorithm is a deterministic technique, the calibration process will terminate based on the inserted initial guess. The final values of Muskingum parameters will be achieved at the end of this step.

Performance evaluation criteria

The utilized performance evaluation criteria adopted for verifying the efficiency of the hybrid MHBMO-GRG algorithm are as follows (Niazkar & Afzali 2015a; Afzali 2016).

Accuracy of procedure consideration: Accuracy of the estimation procedure can be measured by both SSQ and SAD which are given respectively as:

$$SSQ = \sum_{i=1}^N (O_{observed}^t - O_{routed}^t)^2 \quad (12)$$

$$SAD = \sum_{i=1}^N |O_{observed}^t - O_{routed}^t| \quad (13)$$

Table 2 | State variable constraints required for initial population creation in the MHBMO algorithm

Parameters of the new Muskingum model	Minimum value	Maximum value
K	0.001	3.3
x_1	0.001	0.9
α_1	0.001	2.2
x_2	0.001	1.5
α_2	-0.9	0.9
x_3	0.001	1.8
α_3	0.001	2
m	0.001	5
Θ	0	1

where $O_{observed}^t$ = observed outflow at time t ; O_{routed}^t = routed outflow at time t ; and N = total number of time intervals.

Magnitude of peak consideration: The absolute value of the deviations of peak of observed and routed outflows (DPO) is considered as a measure of the accuracy of the amount of peak outflow:

$$DPO = |O_{observed}^{peak} - O_{routed}^{peak}| \tag{14}$$

where $O_{observed}^{peak}$ = peak of observed outflow; O_{routed}^{peak} = peak of routed outflow.

Mean absolute relative error consideration: The mean absolute relative error between the observed and routed outflows (MARE) is considered as the error mode:

$$MARE = \frac{1}{N} \sum_{i=1}^N \frac{|O_{observed}^i - O_{routed}^i|}{O_{observed}^i} \tag{15}$$

Variance explained: The closeness of shape and size of the hydrograph can be measured utilizing the criterion of variance explained in a percentage (VarexQ), which is given as:

$$VarexQ = \left[1 - \frac{\sum_{i=1}^N (O_{observed}^i - O_{routed}^i)^2}{\sum_{i=1}^N (O_{observed}^i - O_{routed}^{mean})^2} \right] \times 100 \tag{16}$$

where O_{routed}^{mean} = mean of observed outflows.

RESULTS AND DISCUSSION

The performance of the new hybrid method, i.e., MHBMO-GRG, in the Muskingum parameter estimation of the new proposed model was demonstrated using two case studies:

(1) experimental (Wilson 1974) and (2) real (Viessman & Lewis 2003) data sets. Since these examples have already been applied in many studies on parameter estimation of Muskingum models, the applicability of the new Muskingum model potentially could be compared with most of the previous models in the literature.

In this research, the utilized objective function is to minimize the residual SSQ between the observed and routed outflows:

$$\min \text{SSQ} = \sum_{i=1}^N \left\{ O_{observed}^i - \left[\frac{1}{x_3} \left(\frac{S_t}{K} \right)^{\frac{1}{m}} - \frac{x_1}{x_3} I_{mod}^{\alpha_1} (1 + x_2 I_{mod}^{\alpha_2}) \right] \right\}^{1/\alpha_3} \tag{17}$$

First case study (smooth flood hydrograph)

This example has been conventionally used for Muskingum parameter estimation, especially for Gill’s nonlinear model, and is traditionally accepted as a standard problem for this purpose. Hence, the new proposed model was compared with numerous Muskingum models in the literature by using this data set. The optimal values of Muskingum parameters of the new nine-parameter model, which were achieved by using both the MHBMO algorithm and the new hybrid method, are shown in Table 3. It should be noted that the results of the MHBMO algorithm can be considered as the final results of the first step of the hybrid algorithm. As Table 3 shows, the hybrid method achieved better SSQ than the MHBMO algorithm. The variation of the inflow-weighted parameter is plotted in Figure 1. This figure obviously depicts the smooth hydrographical-like variation of the inflow-weighted parameter during flood period

Table 3 | Optimum values of Muskingum parameters for the first case study

Algorithm	K	x ₁	α ₁	x ₂	α ₂	x ₃	α ₃	m	θ	SSQ
MHBMO	2.358	0.027	0.730	0.236	0.001	0.763	0.366	4.366	0.065	4.951
Hybrid	2.395	0.024	0.738	0.212	0.000	0.775	0.351	4.526	0.074	4.939
Change in values (%)	1.552	10.764	1.045	9.916	100.000	1.684	4.074	3.674	13.980	0.234

