Storm event-based frequency analysis method

Changhyun Jun, Xiaosheng Qin, Yeou-Koung Tung and Carlo De Michele

ABSTRACT

In this study, a storm event-based frequency analysis method was proposed to mitigate the limitations of conventional rainfall depth–duration–frequency (DDF) analysis. The proposed method takes the number, rainfall depth, and duration of rainstorm events into consideration and is advantageous in estimation of more realistic rainfall quantiles for a given return period. For the purpose of hydraulics design, the rainfall depth thresholds are incorporated to retrieve the rainstorm events for estimating design rainfalls. The proposed method was tested against the observed rainfall data from 1961 to 2010 at Seoul, Korea and the computed rainfall quantiles were compared with those estimated using the conventional frequency analysis method. The study results indicated that the conventional method was likely to overestimate the rainfall quantiles for short rainfall durations. It represented that the conventional method could reflect rainfall characteristics of actual rainstorm events if longer durations (like 24 hours) were considered for estimation of design rainfalls.

Key words | design rainfall, rainfall DDF, rainfall depth threshold, rainstorm event

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INTRODUCTION

Hydrological design for urban infrastructures normally requires determination of design rainfall, which involves analysis of rainfall depth–duration–frequency (DDF) relationship and hyetograph for the design event. The conventional method of establishing design rainfall hyetograph involves (i) frequency analysis of annual maximum rainfall data to estimate rainfall quantiles for different durations of interest, (ii) derivation of a suitable mathematical representation of rainfall DDF relationships, and (iii) establishment of design rainfall hyetograph for a specified rainfall duration and return period according to the derived rainfall DDF relationships (Hershfield 1961; Wenzel 1982; Boni et al. 2006). Over the past years, application of the conventional frequency analysis method to estimate design rainfall has been criticized in many aspects (Palynchuk & Guo 2008). Many studies indicated that the frequency analysis results based on fixed-duration rainfall series could notably deviate from real conditions (Yue 2000b; Yue & Rasmussen 2002; Kao & Govindaraju 2007; Lee et al. 2010). One of the reasons is that the specific rainfalls for fixed time intervals could correspond to any parts of actual rainstorm events, where the rainfall characteristics of independent rainstorm events are overlooked (Wenzel 1982; Park et al. 2014; Yoo et al. 2016).

As an alternative to the conventional method, it has been recommended that independent rainstorm events be directly used for rainfall DDF analysis (EPA 1986; Adams & Papa 2000; Kim & Park 2008; Yoo et al. 2016). A series of rainstorm events can be retrieved from the observed rainfall data and characterized in terms of rainfall depth, peak hourly intensity, average intensity, duration, inter-event time, time to peak, and number of events (Marien & Vandewiele 1986; Salvadori & De Michele 2006; Wu et al. 2006; Knighton & Walter 2016). Previously, several methods

have been used for identifying individual rainstorm events where the rainfall characteristics are generally assumed to be statistically independent (Eagleson 1977; Morris 1978; Yen & Chow 1980; Restrepo-Posada & Eagleson 1982; Bonta & Rao 1988; Ignaccolo & De Michele 2010). There are also storm event-based approaches to improve the current practice in developing rainfall DDF curves. In earlier studies, rainfall characteristics of each rainstorm event were fitted separately using exponential distributions (Adams *et al.* 1986; Guo & Adams 1998; Adams & Papa 2000). Later on, multivariate analyses were performed in order to describe correlations between rainfall event statistics (e.g., rainfall intensity and duration); these analyses could be broadly categorized into multivariate distributions (Yue 2000a; Yue & Wang 2004; Park *et al.* 2014) and copulas (De Michele & Salvadori 2003; Grimaldi & Serinaldi 2006; Salvadori & De Michele 2006; Zhang & Singh 2007; Kao & Govindaraju 2008; Vernieuwe *et al.* 2015). Most of the previous studies on this topic have dealt with rainstorm events and their characteristics in common and focused on statistical methods for modeling rainfall depth, intensity, and duration.

