Using time compression approximation to determine actual infiltration rate from variable rainfall events

Yanyan Cheng, Guotao Cui and Jianting Zhu

ABSTRACT

Understanding infiltration into soils from rainfall events is important for many practical applications. The idea of time compression approximation (TCA) was proposed to simulate infiltration rate, which only requires the relationship between the potential infiltration rate (PIR) and potential cumulative infiltration (PCI). The TCA-based method can be used in any rainfall–runoff models since the PIR vs. PCI relationship can be developed independent of actual rainfall patterns. The main objective of this study is to establish guidelines on when this method can be adequately applied. The results based on the TCA are compared with those from the field observations and the Richards equation numerical solver for observed rainfall events and randomly generated rainfall patterns with prescribed temporal variabilities and hiatuses. For continuous rainfall with potential ponding, the maximum error of infiltration amount using the TCA-based method is less than 5%. The TCA-based method, in general, underestimates the total infiltration amount from variable rainfall events. Variance in rainfall time series does not significantly affect the errors of using the TCA-based method to determine the actual infiltration rate. The TCA-based method can produce reasonable results in simulating the actual infiltration rate for rainfall events with a short hiatus.

Key words | actual infiltration, potential infiltration, rainfall hiatus, rainfall patterns

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INTRODUCTION

Accurately determining rainfall infiltration into soils is required for many practical applications in terms of water budget. Infiltration rate is also one of the principal variables of transport processes in soils and influences environmental quality. Infiltration into a soil is described by Richards (1931) equation for downward movement of water. The Richards equation is a highly nonlinear parabolic partial differential equation. Because of a series of issues intrinsically related to the nature of the infiltration process, numerical simulation of infiltration remains a challenge, and convergence and stability are continuing problems (Kirkland *et al.* 1992). In addition, the corresponding numerical solutions are usually complex for practical applications with issues related to stability and convergence. Therefore, many analytical and quasi-analytical solutions have been developed, such as the time expansion approach (Philip 1969), the integral approach (Parlange 1971), and the space expansion approach (Heaslet & Alksne 1961). However, analytical solutions to the Richards equation for general soil, initial, and boundary conditions remain challenging.

Infiltration rate when there is small ponding in the surface has been observed to decrease over time and gradually approach a steady final rate. This temporally decreasing function can be developed when the surface is always kept saturated. The infiltration rate so determined is referred to as the potential infiltration rate (PIR) and the corresponding cumulative infiltration the potential cumulative infiltration (PCI). The temporal variations of the PIR, $i_c(t)$, can be determined under laboratory or field conditions since the rate is not related to specific patterns of real-world rainfall events.

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When water is applied at the soil surface with low rainfall intensity, all of the water infiltrates into the soil until ponding occurs. After that time, the actual rate of infiltration, i, is controlled by the soil PIR, $i_c(t)$, until the rainfall rate falls below it. Therefore, the temporal history of the actual infiltration rate $i(t)$ is also related to the temporal variations of rainfall rate. When the rainfall rate is intense enough, a portion of the rainfall may be ponded or runs off, and the rest of the rainfall infiltrates at the PIR.

One attractive approximation is that the infiltration rate at any given time after the start of ponding depends only on the cumulative infiltration depth already into the soil, regardless of the previous infiltration or rainfall history (Reeves & Miller 1975; Sivapalan & Milly 1989; Liu *et al.* 1998). This is the main concept behind the time compression approximation (TCA). After the ponding starts, the infiltration rate $i(t)$ is treated as a function of the cumulative infiltration depth only. Therefore, the relationship $i(I)$ is a unique function independent of the rainfall history, the same as the $i_c(I_c)$ relationship, where I_c represents the PCI depth.

For relatively constant rainfall rates, the TCA can be used to accurately simulate the actual infiltration rate i since the $i(I)$ relationship has been shown to be relatively independent of the rainfall rate $r(t)$ (Parlange *et al.* 2000; Smith *et al.* 2002). However, the TCA may lead to errors when $r(t)$ varies significantly with time (Parlange *et al.* 2000; Assouline *et al.* 2007; Hogarth *et al.* 2011; Assouline 2013). Reeves & Miller (1975) examined the performance of the TCA using a simple infiltration and redistribution scenario, in which the surface was first assumed to be suddenly wetted and then kept at saturation for a time period. The rainfall rate was then dropped abruptly to zero for a given period of time. Finally, saturation at the surface was re-established through another time period. Their results showed that the TCA-based approach underestimates the infiltration rate compared to the numerical solutions. When rainfall rate varies with time, Assouline *et al.* (2007) investigated the effect of the time interval used for averaging the rainfall rate data and the impact of soil surface sealing on the ponding time and found that averaging the actual rainfall records over periods of as little as 5 minutes may significantly reduce the predicted runoff and hourly averaging the actual rainfall records resulted in a complete lack of prediction of ponding.

To summarize, previous studies demonstrated that it is reasonable to use the TCA approach to determine the actual infiltration rate when the rainfall rate is constant and the errors of using the TCA approach are related to the way the rainfall rate data are averaged over time. The TCA-based method is attractive and can be used in any rainfall–runoff models since it only requires the relationship between the PIR and PCI, which can be developed independent of actual rainfall patterns for given soils and field conditions. However, it is necessary to establish guidelines on when and how this method can be adequately applied for complex variable rainfall events. In this study, the first objective is to investigate how complex rainfall variability as measured by the variance of temporally variable rainfall rate may affect the performance of the TCA-based method. The second objective is to examine the appropriateness of using the TCA-based method to determine the actual infiltration rate from multiple variable rainfall events with hiatuses and investigate how the errors of using the TCA-based method evolve with time and are related to soil types.