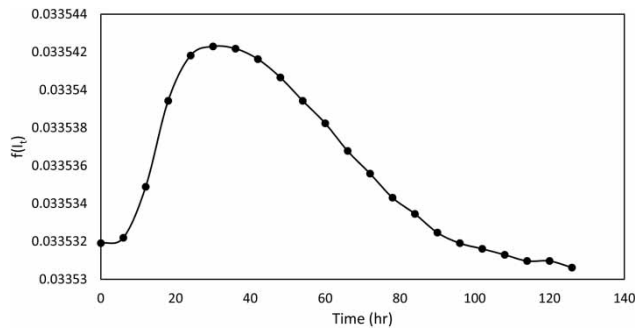


Figure 1 | Variation of the inflow-weighted parameter for the first case study.

for this case study. Figure 2 compares the absolute difference between the observed and routed outflows obtained by best techniques for different versions of Muskingum models for the first case study. This figure indicates the superiority of the proposed model in comparison with others. Figure 3 shows the inflow hydrograph and observed and routed outflow hydrographs using the hybrid MHBMO-GRG algorithm. Table 4 presents a comprehensive comparison between the proposed Muskingum models as well as the utilized optimization techniques for parameter estimation for the first case study. In this table, the performance of 37 different scenarios for three- to twelve-parameter versions of the Muskingum flood routing models are compared using five performance evaluation criteria. At the top of this table, for instance, the three-parameter Muskingum models were sorted from the smallest to the largest SSQ values. Hence, the GRG (Hirpurkar & Ghare 2014) and the MHBMO algorithms (Niazkar & Afzali 2015a) achieve

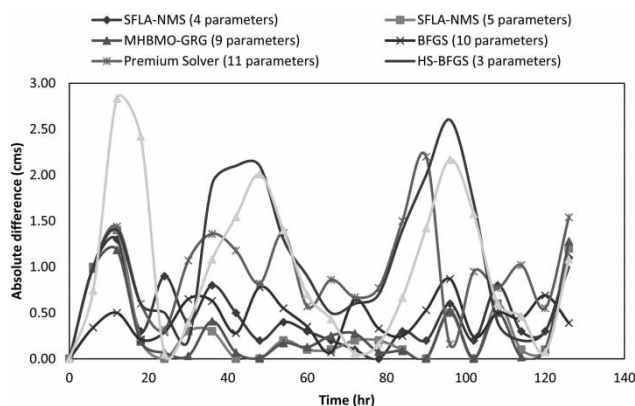


Figure 2 | Comparison of absolute difference between the observed and routed outflows obtained by best techniques for different versions of Muskingum models.

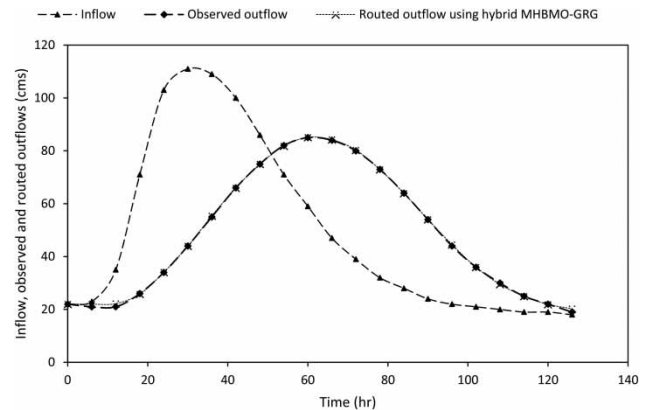


Figure 3 | Observed and routed hydrographs using the new hybrid MHBMO-GRG method for the first case study.