Generally, they hardly examine the occurrence probability of entire rainstorm events throughout the record period and only consider the annual maximum rainfall series for the pre-selected time intervals (Jun *et al.* 2017). This is consistent with the methodology of Yoo *et al.* (2016), which considered hourly rainfall data at Seoul rain gauge station in Korea and evaluated the concept of critical rainfall durations from bivariate frequency analysis of annual maximum independent rainstorm event series. To mitigate this limitation in the design rainfall from the rainfall DDF relationships, continuous observed streamflow/precipitation records have been used to estimate return periods for responses of hydraulic and hydrologic systems (Grimaldi *et al.* 2012; Rogger *et al.* 2012). Specifically, Knighton & Walter (2016) pointed out that the conventional design storm can be an artificial hyetograph, which may not be able to describe precipitation event statistics and patterns. A stochastic rainfall generation approach has been applied to provide strategies for hydrological model calibration and estimation of design floods (Haberlandt & Radtke 2014), examine impact of rainfall patterns on the flood response (Paschalis *et al.* 2014), and reveal dependences among storm variables (Vandenberghe *et al.* 2010; Wang *et al.* 2010; Balistrocchi & Bacchi 2011; Vernieuwe *et al.* 2015).

Thus, the objective of this study is to develop an improved rainfall frequency analysis method based on the stochastic behavior of independent rainstorm events themselves. The storm event analysis with rainfall depth thresholds is applied to offer an alternative way in developing design rainfalls. By using the number of rainstorm events and their rainfall characteristics, it is possible to identify the relationships between rainfall intensity, duration, and frequency. The proposed method is applied to Seoul rain gauge station, Korea, and the derived design rainfalls are compared with those obtained from the conventional frequency analysis method.

METHODOLOGY

Determination of rainstorm event

This study retrieved independent rainstorm events from hourly rainfall data in order to statistically analyze rainfall characteristics of rainstorm events. To be qualified as an independent event, the minimum no-rain period should be adopted to distinguish the independent rainstorm events; this is defined as inter-event time definition (IETD), given by Adams *et al.* (1986), and can be selected to adequately describe responses of a hydrologic system (Yoo & Park 2012). The selected IETD is used to determine rainstorm events in relation to each no-rain period between a series of consecutive rainfall data. If the no-rain period between two consecutive rainfalls is shorter than IETD, these consecutive rainfalls would be regarded as one rainstorm event. If the period is longer than IETD, each consecutive rainfall is considered as an independent rainstorm event. More detailed information can be referred to Yoo *et al.* (2016) and Jun *et al.* (2017).

Storm event-based frequency analysis method: The number and rainfall depth of rainstorm events

For rainstorm events' identification, a rainfall intensity threshold (i°) and an IETD are first chosen. Then, the resulting rainfall intensity–duration pair data series are identified

based on those rainstorm events with rainfall intensity being greater than the threshold (i.e., $i \geq i^\circ$) and the rainfall duration being τ. The rainstorm events selected are defined as follows:

$$\left\{ \left(\bar{i}_{j,y}, \ \tau_{j,y} \right) \right\}_{j=1, 2, \ldots, n_y^\circ; y=1, 2, \ldots, N_y} \tag{1}$$

where $\bar{i}_{j,y}$ and $\tau_{j,y}$ are the average rainfall intensity with $\bar{i}_{j,y} \geq i^\circ$ and the corresponding rainfall duration of the jth rainstorm event in the yth year; n_y° is the number of rainstorm events during the yth year having $\bar{i}_{j,y} \geq i^\circ$, and N_y is the number of years throughout the record period.

Let us denote generically the event, E_*° and its probability of occurrence, P_*°. Then, the probability of occurrence of at least one event E_*° in n independent rainstorm events can be defined as follows:

$$P_{*,n}^\circ = \sum_{k=1}^{n} \binom{n}{k} \left(P_*^\circ \right)^k \left(1 - P_*^\circ \right)^{n-k} = 1 - \left(1 - P_*^\circ \right)^n \tag{2}$$

