Explicit prediction of expanding channels hydraulic jump characteristics using gene expression programming approach

Kiyoumars Roushangar and Roghayeh Ghasempour

ABSTRACT

Hydraulic jump is a useful means of dissipating excess energy of a supercritical flow so that objectionable scour downstream is minimized. The present study applies gene expression programming (GEP) to estimate hydraulic jump characteristics in sudden expanding channels. Three types of expanding channels were considered: channels without appurtenances, with a central sill, and with a negative step. 1,000 experimental data were considered as input data to develop models. The results proved the capability of GEP in predicting hydraulic jump characteristics in expanding channels. It was found that the developed models for channel with a central sill performed better than other channels. In the jump length prediction, the model with input parameters Fr_1 and $(y_2—y_1)/y_1$, and in the sequent depth ratio and relative energy dissipation prediction the model with input parameters Fr_1 and y_1/B led to more accurate outcomes (Fr_1, y_1, y_2, and B are Froude number, sequent depth of upstream and downstream, and expansion ratio, respectively). Sensitivity analysis showed that Fr_1 had the key role in modeling. The GEP models were compared with existing empirical equations and it was found that the GEP models yielded better results. It was also observed that channel and appurtenances geometry affected the modeling.

Key words | central sill, energy dissipator channels, GEP, hydraulic jump characteristics, negative step

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INTRODUCTION

The correct estimation of hydraulic jump characteristics is of great importance in hydraulic engineering, as it directly affects the planning, design and management of hydraulic structures. The hydraulic jump is a natural phenomenon that occurs when supercritical flow is forced to change to subcritical flow by an obstruction to the flow. Hydraulic jump has been used to prevent scouring downstream from the hydraulic structures by dissipating excess energy in water flowing over these structures. It is also used to raise the water level on the downstream to provide the requisite head for diversion into channels. Depending on the geometry of the channel and tailwater conditions, the hydraulic jump can assume several distinct forms. So far, various studies have been conducted to explain the complex phenomenon of the hydraulic jump and to estimate its characteristics. Hughes & Flack (1983) investigated the effect of various roughness designs on the hydraulic jump properties in stilling basins. Hager & Bremen (1989) investigated the influence of wall friction on the sequent depths ratio. Bhutto *et al.* (1989) developed analytical solutions for computing sequent depth and relative energy loss for free hydraulic jump in sloping and horizontal rectangular channels. Finnemore & Franzini (2002) stated that the characteristics of the hydraulic jump depend on the Froude number. Negm (2000) studied the hydraulic performance of rectangular and radial stilling basins, where the latter stand for the diverging channels. Ayanlar (2004) studied the effect of corrugated beds on hydraulic jump

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properties with altering flow characteristics. Bilgin (2005) studied the correlation and distribution of shear stress for turbulent flow in a smooth rectangular basin. However, the existing equations rely on a limited database, untested model assumptions, and a general lack of field data, and do not show the same results under variable flow conditions. These issues cause uncertainty in the prediction of hydraulic jump phenomenon; therefore, it is critical to utilize methods which are capable of predicting hydraulic jump characteristics within the basins under varied hydraulic conditions.

In recent years artificial intelligence tools (e.g. artificial neural networks, neuro-fuzzy models, genetic programming (GP) and support vector machine) have been used for the assessment of the accuracy of hydraulic and hydrologic complex phenomena, such as prediction of suspended sediment concentration (Kisi & Shiri 2012), prediction of scour caused by 2D horizontal jets (Karbasi & Azamathulla 2017), modeling the rainfall–runoff process (Nourani *et al.* 2011), predicting total bed material load (Roushangar *et al.* 2014a), prediction of lake level (Aytek *et al.* 2014), and predicting the mean flow velocity in alluvial channels (Kitsikoudis *et al.* 2015a, 2015b).

Genetic expression programming (i.e. gene expression programming (GEP)) approach has been applied in modeling various components of water resources systems including: developing stage-discharge curves (Azamathulla *et al.* 2011), forecasting daily lake levels (Kisi *et al.* 2012a), predicting energy dissipation over spillways (Roushangar *et al.* 2014b), rainfall–runoff process (Kisi *et al.* 2013), modeling bridge pier scour (Azamathulla *et al.* 2010), estimation of characteristics of a hydraulic jump over a rough bed (Karbasi & Azamathulla 2016a), estimation of daily suspended sediment load (Shiri & Kisi 2012) and suspended sediment modeling (Kisi *et al.* 2012b).

This study aimed to assess the capability of the GEP approach for modeling hydraulic jump properties in sudden expanding channels. The models were prepared under various input combinations (based on the hydraulic characteristics and geometry of the channels and applied appurtenances) in order to find the most appropriate input combination for modeling hydraulic jump characteristics in sudden expanding channels. Then, the accuracy of the best GEP model was compared with the accuracy of several existing empirical equations.

MATERIALS AND METHODS

Data sets

The data sets of laboratory experiments of hydraulic jump performed by Bremen (1990) and Gandhi (2014) were used in the present study, and were collected for three types of sudden expanding channels. The ranges of various parameters used in the experiments are listed in Table 1. The experiments of Bremen (1990) were carried out at the Laboratoire de Constructions Hydrauliques of the Ecole Polytechnique Federale de Lausanne (EPFL), which were intended for expanding channels without appurtenances, with a central sill, and with a negative step (see Figure 1). During experiments, the 17 m^3 upstream basin was supplied by two conduits of 0.30 m diameter (max. total flow discharge $Q = 375$ L/s). A 0.5 m wide and 10.8 m long prismatic rectangular and horizontal channel was connected to the basin. At the upstream channel extremity, a standard shaped 0.50 m wide and 0.70 m high spillway of design head $H_0 = 0.2$ m controlled the channel inflow. According to Figure 1(c) two kinds of symmetric and asymmetric abrupt expansions were considered in experiments. In Figure 1 the parameters of b_s, x_1, xe, x_2, indicate the length of central sill, non-dimensional toe position, the end section of the shorter and the longer lateral eddy starting from the expansion section, respectively. The length x_j corresponds to the distance from the expansion section to the jump end section. Adding this value to x_1 gives the jump length L_j. Also y_{a1}, and y_{ar} are flow depths along the expansion side walls. y_1 and y_2 are the sequent depth of upstream and downstream, b_1 and b_2 are the approach channel and expanded channel width, x_s shows the position of the sill, and s is the height of the step or sill.

The experiments of Gandhi (2014) were carried out at a rectangular channel made up of Perspex sheets and setup consisted of a constant head tank with volume of $3.6 \times 3.6 \times 3$ m^3. The symmetric shaped channel, sized $2.1 \times 0.445 \times 1.2$ m and the expansion ratio between 0.4 and 0.8 and flow Froude number ($2 < Fr < 9$) was used for experiments.