the worst and best SSQ values among all the applied techniques for three-parameter Muskingum model in Table 4. Moreover, the latter scenario performs the best based on the SAD and VaerxQ criteria among three-parameter Muskingum models. Among the utilized techniques for parameter estimation of four-parameter versions of the Muskingum model, both GA-GRG (Barati 2012) and SFLA-NMS (Haddad *et al.* 2015a) perform similar and better than the premium solver (Easa 2013b). Additionally, the premium solver (Easa *et al.* 2014) performs the best in calibration of the Muskingum model with five parameters. Although the lowest SSQ value was obtained by AGA (Zhang *et al.* 2016) for $L=5$ with twelve parameters, this result was achieved with the highest number of parameters. Furthermore, this model achieved larger SSQ values for four-parameter Muskingum in comparison with other four-parameter models. It should be noted that not only the optimization techniques differ in the compared scenarios for the five-parameter Muskingum models, but also these models have different formations as well. For instance, the CS algorithm (Karahan *et al.* 2015) was applied to the modified version of Muskingum which incorporates lateral flow (the 14th model in Table 1) while premium solver (Easa *et al.* 2014) was utilized for the 11th model in Table 1 for $L=1$. Although there is one model with seven-, eight-, nine-, eleven-, and twelve-parameter models in Table 4, these models can be relatively compared with other models that possess a similar number of parameters. For instance, AGA (Zhang *et al.* 2016) scenario for the four-, six-, eight-, ten-, and twelve-parameter models did not

Table 4 | Comparison of the routing results of various methods for the first case study

Method	No. of parameters	SSQ	SAD	DPO	MARE	VarexQ
GRG (Hirpurkar & Ghare 2014)	3	178.822	48.96	1.09	0.053	98.537
LSM (Gill 1978)	3	143.6	46.4	1.8	0.056	98.825
LMM (Das 2004)	3	130.487	43.2	0	0.055	98.938
HJ + CG (Tung 1985)	3	49.64	25.2	0.5	0.03	99.594
HJ + DEP (Tung 1985)	3	45.54	24.8	0.3	0.03	99.627
NONLR (Yoon & Padmanabhan 1993)	3	41.28	25.2	0.7	0.033	99.662
GA (Mohan 1997)	3	38.23	23	0.7	0.025	99.702
PSO (Chu & Chang 2009)	3	36.89	24.1	0.6	0.03	99.695
ICSA (Luo & Xie 2010)	3	36.801	23.4	0.898	0.025	99.699
HS (Kim <i>et al.</i> 2001)	3	36.78	23.4	0.92	0.031	99.627
ES (Barati 2013)	3	36.771	23.46	0.892	0.025	99.699
DE (Xu <i>et al.</i> 2011)	3	36.77	23.46	0.9	0.026	99.693
PSF-HS (Geem 2010)	3	36.768	23.7	0.9	0.026	99.693
BFGS (Geem 2006)	3	36.768	23.46	0.9	0.026	99.693
HS-BFGS (Karahana <i>et al.</i> 2013)	3	36.768	23.4	0.9	0.025	99.701
GRG (Barati 2013)	3	36.765	23.47	0.901	0.025	99.699
NMS (Barati 2011)	3	36.765	23.46	0.899	0.025	99.699
MHBMO (Niazkar & Afzali 2015a)	3	36.242	21.88	0.699	0.028	99.703
Premium solver (Easa 2013b)	4	7.67	10.32	0.31	0.015	99.937
GA-GRG (Barati 2012)	4	7.54	10.2	0.3	0.015	99.938
SFLA-NMS (Haddad <i>et al.</i> 2015a)	4	7.54	10.2	0.3	0.015	99.938
AGA (Zhang <i>et al.</i> 2016)	4	10.541	13.05	0.20	0.017	99.91
GRG (Vatankhah 2014a)	5	26.78	21.8	0.7	0.028	99.781
GRG (Easa 2015)	5	16.57	14.9	0.3	0.02	99.864
CS (Karahana <i>et al.</i> 2015)	5	9.823	12.6	0.22	0.016	99.92
GA-GRG (Barati 2012)	5	5.53	6.7	0.1	0.011	99.955
SFLA-NMS (Haddad <i>et al.</i> 2015a)	5	5.4	6.6	0.1	0.011	99.956
Premium solver (Easa <i>et al.</i> 2014)	5	5.35	NM ^a	NM ^a	NM ^a	NM ^a
Premium solver (Easa <i>et al.</i> 2014)	6	5.44	NM ^a	NM ^a	NM ^a	NM ^a
AGA (Zhang <i>et al.</i> 2016)	6	7.475	10.58	0.31	0.014	99.94
SFLA-NMS (Haddad <i>et al.</i> 2015b)	7	6.348 ^b	7.61 ^b	0.05	0.013	99.948
AGA (Zhang <i>et al.</i> 2016)	8	5.730	8.58	0.10	0.012	99.95
MHBMO-GRG (this study)	9	4.944	6.36	0.128	0.011	99.96
BFGS (Karahana 2014)	10	5.178	9.53	0.35	0.011	99.958
AGA (Zhang <i>et al.</i> 2016)	10	4.911	7.79	0.03	0.012	99.96
Premium solver (Easa 2013a)	11	24.904	20.72	0.57	0.025	99.796
AGA (Zhang <i>et al.</i> 2016)	12	4.535	7.35	0.26	0.010	99.96

^aThe routed outflow was not mentioned in the corresponding paper.^bThe SSQ and SAD values are reported equal to 5.44 and 6.69 in the original paper while the corresponding values in this table were computed based on the reported outflow values in the original paper.