If the number of E_*° follows a Poisson distribution (Haight 1967), then the probability of at least one occurrence of E_*° in a period of D years can be derived as:

$$P\left[E_*^\circ \, in \, (0, \ D) \right] = \sum_{n=1}^{\infty} \frac{e^{-\lambda^\circ D} (\lambda^\circ D)^n}{n!} \times \left[1 - \left(1 - P_*^\circ \right)^n \right]$$
$$= 1 - e^{\lambda^\circ D P_*^\circ} \tag{3}$$

where λ° is the annual average number of rainstorms. It can be estimated as:

$$\lambda^\circ = \frac{\sum_{y=1}^{N_y} n_y^\circ}{N_y} \tag{4}$$

If $D =$ one year, Equation (3) becomes

$$P\left[E_*^\circ \, in \, (0, \ 1) \right] = 1 - e^{-\lambda^\circ P_*^\circ} \tag{5}$$

The corresponding return period of the rainstorm event of interest E_*°, $T\left(E_*^\circ \right)$ can be defined by:

$$T\left(E_*^\circ \right) = \frac{1}{\left(1 - e^{-\lambda^\circ P_*^\circ} \right)} \tag{6}$$

To relax the restriction that the mean and variance of the Poisson distribution are equal, Consul & Jain (1973)

proposed the generalized Poisson distribution (GPO) defined as (Tung *et al.* 2006):

$$\prod_{GPO} (n|\lambda, \ \theta) = \frac{e^{-(\lambda + n\theta)} \lambda (\lambda + n\theta)^{n-1}}{n!}, \quad n = 0, \ 1, 2, \ \cdots \tag{7}$$

where λ and θ are two with $\lambda > 0$ and $0 \leq \theta < 1$. These parameters can be estimated from the first two moments (Consul 1989) as the mean and the variance. Then, the annual occurrence of E_*° can be expressed as:

$$P\left[E_*^\circ \, in \, (0, \ 1) \right] = \sum_{n=1}^{\infty} \prod_{GPO} (n|\lambda, \ \theta) \left[1 - \left(1 - P_*^\circ \right)^n \right]$$
$$= \left(1 - e^{-\lambda} \right) - \sum_{n=1}^{\infty} \left[\frac{\lambda' (\lambda' + n\theta')^{n-1}}{n!} e^{-(\lambda' + n\theta')} \right] e^{(\lambda' - \lambda) + n(\theta' - \theta)} \tag{8}$$

where $\lambda' = \lambda (1 - P_*^\circ)$ and $\theta' = \theta (1 - P_*^\circ)$. Likewise, the corresponding return period of E_*°, $T\left(E_*^\circ \right)$, can be calculated as the reciprocal of Equation (8).

Similar to the widely used peaks-over-threshold approach, this study considers a rainfall depth exceeding certain specific rainfall depth thresholds. However, the proposed method first determines independent rainstorm events corresponding to a fixed storm duration and then extracts rainstorm events with the rainfall depth threshold for a storm event-based frequency analysis. Then, a new concept for rainfall frequency analysis is proposed to estimate design rainfalls considering occurrence probabilities of each rainstorm event, which could better reflect the stochastic nature of storms and actual durations of rainstorm events.

Case-I: Rainfall depth with fixed rainfall durations of $\tau*$

Consider a fixed rainfall duration to find P_*°, which is the same as the specified time interval used for the conventional frequency analysis method. From the retrieved rainstorm events obtained above, one can extract rainstorm events with a rainfall depth exceeding $d^\circ = i^\circ \times \tau*$, i.e., $d \geq d^\circ$ within a specified time period (duration) of $\tau*$. The resulting rainfall depth data series are:

$$\left\{ d_{j,y} \middle| d_{j,y} \geq d^\circ \, in \, \tau* \right\}_{j=1, 2, \ldots, n_y^\circ; y=1, 2, \ldots, N_y} \tag{9}$$

where $d_{j,y}$ is the maximum rainfall depth of the jth rainstorm event in the yth year within a rainfall duration of $\tau*$, namely:

$$d_{j,y} = Max_t \left\{ \sum_{t \in \tau*} d_{t,j,y} \right\} \tag{10}$$

From the rainfall depth data series in Equation (9), one can estimate the probability density function (PDF) of any rainstorm events with rainfall depth $d \geq d^\circ$ and rainfall duration being equal to $\tau*$. Here, E_*° and P_*° can be defined based on a design rainfall depth of interest ($d*$):

$$E_*^\circ = (d \geq d* \geq d^\circ) \tag{11}$$

$$P_*^\circ = P(d \geq d* \mid d \geq d* \geq d^\circ, \ \tau = \tau*) \tag{12}$$

