Infiltration-runoff model for layered soils considering air resistance and unsteady rainfall
Yongde Gan, Huan Liu, Yangwen Jia, Siyuan Zhao, Jiahua Wei, Hongwei Xie and Dongzhu Zhaxi

ABSTRACT
A modified Green–Ampt model (MGAM) was proposed to simulate infiltrations into layered soil profiles with the entrapped air under unsteady rainfall conditions. To account for the effects of the air resistance, the saturation coefficient, actual water content, air bubbling pressure, and water bubbling pressure were introduced in the model. One-dimensional infiltration-runoff experiments were then conducted in multi-layered soil columns, under unsteady rainfall conditions, to evaluate the performance of the MGAM model. The cumulative infiltration, runoff rate, and water content of the soil, calculated by MGAM, were compared with the observed data and the results, calculated by the traditional Green–Ampt model (TGAM), the Bouwer Green–Ampt model (BGAM), and the Mein–Larson model (MLGAM), respectively. The results indicated that the cumulative infiltration, runoff rate, and soil water content, calculated by MGAM, were in better agreement with the observed results than previous models. A parameter sensitivity of MGAM was also analyzed. It was found that the sensitivity of the saturated coefficient was high in the first soil layer, and those of the air bubbling pressure and initial moisture deficit were high or medium in the first and second layers, while those of the other parameters were relatively low.

Key words | air resistance, Green–Ampt model, infiltration-runoff, multi-layered soil

INTRODUCTION
Infiltration and runoff are the dynamic processes of water movement at the soil-air interface during rainfall periods. The accurate performance of the infiltration and runoff is of critical importance in hydrological modeling. As one of the most usable models for practical field problems, the Green–Ampt model can reasonably estimate the depth of the wetting front, infiltration capacity, and cumulative infiltration. It has been subjected to many modifications (Mein & Larson 1973; Jia & Tamai 1997; Chu & Marino 2005) in many semi-distributed hydrological models, such as the SWAT (Neitsch et al. 2002a, 2002b) and WEP (Jia et al. 2006).

The Green–Ampt model (Green & Ampt 1911) was originally used as an infiltration model for dry and uniform soil with surface ponding. Due to its simplicity, researchers have developed many modifications, such as Mein & Larson (1973), Ali et al. (2013, 2016), and Vatankhah (2015), etc. All these modified versions have been found to be suitable for simulating the infiltration in homogeneous soil under ponding or steady rainfall conditions. Moreover, it has been extended to model the infiltration in non-uniform soil (Bouwer 1969), during unsteady rainfall periods (Chu 1978), in two-layered soil profiles (Moore & Eigel 1981), and in multi-layered soil profiles with unsteady rainfall (Jia & Tamai 1997).

The soil profile above the wetting front, in the Green–Ampt model, is assumed to be fully saturated, indicating that the air in the soil can escape freely from the soil...
pores and the air resistance can be abstractly neglected in the water infiltration process (Hammekger et al. 2005). However, many previous studies showed that the air may be trapped and compressed, and the infiltration rate was apparently reduced in the assumed saturated soil profile (Morel-Seytoux & Khanji 1975; Grismer et al. 1994; Wang et al. 1997, 1998; Hammekger et al. 2003). Wang et al. (1998) found that the volume of the residual entrapped air on the condition of air-confining, increased by 7% on average. The infiltration rate was also decreased between three-fold and ten-fold compared to the air-draining condition. Ma et al. (2011) proposed that the ratios of the actual water content to the saturated water content in the soil profile above the wetting front were 0.80 and 0.86 for the laboratory and field infiltration experiments, respectively. The actual water pressure at the wetting front was determined by the capillary and soil air pressures (Morel-Seytoux 1973). Peck (1965a, 1965b) speculated that the required initiation of the escaping air was expected to be equal to the water pressure at the bottom depth of the saturated zone plus the air entry pressure of the material. The escaping air would cease when the value of pressure infinitely closed to 0 to allow the air escape route to be sealed by effective saturation. At that time, the air pressure would start to increase again with further water penetration. Wang et al. (1997) confirmed Peck’s speculation and investigated the relationship of two extreme air pressure relationships in a porous medium, which are referred to as the ‘air-breaking value’ and ‘air-closing value’, respectively. The air pressure was found to be constantly increased with the rising of the wetting front (Wang et al. 1997).

Furthermore, based on the indoor soil column experiments, the changing process of the air pressure in the soil, during water infiltration under air confined conditions, was studied by Grismer et al. (1994), Grismer (2016), Li & Fei (2005), and Li (2007). The results showed that the changing process was divided into three stages: the initial stage, transitional stage, and steady stage. During the initial stage and transitional stage, the soil air pressure automatically changed between the closure and breakthrough pressures. During the steady stage, the soil air pressure was relatively fixed. In addition, certain zones above the wetting front were found to be saturated, while the air pressure under the saturated zone was equal to air pressure of the soil under the wetting front.

To account for the effects of the air resistance, the effective hydraulic conductivity, in place of the saturated value, was applied in some of the Green-Ampt models in many previous studies. Bouwer (1966) suggested that the effective hydraulic conductivity was half of the saturated value. Grismer et al. (1994), Morel-Seytoux & Khanji (1975), and Hammekger et al. (2003) developed a simple flow model based on the Green-Ampt model, taking the air compression and air counter-flow into consideration, while it was only tested to be suitable for a homogeneous soil profile. Ma et al. (2011) developed a simple model for layered soil profiles with constant ponding heads, and introduced a saturated coefficient \(S_a\) into the model to quantitatively demonstrate the air compression and air counter-flow. Wang et al. (1997) proposed an infiltration model which considered the potential effects of the air compression and air counter-flow during water infiltration in a porous medium, and the air pressure was determined as the sum of the ponding depth, wetting front distance and half of air-bubbling pressure and water-bubbling pressure. Nevertheless, the air pressure tends to infinity as wetting front distance tends to infinity, which is not in compliance with the fact.

In summary, with the development of the basin hydrological models, such as SWAT (Neitsch et al. 2002a, 2002b) and WEP (Jia et al. 2006), the simulations of the soil infiltration runoff process gradually took more factors into consideration in the hydrological models, including soil stratification characteristics, air resistance, and unstable rainfall process, to improve the accuracy of the models and solve problems such as the simulation distortions of the models. However, certain deficits were found in the models currently applied and the applicability of the models was limited, and these are systematically summarized in this paper. Based on these deficits, a stratified soil infiltration runoff model under unsteady rainfall, considering the effects of air resistance, was proposed and proven to be practical via indoor experimental testing.

The objectives of this study were as follows: (1) to develop a modified Green-Ampt model to simulate water infiltration in layered soil profiles under unsteady rainfall conditions, considering the influence of air resistance; (2) to test the applicability of the proposed model with
one-dimensional infiltration-runoff experiments with different combinations of soil layered patterns during rainfall events in multi-layered soil columns.

**MATERIAL AND METHODS**

**Green–Ampt model and main extended version**

In Table 1, the summaries of the Green–Ampt model and other primary modified models are shown, indicating that the previous models have many deficiencies and need to be further improved. In this study, to make up for the deficiency of the above model research, based on the indoor experimental research, the new stratified soil unsteady rainfall, the Green–Ampt infiltration runoff model considering air resistance effect (MGAM) is proposed.

**A newly modified GA model considering air resistance for the multi-layered soil during an unsteady rain event (MGAM)**

In Figure 1, the schematic diagram of infiltration into a layered soil profile is illustrated. The soil profile under consideration consists of \( n \) layers with thickness \( Z_i \) (\( i = 1, 2, \ldots, n \)), and an initial water content \( \theta_{0,i} \). The saturated hydraulic conductivity and the saturated water content of each layer of the soil are \( s_{a,i} \) and \( \theta_{s,i} \), respectively. \( Z_i \) represents the thickness of each layer. The origin of the coordinate system is set at the surface of soil, and the coordinate system is positively downward.

In view of the deficiency of the above models, the improvement of the modified Green–Ampt model (MGAM) is illustrated in Figure 2. First, GGAM was originally used as an infiltration model for the uniform soil. In this case, it extends to model infiltration in the layered soil profiles. As performed in the traditional Green–Ampt model (TGAM), the water content of each soil layer below or above the wetting front is assumed to be uniform, and the soil profile is separated into an upper wetted zone and a lower unwetted zone by wetting front. In the unwetted zone, the soil water content is assumed to be the initial value. \( H_0 \) and \( H_L \) are the hydraulic heads at the surface and wetting front of the soil, respectively. \( Z \) represents the depth of wetting front. Second, as a result of the air entrapment, the soil pores in the water content profile of the soil above the wetting front cannot be completely filled with water (Bouwer 1966; Hammecker et al. 2003). Therefore, it is necessary to take the actual water content \( \theta_{a,i} \) and the actual hydraulic conductivity \( k_{a,i} \) of the \( i \)th layer in the soil water content profile above the wetting front as the water content and hydraulic conductivity at the residual air saturation (Hammecker et al. 2003). Previously, \( \theta_{a,i} \) and \( k_{a,i} \), which are less than \( \theta_{s,i} \) and \( k_{s,i} \), have been used in the TGAM. As in Bouwer Green–Ampt model (BGAM) and GGAM, a saturated coefficient \( S_{a,i} \) (\( 0 \leq S_{a,i} \leq 1 \)) is also introduced to determine the proportion between \( \theta_{a,i} \) and \( \theta_{s,i} \) in the \( i \)th soil layer \( (\theta_{a,i} = S_{a,i} \cdot \theta_{s,i}) \). In regard to the relationship between \( k_{a,i} \) and \( k_{s,i} \), \( S_{a,i} \cdot k_{a,i} = S_{s,i} \cdot k_{s,i} \), representing the relative water conductivity, is adopted. However, in this case, the parameters’ calculation methods are improved to be suitable for layered soil profiles.