Table 5 | Optimum values of Muskingum parameters for the second case study

Algorithm	K	x_1	α_1	x_2	α_2	x_3	α_3	m	θ	SSQ
MHBMO	3.300	0.253	0.625	1.900	0.374	0.629	0.994	1.007	0.000	22.274
Hybrid	3.501	0.313	0.410	1.558	0.582	0.594	0.994	1.007	0.000	22.130
Change in values (%)	6.093	23.860	34.391	17.982	55.862	5.595	0.039	0.053	–	0.645

perform better than other similar models, except for the ten-parameter model. Therefore, this table not only compares different versions of Muskingum models for the first case study, but also demonstrates the performance of different optimization techniques applied for the related Muskingum models in the literature. According to Table 4, the hybrid MHBMO-GRG obtains the lowest value for the SSQ, SAD, MARE, and VorexQ criteria among all other scenarios while several other techniques performs as well as the recommended hybrid model based on MARE criteria. Additionally, LMM (Das 2004) performs the best in predicting the value of the maximum outflow value since it has the lowest value of DPO among all scenarios in Table 4. According to this table, the new proposed model accompanied by the new hybrid method reduced by 4.52% the SSQ value of Karahan's model (Karahan 2014) which has ten parameters. All in all, it can be concluded that the new recommended model performs the best among all other models in the literature for Wilson's data.

Second case study (multi-peak flood hydrograph)

A multimodal data set (Viessman & Lewis 2003) was also applied for Muskingum parameter estimation. The new hybrid model was successfully utilized to this data set. The optimum values of the parameters of the new Muskingum model using the MHBMO and MHBMO-GRG algorithms are listed in Table 5 for this case study. The SSQ values obtained by the hybrid algorithm in the parameter estimation process for both case studies (Tables 3 and 5) are lower (better) than the results achieved by the MHBMO algorithm. Although the SSQ improvement may seem not relatively significant, the change in the optimum values for Muskingum parameters are quite considerable. To be more specific, the optimum values of x_1 , α_1 , x_2 , α_2 , and x_3 are considerably changed in the second phase of the hybrid algorithm. Consequently, the routed outflow may vary

because of the differences provided in the optimum values of the Muskingum model. Finally, it can be concluded that the proposed hybrid performs better than the MHBMO algorithm for both case studies considered (Tables 3 and 5). The inflow, observed, and routed outflow hydrographs using the hybrid MHBMO-GRG technique for these data are shown in Figure 4. The closeness of observed and routed outflow hydrographs in this figure obviously implies the accuracy of the utilized model.

The step-by-step routing procedure of the proposed model for this case study is shown in Table 6. In this table, the first, second, and third columns are flood time-interval, the measured inflows, and outflows, respectively. The fourth, fifth, sixth, and seventh columns are the rate of change of storage, absolute storage value, modified inflow, and routed outflows, respectively. This table technically shows the implementation of the simulation process for the new Muskingum model for the second case study. Moreover, the variation of the inflow-weighted parameter is plotted for this example in Figure 5. The wide range of change of the inflow-weighted parameter clearly demonstrates the need for applying a version of Muskingum model which possesses variable inflow-weighted parameter, just like the proposed model. Furthermore, the achieved