Case-II: Rainfall depth with any rainfall duration

This section considers only the marginal probability of rainfall depth with any rainfall duration. With the proper IETD and rainfall depth threshold, the rainfall data series can be retrieved and expressed as follows:

$$\left\{ d_{j,y} \middle| d_{j,y} \geq d^\circ \right\}_{j=1,2,\dots,n_y^\circ; y=1,2,\dots,N_y} \tag{13}$$

The definition of E_*° is the same as that in Equation (11) and the definition of P_*° in Equation (12) can be simplified into Equation (14) as:

$$P_*^\circ = P(d \geq d* \mid d \geq d* \geq d^\circ) \tag{14}$$

From the retrieved rainstorm events, one can determine λ°, λ, and θ, which are parameters corresponding to the annual average number of E_*°.

CASE STUDY

Statistics of rainstorm events in Seoul, Korea

In Korea, systematic rainfall measurements have been available since 1961. The rainfall record of Seoul, Korea was obtained by an automatic weather station (AWS), operated by the Korea Meteorological Administration (KMA). The tipping bucket rain gauge was used to measure the precipitation in millimeters with no missing data at Seoul, Korea. KMA (2004) can be referred to for more detailed information. These rainfall data have been used in previous studies on trend analysis (Jung *et al.* 2001; Wang *et al.* 2006) and data assimilation (Xiao *et al.* 2008; Sohn *et al.* 2010). In this study, a series of rainstorm events were retrieved from 50 years of hourly rainfall data during 1961–2010 at the Seoul rain gauge station, based upon IETD of 10 hours (Lee & Jeong 1992; Kwon 2004). The statistics for retrieved rainstorm events are summarized in Figure 1. It contains a total number of 3,228 rainstorm events and an approximately annual average number of 64.6 rainstorm events. As shown in Figure 1 (a), the annual average rainfall amount appears to remain relatively constant over the 50-year record period. From Figure 1(b), it is noted that average annual number of rainstorm events has increased since the 1980s with a larger degree of variation and this results in a decreasing trend in average rainfall depth per storm as revealed in Figure 1(a). About 20% of rainstorm events have rainfall duration around 1 hour and average rainfall intensity of 0.7 mm/hr (please refer to Table S1, Supplementary material for details, available with the online version of this paper).

When it comes to the determination of design rainfalls, it appears appropriate to adopt a suitable rainfall depth threshold (per storm) by focusing on significant rainfall amounts in frequency analysis. This is because there is a large portion of retrieved rainstorm events having a small amount of rainfall depth (say, <1 mm). This study analyzed annual statistics of rainstorm events and the impact of rainfall depth thresholds on the number of rainstorm events (please refer to Table S2, Supplementary material for details, available online). Under the rainfall depth threshold of 10 mm, the total number of rainfall events is 1,102 which is about 35% of the total.