The wetting front is assumed to be located in the \( m \)th soil layer, and the average actual hydraulic conductivity \( \bar{k}_a \) in the transmission zone behind the wetting front can also be given by the harmonic mean as follows (Bouwer 1969; Moore & Eigel 1981):

\[
\bar{k}_a = \frac{1/m}{\sum_{i=1}^{m} \frac{Z_i}{k_{a,i}}}
\]

(1)

In addition, due to the effects of the air and capillary pressures at the wetting front, the actual water pressure \( SW_{af,i} \) at the wetting front is given by

\[
SW_{af,i} = SW_{cf,i} - SW_{ad,i}
\]

(Morel-Seytoux 1973), where \( SW_{af,i} \) represents the instantaneous air pressure below the wetting front, and \( SW_{cf,i} \) represents the average capillary pressure at the wetting front.

During the rainfall infiltration period, the air pressure in the soil is changed between breakthrough pressure (maximum pressure), and the closure pressure (minimum pressure). Wang et al. (1997) proposed that the maximum air pressure \( SW_{ad} \) at the time when the air erupts from the soil surface could be referred to as the ‘air-breaking value’, and can be defined as follows:

\[
H_b = h_0 + Z + h_{ab}
\]

(2)
<table>
<thead>
<tr>
<th>Model</th>
<th>Author</th>
<th>Air resistance</th>
<th>Soil layers</th>
<th>Rainfall</th>
<th>Verification</th>
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</thead>
<tbody>
<tr>
<td>GAM</td>
<td>Green &amp; Ampt (1911)</td>
<td>Not considered</td>
<td>Homogeneous soil</td>
<td>Ponding infiltration</td>
<td>Verified by ponding infiltration experiment (Green &amp; Ampt 1911)</td>
<td>–</td>
</tr>
<tr>
<td>MLGAM</td>
<td>Mein &amp; Larson (1975)</td>
<td>Not considered</td>
<td>Homogeneous soil</td>
<td>Infiltration-runoff under steady rainfall</td>
<td>Verified by fallow plots data (Nearing et al. 1996) with a homogeneous soil profile</td>
<td>The model has not passed the validation of layered soil infiltration-runoff experiments, especially layered soil infiltration-runoff experiment under the effects of air resistance with combinations of different soil types</td>
</tr>
<tr>
<td>TGAM</td>
<td>Jia &amp; Tamai (1997)</td>
<td>Not considered</td>
<td>Multi-layer soil</td>
<td>Infiltration-runoff under unsteady rainfall</td>
<td>The TGAM model has passed the following experimental validation: (1) one-layer soil unstable rainfall runoff process (Chu 1978); (2) dual-layer soil water infiltration process (Moore &amp; Eigel 1988); and (3) four-layer soil stable rainfall runoff process (Chu &amp; Marino 2005)</td>
<td>The model has not passed the validation of three-layered soil infiltration-runoff experiments, especially with combinations of different soil types</td>
</tr>
<tr>
<td>GGAM</td>
<td>Grismer et al. (1994); Grismer (2006); Wang et al. (1997); Li (2007)</td>
<td>Considered</td>
<td>Homogeneous soil</td>
<td>Ponding infiltration</td>
<td>The model has passed the validation of the one-layer soil water infiltration process test (Grismer et al. 1994); Grismer (2006); Wang et al. 1997; Li 2007)</td>
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</tr>
</tbody>
</table>
where $H_b$ is the air-breaking value, $h_{ab}$ is the air-bubbling capillary pressure value of the material, $Z$ is the wetting front, $h_0$ is the surface water ponding depth. The minimum ‘low enough, but not zero’ $SW_{af}$ immediately after the escaping of the air is referred to as the ‘air-closing value’, and can be defined by the following equation:

$$H_c = h_0 + Z + h_{wb}$$  \hspace{1cm} (3)

where $H_c$ is the air-closing value, $h_{wb}$ represents the water-bubbling value of the material. This model showed that \( \lim_{SW_{cf}} = \infty \), which was found to be inconsistent with the actual case, since not all of the water in the soil water profile above the wetting front produced static water pressure on the entrapped air (Grismer et al. 1994; Li 2007). Li (2007) proposed that $SW_{af} \approx 2.1h_0$. Grismer et al. (1994) proposed that $SW_{af} = h_0 + SW_{cf}$. It can be seen that these research conclusions are not consistent with each other. Based on the existing model, the advantages are systematically summarized, and then the new method is proposed as follows.

In order for an air bubble to escape from a ponded soil surface, the entrapped air phase needs a sufficiently high air pressure to overcome the static water pressure (Wang et al.)
During the infiltration process, the soil water content profile above the wetting front can be divided into four zones (Bodman & Colman 1944, 1945; Shao et al. 2006), namely, the saturated, transitional, transmitting, and wetting zones (Figure 3). In the saturated zone, the soil pores are completely filled with water, and no pores are interconnected except when the air eruption occurs. Therefore, the air in the soil cannot move freely, and \( SW_{af} = 0 \). The remaining zones cannot be completely filled with water, and the macro pores are interconnected. Hence, the air in the soil can move freely, and \( SW_{af} > 0 \) (Grismer et al. 1994; Li 2007).

During the infiltration process, due to the interconnectedness of the pore, no static water pressure is generated by the water in the soil of the transitional, transmitting, and wetting zones. Only the surface ponding water and the water in the saturated zone are able to produce static water pressure on the entrapped air. According to the research data from Peck (1965b), Wang et al. (1997, 1998); Bodman & Colman (1944, 1945), Grismer et al. (1994), and Li (2007), the maximum \( SW_{af} \) should be defined by:

\[
H_b = h_0 + Z_0 + h_{ab}
\]

and the minimum \( SW_{af} \) should be defined by:

\[
H_c = h_0 + Z_0 + h_{wb}
\]

where \( Z_0 \) is the thickness of the saturated zone, \( h_0 \) is the ponding depth, \( h_{ab} \) is the air-bubbling capillary pressure value of the material, \( h_{wb} \) is the water-bubbling value of the material. This new method has been proved to achieve better results.

Due to the entrapped air in the water content profile of the soil above the wetting front (Ma et al. 2011), the thickness of the saturated zone is far less than \( h_{ab} \) and \( h_{wb} \), which can be neglected. During the initial and transitional stages, the \( H_b \approx h_0 + h_{ab} \), and the \( H_c \approx h_0 + h_{wb} \). During the steady stage, it has been found that the \( H_b \approx h_c \approx h_0 + 0.5(h_{ab} + h_{wb}) \) (Li 2007). By assuming that the changes in the air pressure are linear from the air-breaking pressure to air-closing pressure during the initial and transitional stages, the average air pressure can be given by:

\[
SW_{af} = h_0 + 0.5(h_{ab} + h_{wb})
\]

Wang et al. (1997) suggested that the \( h_{ab} \) and \( h_{wb} \) can be evaluated by \( h_{ab} = 1/\alpha_d \) and \( h_{wb} = h_{ab}/2 - \delta \) (\( \delta \) is equal to 0 to approximately 2 cm for sandy soil; 2 to 5 cm for loamy soil; and 8 to 10 cm for clay soil), where \( \alpha_d \) is a parameter of the main drainage curve.

In the research conducted by Whisler & Bouwer (1970), it was proposed that \( SW_{cf} \), the average capillary pressure at the wetting front, could be replaced by the air-bubbling capillary pressure \( h_{ab} \) in the model presented by Brooks & Corey (1964, 1966).

**Wetting front in Layer 1**

In regard to the infiltration during unsteady rainfall events, the rainfall consists of a series of time intervals of equal length, within which the rainfall rate remains constant. The infiltration process at time \( t_k \) can be determined simultaneously by the ponding depth \( h_0 \) of the initial time (ponding water depth in the early stages of the period), rainfall rate \( P \), and ponding infiltration rate \( F_{pt} \), where \( t_k \) is the time at \( x \) (\( x \in \{0, 1, \ldots, n\} \), and \( n \) represents the number of times the discrete time periods occurred in the rainfall record. Since the ponding water depth \( h_0 \) has a different unit from the rainfall intensity \( P \), it is necessary to first divide the ponding water depth in the early stage of this period by time \( \Delta t \) during the calculation process (\( \Delta t = t_k - t_{k-1} \)), and then substitute this into the model for the calculation.
As illustrated in Figure 4, according to the ponding depth $h_0$ in the early stage of the period, along with the rainfall intensity $P'$, rainfall rate $P$, and ponding infiltration rate $f_{pt}$, the infiltration process can be divided as follows.

Case a: $h_0 = 0$, $P > f_{pt} \geq k_a$, where $k_a$ is the average actual hydraulic conductivity in the transmission zone behind the wetting front. In this case, ponding will occur, and the infiltration process can be divided into non-ponding infiltration $f_{npt}$ (before $t_p$) and ponding infiltration $f_{pt}$ (after $t_p$), where $f_{npt}$ is the non-ponding infiltration rate and $t_p$ is the time to surface ponding.