To achieve these objectives, we first develop an iterative procedure to determine infiltration rate based on the concept of TCA for any temporal patterns of rainfall rate sequences. To demonstrate the applicability of the approach, we then compare the runoff results from the developed iterative procedure with field observations from complex rainfall events on a crust (heterogeneous) and sloping surface. After the comparison, the main focus is then devoted to examine the appropriateness of the TCA-based method in determining the actual infiltration and runoff from variable rainfall patterns with a range of temporal variabilities, hiatuses, and soil types.

MATERIALS AND METHODS

Iterative procedure based on the TCA

The infiltration rate is equal to the rainfall rate before the start of runoff. At the starting time of runoff, t_p, the actual cumulative infiltration depth, I, is smaller than the PCI depth at that time, I_c, $I(t_p) < I_c(t_p)$. To provide a similar cumulative infiltration depth under these two conditions, a time shift t_c, with $t_c < t_p$, is introduced so that $I(t_p) = I_c(t_c)$.

Once t_p and t_c are determined, the actual infiltration rate $i(t)$ can be estimated as:

$$i(t) = i_c\left[t - \left(t_p - t_c\right)\right] \tag{1}$$

This is the basic concept of TCA. Approximate expressions have been developed to estimate t_p and t_c assuming a constant rainfall rate (Brutsaert 2005).

In this study, we develop a general procedure to determine infiltration rate at any time based on the idea of TCA. The time to ponding is determined by an iterative approach, which is not necessarily expressed as an explicit analytical form. The time to ponding is the time when the actual infiltration rate starts to fall just below the given rainfall rate at that time. In other words, there is no need to compress time although the developed approach is based on the concept of TCA. This general iterative approach can be implemented in the same way for both constant rainfall rate and temporally variable rainfall rate.

Since both the actual infiltration rate $i(t)$ and the actual cumulative infiltration $I(t)$ are functions of time, we can obtain the inverse functions $t = t(i)$ and $t = t(I)$. Similarly, the inverse functions $t = t(i_c)$ and $t = t(I_c)$ can also be expressed for the PIR and PCI, respectively. The PCI depth at any time t can be integrated as:

$$I_c(t) = \int_0^t i_c(t)dt \tag{2}$$

From this equation, we can establish the relationship between i_c and I_c, $i_c = i_c(I_c)$.

The actual infiltration rate is then given by:

$$i(t) = min[i_c(I), r(t)] \tag{3}$$

where the actual cumulative infiltration depth I at time t can be expressed by:

$$I(t) = \int_0^t i(t)dt = \int_0^t min\{i_c[I(t)], r(t)\}dt \tag{4}$$

Equation (4) represents the key concept of using the iterative approach to determine the actual infiltration rate $i(t)$ and actual cumulative infiltration $I(t)$. Since $I(t)$, which is directly related to $i(t)$, appears on both sides of Equation (4) and $i(t)$ has the constraint expressed in Equation (3), it is possible to iteratively determine both $i(t)$ and $I(t)$ for any given rainfall rate patterns.

It is important to note that if we express $i_c(t)$ as a function of t, we are not able to directly use it to determine the actual infiltration rate since prior to ponding the cumulative rainfall depth has not reached the PCI. However, if we relate i_c to I_c, we can use this relation to determine the actual infiltration rate at any time. It means the actual infiltration rate at any time is dictated by the cumulative infiltration that has already infiltrated into the soil.

Figure 1 is a schematic diagram of the general iterative approach. The relationship between the PIR and the PCI is shown in Figure 1(a). If the rainfall rate is high enough, only part of the rainfall could infiltrate starting at time t_1, (i.e., t_1 shown in Figure 1(b) is the time when the runoff

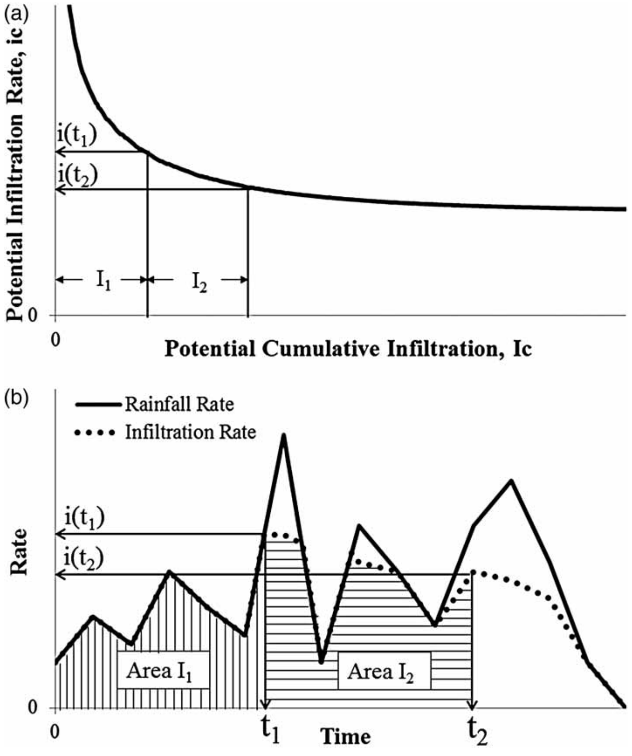

Figure 1 | Schematic illustration of how the actual infiltration rate *i*(*t*) can be determined (indicated in (b)) from the relationship between the PIR and the PCI given in (a) at any time. In the figure, the solutions at two times (t_1 and t_2) are shown. t_1 shown in the figure is the start of runoff generation (i.e., the actual infiltration rate is less than the given rainfall rate).