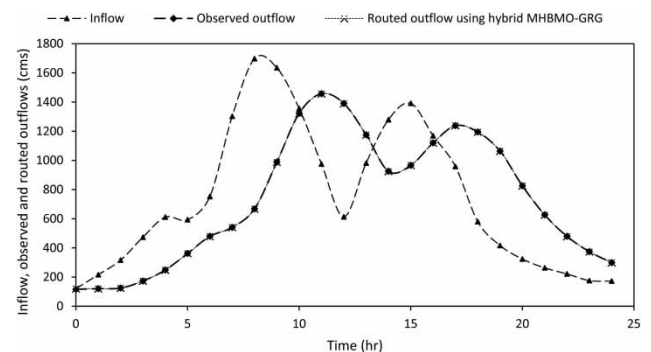
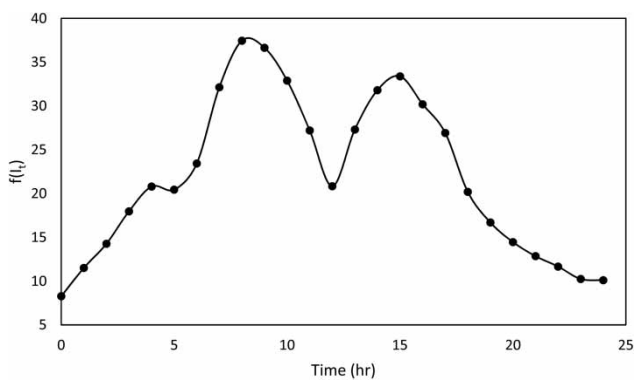
**Figure 4** | Observed and routed hydrographs using the new hybrid MHBMO-GRG method for the second case study.

Table 6 | The step by step simulation routing procedure for the second case study

t (hr)	I (m ³ /s)	O (m ³ /s)	$\Delta S/\Delta t$	S	I _{mod}	O _{routed}
0	121	116	–	–	–	120.63
1	217	121	174.64	466.55	120.63	120.63
2	316	125	272.68	641.19	216.51	124.91
3	474	173	428.80	913.88	316.21	173.06
4	611	249	474.83	1,342.67	473.74	248.43
5	593	362	216.39	1,817.50	611.36	362.08
6	752	480	402.21	2,033.89	593.24	479.82
7	1,303	541	1,206.63	2,436.10	752.38	541.42
8	1,698	668	1,350.05	3,642.73	1,302.58	667.66
9	1,635	988	595.80	4,992.78	1,697.88	988.21
10	1,356	1,323	–193.30	5,588.58	1,635.02	1,322.39
11	976	1,457	–792.09	5,395.27	1,356.09	1,457.42
12	613	1,391	–1,074.30	4,603.19	975.80	1,390.66
13	982	1,176	107.83	3,528.89	613.34	1,175.57
14	1,279	925	595.70	3,636.73	982.03	925.51
15	1,391	966	515.84	4,232.43	1,279.36	966.78
16	1,169	1,120	–132.67	4,748.27	1,391.49	1,120.86
17	958	1,238	–453.19	4,615.60	1,169.20	1,238.74
18	581	1,195	–923.50	4,162.41	957.96	1,195.36
19	417	1,064	–782.03	3,238.90	580.78	1,064.08
20	324	826	–578.67	2,456.88	416.82	825.91
21	263	627	–413.30	1,878.20	323.83	626.46
22	222	479	–291.73	1,464.90	263.18	479.28
23	176	374	–235.43	1,173.17	221.75	374.26
24	172	299	–130.55	937.74	176.36	299.41

**Figure 5** | Variation of the inflow-weighted parameter for the second case study.

result using the hybrid model was compared with other techniques in Table 7. In this table, 15 scenarios with different optimization techniques and different versions of

Muskingum model were compared using the prescribed evaluation criteria. The MHBMO algorithm (Niazkar & Afzali 2015a) achieves the best SSQ value in three-parameter nonlinear Muskingum models. The SFLA-NMS (Haddad *et al.* 2015a), GRG (Easa 2015), GRG (Easa 2014), and GRG (Easa 2014) algorithms perform the best based on the SSQ value in four-, five-, six-, and seven-parameter Muskingum models in Table 7. Among all these scenarios considered in this table, the hybrid MHBMO-GRG with the new proposed version of Muskingum model performs the best since it significantly reduced the best SSQ available in the literature, i.e., the MHBMO algorithm (Niazkar & Afzali 2015a). Since most previous studies did not mention the routed outflow values, the comparison of scenarios in Table 7 based on other criteria may not be applicable,

Table 7 | Comparison of the routing results of various methods for the second case study