Case b: $h_0 > 0$, $P' + P < k_a \leq f_{pt}$. In this case, no ponding will occur, and the infiltration process can be divided into ponding infiltration $f_{pt}$ (before $t_p$) and non-ponding infiltration $f_{npt}$ (after $t_p$).

Case c: $h_0 > 0$, $P' + P \geq f_{pt} \geq k_a$. In this case, the infiltration process is under the ponding condition $f_{pt}$.

Case d: $h_0 = 0$, $P < k_a \leq f_{pt}$. In this case, the infiltration process is under the non-ponding condition $f_{npt}$.

The wetting front for the infiltration in the first layer is illustrated in Figure 5. Before ponding:

$$f_{npt} = P_t$$
$$F_t = F_{t-1} + (t - t_{n-1})P_t$$

where $f_{npt}$ is the non-ponding infiltration rate, $P_t$ is the rain intensity, $F_t$ represents the accumulated infiltration, $t$ is the time ($t_{n-1} < t \leq t_n$), $t_n$ is the time at $x$ ($x \in 0, 1, \ldots, n$), and $n$ is the number of the time discrete time periods in the rainfall record.

![Figure 4](http://iwaponline.com/hr/article-pdf/50/2/431/548938/nh0500431.pdf)  
Figure 4 | Shift between the ponding and non-ponding conditions: cases a, b, c, and d represent the different infiltration processes under the ponding and non-ponding conditions.
Then, after ponding occurs (neglecting ponding depth $h_0$):

$$\frac{dF_{tx}}{dt} = f_{pt} = k_{a,1} \left( Z + SW_{cf,1} - SW_{aw,1} \right) \frac{Z}{Z}$$

$$= k_{a,1} \left( Z + SW_{cf,1} - 0.5(h_{ab,1} + h_{aw,1}) \right)$$

(9)

where $F_{tx}$ represents the accumulated infiltration, $f_{pt}$ is the infiltration rate after ponding, $k_{a,1}$ is the actual hydraulic conductivity of Layer 1, $SW_{cf,1}$ denotes the average capillary pressure at the wetting front in the first layer, $h_{ab,1}$ is the air-bubbling capillary value of Layer 1, $h_{aw,1}$ represents the water-bubbling value of Layer 1, $SW_{aw,1}$ is the air pressure of the soil at the wetting front in the first layer and $Z$ is the wetting front distance.

Since:

$$Z = \frac{F_{tx}}{\Delta \theta_1}$$

then let:

$$A = [SW_{cf,1} - 0.5(h_{ab,1} + h_{aw,1})] \Delta \theta_1$$

(10)

(11)

where $\Delta \theta_1 = \theta_{a,1} - \theta_{a,0,1}$, $\theta_{a,0,1}$ is the initial moisture content of Layer 1, $\theta_{a,1}$ is the actual moisture content of Layer 1, and $A$ represents the parameters.

Then:

$$\frac{dF_{tx}}{dt} = f_{pt} = k_{a,1} \left( 1 + \frac{A}{F_{tx}} \right)$$

(12)

and after the integration of Equation (12), the following can be obtained:

$$F_{tx} - F_{tp} = k_{a,1}(t_x - t_p) + A \ln \left( \frac{A + F_{tx}}{A + F_{tp}} \right)$$

(13)

Also, based on Equation (12), the following can be obtained:

$$P_{tp} = f_p = k_{a,1} \left( 1 + \frac{A}{F_{tp}} \right), F_{tp} = \frac{A}{P_{tp} / k_{a,1} - 1}$$

(14)

then:

$$F_{tp} - F_{tx} = P_{tp}(t_p - t_x - 1), t_p - t_x - 1 + \frac{F_{tp} - F_{tx}}{P_{tp}}$$

(15)

where $F_{tp}$ denotes the accumulated infiltration at the instant of the surface ponding, $t_p$ is the time to the surface ponding, $P_{tp}$ represents the rain intensity during the $x$th time step when the surface ponding occurs, $f_p$ denotes the infiltration rate at the instant of the surface ponding, $t_x$ is the time at $x$ ($x \in 0, 1, \ldots, n$), and $n$ represents the number of time discrete time periods in the rainfall record, which is the constant rainfall rate during the time period.

**Wetting front in Layer $m$ ($2 \leq m \leq n$)**

Water infiltration process of the soil under ponding conditions. In the cases where the surface ponding occurred from the beginning of the rain events and the ponding was continuous, Equations (12) and (13) can be applied, before the wetting front enters the second soil layer ($Z \leq Z_1$). After it enters the $m$th layer (Figure 6), then $m_{th} = 2, 3, \ldots, m$, and ($Z > Z_1$), the following can be obtained:

$$\frac{dF_{tx}}{dt} = f_{pt} = k_{a,m} \left[ 1 + \frac{SW_{cf,m} - 0.5(h_{ab,m} + h_{aw,m})}{Z} \right]$$

(16)
where \( k_a \) is the average actual hydraulic conductivity in the transmission zone behind the wetting front, \( SW_{cf,i} \) represents the average capillary pressure head of Layer \( i \), \( SW_{cf,i} \) is the air-bubbling capillary value of Layer \( i \), \( SW_{hwb,i} \) is the water-bubbling value of Layer \( i \).

Due to the fact that:

\[
F_{tx} = \sum_{i=1}^{m-1} Z_i \Delta \theta_i + \left( Z - \sum_{i=1}^{m-1} Z_i \right) \Delta \theta_m,
\]

\[
Z = \frac{\sum_{i=1}^{m-1} Z_i \Delta \theta_i}{\Delta \theta_m} + \sum_{i=1}^{m-1} Z_i
\]

where \( Z \) is the wetting front distance, \( Z_i \) is the thickness of the \( i \)th soil layer, \( \Delta \theta_i = \theta_{a,i} - \theta_{0,i} \), \( \theta_{a,i} \) represents the actual moisture content of Layer \( i \), \( \theta_{0,i} \) is the initial moisture content of Layer \( i \), \( \Delta \theta_m = \theta_{a,m} - \theta_{0,m} \), \( \theta_{a,m} \) represents the actual moisture content of Layer \( m \), \( \theta_{0,m} \) is the initial moisture content of the layer \( m \), the subscripts \( i \) and \( m \) represent the number of soil layers, respectively.

Then let:

\[
B = \sum_{i=1}^{m-1} \frac{Z_i}{k_{a,m}} + 0.5 \left( SW_{hwb,i} + h_{wbi} \right), \quad C = \sum_{i=1}^{m-1} \frac{Z_i \Delta \theta_i}{k_{a,m} \Delta \theta_m} \quad D = \sum_{i=1}^{m-1} \frac{Z_i \Delta \theta_i}{k_{a,m} \Delta \theta_m}
\]

where \( B, C, \) and \( D \) are the parameters, \( k_{a,i} \) is the actual hydraulic conductivity of Layer \( i \), and the subscripts \( i \) and \( m \) represent the number of soil layers, respectively.

By assuming:

\[
B_{m-1} = B, \quad C_{m-1} = C, \quad D
\]

the following can be obtained:

\[
f_{pt} = \frac{C_{m-1} + F_{tx}}{C_m + F_{tx}} \quad F_{tx} = F_{m-1} = k_{a,m} (t_m - t_{m-1}) + B_{m-1} \cdot \ln \left( \frac{B_{m-1} + C_{m-1} + F_{tx}}{B_{m-1} + C_{m-1} + F_{m-1}} \right)
\]

where:

\[
B_{m-1} = \sum_{i=1}^{m-1} Z_i + SW_{cf,i} - 0.5 (h_{wbi} + h_{wbi}) - \sum_{i=1}^{m-1} Z_i \Delta \theta_m\]

\[
C_{m-1} = \sum_{i=1}^{m-1} \frac{Z_i k_{a,m} / k_{a,i}}{Z_i \Delta \theta_i} \quad D = \sum_{i=1}^{m-1} \frac{Z_i \Delta \theta_i}{k_{a,m} \Delta \theta_m}
\]

in the equation, \( t_m \) is the time when the wetting front reaches the interface between the \( (m - 1) \)th and \( m \)th layers. This can be solved successively from \( t_1, t_2, \ldots \), to \( t_m \); \( F_{m-1} \) denotes the accumulated infiltration above the \( m - 1 \) layer; and \( B_{m-1} \) and \( C_{m-1} \) are the parameters, respectively.

During the ponding infiltration process in the condition of rainfall, the excess rainfall will first fill the surface storage, then contribute to the surface runoff. Next, according to the mass balance equation, at the time step \( t \), the cumulative surface runoff (neglecting the evaporation/evapotranspiration during the rainfall period) can be defined as follows:

\[
R_{ts} = Q_{ts} - F_{ts} - h_0
\]
Water infiltration in the soil when ponding occurs at \( t > 0 \). If the surface ponding first occurs in the \( m \)th layer, according to Equation (20), the following will be obtained:

\[
P_{tp} = f_p = k_{a,m} \left( 1 + \frac{B_{m-1}}{C_{m-1} + F_{tp}} \right) (24)
\]

\[
F_{tp} = \frac{B_{m-1}}{P_{tp}/k_{a,m} - 1} - C_{m-1}
\]

then:

\[
t_p = t_{x-1} + \frac{F_{tx} - F_{tx-1}}{P_{tp}} (25)
\]

where \( k_{a,m} \) is the actual hydraulic conductivity of Layer \( m \), \( F_{tp} \) is the accumulated infiltration at the instant of the surface ponding, \( t_p \) represents the time to surface ponding, \( P_{tp} \) is the rain intensity during the \( x \)th time step when the surface ponding occurs, \( f_p \) denotes the infiltration rate at the instant of surface ponding, \( B_{m-1} \) and \( C_{m-1} \) are the parameters, respectively.