Gene expression programming

Ferreria (2001) developed GEP algorithm using fundamental principles of the genetic algorithms (GA) and GP. GAs are the heuristic search and optimization techniques that mimic

Table 1 | The range of experimental data

Series	Channel shape	Parameters: Fr_1 Froude number	B (b2/b1) Expansion ratio	Q (l/s) Water discharge	Y (y_2/y_1) Sequence depth ratio	Xs (cm) (sill position)	S (relative height of sill or step)	No. of data	Researcher
Channel without appurtenances									
1	asym	2.63–8.12	5	2–18.1	2.02–11.26	–	–	162	Bremen (1990)
2	sym	2.63–8.05	3	5.1–26.9	2.18–10.28	–	–	178	
3	asym	2.65–8.13	1.5	10–40	3.46–12.05	–	–	91	
1	sym	2–9	0.4	6.44–15	2–8	–	–	8	Gandhi (2014)
2	sym	2–9	0.5	12.3–12.8	2–9	–	–	8	
3	sym	2–9	0.6	6.44–10.6	2–9.5	–	–	8	
4	sym	2–9	0.8	6.3–9.8	2–10	–	–	8	
Channel with negative step									
1	asym	3.2–8.43	1.5	11–39.5	4.15–12.59	–	0.61–1.64	105	Bremen (1990)
2	asym	2.68–8.12	2	11–39.5	4.15–12.59	–	0.59–1.61	112	
3	sym	2.05–7.37	3	4.3–24	1.45–11.05	–	0.49–1.66	129	
Channel with central sill									
1	sym	3,5,7,9	3	31–225	2.89–8.81	20–80	0.6–3	78	Bremen (1990)
2	sym	3,5,7,9	2	31–225	3.07–9.88	20–80	0.6–3	75	
3	sym	3,5,7,9	1.5	31–225	3.15–10.63	20–80	0.6–3	60	

the process of natural evolution. Thus, GA implement the optimization strategies by simulating evolution of species through natural selection (Bhattacharjya 2012). GP, a branch of GA, is a method for learning the most 'fit' computer programs by means of artificial evolution, and GEP is an extension to GP that evolves computer programs of different sizes and shapes encoded in linear chromosomes of fixed length. The fundamental difference between the three algorithms resides in the nature of the individuals: in GAs the individuals are symbolic strings of fixed length (chromosomes); in GP the individuals are non-linear entities of different sizes and shapes (parse trees); and in GEP the individuals are encoded as symbolic strings of fixed length (chromosomes) which are then expressed as non-linear entities of different sizes and shapes (expression trees (ETs)).

One strength of the GEP approach is that the creation of genetic diversity is extremely simplified as genetic operators work at the chromosome level. Another strength of GEP consists of its unique, multigenic nature, which allows the evolution of more complex programs composed of several subprograms. As a result, the GEP surpasses the old GP system by a factor of 100–60,000 (Guven 2009). GEP mimics the biological evolution to create a computer program for simulating a specified phenomenon. The problems are encoded in linear chromosomes of fixed-length as a computer program (Ferreria 2001) which are then expressed or translated into ETs.

There are five major steps in the GEP algorithm: the terminal set, the function set, fitness function, control parameters, and stop condition. The first major step is to identify the set of terminals to be used in the individual computer programs. The major types of terminal sets contain the independent variables of the problem, the state variables of the system and the functions with no arguments. The second step is to determine the set of functions (+, −, ×, /, x^a, log(x), ln(x), sqrt(x), etc.). The third step is fitness measure, which identifies the way of evaluating how good a given program solves a particular problem. In the current study, root mean square error (RMSE) is taken as fitness function. The fourth step is the selection of certain parameters to control the runs. The control parameters contain the size of the population, the rate of crossover, etc. The last step is the determination of the criteria to terminate the run. There is a comparison between predicted values and actual values in subsequent steps. When the desired results in accord with

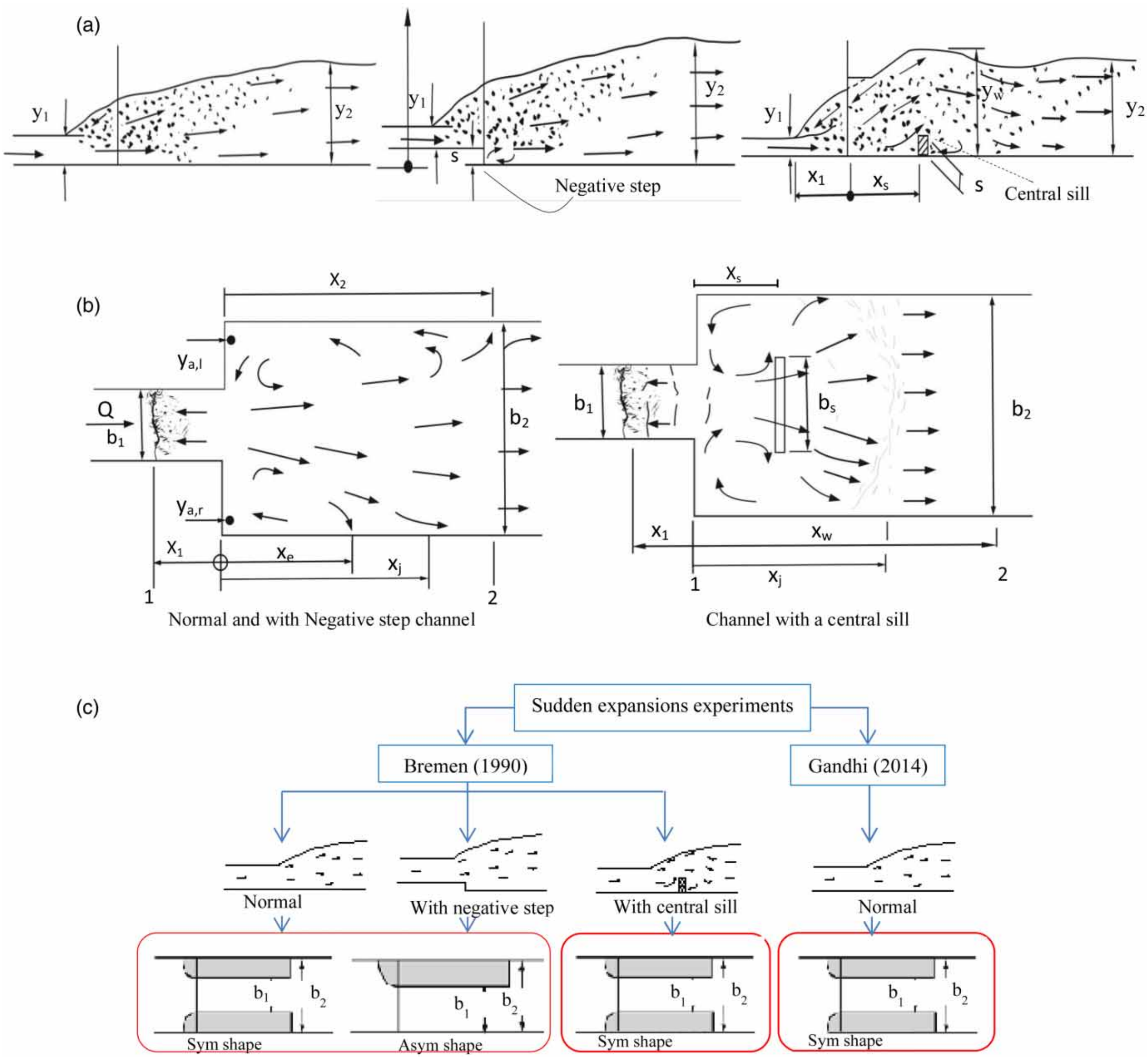

Figure 1 | Hydraulic jump in sudden expansion basin: (a) side view, (b) plan view (Bremen 1990), and (c) schematic view of the experiments.

error criteria initially selected are found, the GEP process is terminated. If desired error criteria could not be achieved, some chromosomes are chosen by a method called roulette wheel sampling and they are mutated to obtain new chromosomes. After the desired fitness score is found, this process terminates and then the chromosomes are decoded for the best solution of the problem (Teodorescu & Sherwood 2008).