starts) and the difference between $r(t)$ and $i(t)$ represents the excess rainfall rate that produces runoff. The solid curve in Figure 1(b) represents the given rainfall rate, and the dash curve is the actual infiltration rate obtained iteratively by considering both the PIR vs. PCI relation constraint in Figure 1(a) and the constraint given by Equation (3). The main idea of the TCA-based method is briefly explained by the following three basic steps.

Step 1: Establish the PIR vs. PCI relation, shown in Figure 1(a). This relationship may be established in advance in different ways, such as experimental observations or numerical simulations for given soils and field conditions since it is independent of rainfall patterns. From this relationship, we can determine how the maximum infiltration rate is related to the cumulative infiltration already into the soil.

Step 2: Determine the actual infiltration rate $i(t)$ shown in Figure 1(b) by an iterative approach step by step in time, with the constraint of Equation (3). The actual infiltration rate and the actual cumulative infiltration can then be established for every time step.

Step 3: Repeat Steps 1 and 2 for all time steps. After the actual infiltration rates and the cumulative infiltration depths have been determined for all time steps, we can then determine the runoff rates and the cumulative runoff depths for all time steps.

Observations and generations of rainfall events

To examine the applicability of the TCA-based approach, we compare with field observations of runoff in the study of Esteves *et al.* (2000) using two rainfall events as reported in the same study. Event 1 is used to inversely optimize the hydraulic parameters. In particular, α and n parameters in the van Genuchten model (van Genuchten 1980) are inversely determined from the program imbedded in the Hydrus-1D solver (Simunek *et al.* 2013). The optimized hydraulic parameters are then used to construct the PIR vs. PCI relation for the field conditions using the Hydrus-1D, which is then used to determine the infiltration and runoff for Event 9 in Esteves *et al.* (2000) using the TCA-based method. The other required hydraulic parameters are the same as those given in Table 2 of Esteves *et al.* (2000) (p. 272 of their paper).

We also try to extend the TCA-based method to multiple variable rainfall events with a range of hiatuses and investigate how the errors of using the TCA-based method may evolve in simulating the actual infiltration rates for multiple rainfall events. For this purpose, we use the rainfall rate data observed in the SIRG Rain Gage 105 near Jupiter, Florida (26° 56′ 2″ N and 80° 5′ 39″ W) from June 1 to June 15, 2015 with a 0.25 hr (15 min) temporal resolution. The rainfall rates were downloaded from the Southwest Florida Water Management District Hydrologic Data website, https://www.swfwmd.state.fl.us/data/hydrologic/.

In addition to the observed rainfall events, we also generate various rainfall rate time series embracing a wide range of temporally variable patterns to more comprehensively investigate the influence of rainfall rate temporal patterns on the appropriateness of using the TCA-based method to simulate the actual infiltration rate. The rainfall rate patterns with given statics are generated by using the spectral method developed by Robin *et al.* (1993). The temporal patterns of rainfall rate can be generated with any prescribed mean rate, standard deviation, and auto-correlation length in temporal scale. With the generated rainfall rate series, we can also examine the impact of rainfall hiatuses, which represent the discontinuity in the rainfall patterns, on the appropriateness of using the TCA-based method. In doing so, we simulate the actual infiltration rate using both the TCA-based approach and compare the results with the Hydrus-1D simulations for rainfall hiatuses ranging from 0.25 to 360 hours.

Hydrus-1D simulations

In this study, we also use the Richards equation numerical solver Hydrus-1D (Simunek *et al.* 2013) to: (1) inversely obtain the optimal hydraulic parameters based on the field observations of Esteves *et al.* (2000); (2) compute the PIR vs. PCI relations under the field conditions of Esteves *et al.* (2000) using the optimal parameters determined in (1); (3) simulate the PIR vs. PCI relations for a variety of soil types with known hydraulic parameters; and (4) simulate the actual infiltration and runoff from rainfall events. In order to simulate water flow in unsaturated soil, the relationship between the water content (θ) and the capillary pressure head (ψ), and the relationship between the unsaturated hydraulic conductivity (K) and the capillary pressure head need to be prescribed. We use the model of van Genuchten (1980), which closely fits measured water-retention data of many types of unstructured soils. The van Genuchten model for the soil water retention curve combined with the hydraulic conductivity function (Mualem 1976) can be expressed as:

$$\theta(\psi) = \theta_r + \frac{\theta_s - \theta_r}{\left[1 + (\alpha|\psi|)^n\right]^m} \tag{5}$$

$$K(\psi) = \frac{K_s\left\{1 - (\alpha|\psi|)^{mn}\left[1 + (\alpha|\psi|)^n\right]^{-m}\right\}^2}{\left[1 + (\alpha|\psi|)^n\right]^{m/2}} \quad m = 1 - 1/n \tag{6}$$

where θ_s [-] and θ_r [-] are the saturated and residual water contents, respectively. α [L^{-1}], n [-], m [-] are parameters that determine the shape of the soil water retention curve, and K_s [LT^{-1}] is the soil saturated hydraulic conductivity.