Then, according to Equation (21), if the ponding had occurred from the beginning and continued, then the pseudo-time \( t_p \) to infiltrate \( F_{tp} \) can be calculated as follows:

\[
F_{tp} - F_{m-1} = k_{a,m}(t_p - t_{m-1}) - B_{m-1}
\]

\[
\cdot \ln \left( \frac{B_{m-1} + C_{m-1} + F_{tx}}{B_{m-1} + C_{m-1} + F_{tx-1}} \right) (26)
\]

Also, by considering the differences between the actual time coordinates and the assumed ones, Equation (26) should be modified as follows:

\[
F_{tx} - F_{m-1} = k_{a,m}(t_x - t_{m-1} - (t_p - t_p')) + B_{m-1}
\]

\[
\cdot \ln \left( \frac{B_{m-1} + C_{m-1} + F_{tx}}{B_{m-1} + C_{m-1} + F_{tx-1}} \right) (27)
\]

By combining the above equations, the following can be obtained.

Before ponding:

\[
f_{pns} = P_{tx} (28)
\]

\[
F_{tx} = F_{tx-1} + (t - t_{x-1})P_{tx} (29)
\]

After ponding:

\[
f_{pt} = k_{a,m} \left( 1 + \frac{B_{m-1}}{C_{m-1} + F_{tx}} \right) (30)
\]

\[
F_{tx} - F_{tp} = k_{a,m}(t_x - t_p) + B_{m-1} \ln \left( \frac{B_{m-1} + C_{m-1} + F_{tx}}{B_{m-1} + C_{m-1} + F_{tx-1}} \right) (31)
\]

\[
F_{tp} = \frac{B_{m-1}}{P_{tp}/k_{a,m} - 1} - C_{m-1}, t_p = t_{x-1} + \frac{F_{tp} - F_{tx-1}}{P_{tx}} (32)
\]

During the non-ponding infiltration process under rainfall conditions, all of the rainfall infiltrates into the soil layers, and no surface runoff occurs. During the ponding infiltration process, the surface runoff can be calculated by the mass balance equation (Equation (23)). The infiltration can then be calculated by Equations (12) and (13) before the wetting front enters the second soil layer (\( Z \leq Z_1 \)), and by using Equations (30) and (31) after it enters the \( m \)th layer.

**Laboratory rainfall infiltration-runoff experiments**

To the authors’ knowledge, infiltration-runoff experiments regarding three-layered soil under unsteady rainfall conditions have not yet been reported. The combinations of the layered patterns of the different soil types are known to have great effects on the infiltration process when the wetting front enters the interface between the different soil types. Therefore, further investigations via infiltration experiments are required. In addition, during the process of rainfall infiltration, the air in the soil is difficult to discharge, as it is sealed within the soil body. The air pressure produced by the enclosed air hinders the soil infiltration process. The influence of the air resistance on the rainfall infiltration-runoff process also requires further experimental researches.

**Figure 7** shows the schematic of this study’s experimental setup. The experiments were performed on a uniformly packed column. The length and inner diameter of this column were 50 cm and 29 cm, respectively. The soil column bottom was sealed to prevent the air in the soil from emitting from the bottom. A Marriott tube (with an inner diameter of 10 cm, and height of 50 cm) was used to supply water, and the rainfall intensity was controlled by a
medical needle with a No. 5 pinhead simulating 0.10 to 0.32 mm/min; a No. 7 pinhead simulating 0.32 to 1.9 mm/ min; and a No.12 pinhead simulating larger than 1.9 mm/ min. The rainfall simulator was kept at a certain distance (approximately 2.0 m from the surface of the soil).

In this study, three types of air drying soil, which had been obtained from China’s Hunan Province, were contained in the aforementioned column, and formed the clay, loam, and sand layers. In order to obtain sand, clay, and loam, three sampling fields were selected, and three sampling sites were set up. The soil samples (three replications for each type of soil) were collected with a soil wreath knife (100 cm³) at sampling sites to determine soil dry bulk density.

The loading patterns are shown in Table 2. We chose these three-layer soil loading patterns rather than other kinds of soil loading patterns based on two considerations: (1) to save the experimental work task because there is a total of six loading patterns for the three kinds of soil; (2) the three soil loading patterns in Table 2 are believed to be more frequently encountered in the field than other kinds, for example, the sand/loam/clay loading pattern and the sand/loam/clay loading pattern in farmlands because of land reclamation and tillage activities.

The air-dried soil was filled in layers at a certain bulk density (the same as sampling sites), with a number of small increments in the vertical cylinder. Each increment was set at 5 cm, and each soil layer was 15 cm thick. The dry soil was divided into layers according to volume weight (5 cm/layer), as shown in Table 2, and placed into the soil column. During the loading process, in order to avoid artificial effects on soil layer interface, when the upper soil samples were being loaded, the lower compacted soil samples were first loosened, and then filled by the upper soil samples. Then, in accordance with the test settings, the filling thickness of each type of soil was 15 cm, and the total filling thickness 45 cm. During the compacting process, five TDR probes were installed into the column to measure the water content of the soil. Considering the soil layered characteristic and soil water content change with increase of soil depths, the TDR probes were installed at the depths of 5 cm, 15 cm, 25 cm, 35 cm, and 45 cm. The infiltration in the layered soil experiments was designed with three replications of three treatments, and each treatment included a three-layered soil profile.

The particle-size distribution of the soil was measured by the sieve-pipette method. The saturated soil water content was determined by a soak test (Klute 1986). Then, the tested samples were prepared using the same procedure relative to packing, and had the same initial water content and bulk density as those in the column. The saturated soil conductivity was determined by a constant-head method, as illustrated in Figure 8, with a head difference of 10 cm (Huo et al. 2010). All of the soil samples were slowly saturated from the bottom for approximately 1 day, in order to ensure that they became fully saturated before the saturated water content and hydraulic conductivity were measured. The initial water content and bulk density of the soil were determined by an oven drying method with three
replications. The field moisture capacity was determined by the Colman method (Colman 1947). The physical properties of the soil are shown in Table 3. A pressure plate method was used to determine the water characteristic curve (US-made 1500-type pressure membrane instrument) of the soil, and the van Genuchten (1980) model was adopted for fitting (Figure 9). The fitting parameters are shown in Table 4.

The infiltration-runoff experiments in this study were conducted with a rainfall process. During the experiment, the water table was a Mariotte bottle, and the wetting front distance and runoff were measured to calculate the cumulative infiltration, infiltration rate, and wetting front distance movement. The experiment was terminated when the wetting front reached the bottom of the last soil layer. Then, after the infiltration experiment was completed, the soil cores at different depths were sampled to measure the water content using an oven-dry method. The water table remained at 25 ± 1 °C and the evaporation relevant to the infiltration was minimal, and thus neglected.

The van Genuchten (1980) model can be calculated as follows:

$$S = \frac{\theta - \theta_r}{\theta_s - \theta_r} = \left[\frac{1}{1 + (ah)^n}\right]^m$$

(33)

where $h$ is the soil suction, $\theta$ is the soil water content, $\theta_r$ is the residual water content, and $m$, $n$, and $S$ represent the parameters, $m = 1-1/n$.

Estimation and simulation of the parameters

The successful performance of the MGAM is dependent on the determination of the appropriate values of actual hydraulic conductivity ($k_{a,i} = S_{a,i} \cdot k_s$), actual soil water content ($\theta_{a,i} = S_{a,i} \cdot \theta_s$), average capillary pressure at the wetting front ($SW_{cf}$), air-bubbling capillary pressure ($h_{ab}$), and water-bubbling pressure ($h_{wb}$).

In the soil water profile above the wetting front, the soil pores consist of flowing water, residual air, and residual water. The saturated coefficient ($S_{aw}$) reflects the saturation degree of the soil water profile above the wetting front. In the condition of full saturation, the actual water content in the soil water profile above the wetting front is found to be equal to the total porosity of soil, and the $S_{aw}$ is equal to 1 ($S_{aw} = 1$). Also, as a result of the air entrapment, the actual water content in the zone is equal to the difference between the saturated water content plus the residual air and residual water content. Therefore, the saturation coefficient can be determined by the following equation:

$$S_{aw} = 1 - \frac{\theta_{aw} - \theta_{rw}}{\theta_s}$$

(34)

<table>
<thead>
<tr>
<th>Weight of particle size classes (%)</th>
<th>Bulk density/(g/cm³)</th>
<th>Saturated water content/(cm³/cm³)</th>
<th>Field moisture capacity/(cm³/cm³)</th>
<th>Residual water content/(cm³/cm³)</th>
<th>Saturated hydraulic conductivity/(cm/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sand</td>
<td>&gt; 50 μm</td>
<td>0.00</td>
<td>0.00</td>
<td>1.42</td>
<td>0.41</td>
</tr>
<tr>
<td>Clay</td>
<td>2-50 μm</td>
<td>0.00</td>
<td>0.00</td>
<td>1.44</td>
<td>0.46</td>
</tr>
<tr>
<td>Loam</td>
<td>&lt; 2 μm</td>
<td>0.00</td>
<td>0.00</td>
<td>1.43</td>
<td>0.43</td>
</tr>
</tbody>
</table>
where $\theta_{ra}$ is the residual air content, $\theta_{rw}$ is the residual water content.