Classical equations

The majority of research work on hydraulic jump has been concentrated on channels and flumes to develop the governing equations describing the complex phenomenon of the hydraulic jump process. These proposed methods are based on statistical correlations, a combination of the theoretical models, logical assumptions, and the experimental information. Under variable hydraulic conditions, the obtained results of these formulas often differ from each other. The semi-theoretical formulas used in this study are listed in Table 2.

Performance criteria

In this study, the total data were divided into three sets: the training, validation, and testing set (see Table 3). 60% of the

Table 2 | Utilized equations in the study

Researcher	Equation	Consideration	Equation number
Sequent depth ratio			
Kusnetzow (1958)	$Y = \frac{0.5}{B} K_K \left[\sqrt{1+8BFr_1^2} - 1\right]$	$K_K = 0.8 - \left(0.9 - \frac{1}{B}\right) \times 0.15$ For free jump in expanding channel without appurtenances	(1)
Hager (1985)	$\frac{Y^* - Y}{Y^* - 1} = \left(1 - \frac{1}{\sqrt{B}}\right) \times [1 - th(1.9X_1)]$	$Y^* = 0.5\left[\sqrt{1+8Fr_1^2} - 1\right]$ For free jump in expanding channel without appurtenances	(2)
Herbrand (1973)	$Y = Fr_1 \sqrt{\frac{2}{B} - \frac{1}{2B}}$	$Y = \frac{y_2}{y_1}$ For free jump in expanding channel without appurtenances	(3)
Hydraulic jump's length			
Smetana (1934)	$Lj = 6 \times Hj$	$Hj = y_2 - y_1$ For free jump in horizontal smooth bed	(4)
Safranez (1929)	$Lj = 6y_1 \times Fr_1$	For free jump in horizontal smooth bed	(5)
Bakhmeteff & Matezke (1936)	$Lj = 5 \times Hj$	For free jump in horizontal smooth bed	(6)
Hager (1985)	$Lj = \left\{1 + \left(1 - \frac{1}{\sqrt{B}}\right) \times [1 - th(1.9X_1)]\right\} Lj^*$	$Lj^* = y_1 \times 220 \times th\left(\frac{Fr_1 - 1}{22}\right)$ th(i) = [exp(i) – exp(– i)]/[exp(i) + exp(– i)].	(7)
Silvester (1964)	$Lj = 6.02 \times Hj$	For free jump in horizontal smooth bed	(8)
Loss of energy			
Agarwal (2001)	$\frac{\Delta E}{E_1} = \frac{Fr_1^2[1 - B^2 \times (y_1/y_2)^2] - 2((y_2/y_1) - 1)}{2 + Fr_1^2}$	For free jump in expanding channel	(9)

whole data were used for training, 20% of data were used for validating, and the rest 20% of data were used to test the model. The training set trains the scheme on the basis of a minimization criterion and the validation set is used as a stopping criterion for training to avoid overfitting to the data. The testing set is used to evaluate the generated model and assess its generalization capability (Kitsikoudis *et al.* 2015a, 2015b). It should be noted that the order of the data sets was selected in a way such that the training data set contains a representative sample of all the behavior in the data in order to obtain a model with higher accuracy. One method for finding a good training set which can give good accuracy both in training and testing sets, is an instance exchange which starts with a random selected training set (Bolat & Yildirim 2004). Table 3 shows several statistical measures for the training, validation, and testing sets. Evaluating the performance of a model is commonly done using different statistical indexes. In this study, the efficiency of the GEP models was evaluated using four statistical indexes: the determination coefficient (DC), correlation coefficient (R), mean absolute error (MAE), and RSMEs. The DC describes the relative assessment of the model performance in dimensionless measures; R indicates the linear dependence between observation and predicted values and should not be applied alone as a performance criterion (Legates & McCabe 1999); MAE is a quantity used to measure how close predictions are to the measured values; and RMSE is the coefficient used to point the average difference between predicted values and

Table 3 | Statistical measures for the training, validation, and testing sets

Statistical measures		Parameters				
		Fr_1	Y	y_1/B	S/y_1	L_j/y_1
Channel without appurtenances (sym shape)						
Train set	Minimum value	2	2	4.4	–	1.174
	Maximum value	9	10.28	20.2	–	6.54
	Mean value	4.137	4.06	11.19	–	3.35
	Standard deviation	1.137	1.12	8.12	–	1.82
	Skewness coefficient	−0.177	0.31	0.048	–	0.37
Validation set	Minimum value	2.60	2.15	4.45	–	1.18
	Maximum value	7.97	10.17	19.9	–	6.477
	Mean value	4.12	4.14	11.42	–	3.417
	Standard deviation	1.123	1.106	8.93	–	1.797
	Skewness coefficient	−0.135	0.236	0.042	–	0.2827
Test set	Minimum value	2.35	2.21	4.45	–	1.82
	Maximum value	7.95	10.15	19.85	–	6.45
	Mean value	4.13	4.19	11.38	–	3.424
	Standard deviation	1.126	1.10	8.96	–	1.792
	Skewness coefficient	−0.151	0.252	0.0421	–	0.405
Channel with negative step (asym shape)						
Train set	Minimum value	2.68	4.15	4.6	0.59	1.49
	Maximum value	8.43	12.59	27.21	1.64	8.84
	Mean value	5.53	7.61	14.57	0.725	4.9
	Standard deviation	1.26	1.96	5.78	0.211	1.49
	Skewness coefficient	−0.27	0.907	1.57	0.00061	0.91
Validation set	Minimum value	2.72	4.23	4.68	0.598	1.51
	Maximum value	8.31	12.41	26.82	1.616	8.76
	Mean value	5.38	7.4	14.17	0.74	4.767
	Standard deviation	1.13	1.7578	5.1839	0.189	1.336
	Skewness coefficient	−0.268	0.9	1.558	0.00061	0.905
Test set	Minimum value	2.72	4.21	4.66	0.592	1.51
	Maximum value	8.35	12.42	26.9	1.624	8.75
	Mean value	5.34	7.35	14.09	0.70009	4.735
	Standard deviation	1.128	1.74	5.17	0.188	1.34
	Skewness coefficient	−0.26	0.902	1.511	0.00058	0.8762
Channel with central sill (sym shape)						
Train set	Minimum value	3	2.89	1.23	0.6	10
	Maximum value	9	10.63	5	3	29.7
	Mean value	6.06	6.33	2.518	1.38	14.81
	Standard deviation	2.022	2.18	1.18	0.71	6.57
	Skewness coefficient	0.00017	0.0006	0.668	0.0008	0.0104
Validation set	Minimum value	3.05	3.13	1.25	0.61	10.16
	Maximum value	8.68	10.13	4.95	2.89	27.6
	Mean value	5.62	5.82	2.3351	1.279	13.734
	Standard deviation	1.013	1.092	1.01	0.355	5. 98
	Skewness coefficient	0.00012	0.00042	0.471	0.00056	0.00734
Test set	Minimum value	3.12	3.056	1.279	0.624	10.4
	Maximum value	8.65	9.98	4.805	2.8833	28.545
	Mean value	5.51	5.85	2.289	1.254	13.465
	Standard deviation	1.12	1.207	0.9536	0.393	5.36
	Skewness coefficient	0.00013	0.00044	0.5108	0.00061	0.0075

measured values as depicted in Equation (9). The smaller the RMSE and MAE and the larger the DC and R, the higher the accuracy of the model.