To systematically examine the performance of the TCA-based method for various soil types, we test 12 major soil textural groups (sand, loamy sand, sandy loam, loam, silt, silt loam, sandy clay loam, clay loam, silty clay loam, sandy clay, silty clay, and clay) as defined by Carsel & Parrish (1988). The van Genuchten hydraulic parameters for all 12 soil groups are taken from Carsel & Parrish (1988). The boundary conditions we used in the Hydrus-1D simulations can be summarized as follows. In developing the PIR vs. PCI relations, the surface is always saturated with $\theta = \theta_s$, which is independent of rainfall patterns. For the simulations of infiltration and runoff with given rainfall events, the surface condition varies between flux condition and saturated condition. The lower boundary is set to be free drainage condition. The depth of the soil profile is set to be 300 cm. While other depths can also be used easily, they do not change the general conclusions. The initial water content conditions in the Hydrus-1D simulations for each soil group to construct the PIR vs. PCI relation required in the iterative approach as well as in the infiltration and runoff simulations for the 12 soil groups are 0.045, 0.058, 0.066, 0.097, 0.049, 0.076, 0.102, 0.113, 0.117, 0.119, 0.246, and 0.166 (m^3/m^3), respectively. In general, we use relatively dry soil conditions initially, which means the initial soil moisture condition is close to the residual water content for each soil group. Note that although the total amount of infiltration will be influenced by using other wetter initial conditions, the main conclusions related to relative errors presented and discussed later remain similar.

Performance criteria

We use two indices to examine the performance of the TCA-based method. The relative error, δ, is defined in Equation (7):

$$\delta = \left(\frac{X_{\text{TCA based}} - X_{\text{O}}}{X_{\text{O}}}\right) \times 100\% \tag{7}$$

where X is either the cumulative infiltration or the runoff. Obviously $\delta > 0$ means that the TCA-based method overestimates X while $\delta < 0$ indicates that the method underestimates X. The subscript 'O' can represent the quantity either simulated by the Hydrus-1D solver or observed in the field experiments, depending on the intent of comparison.

The goodness of fit for the TCA-based method is also examined by the Nash and Sutcliffe efficiency (NSE) (Nash & Sutcliffe 1970) defined below:

$$NSE = 1 - \frac{\sum_{i=1}^{k}\left(X_{\text{TCA based,i}} - X_{\text{O,i}}\right)^2}{\sum_{i=1}^{k}\left(X_{\text{O,i}} - \overline{X_{\text{O}}}\right)^2} \tag{8}$$

where i is the time step index, k is the number of time steps under consideration. $\overline{X_{\text{O}}}$ is the observed or Hydrus-1D simulated mean X from time step 1 to step k.

RESULTS AND DISCUSSION

To illustrate the applicability of the TCA-based method, we first compare with the runoff results from the field observation studies conducted in Niger (West Africa) (Esteves *et al.* 2000) on a sandy hillslope of 14.25 m long by 5 m wide. The soil surface is crusted with average slopes of 1.96% and 6.4% in the two directions. The data used for the inverse approach in the Hydrus-1D are the cumulative infiltration depth calculated from Figure 4 of Esteves *et al.* (2000) for Event 1. The optimized α values for the crust and the soil are 1.02×10^{-6} (1/mm) and 0.00167 (1/mm), respectively. The n values for the crust and the soil are 1.91 and 1.44, respectively. Compared to the measured runoff in the experimental plot, the RMSE of the runoff rate is 0.113 (mm/min) and the NSE of total runoff for Event 1 is 0.899 using these optimized hydraulic parameters.

After the van Genuchten hydraulic parameters are determined, we then simulate the PIR vs. PCI relation curve for the field conditions when the surface is set to be saturated all the time. The Hydrus-1D simulated PIR vs. PCI relation curve is shown in Figure 2(a). Based on this curve, we simulate the infiltration and runoff for Event 9 in Esteves *et al.* (2000) using the TCA-based method and compare with the observed data, which are shown in Figure 2(b) and 2(c). The relative difference of the cumulative runoff depth for the event, δ, is -0.566%, while the *NSE* is 0.996. These statistical measures clearly demonstrate that the TCA-based method captures very well the runoff response from the complex time-variable rainfall event on a crust (heterogeneous) and sloping surface. The complex response seen from the observed hydrograph is well represented by the TCA-based method under the complex field conditions.