Method	No. of parameters	SSQ	SAD	DPO	MARE	VarexQ
Premium solver (Easa 2013a)	3	71,114	NM ^a	NM ^a	NM ^a	NM ^a
MHBMO (Niazkar & Afzali 2015a)	3	7,751.662	304.675	8.717	0.037	99.837
GA-GRG (Barati 2012)	4	76,785	NM ^a	NM ^a	NM ^a	NM ^a
SFLA-NMS (Haddad <i>et al.</i> 2015a)	4	73,379	1,034	NM ^a	NM ^a	NM ^a
GA-GRG (Barati 2012)	5	70,738	1,018	NM ^a	NM ^a	NM ^a
SFLA-NMS (Haddad <i>et al.</i> 2015a)	5	69,860	993	NM ^a	NM ^a	NM ^a
Premium solver (Easa <i>et al.</i> 2014)	5	69,726.75	NM ^a	NM ^a	NM ^a	NM ^a
GRG (Easa 2015)	5	57,391	NM ^a	NM ^a	NM ^a	NM ^a
Premium solver (Easa <i>et al.</i> 2014)	6	68,015	NM ^a	NM ^a	NM ^a	NM ^a
GRG (Easa 2014)	6	52,748	NM ^a	NM ^a	NM ^a	NM ^a
SFLA-NMS (Haddad <i>et al.</i> 2015b)	7	73,399	1,034	50	NM ^a	NM ^a
GRG (Easa 2014)	7	38,823	NM ^a	NM ^a	NM ^a	NM ^a
MHBMO-GRG (this study)	9	22.13	9.293	0.03	0.002	100
BFGS (Karahan 2014)	10	1,277,047.73 ^b	4,664.23	24.23	0.37	73.16
Premium solver (Easa 2013a)	11	44,894	NM ^a	NM ^a	NM ^a	NM ^a

^aThe routed outflow was not mentioned in the corresponding paper.

^bThe SSQ value is reported equal to 26,185 in the original paper while the corresponding value in this table was computed based on the reported outflow values in the original paper.

although the hybrid MHBMO-GRG obtains the best values for other criteria in Table 7. Finally, the application of the new proposed Muskingum model for the second case study clearly shows the superiority of this model over others for double-peak flood hydrographs.

Sensitivity analysis

In order to conduct sensitivity analysis of new Muskingum parameters in terms of SSQ, the ranges presented in Table 2 were considered using 0.001 intervals. In the achieved results, the minimum value for the selected parameter is obtained equal to the reported optimum values in Tables 3 and 5 for the first and second case studies, respectively. SSQ is relatively less sensitive to θ and more sensitive to x_1 and α_1 in the sensitivity analysis for the first case study. Furthermore, the result of sensitivity analysis for the second case study shows that SSQ is relatively less sensitive to α_2 and more sensitive to α_3 . According to the results of sensitivity analysis for the two case studies, the greatest and the least parameter to which SSQ is sensitive differs for the selected case studies. Finally, it is necessary to note that the obtained results from this analysis clearly

demonstrate that the parameters to which the final results are more sensitive depends upon the case study under consideration, and no specific parameter can be reported as the one to which the final results are mostly sensitive.

CONCLUSION

The Muskingum model is one of the most widely utilized hydrologic flood routing approaches although calibration of its parameters is considered to be challenging. In this study, a new hybrid optimization technique is introduced as a new tool to address this challenge. This technique combines the MHBMO and the GRG algorithms in two successive steps. In the first step, the MHBMO algorithm, as a zero-order method, commences the estimation process of Muskingum parameters for several specific iterations. Afterwards, the GRG algorithm, as a first-order approach, continues the calibration procedure using the obtained result from the first step as an improved initial guess for Muskingum parameters. This hybrid technique not only overcomes the requirement for appropriate initial guess for deterministic Excel-embedded optimization technique, but

it also brings about more efficiency in the whole calibration process while it prevents sticking in probable local optimums by stochastic optimization techniques like the MHBMO algorithm. Additionally, a new improved version of Muskingum model, which comprises nine constant parameters, is proposed. Since the inflow-weighted parameter in this new Muskingum model varies with inflow of flood hydrograph, this model can be allocated as a variable-parameter Muskingum model, too. The new hybrid model was applied for parameter estimation of the new nonlinear Muskingum model using a simulation process based on Euler's method. The proposed hybrid algorithm was utilized to minimize the most common objective function for Muskingum calibration procedure for two case studies including smooth and double-peak flood hydrographs. The performance of the recommended algorithm for the new improved Muskingum model was compared with various utilized techniques and different Muskingum versions in the literature by applying several common criteria in flood routing studies. The hybrid MHBMO-GRG algorithm reached the best values for SSQ, SAD, MARE, and VAREXQ among all other techniques for both case studies. For instance, this new hybrid method particularly reduced the best SSQ value available in the literature by 99.7% for the utilized case study. The obtained results demonstrate that the new hybrid method can be efficiently utilized for rigorous nonlinear Muskingum parameter estimation. Furthermore, the proposed modified Muskingum flood routing model appeared to be the most reliable version of Muskingum model among all versions which were considered in this study.

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