The values of $\theta_{ra}$ and $\theta_{rw}$ are difficult to determine, and complex laboratory infiltration experiments and precise setups are required, such as the use of a tension-pressure infiltrometer and an air flowmeter (Wang et al. 1998). Chong et al. (1981) proposed that the saturation coefficient of a soil layer could be readily calculated with the measured water content in the soil water profile above the wetting front at the termination of the infiltration experiment ($\theta_a$) by $S_{sat} = \theta_a/\theta_s$. However, it was found to be more effective to develop an independent approach which only required the physical properties and hydraulic parameters of the soil to determine the saturation coefficient.

During the infiltration process, the soil water content profile above the wetting front can be divided into four zones (Bodman & Colman 1944, 1945; Shao et al. 2006), including the saturated, transitional, transmitting, and wetting zones. The water content of the soil in the saturated zone was found to be equal to the saturated water content. In the transitional zone, the water content was determined to be close to the saturated water content, and the transmitting and wetting zones were close to the field moisture capacity (Gan et al. 2012). The water content of the soil above the wetting front was also divided into a saturated zone and a transitional zone. The saturated zone was approximately half the area of the soil water profile above the wetting front (Wang et al. 2002). In this study, for the sake of simplicity, the soil water content profile above the wetting front was also divided into saturated and transitional zones. The saturated zone was half of the soil water content profile above the wetting front: $\theta_a = \theta_s$ ($S_{sat} = 1$) in the saturated zone, and $\theta_a = \theta_h$ ($S_{sat} = \theta_h/\theta_s$) in the transitional zone, where $\theta_h$ is the saturated water content and $\theta_h$ is the field moisture capacity.

The relative water conductivity ($S_{ak}$) reflects the influence of $S_{sat}$ on soil hydraulic conductivity. Therefore, the saturation coefficient can be determined by the following equation (van Genuchten 1980, van Genuchten et al. 1991):

$$S_{ak} = S_{sat}^{m} \left[1 - \left(1 - S_{sat}^{1/m}\right)^{m}\right]^{2}$$

(35)

The $S_{ak}$ is difficult to determine, as the values of $\theta_{ra}$ and $\theta_{rw}$ are difficult to determine. The $S_{ak}$ can also be determined by an independent approach which only requires the physical properties and hydraulic parameters of the soil. Bouwer (1966) suggested that $k_a$ was half of $k_s$, and $S_{ak} \approx 0.5$ (BGAM). Ma et al. (2011) proposed that the $S_{ak}$
was approximately equal to one minus the plus of the residual air $\theta_r$ and residual water saturation degree $\theta_{rw}$, which could be determined from the water retention curve equation of the soil. However, this study showed the following. (1) The use of the residual soil water content $\theta_r$ instead of adding the residual air $\theta_r$ and residual water saturation degree $\theta_{rw}$ was found to be suitable for loam and clay. However, it was not suitable for sand. The reasons are that the $(\theta_r-\theta_s)/\theta_r$ for loam and sand (0.82 and 0.83) are approximately equal to $\theta_g/\theta_s$ (0.89 and 0.77); however, for sand (0.90) it is much larger than $\theta_g/\theta_s$ (0.44), approximate to 1.0. Also, using the saturated soil water content $\theta_s$ minus the field moisture capacity $\theta_h$ instead of adding the residual air $\theta_r$ and residual water saturation degree $\theta_{rw}$, was found to be suitable for loam, sand, and clay. (2) The $S_{sh}$ was determined by the combination of the soil’s layered pattern, and the $S_{sh}$ could be calculated with the saturated soil water content $\theta_s$ minus the field moisture capacity $\theta_h$ for the first layer ($S_{sh} \approx \theta_g/\theta_s$), and 0.5 ($S_{sh} \approx 0.5$) (Bouwer 1966) for the remaining layers ($i > 1$).

The air pressure ($SW_{ad}$) at the wetting front varied dynamically from the air-closing value ($H_s$) to the air-breaking value ($H_b$). Also, the corresponding value of the average capillary pressure ($SW_{cf}$) at the wetting front automatically varied following a scanning drainage curve towards a scanning wetting curve. Whisler & Bouwer (1970) proposed that the average capillary pressure $SW_{cf}$ could be replaced by the air-bubbling capillary pressure in the wetting retention model of Brooks & Corey (1964, 1966). For example, $SW_{cf} = h_{ab} = 1/\alpha_{d}$, where the air pressure $SW_{ad} = h_0 + 0.5 (h_{ab} + h_{wb})$, and $h_0$ is the ponding depth. The value of $h_{ab} = 1/\alpha_{d}$, where $\alpha_{d}$ is the parameter of the main drainage curve and $h_{wb} = h_{ab}/2\delta$ ($\delta$ equals approximately 0 to 2 cm for sandy soil; 2 to 5 cm for loamy soil, and 8 to 10 cm for clay soil).

The $\alpha_{d}$ is determined by the BC model (Brooks & Corey 1964):

$$\frac{\theta - \theta_r}{\theta_h - \theta_s} = \left( \frac{h_0}{h} \right)^{\lambda} = \left( \frac{1}{\alpha_d h} \right)^{\lambda} \quad \alpha_d h > 1$$

$$\theta - \theta_s \quad \alpha_d h \leq 1$$

(36)

where $h$ is the soil water pressure head, $\alpha_d$ is an empirical parameter, $h_e$ is air entry value, $\lambda$ is the pore-size distribution parameter. $\theta_r$ and $h_a$ can be determined by fitting the measured soil water retention curve (RETC) with the RETC code developed by van Genuchten et al. (1991).

The soil suction head $SW$ at the wetting front for the TGAM model was calculated with Equation (36) as follows (Bouwer 1969):

$$SW = h_{ab}/2$$

(37)

where $SW$ is the average soil suction head at the wetting front.

The Mein–Larson model (MLGAM) was developed for infiltration-runoff under a constant intensity rainfall in homogeneous soil with uniform initial moisture content. Nearing et al. (1996) proposed an effective hydraulic conductivity for the MLGAM model, which was calculated with Equation (38) as follows:

$$k_e = \frac{56.82 \cdot h_0^{2.86}}{1 + 0.051 \cdot \exp(0.062 \cdot CN)} - 2$$

(38)

$$CN = \frac{25400}{234 + S}$$

$$S = 5[P + 2R - (4R^2 + 5PR)^{1/2}]$$

(39)

(40)

where $k_e$ represents the effective hydraulic conductivity, $k_s$ is the saturated hydraulic conductivity, $CN$ denotes the curve number of the SCS-CN model, $S$ is the potential maximum retention, $P$ represents the total rainfall (cm), and $R$ is the runoff volume (cm).

The MLGAM was found to be suitable for simulating the infiltration-runoff process in a homogeneous soil with uniform initial moisture content. However, when the MLGAM model was applied for the simulation of the infiltration-runoff process in a layered soil profile, it was found that the uniform soil-water movement parameters were difficult to obtain. Therefore, in order to obtain the uniform soil-water movement parameters of a layered soil profile, a harmonic mean was applied to calculate the average soil saturated hydraulic conductivity along with the average suction head of the wetting front. Then, a mathematical mean was applied to calculate the saturated water and soil initial water contents of the soil. The effective hydraulic conductivity was then
calculated with Equation (38). The total rainfall $P$ and runoff volume $R$ were the measured values. The values of $P$ and $R$ were different between different loading patterns. Thus, $CN$ and $k_e$ have different values for different loading patterns.

In all of the models (with the exception of the MLGAM), the soil saturated water content $\theta_s$, field moisture capacity $\theta_h$, and saturated hydraulic conductivity $k_s$ were measured. Also, the relative water conductivity ($S_{ak}$), soil suction head $SW_{cf}$ at the wetting front, air-bubbling capillary pressure $h_{ab}$, and water-bubbling pressure $h_{wb}$ were determined by the soil properties. The models did not conduct any parameter adjustments, with the exception of the MLGAM. The soil hydraulic parameters, and the model parameters for the laboratory experiments, are summarized in Tables 5–7, respectively.