Additionally, this study compares the statistics of rainstorm characteristics during 1961–2010 in Seoul, Korea with respect to three rainfall depth thresholds: 0 mm, 5 mm, and 10 mm (see Table 1). It is observed that the average rainfall depth/duration/intensity per storm increase although the total rainfall depth decreased with a large amount of rainfall depth threshold. It implies that the rainfall depth threshold significantly affects the characteristics of rainstorm events retrieved from the observed rainfall data.
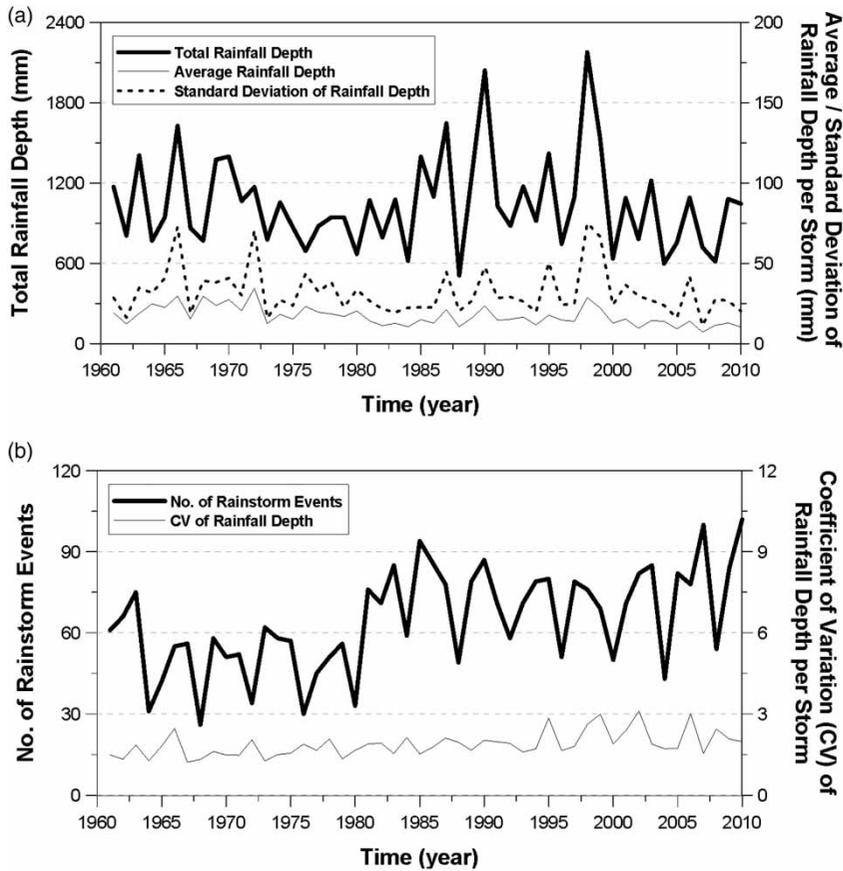
Figure 1 | Time series of annual statistics for independent rainstorm events: (a) total rainfall depth (mm), average rainfall depth per storm (mm), and standard deviation of rainfall depth per storm (mm); (b) number of storm events and coefficient of variation of rainfall depth per storm.

Table 1 | Characteristics of rainstorm events during 1961–2010 in Seoul, Korea

Rainfall characteristics	Annual statistics	Rainfall depth threshold (mm/storm)		
		0	5	10
Total rainfall depth (mm)	Mean	1,045.86	998.06	941.26
	Coefficient of variation	0.34	0.35	0.36
No. of rainstorm events	Mean	64.56	30.08	22.02
	Coefficient of variation	0.29	0.28	0.26
Average rainfall depth per storm (mm)	Mean	17.09	33.63	42.83
	Coefficient of variation	0.35	0.25	0.23
Average rainfall duration per storm (hr)	Mean	12.12	20.30	23.86
	Coefficient of variation	0.17	0.13	0.14
Average rainfall intensity per storm (mm/hr)	Mean	1.15	1.90	2.14
	Coefficient of variation	0.24	0.19	0.22

Result analysis

This study used the generalized Pareto distribution (GPD) to characterize rainfall depths of rainstorm events. This

distribution has been applied to model the behavior of random variables greater than specific threshold values (Pickands 1975; Davison & Smith 1990; Coles 2001; De Michele & Salvadori 2003). The exceedance probability of

rainfall depth d being larger than a threshold $d°$ can be obtained from GPDs in relation to the entire set of rainstorm events with rainfall depths above a given threshold. The PDF is shown as (Pickands 1975):

$$f_{(k,\mu,\sigma)}(x) = \frac{1}{\sigma}\left(1 + k\frac{x-\mu}{\sigma}\right)^{-1-\frac{1}{k}} \qquad (15)$$

where k, μ, and σ are shape, location, and scale parameters, respectively. This study used the Anderson–Darling (AD) test (Anderson & Darling 1954) and the Kolmogorov–Smirnov (KS) test (Stephens 1979) to evaluate the goodness-of-fit of the GPD and calculated the test statistics for the retrieved rainstorm events with a given threshold. At a significance level of 5%, the GPD model was considered acceptable for the analysis of rainfall characteristics.