$$DC = 1 - \frac{\sum_{i=1}^{N}(l_o - l_p)^2}{\sum_{i=1}^{N}(l_o - \overline{l_p})^2},$$
$$R = \frac{\sum_{i=1}^{N}(l_o - \overline{l_o}) \times (l_p - \overline{l_p})}{\sqrt{\sum_{i=1}^{N}(l_o - \overline{l_o})^2 \times (l_p - \overline{l_p})^2}},$$
$$\mathrm{RMSE} = \sqrt{\sum_{i=1}^{N}\frac{(l_o - l_p)^2}{N}},$$
$$\mathrm{MAE} = \frac{1}{N}\sum_{i=1}^{N}|l_o - l_p| \quad (10)$$

where l_o, l_p, $\overline{l_o}$, $\overline{l_p}$, N are the measured values, predicted values, mean measured values, mean predicted values and number of data samples, respectively.

Predicting the intended parameter using non-normalized data may lead to undesirable results, therefore, in this study, all utilized data were normalized before modeling by scaling between 0.1 and 1. This will increase the training speed and the capability of the model. The following equation was used to normalize the utilized data in this study:

$$x_n = 0.1 + 0.9 \times \left(\frac{x - x_{min}}{x_{max} - x_{min}}\right) \quad (11)$$

where x_n, x, x_{max}, and x_{min} are the normalized value of variable x, the original value, the maximum and minimum of variable x, respectively.

Simulation and model development

Input variables

The efficiency of the models throughout the modeling process can be affected using various parameters as input combinations. Therefore, the appropriate selection of parameters is an important step during modeling process. Figure 1(b) demonstrates the quantities measured for hydraulic jumps in sudden expanding channels with and without appurtenances. The jump pattern and energy dissipation can be affected by the following parameters:

$$f(\mathrm{y_1, y_2, V_1, L_j, E_L}, \mu, \mathrm{g}, \rho, \mathrm{b_1, b_2, s}) = 0 \quad (12)$$

where V_1 is pre-jump velocity, μ is dynamic viscosity of water, g is acceleration due to gravity, L_j is length of jump, ρ is density of water, E_L is $E_1 - E_2$ in which E_1 and E_2 are energy per unit weight before and after the jump.

From dimensional analysis and using parameters y_1, g and μ as repeating variables, Equation (12) can be represented as following:

$$f\left(\frac{y_2}{y_1}, \frac{E_L}{E_1}, \frac{L_j}{y_1}, \frac{b_1}{y_1}, \frac{b_2}{y_1}, \frac{v_1^2}{gy_1}, \frac{\rho v_1 y_1}{\mu}, \frac{s}{y_1}\right) = 0 \quad (13)$$

Equation (13) can be expressed as:

$$f\left(\frac{y_2}{y_1}, \frac{E_L}{E_1}, \frac{L_j}{y_1}, B, Fr_1, R_n, \frac{s}{y_1}\right) = 0 \quad (14)$$

where Fr_1, R_n and $B(=(\mathrm{b_1/b_2}))$ are flow Froude number, flow Reynolds number and ratio of expansion, respectively. Experimental studies by Elevatorski (2008) and Ranga Raju *et al.* (1980) revealed that hydraulic jump characteristics can be affected by Froude number only and Reynolds number has no significant impact on prediction of the hydraulic jump characteristics. Also, Hager (1992) showed that the length of hydraulic jump depends on the height of jump and flow Froude number. Therefore, in this study, several models based on channel and appurtenances geometry and characteristics of upstream flow were considered for prediction of hydraulic jump characteristics (see Table 4).

RESULTS AND DISCUSSION

GEP models

In the present study a soft computing approach (GEP) has been trained for hydraulic jump characteristics prediction in sudden expanding channels. Basic arithmetic operators (+, −, *, /) and several mathematical functions (exp, X^2, X^3, $\sqrt{}$, $\sqrt[3]{}$) were used as the GEP function set. The architecture of the chromosomes including number of chromosomes (25-30-35), head size (7–8) and number of genes (3–4), were selected and different combinations of the mentioned parameters were tested. The model was run for a number of generations and was stopped when there was no significant change in the fitness function value and coefficient of correlation. It is

Table 4 | GEP developed models

Hydraulic jump characteristics					
Sequent depth ratio Y		**Length of hydraulic jump L_j/y_1**		**Loss of energy $\Delta E_L/E_1$**	
Model	**Input variables**	**Model**	**Input variables**	**Model**	**Input variables**
S(I)	Fr_1	L(I)	Fr_1	E(I)	Fr_1
S(II)	Fr_1, y_1/B	L(II)	Fr_1, $(y_2\text{-}y_1)/y_1$	E(II)	Fr_1, y_1/B
S(III)	Fr_1, S/y_1	L(III)	Fr_1, y_2/y_1	E(II)	Fr_1, S/y_1
S(IV)	Fr_1, B	L(IV)	Fr_1, S/y_1		
		L(V)	Fr_1, B		
		L(VI)	Fr_1, y_1/B		

observed that the model with number of chromosomes of 30, head size of 7, and number of genes of 3 yielded better results. Also addition and multiplication were tested as linking functions and it was found that linking the sub-ETs by addition represented better fitness values. One of the important steps in preparing the GEP model is to choose the set of genetic operators. In the current study a combination of all genetic operators (recombination, mutation, transposition, and crossover) was used for this aim. Mutations can occur anywhere in the chromosome. However, the structural organization of chromosomes must remain intact. In GEP there are three kinds of recombination: one-point, two-point, and gene recombination. In all cases, two parent chromosomes are randomly chosen and paired to exchange some material between them. The transposable elements of GEP are fragments of the genome that can be activated and jump to another place in the chromosome. There are three kinds of transposable elements in GEP model: short fragments with a function or terminal in the first position that transpose to the head of genes, except to the root (IS elements), short fragments with a function in the first position that transpose to the root of genes (root IS elements or RIS elements), and entire genes that transpose to the beginning of chromosomes (Ferreria 2001). Also, the inversion operator is restricted to the heads of genes. Here any sequence might be randomly selected and inverted. The certain rates of genetic operators which define a certain probability of a chromosome were determined based on a trial and error process. Each GEP model was evolved until there was no significant change in the fitness function value, and then the program was stopped. The parameters related to trained GEP models were then tuned using refining process. Refining process is a method to optimize the parameters and size of developed GEP-based models. Parameters of the optimized GEP model are shown in Table 5.