With the previously determined parameters, the MGAM, TGAM, BGAM, and MLGAM were applied to simulate the runoff rate, water distribution, and cumulative infiltration of the soil. In order to compare the observed and measured values, the average relative error ($ARE$), mean absolute error ($MAE$), Nash–Sutcliffe efficiency coefficient ($NSE$), and root mean square error ($RMSE$) were used as the four criteria by which to reflect the effectiveness of the

### Table 5 | Soil hydraulic and model parameters for the MGAM

<table>
<thead>
<tr>
<th>Treatments</th>
<th>Depth/cm</th>
<th>$\theta_0/(\text{cm}^3/\text{cm}^3)$</th>
<th>$\theta_h/(\text{cm}^3/\text{cm}^3)$</th>
<th>$\theta_s/(\text{cm}^3/\text{cm}^3)$</th>
<th>$k_s/(\text{cm/min})$</th>
<th>$S_{ak}$</th>
<th>$\alpha_{dcm}/\text{cm}^3$</th>
<th>$h_{ab}$ cm</th>
<th>$h_{wb}$ cm</th>
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</thead>
<tbody>
<tr>
<td>Clay-loam-sand</td>
<td>0–15</td>
<td>0.09</td>
<td>0.41</td>
<td>0.46</td>
<td>0.003</td>
<td>0.89</td>
<td>0.008</td>
<td>125</td>
<td>52</td>
</tr>
<tr>
<td></td>
<td>15–30</td>
<td>0.07</td>
<td>0.31</td>
<td>0.43</td>
<td>0.07</td>
<td>0.50</td>
<td>0.016</td>
<td>63</td>
<td>31</td>
</tr>
<tr>
<td></td>
<td>30–45</td>
<td>0.05</td>
<td>0.18</td>
<td>0.41</td>
<td>1.43</td>
<td>0.50</td>
<td>0.145</td>
<td>7</td>
<td>2</td>
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<tr>
<td>Sand-clay-loam</td>
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<td>0.05</td>
<td>0.18</td>
<td>0.41</td>
<td>1.43</td>
<td>0.44</td>
<td>0.145</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>15–30</td>
<td>0.09</td>
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<td>0.50</td>
<td>0.145</td>
<td>63</td>
<td>31</td>
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<td>15–30</td>
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<td>0.18</td>
<td>0.41</td>
<td>1.43</td>
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<td>0.145</td>
<td>7</td>
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<tr>
<td></td>
<td>30–45</td>
<td>0.09</td>
<td>0.41</td>
<td>0.46</td>
<td>0.005</td>
<td>0.5</td>
<td>0.008</td>
<td>125</td>
<td>52</td>
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</table>

### Table 6 | Soil hydraulic and model parameters for the TGAM

<table>
<thead>
<tr>
<th>Treatments</th>
<th>Depth/cm</th>
<th>$\theta_0/(\text{cm}^3/\text{cm}^3)$</th>
<th>$\theta_h/(\text{cm}^3/\text{cm}^3)$</th>
<th>$\theta_s/(\text{cm}^3/\text{cm}^3)$</th>
<th>$k_s/(\text{cm/min})$</th>
<th>$SW/cm$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay-loam-sand</td>
<td>0–15</td>
<td>0.09</td>
<td>0.46</td>
<td>0.46</td>
<td>0.003</td>
<td>62</td>
</tr>
<tr>
<td></td>
<td>15–30</td>
<td>0.07</td>
<td>0.43</td>
<td>0.43</td>
<td>0.07</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>30–45</td>
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<td>0.41</td>
<td>0.41</td>
<td>1.43</td>
<td>3.5</td>
</tr>
<tr>
<td>Sand-clay-loam</td>
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<td>0.41</td>
<td>0.41</td>
<td>1.43</td>
<td>3.5</td>
</tr>
<tr>
<td></td>
<td>15–30</td>
<td>0.09</td>
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<td>0.46</td>
<td>0.003</td>
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<td></td>
<td>30–45</td>
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<td>0.43</td>
<td>0.43</td>
<td>0.07</td>
<td>32</td>
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<tr>
<td>Loam-sand-clay</td>
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<td>0.43</td>
<td>0.43</td>
<td>0.07</td>
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<td>1.43</td>
<td>3.5</td>
</tr>
<tr>
<td></td>
<td>30–45</td>
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<td>0.46</td>
<td>0.003</td>
<td>62</td>
</tr>
</tbody>
</table>

### Table 7 | Soil hydraulic and model parameters for the MLGAM

<table>
<thead>
<tr>
<th>Depth cm</th>
<th>$\theta_0/(\text{cm}^3/\text{cm}^3)$</th>
<th>$\theta_h/(\text{cm}^3/\text{cm}^3)$</th>
<th>$CN/cm$</th>
<th>$k_s/(\text{cm/min})$</th>
<th>$k_e/(\text{cm/min})$</th>
<th>$SW/cm$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–45</td>
<td>0.07</td>
<td>0.43</td>
<td>63</td>
<td>0.009</td>
<td>0.04</td>
<td>9.0</td>
</tr>
<tr>
<td>0–45</td>
<td>0.07</td>
<td>0.43</td>
<td>59</td>
<td>0.009</td>
<td>0.05</td>
<td>9.0</td>
</tr>
<tr>
<td>0–45</td>
<td>0.07</td>
<td>0.43</td>
<td>62</td>
<td>0.009</td>
<td>0.04</td>
<td>9.0</td>
</tr>
</tbody>
</table>
simulation, as follows:

\[
ARE = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{Q_{p,i} - Q_{o,i}}{Q_{o,i}} \right) \times 100
\]

(41)

\[
MAE = \frac{1}{n} \sum_{i=1}^{n} |Q_{p,i} - Q_{o,i}|
\]

(42)

\[
NSE = 1 - \frac{\sum_{i=1}^{n} (Q_{p,i} - Q_{o,i})^2}{\sum_{i=1}^{n} (Q_{o,i} - \bar{Q}_o)^2}
\]

(43)

\[
RMSE = \sqrt{\frac{\sum_{i=1}^{n} (Q_{p,i} - Q_{o,i})^2}{n}}
\]

(44)

where \( n \) is the number of observations, \( Q_{o,i} \) and \( Q_{p,i} \) are the observed and predicted values, respectively, \( \bar{Q}_o \) represents the mean values of \( Q_{o,i} \).

Parameter sensitivity analysis

The successful performance of the MGAM was dependent on determining the appropriate input parameters. For the MGAM, the following factors needed to be considered: the determination of which parameter was dominant; the reduction of the calibration uncertainty; and the enhancement of the optimization efficiency. However, the means by which to achieve these still remained unclear. Therefore, the MGAM required further examination by a parameter sensitivity analysis. An LH-OAT parameter sensitivity analysis was adopted in the MGAM model. The LH-OAT is a type of new method which integrates the LH (Latin-Hypercube) sampling, and the OAT (one-factor-at-a-time) sensitivity analysis. Therefore, it has the advantages of both the LH sampling and the OAT sensitivity analysis.

The LH-OAT method was used as follows: first of all, it divided the entire parameter space into \( N \) layers. Then, one sample was separately taken in every layer, for one LH sampling point (including the parameter set of the \( P \) parameters). Finally, changes were made in the parameters for the \( P \) times for every LH sampling point, and only one parameter was changed each time. The LH-OAT method was able to be fulfilled within several circulations. The starting point of every circulation was the LH sampling point, and the partial influence value \( S_{ij} \) of every parameter \( e_i \) could be calculated on the basis of the following formula:

\[
S_{ij} = \left. \frac{100}{\left( M(e_1, \ldots, e_i+1+f_i, \ldots, e_p) - M(e_1, \ldots, e_i, \ldots, e_p) + M(e_1, \ldots, e_i, \ldots, e_p)/2 \right)} \right| f_i
\]

(45)

where \( M(\cdot) \) is the model objective function, \( f_i \) represents the slight disturbance incurred by the parameter change \( e_i \), and \( j \) is a sampling point of LH. Equation (45) shows that a \( f_i \) change can result in increasing or decreasing the parameters. The circulation was carried out for \( P+1 \) times. The final influence value was calculated from the average partial influence of every circulation of all of the LH sampling points. The LH method had \( N \) intervals, and therefore the model had to be operated for \( N^*(P+1) \) times.

A final effect (sensitivity degree \( SA \)) was calculated by averaging the partial effects of each loop for all of the Latin Hypercube points. The final effects could then be ranked with the calculated sensitivity degree, as shown in Table 8.

In order to analyze the sensitivity of the input parameters of the MGAM model, it was applied to the infiltrations in a three-layered soil profile during rainfall conditions. Then, 12 key parameters of the MGAM were selected for the sensitivity analysis. The observed cumulative infiltration and runoff rate were selected as the targeted output variables. Table 9 lists the name, range, and description of each input parameter which was investigated in this study. The MGAM model parameters were the relative water conductivity \( (S_{ah,1}, S_{ah,2}, S_{ah,3}) \), air-bubbling pressure

<table>
<thead>
<tr>
<th>Class</th>
<th>Sensitivity degree</th>
<th>Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.00 \leq</td>
<td>SA</td>
</tr>
<tr>
<td>II</td>
<td>0.05 \leq</td>
<td>SA</td>
</tr>
<tr>
<td>III</td>
<td>0.20 \leq</td>
<td>SA</td>
</tr>
<tr>
<td>IV</td>
<td></td>
<td>SA</td>
</tr>
</tbody>
</table>
values \((h_{ab1}, h_{ab2}, h_{ab3})\), parameters \((\delta_1, \delta_2, \delta_3)\), and initial water content deficit \((\Delta \theta_1, \Delta \theta_2, \Delta \theta_3)\).

The sampling process divided the entire parameter space into 20 layers, and 12 parameter sets were obtained through the sampling. An OAT sampling of each parameter set was made in each parameter space LH sampling. Since each OAT sampling only changed one parameter value, and there were 12 parameters in each group, then 12 samplings were made of all the OAT samplings of the LH parameter set in the group. Therefore, the LH-OAT sampling number was \(260 = 20 \times (1 + 12)\). Also, the increment of each input parameter was set at 5%.