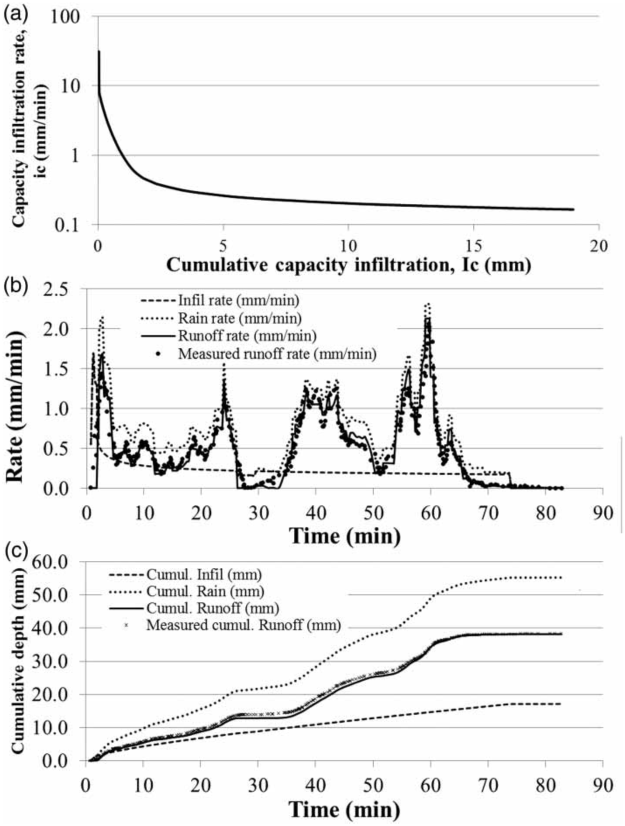

Figure 2 | Comparison of the TCA-based method with field observations of Esteves *et al.* (2000). (a) Relationship between the PIR and the cumulative infiltration depth, (b) comparison of the TCA-based runoff rate and the observed runoff rate, and (c) comparison of TCA-based cumulative runoff depth and observed runoff depth.

Next, we investigate the performance of the TCA-based method from multiple variable rainfall events with hiatuses and examine how the errors of using the method evolve with time and are related to the soil types. Since there are no comprehensive field observation data that allow systematic comparison, we compare the results with those from the Hydrus-1D simulations. We use two types of rainfall data. The first one is synthetically generated with a range of rainfall variability and hiatus. The main objective is to assess the effects of variability and the length of hiatus between rainfall events on the performance of the TCA-based method. The second type of rainfall data is the observed rainfall sequence with multiple events. The main objective is to investigate how the performance of the TCA-based method evolves with time, especially after rainfall hiatuses.

Figure 3 shows the time series results of the dimensionless actual infiltration rate (i.e., $i(t)/r(t)$) for all 12 soil groups from a rainfall event lasting 3 hours with a constant rainfall rate. In order to have runoff for all soil groups, we use rainfall rate that is higher than the saturated hydraulic conductivity for all soil groups although the given rainfall rate is unrealistically high for some soil groups such as sand and loamy sand. Note that $r/K_s > 1$ is to ensure runoff generations at a certain point in time. However, in order to generate runoff during the 3-hour rainfall event, r/K_s needs to be set at different levels for different soil groups. For some soil groups, r/K_s could be substantially larger than 1 in order to have runoff generation within 3 hours. The results simulated from the Hydrus-1D are also included in Figure 3 for comparison. For constant rainfall rate events, the TCA-based method produces very good results for all soil groups. In Table 1(a), we list the relative errors in percent, δ, for all soil groups, which demonstrate that the maximum relative errors are less than 5%. For the soil groups that demonstrate noticeable differences, the TCA-based method