Table 5 | Optimized parameters of GEP models used in this study

Description of parameter	Setting of parameter
Function set	+, −, *, /, exp, X^2, X^3, $\sqrt{}$, $\sqrt[3]{}$
Chromosomes	30
Head size	7
Number of genes	3
Linking function	Addition
Fitness function error type	Root mean square error (RMSE)
Mutation rate	0.044
Inversion, IS and RIS transposition rate	0.1
One and two-point recombination rate	0.3
Gene recombination and transposition rate	0.1

Expanding channels without appurtenances

For predicting the hydraulic jump characteristics in expanding channel without any appurtenances, several models were developed using the upstream flow characteristics and expanding channel geometry as input variables. The results of the GEP models are shown in Table 6 and Figure 2. From the obtained results of statistical parameters (MAE,

Table 6 | Statistical parameters of the GEP models: channel without appurtenances

		Performance criteria							
		Validation				Test			
Channel shape	GEP models	R	DC	RMSE	MAE	R	DC	RMSE	MAE
Sequent depth ratio Y									
sym channel	S(I)	0.921	0.825	0.067	0.051	0.912	0.817	0.068	0.052
	S(II)	0.942	0.885	0.048	0.035	0.934	0.876	0.049	0.035
asym channel	S(I)	0.846	0.739	0.101	0.087	0.837	0.731	0.103	0.088
	S(II)	0.904	0.798	0.088	0.073	0.895	0.786	0.09	0.075
Length of hydraulic jump L_j/y_1									
sym channel	L(I)	0.772	0.612	0.102	0.079	0.762	0.608	0.109	0.081
	L(II)	0.840	0.705	0.091	0.063	0.832	0.697	0.093	0.064
	L(III)	0.792	0.689	0.094	0.071	0.784	0.676	0.095	0.070
	L(VI)	0.833	0.701	0.092	0.067	0.824	0.693	0.094	0.069
Loss of energy E_L/E_1									
sym channel	E(I)	0.895	0.846	0.059	0.042	0.886	0.838	0.060	0.044
	E(II)	0.944	0.901	0.046	0.033	0.936	0.891	0.047	0.034
asym channel	E(I)	0.837	0.705	0.121	0.096	0.828	0.694	0.123	0.097
	E(II)	0.868	0.767	0.096	0.073	0.859	0.758	0.098	0.075

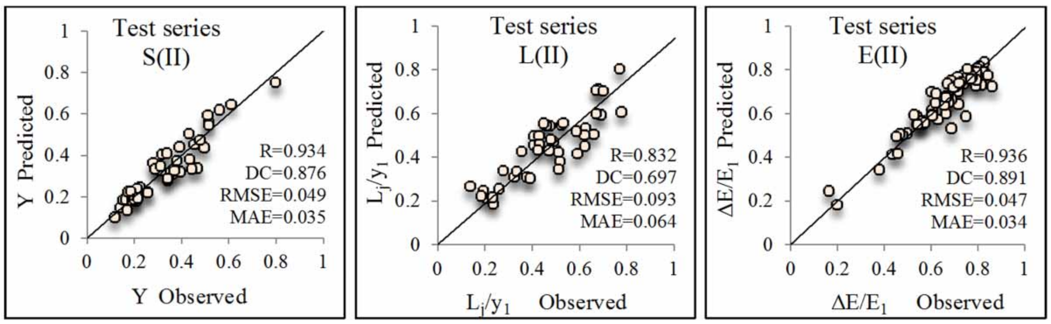

Figure 2 | Comparison of observed and predicted hydraulic jump characteristics: channel without appurtenances.

RMSE, R, and DC) it can be stated that among developed models, the model $S(II)$ with input parameters Fr_1 and y_1/B for modeling the sequent depth ratio and relative energy dissipation performed more successfully than the other models. It was deduced that using y_1/B as an input parameter improved the efficiency of the models, which confirms the importance of the expansion ratio in hydraulic jump characteristics estimating process in expanding channels. For the jump length, the superior performance was obtained for the model $L(II)$ with input parameters Fr_1, $(y_2{-}y_1)/y_1$. It was also observed that the obtained results from model $L(VI)$ with variables of Fr_1 and y_1/B were approximately similar to the superior model. It could be inferred that the applied method can successfully predict the jump length using only the upstream flow characteristic as input data. According to the results of Table 6, for the channel without appurtenances, the models of a symmetric channel yielded better predictions than an asymmetric channel. Figure 2 shows the verification between measured and estimated values of test series for the best models.

Expanding channels with appurtenances

Expanding channels with a negative step

The results of GEP models for predicting the hydraulic jump characteristics in channel with a negative step are shown in Table 7 and Figure 3. The results of Table 6 indicated that for

Table 7 | Statistical parameters of the GEP models: channel with appurtenances

Channel shape	GEP models	Performance criteria							
		Validation				Test			
		R	DC	RMSE	MAE	R	DC	RMSE	MAE
Channel with negative step									
Sequent depth ratio Y									
sym channel	S(I)	0.837	0.723	0.109	0.069	0.830	0.720	0.110	0.069
	S(II)	0.932	0.852	0.059	0.044	0.924	0.848	0.061	0.045
	S(III)	0.897	0.852	0.092	0.053	0.894	0.848	0.094	0.055
asym channel	S(I)	0.923	0.877	0.075	0.056	0.915	0.873	0.077	0.055
	S(II)	0.961	0.925	0.048	0.036	0.953	0.921	0.050	0.039
	S(III)	0.938	0.888	0.070	0.050	0.931	0.884	0.072	0.051
	S(IV)	0.921	0.878	0.074	0.053	0.914	0.874	0.076	0.055
Length of hydraulic jump L_j/y_1									
sym channel	L(I)	0.860	0.743	0.110	0.069	0.853	0.740	0.112	0.071
	L(II)	0.897	0.755	0.091	0.063	0.891	0.748	0.095	0.062
	L(III)	0.883	0.751	0.096	0.066	0.875	0.744	0.097	0.068
	L(IV)	0.904	0.767	0.087	0.058	0.896	0.758	0.089	0.059
	L(VI)	0.896	0.752	0.094	0.064	0.892	0.747	0.096	0.063
asym channel	L(I)	0.888	0.806	0.095	0.053	0.880	0.803	0.097	0.054
	L(II)	0.906	0.818	0.068	0.051	0.898	0.814	0.070	0.052
	L(III)	0.859	0.755	0.106	0.058	0.852	0.751	0.108	0.059
	L(IV)	0.867	0.794	0.098	0.054	0.860	0.792	0.100	0.055
	L(V)	0.814	0.719	0.110	0.060	0.807	0.716	0.112	0.061
	L(VI)	0.901	0.812	0.069	0.051	0.892	0.808	0.071	0.050
Loss of energy E_L/E_1									
sym channel	E(I)	0.719	0.697	0.112	0.061	0.714	0.694	0.114	0.062
	E(II)	0.935	0.882	0.050	0.042	0.927	0.878	0.053	0.043
	E(III)	0.758	0.708	0.092	0.056	0.752	0.705	0.094	0.057
asym channel	E(I)	0.724	0.703	0.096	0.054	0.717	0.700	0.098	0.055
	E(II)	0.974	0.938	0.041	0.036	0.966	0.933	0.043	0.038
	E(III)	0.946	0.900	0.055	0.041	0.938	0.896	0.057	0.042
Channel with central sill									
Sequent depth ratio Y									
sym channel	S(I)	0.951	0.915	0.061	0.044	0.943	0.908	0.063	0.043
	S(II)	0.972	0.955	0.046	0.034	0.964	0.948	0.047	0.035
	S(III)	0.951	0.928	0.068	0.041	0.942	0.923	0.070	0.042
	S(IV)	0.950	0.912	0.062	0.050	0.942	0.906	0.064	0.051
Length of hydraulic jump L_j/y_1									
sym channel	L(I)	0.868	0.753	0.094	0.063	0.861	0.748	0.096	0.064
	L(II)	0.896	0.829	0.062	0.049	0.886	0.821	0.064	0.051
	L(III)	0.867	0.756	0.095	0.062	0.860	0.751	0.097	0.063
	L(IV)	0.875	0.765	0.091	0.061	0.868	0.760	0.093	0.064
	L(V)	0.854	0.738	0.108	0.070	0.847	0.733	0.110	0.073
	L(VI)	0.881	0.778	0.073	0.054	0.873	0.773	0.075	0.056
Loss of energy E_L/E_1									
sym channel	E(I)	0.955	0.938	0.057	0.037	0.947	0.932	0.059	0.039
	E(II)	0.981	0.948	0.039	0.033	0.973	0.942	0.041	0.035
	E(III)	0.961	0.942	0.063	0.040	0.953	0.936	0.065	0.041