Since the MGAM model is suitable to simulate the infiltration-runoff process during unsteady rainfall, the parameters' values change according to soil properties, initial water content, etc. Therefore, the default values of these parameters are changed in a relative way over a certain range. The soil types can be divided into three major types of soil (clay, loam, and sand) according to soil particle-size distribution, thus the default parameters' values change among three major soil types of soil physical properties, for instance over the range 5.0 cm (sand) \(\sim130\) cm (clay) for air-bubbling pressure.

\(S_{sk}\) is a dimensionless coefficient that reflects the influence of saturated coefficient \((S_{sk})\) on soil hydraulic conductivity. Moreover, the saturated coefficient reflects the saturation degree of the soil water profile above the wetting front. \(h_{ab}\) is the air-bubbling capillary pressure value of the material, cm; \(\delta\) is a parameter for calculating the water bubbling pressure \((h_{wb})\), which equals approximately 0 to 2 cm for sandy soil, 2 to 5 cm for loamy soil, and 8 to 10 cm for clay soil (Wang et al. 1997); \(\Delta \theta\) is the initial water content deficit, cm\(^3\)/cm\(^3\).

## RESULTS AND DISCUSSION

### Parameter sensitivity

The results of the sensitivity analysis for the infiltration-runoff process are shown in Table 10. It can be seen that the parameter sensitivity displayed clear differences between the different layers. For the soil's cumulative infiltration and runoff rate, the sensitivity of relative water conductivity values in the first layer were found to be very high (3.42 and 3.18, respectively). Those of the air-bubbling pressure in the first and second layer were high, and all the values ranged from 0.26 to 0.40. The parameters \(\delta\) in the first and second layers were determined to be medium, and all the values ranged from 0.09 to 0.18. The initial moisture deficits in the first and second layers were found to be high or medium, and all of the values ranged from 0.13 to 0.28. The sensitivities of the other parameters were determined to be

<table>
<thead>
<tr>
<th>Name</th>
<th>Infiltration</th>
<th>Runoff rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>(S_{sk,1})</td>
<td>3.42 IV</td>
<td>3.18 IV</td>
</tr>
<tr>
<td>(S_{sk,2})</td>
<td>0.03 I</td>
<td>0.04 I</td>
</tr>
<tr>
<td>(S_{sk,3})</td>
<td>0.00 I</td>
<td>0.00 I</td>
</tr>
<tr>
<td>(h_{ab1})</td>
<td>0.40 III</td>
<td>0.33 III</td>
</tr>
<tr>
<td>(h_{ab2})</td>
<td>0.26 III</td>
<td>0.29 III</td>
</tr>
<tr>
<td>(h_{ab3})</td>
<td>0.00 I</td>
<td>0.00 I</td>
</tr>
<tr>
<td>(\delta_1)</td>
<td>0.14 II</td>
<td>0.18 II</td>
</tr>
<tr>
<td>(\delta_2)</td>
<td>0.09 II</td>
<td>0.11 II</td>
</tr>
<tr>
<td>(\delta_3)</td>
<td>0.00 I</td>
<td>0.00 I</td>
</tr>
<tr>
<td>(\Delta \theta_1)</td>
<td>0.28 III</td>
<td>0.24 III</td>
</tr>
<tr>
<td>(\Delta \theta_2)</td>
<td>0.20 III</td>
<td>0.13 III</td>
</tr>
<tr>
<td>(\Delta \theta_3)</td>
<td>0.00 I</td>
<td>0.00 I</td>
</tr>
</tbody>
</table>
small for the infiltration-runoff process, and all sensitivity values were less than 0.05.

For the cumulative infiltration and runoff rate, the parameter sensitivities were also reflected in the class index, and the sensitivity values of relative water conductivity in the first layer were in class IV, and for the second and third layers, the values were in class I. The sensitivity values of the air-bubbling capillary pressure of the first and second layers were in class III, and were in class I for the third layer. The sensitivity values of the parameters $\delta$ in the first and second layers were in class II, and in class I for the third layer. The sensitivity values of the initial moisture deficit in the first and second layers were in class III or II, and for the third layer were in class I.

In summary, the parameter sensitivity analysis showed that the sensitivity of relative water conductivity was very high in the first layer, and those of the air bubbling pressure and initial moisture deficit were high or medium in the first and second layers, whereas those of the other parameters were found to be small.

**Soil water distribution**

The comparison of the measured and simulated soil moisture distribution, by the MGAM and BGAM (Bouwer 1966); TGAM (Jia & Tamai 1997), and MLGAM (Mein & Larson 1973), at the times when the wetting front entered 15, 30, and 45 cm, are shown in Figure 10. With the exception of the results from the MLGAM, the observed and the simulated soil water distribution were determined by the combination of the soil layered patterns.

The TGAM significantly overestimated the soil water content, and the BGAM underestimated those values for the fine over coarse layered patterns. The MGAM results were in better agreement with those of the measured values. The simulated values by the MLGAM were determined to be larger than those of the observed values in transitional zones, less than those of the observed values in saturated zones for the loading pattern of the clay-loam-sand, and larger than those of the observed values for loading pattern of the sand-clay-loam. Therefore, when compared to the TGAM, BGAM, and MLGAM, the simulated results of the MGAM were found to be in better agreement with the measured soil water content.

The performances of the TGAM, BGAM, MLGAM, and MGAM in the simulations of the soil water content were also reflected in their associated $MAE$, $ARE$, and $RMSE$ values, as shown in Table 11. In regard to the loading patterns of the clay-loam-sand, loam-sand-clay, and sand-clay-loam, most of the $MAE$ values for the BGAM ranged from approximately 0.003 to 0.067 cm$^3$/cm$^3$; for the TGAM, they ranged from approximately 0.026 to 0.043 cm$^3$/cm$^3$; and for the MLGAM, they ranged from approximately 0.065 to 0.073 cm$^3$/cm$^3$, whereas for the MGAM the range was from approximately 0.002 to 0.006 cm$^3$/cm$^3$. In regards to loading patterns of the clay-loam-sand, loam-sand-clay, and sand-clay-loam, it was found that most of the $ARE$ values for the BGAM ranged from approximately −9.72% to 0.46%; for the TGAM, the range was approximately from 3.95% to 10.56%; and for the MLGAM, the range was from approximately 32.29% to 72.95%, whereas for the MGAM the range was determined to be from approximately −0.46% to 3.50%. In regards to the loading patterns of the clay-loam-sand, loam-sand-clay, and sand-clay-loam, the $NSE$ values for the BGAM were found to range from approximately −3.06 to 0.75; for the TGAM, the range was from −5.11 to −0.82; and for the MLGAM, the range was found to be from −21.25 to −8.44, whereas for the MGAM the range was determined to be from approximately 0.75 to 0.89.

The overestimation of the soil water content provided by TGAM assumed that the soil water content profile above the wetting front was fully saturated. However, the soil water content profile above the actual wetting front could not be saturated, due to the entrapment of air during the infiltration process, and the actual water content in the soil water content profile above the wetting front was less than the saturated water content. The underestimation of the soil water content by the BGAM for the fine over coarse layered soil was due to the assumption that hydraulic conductivity of the soil water content profile above the wetting front was only half of the saturated hydraulic conductivity. However, the actual hydraulic conductivity in the first layer above the wetting front was larger than the half of the saturated hydraulic conductivity. Therefore, the BGAM underestimated the soil cumulative infiltration, and provided a smaller depth of wetting front than that of the measured results. The poor performance of the MLGAM for all of the loading patterns
was due to the fact that the MLGAM was suitable for simulating the infiltration-runoff process in homogeneous soil, but not in the layered soil. In addition, to all treatments, the values of saturated water content in MLGAM are obtained by mathematical mean method, so all treatments share the same values above the wetting front.
Cumulative infiltration of the soil

The comparison of the measured and calculated results of the soil cumulative infiltration by the TGAM, BGAM, MGAM, and MLGAM are shown in Figure 11. With the exception of the results from the MLGAM, the observed and simulated soil cumulative infiltrations were determined by the combination of the soil layered patterns.

Also, with the exception of the MLGAM, in the loading patterns of the clay-loam-sand, loam-sand-clay, and sand-clay-loam, the calculated and measured values were found to undergo sudden changes when the wetting front entered 15 cm and 30 cm. The reasons for these changes may have been that the capillary suction and hydraulic conductivity underwent major changes when the wetting front entered the interfaces between the layers.

The simulated soil cumulative infiltration by the TGAM was found to be larger than the measured results, and the differences between them were determined to be related to a combination of soil layered patterns. In regard to the loading patterns of the clay-loam-sand, loam-sand-clay, and sand-clay-loam, the measured total cumulative infiltration values at the termination of the experiments were 14.62, 16.50, and 15.73 cm, respectively. The simulated values by the MGAM were 15.12, 16.60, and 15.53 cm, respectively. However, the simulated values by the BGAM were 10.26, 15.72, and 16.33 cm, respectively; 15.05, 20.28, and 25.21 cm, respectively, by the TGAM; and 58.21, 26.66, and 49.66 cm, respectively, by the MLGAM. Therefore, when compared with the BGAM, TGAM, and MLGAM, the simulated values for the MGAM were found to be in better agreement with the observed values.