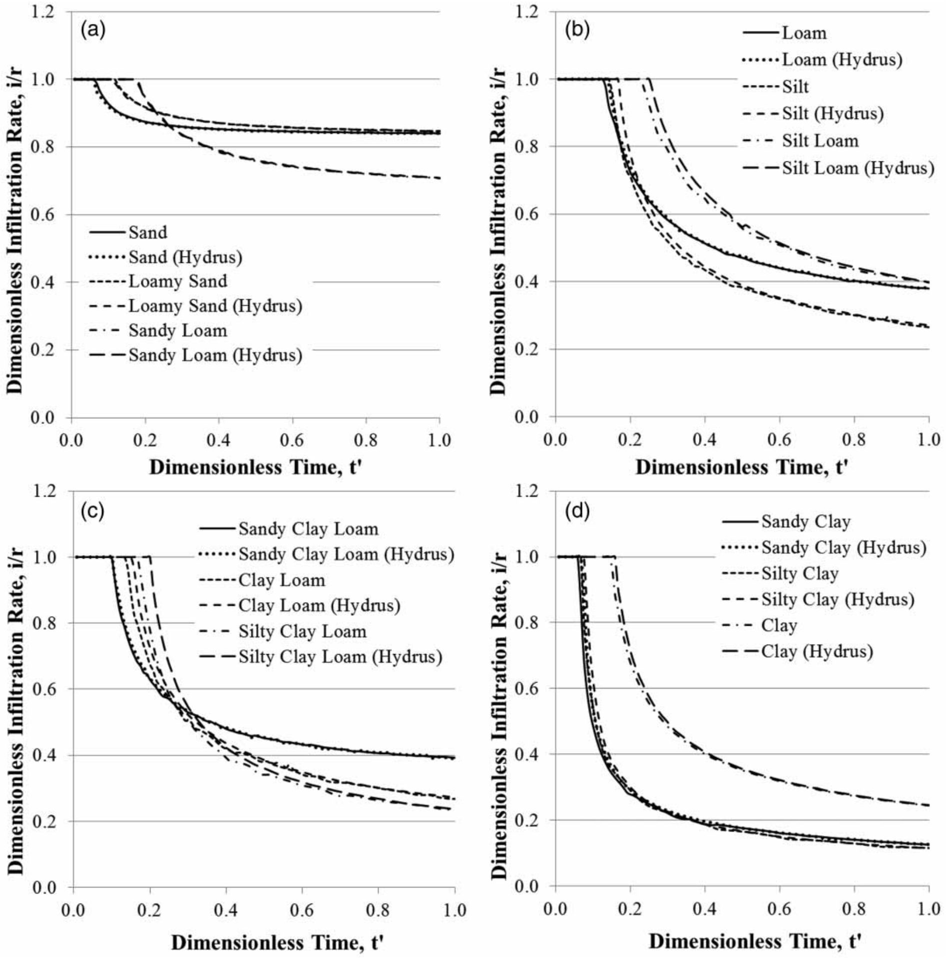

Figure 3 | Dimensionless infiltration rate (*i*/*r*) vs. dimensionless time (*t*/*T*, where *T* is the duration of rainfall event) when rainfall rate is constant. Both the results from the TCA-based method and the results from the numerical solutions of the Richards equation (Hydrus-1D) are included for comparison. The dimensionless rainfall rates (i.e., *r*/*Ks*) are given as: 1.2 for sand, 1.2 for loamy sand, 1.5 for sandy loam, 3.5 for loam, 10.0 for silt, 5.0 for silt loam, 3.0 for sandy clay loam, 8.0 for clay loam, 20.0 for silty clay loam, 20.0 for sandy clay, 50.0 for silty clay, and 10.0 for clay.

produces smaller total infiltration than the Hydrus-1D simulations. This is mainly due to the fact that at the start of infiltration, the soil is unsaturated and the actual infiltration rate is higher than that predicted from the TCA-based method. The TCA-based method, in general, underestimates the actual infiltration rate compared to the Hydrus-1D simulations. However, the difference is negligibly small with the maximum errors of less than 5% for all soil groups.

Figure 4 shows several time series results of the actual dimensionless infiltration rate for four representative soil groups from a temporally variable rainfall event lasting 3 hours when the coefficient of variation (*CV*) of the rainfall rate is about 0.25 and the auto-correlation length of 0.01 hour. The rainfall rate pattern is highly irregular with virtually no auto-correlation (i.e., an auto-correlation length of 0.01 hour for a 3-hour event). We only use rainfall events with small auto-correlation length since this correlation should be small in nature. Similar to producing results in Figure 3, the ratio of the average rainfall rate over the saturated hydraulic conductivity needs to be set at different levels for different soil groups in order to generate runoff during the 3-hour rainfall event. The mean rainfall rates for Figure 4 are the same as those used in Figure 3. The results of the relative errors (in percent) of the cumulative infiltration depths when compared to the Hydrus-1D results are shown in Table 1(b). Similar to the conclusion of the constant rainfall rate cases, the relative errors of using the TCA-based method to simulate the actual infiltration from a variable

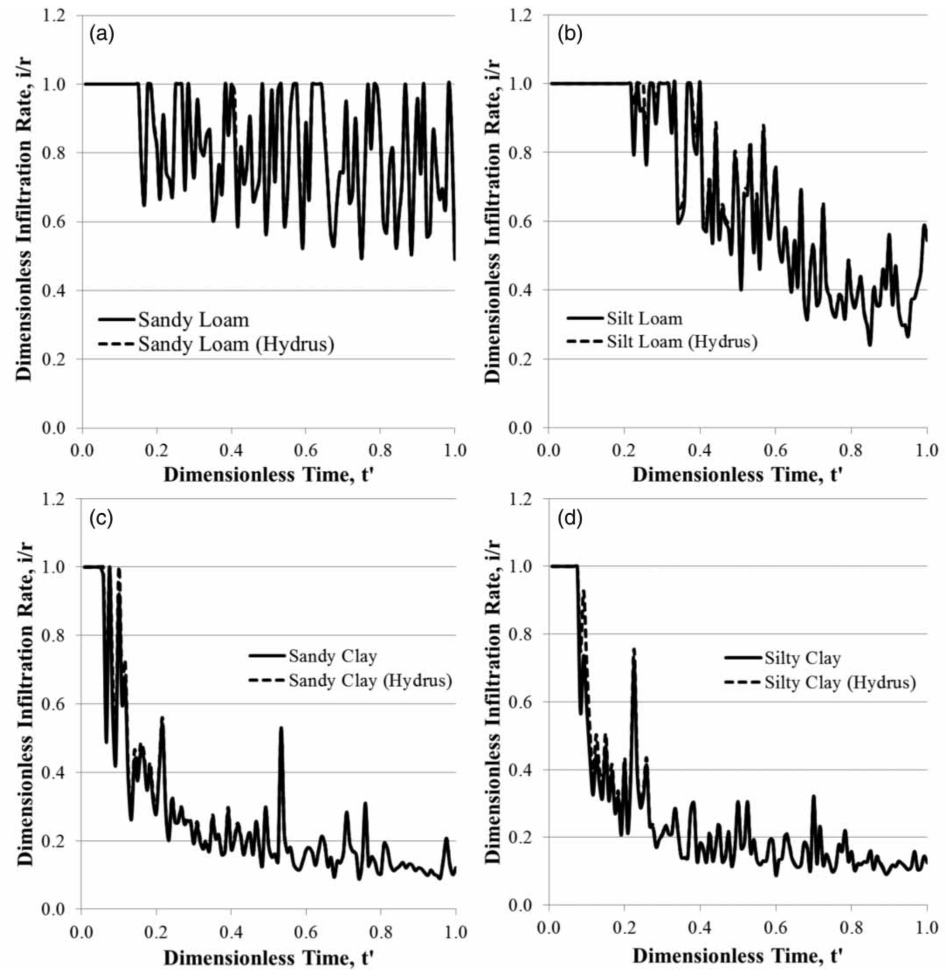

Figure 4 | Dimensionless infiltration rate (*i*/*r*) vs. dimensionless time (*t*/*T*, where *T* is the duration of rainfall event) when rainfall rate is variable. The auto-correlation length of the rainfall rate time series is 0.01 hr for the 3-hr event. The coefficient of variation is 0.25. The dimensionless average rainfall rates (i.e., $\bar{r}/K_s$) are given as: 1.5 for sandy loam, 5.0 for silt loam, 20.0 for sandy clay, and 50.0 for silty clay.