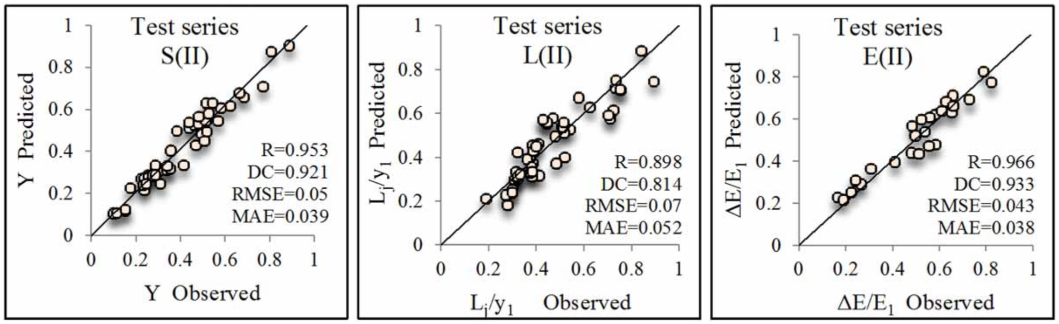

Figure 3 | Comparison of observed and predicted hydraulic jump characteristics: channel with a negative step.

modeling the sequent depth ratio and relative energy dissipation, the model $S(II)$ with parameters Fr_1 and y_1/B was more successful than the other models. It was observed that in the jump length prediction process in expanding channel with a negative step, the model $L(IV)$ with input parameters Fr_1, S/y_1 led to more accurate predictions in the symmetric channel, while for the case of the asymmetric channel the model $L(II)$ with parameters Fr_1, $(y_2-y_1)/y_1$ represented higher efficiency. Based on the results it could be inferred that for modeling the length of hydraulic jump in basins with a negative step, using parameters of y_2/y_1 and S/y_1 increased the accuracy of the models. Parameter S/y_1 confirms the importance of the relative height of the applied step in hydraulic jump characteristics predicting process in channels with negative step. In this case, the developed models represented higher accuracy for the asymmetric channel than the symmetric channel.

Expanding channels with a central sill

For modeling the hydraulic jump characteristics in sudden expanding channel with a central sill, some models based on the flow condition and applied appurtenance geometry were developed. Then, the influence of the central sill in the modeling process was assessed. All of the GEP models were trained and tested to carry out the sequent depth ratio, length of hydraulic jump and relative energy dissipation ratio prediction in expanding basin. The obtained results are given in Table 7 and Figure 4. From the results, it could be deduced that the model $S(II)$ with parameters Fr_1 and y_1/B had more accuracy in predicting the sequent depth ratio and relative energy dissipation, and the model $L(II)$ with parameters Fr_1, $(y_2-y_1)/y_1$ performed more successful in modeling the length of hydraulic jump. Also the model $L(VI)$ with input parameters Fr_1 and y_1/B represented reasonable results. It was observed that adding parameter S/y_1 to input combinations improved the model efficiency which verifies the importance of the relative height of the central sill in hydraulic characteristics estimation. Also parameter y_2/y_1 caused an increment in model accuracy. According to the results of Tables 6 and 7, the channel with a central sill for modeling hydraulic jump characteristics in expanding channels yielded better predictions in comparison with the channel without appurtenance or with a negative

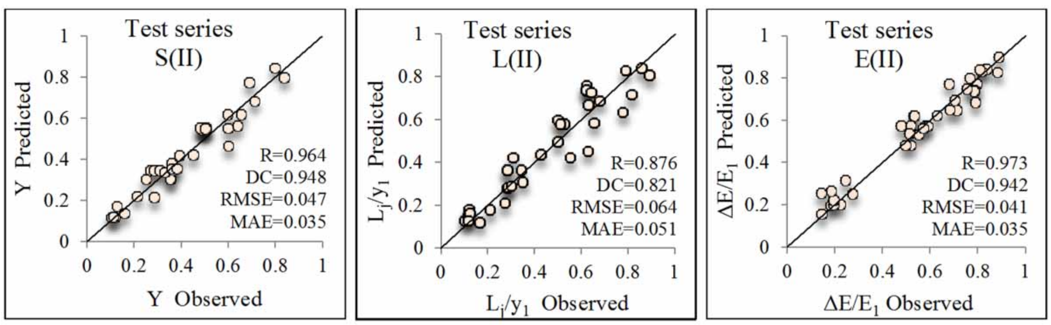

Figure 4 | Comparison of observed and predicted hydraulic jump characteristics: channel with a central sill.