First, consider Case-I where the rainstorm events have rainfall depths with a fixed rainfall duration of $\tau*$. In this study, the values of rainfall duration are fixed at 1, 6, 12, and 24 hours to estimate design rainfall in the storm event-based frequency analysis method. The hourly rainfall data are recorded at fixed-interval of 1 hour and thus it is impossible to find out when rainfall occurs within each hourly measurement, especially the starting and ending time of a rainstorm event. The real duration of each rainstorm event has the value calculated from the selected IETD with a difference up to 2 hours when rainstorm events are retrieved from a time series of hourly rainfall data (Palynchuk & Guo 2008). For this reason, this study considered additional rainstorm events corresponding to the fixed rainfall durations of 2, 5, 7, 11, 13, 23, and 25 hours to estimate the parameters of the GPD.

From the statistics of the retrieved rainstorm events (please refer to Table S3, Supplementary material for details, available online), it is observed that the rainfall depth threshold has more significant impact on rainfall depth statistics of 1- and 2-hour rainstorm events than those of longer-duration rainstorm events. It also reveals that there is only a small fraction of rainstorm events having rainfall depth greater than 20 mm during the record period of 50 years. Furthermore, this study obtains the maximum rainfall depths followed by the fixed rainfall durations from the retrieved rainstorm events and the results are: 21.5 mm (1 hour), 58.0 mm (2 hours), 67.0 mm (5 hours), 45.1 mm (6 hours), 35.8 mm (7 hours), 43.0 mm (11 hours), 84.7 mm (12 hours), 130.3 mm (13 hours), 192.8 mm (23 hours), 180.5 mm (24 hours), and 107. 5 mm (25 hours).

This study uses the method of maximum likelihood (Stedinger *et al.* 1993) to estimate the parameters of GPDs from the rainstorm events having the annual maximum rainfall depths corresponding to the above fixed rainfall durations. The occurrence probabilities of $E_*°$ are also modeled by the Poisson distribution and the GPO in the study (please refer to Table S4, Supplementary material for details, available online). It is observed that the parameters of statistical models are also dependent on the value of rainfall depth threshold, in particular when they are estimated for short-duration rainstorm events. Note that the location parameters of the GPD were set at the rainfall depth thresholds.

In this study, the results for the rainfall duration of 1 hour were based on rainfall characteristics of rainstorm events with the fixed rainfall durations of 1 and 2 hours. Likewise, the parameters for the rainfall durations of 6, 12, and 24 hours were estimated from the entire rainstorm events with the fixed rainfall durations of 5–7, 11–13, and 23–25 hours, respectively. Followed by the adopted rainfall depth thresholds, the number of annual maximum rainstorm events can be considered as the specific value smaller than 50, which is the length of rainfall data from 1961 to 2010. It implies that some years have no rainstorm events having the fixed rainfall durations with rainfall depths over the rainfall depth thresholds. When it comes to the number of rainstorm events under the rainfall depth threshold of 20 mm, it is found there are not enough rainstorm events and the corresponding rainfall depth does not fit well with the GPD. Thus, this study estimates the rainfall depths from the rainfall characteristics of rainstorm events under the depth thresholds of 0, 1, 5, and 10 mm and applied the GPO to characterize the number of rainstorm events because of inequality between the mean and variance. The results are summarized in Table 2.

This study analyzed the rainfall characteristics of the observed rainfall data during 50 years from 1961 to 2010 and reviewed each maximum rainfall depth of the retrieved rainstorm events corresponding to the fixed rainfall durations. The details are as follows: 32.4–56.4 mm (1–2 hours), 69.4–73.7 mm (5–7 hours), 123.1–124.9 mm (11–13 hours), and 187.7–194.9 mm (23–25 hours). It was found