rainfall event are acceptable with a maximum relative error of about −5%. For coarse texture soils (sand, loamy sand, sandy loam, and loam), the relative errors are small with maximum relative differences of less than 2%. The influence of rainfall variability over time (as measured by the *CV* of rainfall rate time series) is also small, which illustrates that as long as rainfall is continuous without hiatus, using the TCA-based method is reasonable. The rainfall rate variability has insignificant impact on the capability of the TCA-based approach in simulating the actual infiltration rate solely based on the PIR vs. PCI relation, which can be constructed for any particular soil independent of the rainfall rate patterns.

The relative errors (δ) and the *NSE*s of cumulative infiltration as functions of rainfall hiatus lengths for four representative soil groups are shown in Figure 5. The magnitude of the relative errors increases with increasing rainfall hiatus. For some soil groups, such as silty clay, the relative errors could be as high as −20% with rainfall hiatus of 360 hours, which means the TCA-based method underestimates the actual infiltration rate significantly. The two indices follow the same trend of performance across the soil groups. In particular, soil groups with large relative errors have smaller NSE, both indicating worse performance of the TCA-based method.

Figures 6 shows the results of the relative errors in percent (δ) and *NSE*s in the cumulative infiltration for the four representative soil groups for the multiple rainfall events observed near Jupiter, Florida. It can be seen that after the

Table 1 | Relative errors in percent (δ) in the cumulative infiltration using the TCA-based method compared to the results from the numerical solutions of the Richards equation (Hydrus-1D): (a) for constant rainfall rate event and (b) for variable rainfall event

r/Ks	δ %	r/Ks	δ %	r/Ks	δ %	r/Ks	δ %
(a) Constant rainfall event							
Sand		*Loamy sand*		*Sandy loam*		*Loam*	
1.1	0.07	1.1	0.01	1.3	−0.21	2.5	−0.62
1.2	0.13	1.2	0.07	1.5	−0.19	3.5	−0.58
1.3	0.20	1.3	0.14	1.7	−0.10	4.0	−0.47
1.5	0.33	1.5	0.24	2.0	0.02	5.0	−0.18
Silt		*Silt loam*		*Sandy clay loam*		*Clay loam*	
4.0	−1.01	3.5	−1.49	1.5	−0.35	5.0	−2.29
10.0	−2.83	5.0	−1.80	3.0	−0.31	8.0	−2.47
15.0	−2.40	7.0	−1.71	4.0	−0.08	12.0	−1.93
20.0	−1.90	10.0	−1.38	5.0	0.30	15.0	−1.62
Silty clay loam		*Sandy clay*		*Silty clay*		*Clay*	
10.0	−3.64	5.0	−3.08	15.0	−3.86	5.0	−1.48
20.0	−4.48	20.0	−3.21	50.0	−3.47	10.0	−1.59
40.0	−3.54	25.0	−2.48	55.0	−3.39	15.0	−0.95
50.0	−2.88	30.0	−2.15	60.0	−3.33	20.0	−0.41
(b) Variable rainfall event							
CV	*Sand*	*Loamy sand*	*Sandy loam*	*Loam*	*Silt*	*Silt loam*	
0.00	0.129	0.074	0.051	−0.611	−2.887	– 1.852	
0.05	0.119	0.084	0.004	−0.555	−2.792	– 1.864	
0.15	−0.044	−0.126	0.007	−0.556	−2.730	– 1.944	
0.25	−0.485	−0.451	−0.195	−0.659	−2.760	– 1.818	
0.35	−1.452	−1.036	−0.773	−0.704	−2.846	– 1.882	
0.45	−2.195	−1.505	−1.374	−0.811	−2.964	– 1.927	
CV	*Sandy clay loam*	*Clay loam*	*Silty clay loam*	*Sandy clay*	*Silty clay*	*Clay*	
0.00	−0.380	−2.299	−4.346	−3.293	−4.247	– 1.626	
0.05	−0.360	−2.363	−4.371	−3.394	−3.493	– 1.601	
0.15	−0.283	−2.287	−4.440	−3.067	−3.643	– 1.515	
0.25	−0.062	−2.559	−4.548	−2.869	−3.710	– 1.409	
0.35	0.194	−2.615	−4.538	−3.030	−3.355	– 1.377	
0.45	0.145	−2.758	−4.880	−3.018	−4.099	– 1.147	

rainfall rate time series experiences a period of hiatus, the relative errors (δ) experience a step increase in magnitude due to the underestimation of the actual infiltration rate. The results demonstrate the relative errors are cumulative and increase with time. After each rainfall hiatus, the actual infiltration rate increases. However, based on the TCA concept, the simulated infiltration rate would stay at the same rate before the hiatus since the cumulative infiltration does not change due to the hiatus. Similar to the case of a single period of hiatus, the magnitude of the relative errors (δ) is highly related to the soil group. As expected, the soil group with larger relative errors with a single hiatus also has larger errors in the rainfall rate series with multiple hiatuses. Once again, coarser soil groups have smaller errors in a relative sense, seen from the results of δ (Figure 6(a)) and NSE (Figure 6(b)). Both