Table 8 | The equations developed by the GEP model

Channel type	Equation	Equation number
Channel without appurtenances	$Y = Fr_1 + \frac{((y_1/B)^2 - (y_1/B))^3}{((y_1/B)/Fr_1) - (-0.2937 + (y_1/B))} + \left(\sqrt{\frac{y_1}{B}} \times Fr_1^2\right)^3 - \left(Fr_1 \times \frac{y_1}{B}\right)$	(15)
	$\frac{L_j}{y_1} = \left(Fr_1 + \left(\frac{y_2 - y_1}{y_1} - Fr_1\right) \times \left(\frac{y_2 - y_1}{y_1} - 6.4104 - Fr_1\right) \times Fr_1^2\right)^2 + \left(\frac{(y_2 - y_1)/y_1}{\sqrt[3]{Fr \times (y_2 - y_1)/y_1}} - Fr_1\right)^3$	(16)
	$\frac{\Delta E}{E_1} = \sqrt[3]{(Fr_1 + Fr_1^{1/4})^2} - Fr_1 + 5.536Fr_1 \times \left(Fr_1 \frac{y_1}{B}\right)^2 \left(\frac{y_1}{B} - Fr_1\right) - \left(Fr_1 \frac{y_1}{B}\right)^{4/3} \times \frac{y_1}{B}$	(17)
Channel with negative step	$Y = \frac{y_1}{B}(-1.173 + Fr_1)\left(\frac{y_1}{B} + Fr_1\right)Fr_1^3 + (2Fr_1 - 1.173)^6 \times \sqrt[3]{Fr_1} + Fr_1 - \left(Fr_1 \times \frac{y_1}{B}\right)\left(\frac{y_1}{B} - Fr_1\right) \times Fr_1^3$	(18)
	$\frac{L_j}{y_1} = Fr_1^2(0.603 - Fr_1)\frac{Fr_1^3}{((y_2 - y_1)/y_1) + 0.603} + \left(\sqrt[3]{\left(\frac{y_2 - y_1}{y_1}\right)^4} - Fr_1\right)^6 + \left(Fr_1 - \frac{y_2 - y_1}{y_1}\right)^3 + Fr_1$	(19)
	$\frac{\Delta E}{E_1} = \left((Fr_1 - \frac{y_1}{B}) + Fr_1\right)^{1/27} - Fr_1 + \left(-0.584\frac{y_1}{B} + 0.584\right)^2\left(\frac{y_1}{B} + 0.584\right)Fr_1 + \left(Fr_1\frac{y_1}{B} - \frac{y_1}{B}\right)^3\left(\frac{y_1}{B} - Fr_1\right)$	(20)
Channel with central sill	$Y = \frac{Fr_1}{(-1.0296(y_1/B)) - (Fr_1 - 0.2369)} + Fr_1 - 0.2369(-0.2369 + Fr_1) \times \sqrt{2Fr_1}$	(21)
	$\frac{L_j}{y_1} = \frac{y_2 - y_1}{y_1}(Fr_1 - 1) \times \left(\frac{Fr_1}{0.97}\right)^2 + \sqrt[3]{\left(\left(\frac{y_2 - y_1}{y_1}\right)^{2/3} - \frac{y_2 - y_1}{y_1}\right)^4} + \frac{y_2 - y_1}{y_1} - \frac{0.002((y_2 - y_1)/y_1)}{Fr_1^2}$	(22)
	$\frac{\Delta E}{E_1} = \left(\left(Fr_1 - \frac{y_1}{B}\right) + (-0.829 + \frac{y_1}{B}\right) \times (-0.829Fr_1) + (0.6872 - Fr_1)\left(-0.44\frac{y_1}{B} \times (-0.829 + \frac{y_1}{B}\right) + Fr_1$	(23)

step. The scatter plots of observed and predicted values for superior models are shown in Figure 4.

The key point of GEP is that it is able to give the explicit expression of the relationship between the variables. The mathematical expressions of GEP for all cases are as Table 8.

Sensitivity analysis

To investigate the impacts of different parameters applied in the hydraulic jump characteristics prediction via GEP, sensitivity analysis was done (for the channel with a central sill). In order to evaluate the effect of each independent parameter, the model was run with all input parameters and then, one of the input parameters was eliminated and the GEP model was re-run. Two error criteria (RMSE and MAE) were used as indications of the significance of each parameter. Table 9 shows the obtained results of sensitivity analysis. In Table 9, ΔRMSE and ΔMAE represent the percentages of added values to the error criteria for each eliminated parameter. Based on the results listed in Table 9, it can be deduced that the variable Fr_1 had the most significant impact on the hydraulic jump characteristics.

Combination of data

For estimating hydraulic jump characteristics in sudden expanding channels with different appurtenances, and for evaluating the applicability of the applied technique for a wide range of data; all data series of hydraulic jump were combined. Then, for predicting the parameters of Y, L_j/y_1 and $\Delta E_L/E_1$, as the dependent variables, the best GEP models were analyzed for the combined data set. The obtained results are given in Table 10 and Figure 5. Comparison between Tables 6, 7 and 10 indicated that the GEP models for combined data set did not show the desired accuracy especially for the jump length. It was found that analyzing data sets separately lead to more accurate results. Figure 5 illustrates the observed values vs. predicted values for test stages.

Comparison of the best GEP models with classical equations

The experimental data of test series were used to evaluate the applicability of several existing equations for hydraulic jump characteristics. The overall performance of each

Table 9 | Relative significance of each of input parameters of the best models (the channel with a central sill)

		Performance criteria Test			
Output parameters	**Input parameters**	**RMSE**	**MAE**	**ΔRMSE%**	**ΔMAE%**
Y	Fr_1, S/y_1, B, y_1/B	0.038	0.029	–	–
	without Fr_1	0.218	0.159	473	448
	without y_1/B	0.055	0.039	45	34
	without S/y_1	0.045	0.035	18	21
	without B	0.041	0.033	8	14
L_j/y_1	Fr_1, $(y_2\text{-}y_1)/y_1$, y_2/y_1, S/y_1, B, y_1/B	0.05	0.039	–	–
	without Fr_1	0.138	0.116	174	197
	without $(y_2\text{-}y_1)/y_1$	0.059	0.048	16	23
	without y_2/y_1	0.056	0.044	10	13
	without S/y_1	0.054	0.043	6	10
	without B	0.052	0.042	2	8
	without y_1/B	0.058	0.047	14	20
$\Delta E_L/E_1$	Fr_1, S/y_1, y_1/B	0.034	0.026	–	–
	without Fr_1	0.211	0.153	520	488
	without y_1/B	0.065	0.041	81	57
	without S/y_1	0.041	0.035	20	35

Table 10 | Statistical parameters of the GEP models for combined data

		Performance criteria							
		Validation				Test			
Output variable	**Input variable**	**R**	**DC**	**RMSE**	**MAE**	**R**	**DC**	**RMSE**	**MAE**
Y	Fr_1, y_1/B	0.874	0.756	0.088	0.0589	0.825	0.658	0.094	0.0607
L_j/y_1	Fr_1, $(y_2\text{-}y_1)/y_1$	0.571	0.309	0.111	0.0937	0.559	0.268	0.134	0.0946
$\Delta E_L/E_1$	Fr_1, y_1/B	0.882	0.775	0.084	0.0926	0.869	0.661	0.091	0.0598

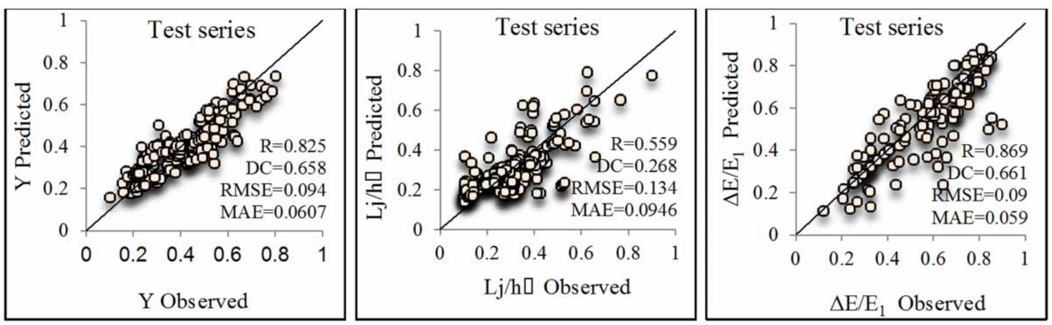

Figure 5 | Comparison of observed and predicted hydraulic jump characteristics; combined data.