Table 2 | Rainfall DDF relationships under rainfall depth thresholds of 0, 1, 5, and 10 mm
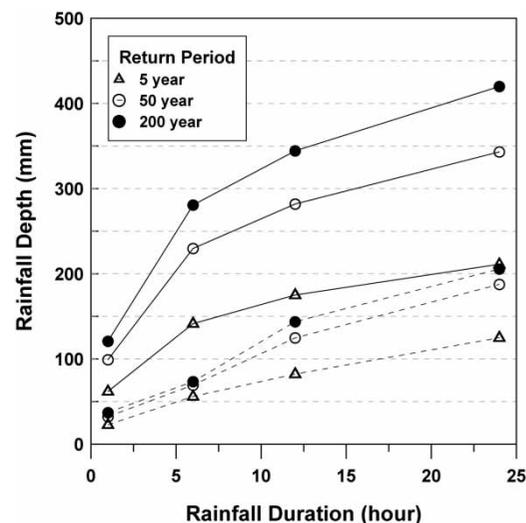
Rainfall depth threshold (mm/storm)	Rainfall duration (hour)	Return period (year)						
		2	5	10	20	50	100	200
0	1	17.41	22.76	25.99	28.88	32.37	34.82	37.10
	6	44.66	56.01	61.43	65.47	69.43	71.67	73.45
	12	56.83	82.38	97.18	110.01	124.86	134.80	143.78
	24	68.79	125.01	150.70	169.51	187.65	197.81	205.72
1	1	20.27	31.29	38.18	44.55	52.49	58.22	63.78
	6	44.10	55.99	61.57	65.69	69.68	71.91	73.66
	12	56.30	82.10	97.04	109.97	124.92	134.95	144.01
	24	69.79	126.01	151.70	170.51	188.65	198.81	206.72
5	1	12.49	26.75	34.99	42.07	50.16	55.51	60.29
	6	39.37	54.61	61.46	66.35	70.93	73.42	75.29
	12	50.37	78.50	94.31	107.77	123.06	133.13	142.15
	24	70.64	127.72	153.90	173.10	191.62	202.01	210.09
10	1	–	24.38	37.05	46.76	56.43	62.00	66.42
	6	31.98	53.73	62.58	68.51	73.71	76.34	78.23
	12	46.97	77.55	94.41	108.40	123.82	133.66	142.22
	24	71.70	129.16	155.99	175.78	194.93	205.69	214.05

that the maximum $\tau*$-hr rainfall depth of actual rainstorm events over the record period of 50 years were similar to the GPD-based rainfall quantiles for the return period of 50 years, which were estimated from a storm event-based frequency analysis method.

In Case-II, the overall procedure of frequency analysis is the same as Case-I, but there is a difference in terms of consideration of rainstorm events corresponding to any rainfall durations under the selected rainfall depth thresholds (please refer to Table S5, Supplementary material for details, available online). Here, rainstorm events having a small amount of rainfall depths corresponding to all the rainfall durations were considered to estimate rainfall depths followed by different return periods. It means that the fitted generalized Pareto distributions could be skewed under the condition of rainfall depth thresholds such as 0, 1, and 5 mm. For example, the rainfall quantiles for different return periods are estimated as 422.8 mm for 10 year, 630.9 mm for 20 years, and 1,056.7 mm for 50 years under the rainfall depth threshold of 5 mm.

To examine the applicability of the proposed methodology, a comparison is made of the results of Case-I and Case-II with those from the conventional frequency analysis method presented in the form of the Rainfall Frequency Atlas of Korea (MOCT 2000), which is based on GEV-

based rainfall DDF relationships. The comparison results are shown in Figures 2 and 3. It can be seen that the rainfall quantiles estimated from the conventional frequency analysis method are considerably higher than those from this study based on rainfall characteristics of actual rainstorm events (as shown in Figure 2). This is expected because the conventional frequency analysis method does not address

Figure 2 | Rainfall depth–duration–frequency relationships by the conventional frequency analysis method (solid line) and this study (Case-I) under the rainfall depth threshold of 10 mm/storm (dashed line).
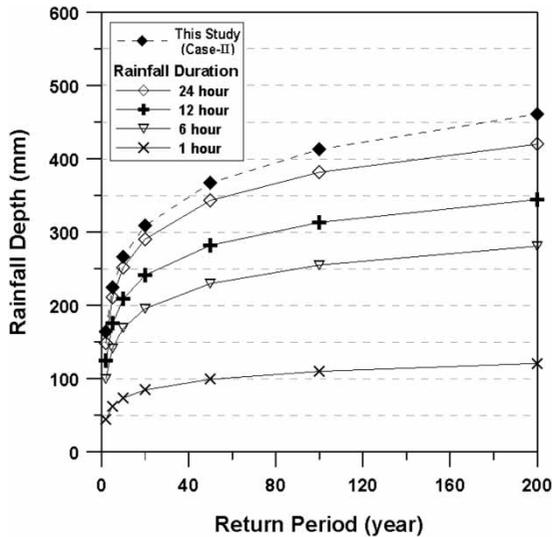
Figure 3 | Comparison results of rainfall quantiles between the conventional frequency analysis method (solid line) and this study (Case-II) with any rainfall durations (dashed line).

the real duration of the rainstorm event but the largest portions of rainfall for the pre-determined, fixed time intervals. As a reference, the results from the conventional frequency analysis method for the return period of 50 years are: 99.2 mm (1 hour), 229.8 mm (6 hours), 282.0 mm (12 hours), and 343.2 mm (24 hours).