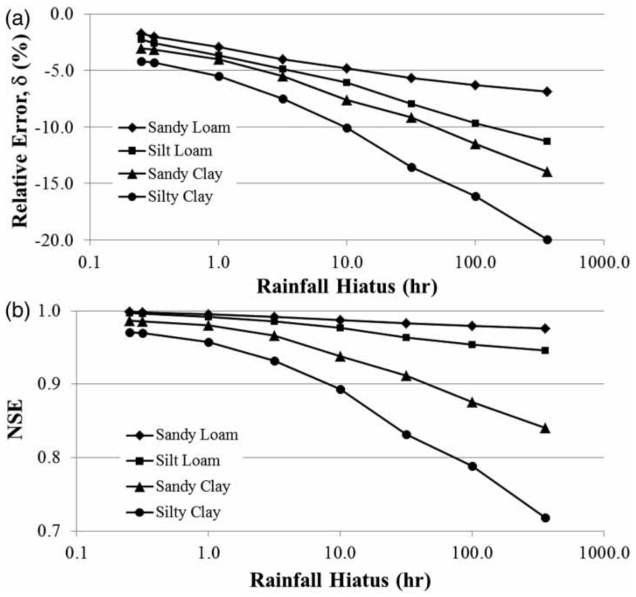

Figure 5 | (a) Relative errors in percent (δ), (b) *NSE*, as a function of the length of rainfall hiatus. The rainfall hiatus is in the middle of two 3-hr rainfall events, which have the same characteristics as those used in Figure 4.

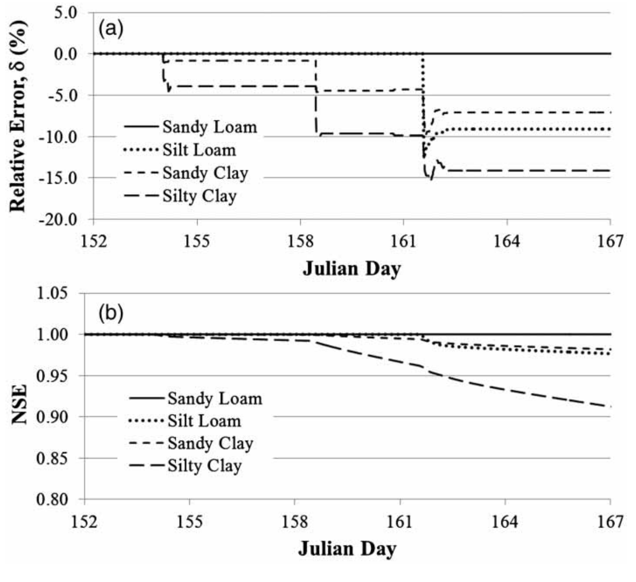

Figure 6 | Evolution of (a) the relative errors in percent (δ), (b) *NSE*, over time using the TCA-based method compared to the results from the Hydrus-1D simulations for the four representative soil groups using the rainfall rate data observed in the SIRG Rain Gage 105 near Jupiter, Florida.

the TCA-based method and Hydrus-1D predict that all the rainfall will infiltrate and therefore the errors for the coarsest group (i.e., sand) are zero since the infiltration potential is so high for the sand group, which dictates the actual infiltration rate. Therefore, the redistribution potential in the case of the sand group is not even relevant in determining infiltration amount after hiatuses. The main reason lies in the fact that a soil group with high redistribution potential also has high infiltration potential. The combining effects of the two competing mechanisms of infiltration and redistribution potentials determine the errors or the appropriateness of the TCA-based method.

In general, the soil with smaller K_s tends to have larger relative errors. For coarse texture soil, the majority of rainfall infiltrates and runoff only occurs after a few big rainfall events. The actual infiltration rate is close to the rainfall rate no matter which approach we use. In other words, there is a small amount of runoff generation regardless of redistribution, which also results in a small difference between considering and not considering redistribution. On the other hand, the infiltration is only a small portion of the rainfall rate for fine texture soil. The TCA-based approach without considering redistribution of the infiltrated water underestimates the actual infiltration rate to a large extent for fine-textured soil.

CONCLUSIONS

In this study, we investigated whether the actual infiltration rate from rainfall events can be accurately determined using the approach based on the PIR and PCI relationship (the TCA-based method), which can be developed independent of the rainfall history. In particular, we examined how rainfall rate patterns and the variabilities of rainfall rate may affect the performance of the TCA-based method. We also extended the TCA-based method to multiple rainfall events with hiatuses and explored the appropriateness of the TCA-based method by examining how the errors of using this method are related to the length of rainfall hiatus. We compared the results with those from both the field observations and the numerical solutions of the Richards equation. Some of the main conclusions are summarized as follows:

- For continuous rainfall with potential ponding, the maximum relative errors of the cumulative infiltration using the TCA-based method are less than 5% for all soil groups from comparison with both field observations and numerical simulations. The TCA-based method, in general, slightly underestimates total infiltration from rainfall events.

- The variance of the rainfall rate does not significantly impact the performance of using the TCA-based method to determine the actual infiltration rate.
- The TCA-based method can produce reasonable results in simulating the actual infiltration rate for rainfall events with short hiatus since the errors of using the approach are proportional to the length of rainfall hiatus.
- For coarse soils, the TCA-based method can also be used to simulate the actual infiltration for variable rainfall events with multiple hiatuses.

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