equation is presented graphically in a plot of observed hydraulic jump characteristics against their computed values. Four evaluation criteria (R, DC, MAE, and RMSE) were used as indications of the accuracy of the equations. Then a comparison was performed among the best GEP models of a channel with a central sill and a channel without appurtenance and those equations. The results of the comparisons are plotted on Figure 6. According to the obtained results, for both types of channels, the utilized equations provided a reasonable fit to the experimental data for the sequent depth ratio and relative energy dissipation, while the hydraulic jump's length equations did not show reasonable agreement between the estimated and observed values. From the results of Figure 6, it could be

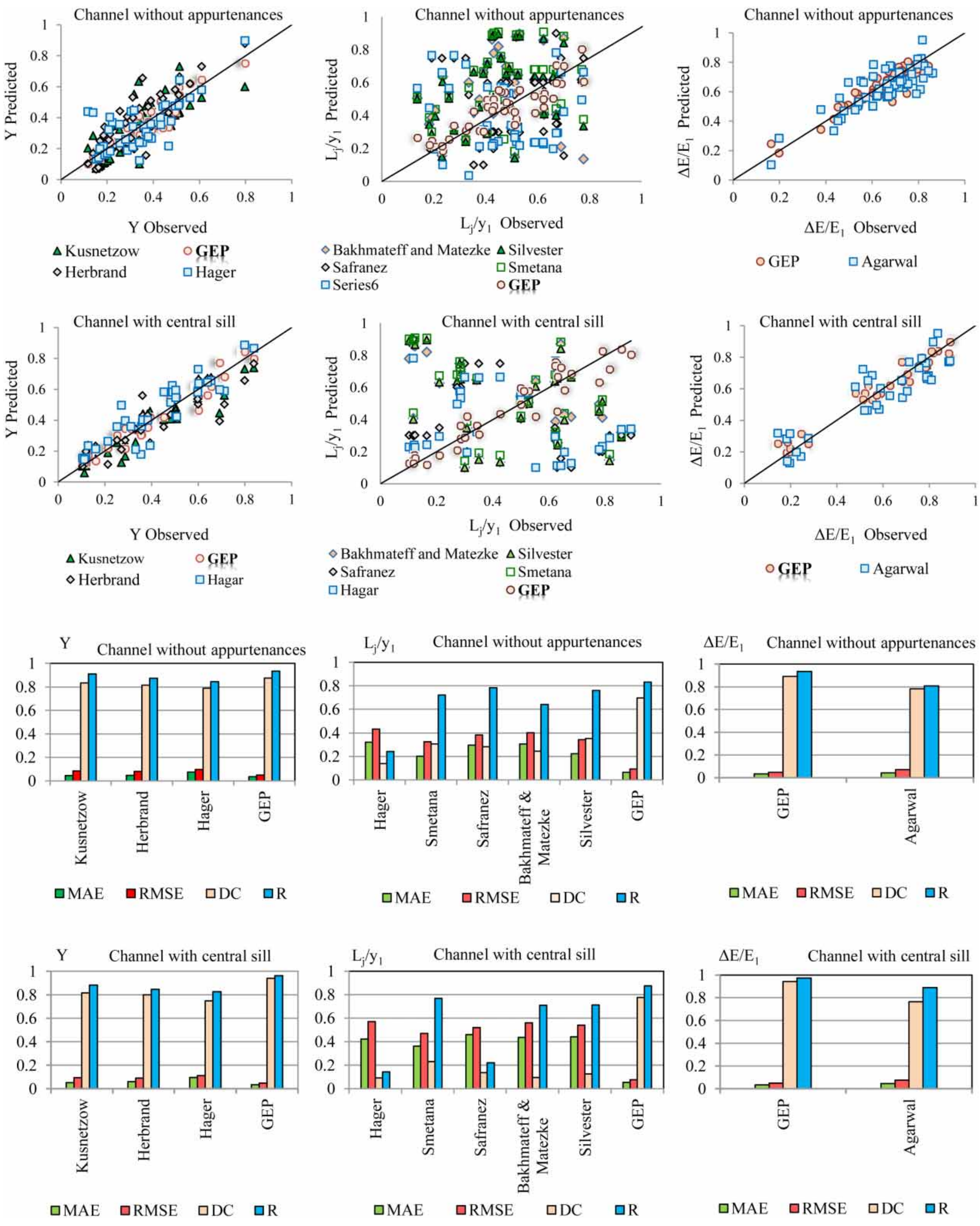

Figure 6 | Comparison of statistical parameters between formula and GEP-best models of channels without appurtenances and with central sill.

inferred that the semi-empirical formula led to better prediction in channel without appurtenance than channel with a central sill. However, for all hydraulic jump characteristics (Y, L_j/y_1, and $\Delta E_L/E_1$) the results obtained by the best GEP models were quietly compatible with the experimental data and they were in good agreement. It should be noted that the existing equations are developed based on special flow conditions, therefore, the application of equations are limited to special cases of their development and did not show uniform results under different conditions. The mentioned issue can be seen in Figure 6, which shows that the obtained results from equations differ from each other, and have less preciseness than proposed models in this study. However, despite the nonlinearity and uncertainties of the hydraulic jump process, the obtained results confirmed the applicability of GEP as an efficient approach in modeling of hydraulic jump characteristics in sudden expanding channels.

CONCLUSION

The modeling of the hydraulic jump characteristics has great importance since it directly affects the designing of hydraulic structures. In the present study, the capability of the GEP approach was verified for predicting hydraulic jump characteristics in sudden expanding channels. The GEP method was applied for three types of expanding channels: channels without appurtenances, with a central sill, and with a negative step. The capability of the selected approach was assessed for combined and separate data sets. A comparison was also made between GEP models and some semi-empirical equations from the literature. The obtained results proved the superior performance of GEP models over all of the semi-empirical equations in prediction of hydraulic jump characteristics in sudden expansion channel. In the jump length prediction the model with input parameters Fr_1 and $(y_2-y_1)/y_1$ and in the sequent depth ratio and relative energy dissipation prediction the model with input parameters Fr_1 and y_1/B were more accurate than the other models. It was also observed that the obtained results from the model *L*(*VI*) with parameters Fr_1 and y_1/B represented reasonable results in the jump length prediction. It could be deduced that the applied method is able to successfully predict the jump length using only the upstream flow characteristics as input data. Comparison between the results obtained from the three expanding basins showed that the developed models led to more accurate outcome in the case of channel with a central sill. It was observed that the accuracy of the models for channel with negative step were higher for a symmetric channel than for an asymmetric channel, while for basin without appurtenances, a symmetric channel represented better results. It showed that adding S/y_1 as an input parameter improved the efficiency of the models, which confirmed the importance of the applied appurtenances geometry in the hydraulic jump characteristics estimation process in expanding channels with appurtenances. The results revealed that analyzing data sets separately led to a more accurate outcome. According to the results obtained from sensitivity analysis it was found that Fr_1 had the most important role in the prediction of hydraulic jump characteristics compared to other parameters.

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