It should be noted that the rainfall DDF relationships from the conventional method are not composed entirely of rainstorm events; rather they tend to capture rainfall accumulations higher than rainfall depths of independent rainstorm events for the pre-selected time intervals. This implies that the time intervals considered in the conventional frequency analysis method are not directly related to real rainstorm durations. Thus, it is likely to overestimate the rainfall quantiles for different rainfall durations and return periods.

When it comes to the case with any rainfall durations, the estimated rainfall quantiles under the rainfall depth threshold of 10 mm appear to be similar to those from the conventional frequency analysis method for the rainfall duration of 24 hours (as shown in Figure 3). This indicates that the conventional method can reflect rainfall characteristics of actual rainstorm events if longer rainfall duration (like 24 hours) is considered, although it ignores real rainfall durations of rainstorm events and simply uses rainfall accumulation within the fixed time intervals. This finding

is consistent with the results from Yoo *et al.* (2016), where longer rainfall duration would have a smaller difference between series of the annual maximum independent rainfall event and that of the annual maximum fixed-duration one. In this sense, it seems that the rainfall quantiles for the rainfall duration of 24 hours could be considered as design rainfalls under the conventional frequency analysis for the purpose of hydraulic design.

CONCLUSIONS

Here, a storm event-based frequency analysis method was developed, and a new approach for determining design rainfall was then proposed from storm event analysis with rainfall depth thresholds. The method is applied to analyze hourly rainfall data from 1961 to 2010 at Seoul rain gauge station, Korea. From the retrieved rainstorm events, the relationships between rainfall intensity, duration, and frequency were identified and rainfall quantiles were estimated. The applicability of the proposed methodology was examined by comparing the estimated rainfall quantiles with those obtained from the conventional frequency analysis method.

The proposed method uses the number of rainstorm events and their rainfall depths and durations. Thus, with the proper rainfall depth thresholds, it was possible to estimate rainfall quantiles based on the occurrence probability of rainstorm events having different rainfall characteristics. The proposed approach for rainfall quantiles was found to improve the accuracy of describing rainfall depths and durations of design rainfalls. Moreover, it was found that the rainfall quantiles estimated from the actual rainstorm events with the fixed rainfall durations were much lower than those derived from the conventional frequency analysis method. However, the rainfall quantiles obtained from this study were more realistic with respect to maximum rainfall depths of actual rainstorm events corresponding to the specific rainfall durations, which were retrieved from the observed rainfall data over a 50-year period. It was also found that the conventional frequency analysis method somewhat described the real rainfall duration of a rainstorm event in the case that the rainfall quantiles for the rainfall duration of 24 hours were considered as design rainfalls.

For the purpose of hydraulics design, a more realistic assessment of design rainfalls and a clearer understanding of actual system performance are especially important. The conventional frequency analysis method simply measures rainfall accumulation within the pre-selected time intervals which are just any portions of rainstorm events and thus cannot consider rainfall characteristics of rainstorm events in the process of developing rainfall DDF curves. This study applied statistical models to mitigate the limitations of conventional rainfall DDF analysis, which could describe the occurrence probability of rainstorm events and their rainfall depths corresponding to the real rainfall durations. It is expected that the new approach offers a viable alternative to the conventional frequency analysis method with respect to various applications of a storm event-based rainfall analysis to hydraulics design. For future extensions of our research works, we will further consider issues of stochastic precipitation generation, dependency of storm characteristics, and parameter uncertainty. It is noted that the estimated design rainfalls with each parameter of the proposed method can be evaluated from a Bayesian point of view (Thiemann *et al.* 2001; Kim *et al.* 2017; Wani *et al.* 2017) and further Markov chain Monte Carlo (MCMC) methods (Wang *et al.* 2017).

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