### Table 11 - Comparison of the MAE, ARE, and RSME of the soil water content between the simulated and observed results

<table>
<thead>
<tr>
<th>Model</th>
<th>Treatments</th>
<th>MAE/cm^3/cm^3</th>
<th>ARE/%</th>
<th>RMSE/cm^3/cm^3</th>
<th>NSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>MGAM</td>
<td>Clay-loam-sand</td>
<td>0.009</td>
<td>3.50</td>
<td>0.013</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>Sand-clay-loam</td>
<td>0.002</td>
<td>0.52</td>
<td>0.005</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>Loam-sand-clay</td>
<td>0.006</td>
<td>0.46</td>
<td>0.008</td>
<td>0.89</td>
</tr>
<tr>
<td>BGAM</td>
<td>Clay-loam-sand</td>
<td>0.037</td>
<td>-9.72</td>
<td>0.082</td>
<td>-3.06</td>
</tr>
<tr>
<td></td>
<td>Sand-clay-loam</td>
<td>0.003</td>
<td>0.46</td>
<td>0.005</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>Loam-sand-clay</td>
<td>0.067</td>
<td>-3.93</td>
<td>0.012</td>
<td>0.75</td>
</tr>
<tr>
<td>TGAM</td>
<td>Clay-loam-sand</td>
<td>0.041</td>
<td>10.12</td>
<td>0.075</td>
<td>-5.11</td>
</tr>
<tr>
<td></td>
<td>Sand-clay-loam</td>
<td>0.028</td>
<td>3.95</td>
<td>0.053</td>
<td>-0.82</td>
</tr>
<tr>
<td></td>
<td>Loam-sand-clay</td>
<td>0.026</td>
<td>10.56</td>
<td>0.061</td>
<td>-5.04</td>
</tr>
<tr>
<td>MLGAM</td>
<td>Clay-loam-sand</td>
<td>0.065</td>
<td>32.29</td>
<td>0.099</td>
<td>-8.44</td>
</tr>
<tr>
<td></td>
<td>Sand-clay-loam</td>
<td>0.073</td>
<td>72.95</td>
<td>0.133</td>
<td>-14.55</td>
</tr>
<tr>
<td></td>
<td>Loam-sand-clay</td>
<td>0.066</td>
<td>68.56</td>
<td>0.136</td>
<td>-21.25</td>
</tr>
</tbody>
</table>
The performances of the TGAM, BGAM, MLGAM, and MGAM in the simulations of the cumulative infiltration were also reflected in their associated MAE, ARE, RSME, and NSE values, as shown in Table 12. The majority of the MAE for the BGAM ranged from 0.39 to 3.39 cm; TGAM ranged from 0.64 to 1.37 cm; and the MLGAM ranged from 10.83 to 52.85 cm, whereas the MGAM was less than 0.40 cm. The majority of the ARE for the BGAM were in the range of –48.0% to 10.4%; in the TGAM they were larger than 25.0%; and in the MLGAM they were larger than 25.63%, whereas for the MGAM the range was from –7.9% to 6.8%. The majority of the RSME for the BGAM were in the range of 0.46 to 3.78 cm; for the TGAM, the range was from 0.73 to 2.84 cm; and for the MLGAM, the RSME was larger than 12.83 cm, whereas the RSME for the MGAM was less 0.60 cm. Also, the majority of the NSE values for the BGAM were 0.69, 0.90, and 0.91, respectively; for the TGAM, they were 0.98, 0.91, and 0.70; and for the MLGAM, the values were –16.02, –11.22, and –0.63. However, the NSE values for the MGAM were all determined to be larger than 0.98.

The overestimation of the soil cumulative infiltration by the TGAM was due to the fact that it ignored the entrapment of air in the upper soil water profile above the wetting front, and also the air compression below the wetting front. In reality, the upper soil water profile above the wetting front was unsaturated. The hydraulic conductivity of the soil water profile above the wetting front was found to be smaller than that of the saturated zone. Therefore, the TGAM overestimated the cumulative infiltration. In regard to the loading pattern of the clay-loam-sand and loam-sand-clay, the underestimation of the cumulative infiltration by the BGAM was due to the fact that it underestimated the actual hydraulic conductivity of the first layer in the soil water profile above the wetting front. For the loading pattern of the sand-clay-loam, the overestimation of the cumulative infiltration by the TGAM was due to the fact that it ignored the entrapment of air in the upper soil water profile above the wetting front, and also the air compression below the wetting front.

### Table 12 | Comparison of the MAE, ARE, and RSME of the cumulative infiltration of the soil between the simulated and observed results

<table>
<thead>
<tr>
<th>Model</th>
<th>Treatments</th>
<th>MAE/cm</th>
<th>ARE/%</th>
<th>RSME/cm</th>
<th>NSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>MGAM</td>
<td>Clay-loam-sand</td>
<td>0.36</td>
<td>–3.1</td>
<td>0.46</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>Sand-clay-loam</td>
<td>0.37</td>
<td>6.8</td>
<td>0.37</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>Loam-sand-clay</td>
<td>0.31</td>
<td>–7.9</td>
<td>0.60</td>
<td>0.99</td>
</tr>
<tr>
<td>BGAM</td>
<td>Clay-loam-sand</td>
<td>3.39</td>
<td>–48.0</td>
<td>3.78</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td>Sand-clay-loam</td>
<td>0.39</td>
<td>10.4</td>
<td>0.46</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>Loam-sand-clay</td>
<td>2.09</td>
<td>–25.3</td>
<td>2.6</td>
<td>0.91</td>
</tr>
<tr>
<td>TGAM</td>
<td>Clay-loam-sand</td>
<td>0.64</td>
<td>29.2</td>
<td>0.73</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>Sand-clay-loam</td>
<td>1.08</td>
<td>34.8</td>
<td>2.84</td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td>Loam-sand-clay</td>
<td>1.37</td>
<td>25.0</td>
<td>1.89</td>
<td>0.91</td>
</tr>
<tr>
<td>MLGAM</td>
<td>Clay-loam-sand</td>
<td>10.83</td>
<td>82.9</td>
<td>12.83</td>
<td>–16.02</td>
</tr>
<tr>
<td></td>
<td>Sand-clay-loam</td>
<td>52.85</td>
<td>47.86</td>
<td>64.65</td>
<td>–11.22</td>
</tr>
<tr>
<td></td>
<td>Loam-sand-clay</td>
<td>46.54</td>
<td>59.62</td>
<td>59.62</td>
<td>–0.63</td>
</tr>
</tbody>
</table>

![Figure 11](http://iwaponline.com/hr/article-pdf/50/2/431/548938/nh0500431.pdf)
infiltration by the BGAM was due to the fact that it overestimated the actual hydraulic conductivity of the first layer in the soil water profile above the wetting front. The overestimation of the cumulative infiltration by the MLGAM was due to the fact that all of the MLGAM parameters experienced no changes during the infiltration process. However, when the wetting front entered the interfaces between two soil layers, the changes of the soil’s physics resulted in changes in the actual hydraulic conductivity, capillary pressure head, actual water content, and initial water content of the soil, which caused changes in the infiltration process.

Runoff rate

The comparison of the measured and calculated results of the runoff rate are shown in Figure 12. The numbers I, II, and III in Figure 12 represent the repetitions of each treatment. With the exception of the results of the MLGAM simulation, the observed and simulated runoff characteristics were determined by a combination of the soil layered patterns.

The mean calculated runoff-yielding time was also found to be related to a combination of the soil layered patterns. In regard to the loading pattern of the clay-loam-sand, the mean simulation runoff-yielding times by the MGAM (10 minutes), BGAM (4 minutes), and TGAM (10 minutes) were less than that of the observed times (29 minutes), and the mean time of the MLGAM (60 minutes) was found to be markedly larger than that of the observed results. For the loading pattern of the loam-sand-clay, the mean simulation runoff-yielding times by the MGAM (26 minutes) and the TGAM (45 minutes) were determined to be larger than that of the observed results (25 minutes). Also, the BGAM (6 minutes) and MLGAM (6 minutes) were found to be less than that of the observed results. In regards to the loading pattern of the sand-clay-loam, the mean simulated runoff-yielding times by the MGAM (5.0 minutes), BGAM (5.0 minutes), and MLGAM (0 minutes) were determined to be less than that of the observed results (6.0 minutes). However, those of the TGAM (11 minutes) were found to be markedly larger than that of the observed results.

The runoff process is known to fluctuate with the fluctuations in rainfall. The simulated values of the TGAM were found to be less than the observed values. The simulated values of the BGAM were larger than those of the observed values for the loading pattern of the clay-loam-sand and loam-sand-clay, and less than those of the observed values for the loading pattern of the sand-clay-loam. The simulated values by the MLGAM were determined to be less than that of the observed values for the loading pattern of the clay-loam-sand, larger than that of the observed values before the wetting front entered the third layer, and less than that of the observed values after the wetting front entered the third layer for the loading pattern of the sand-clay-loam. They were larger than that of the observed values before the wetting front entered the second layer, and less than that of the observed values after the wetting front entered the second layer for the loading pattern of the sand-clay-loam. Therefore, when compared to the TGAM, BGAM, and MLGAM, the simulated results of the MGAM were found to be in better agreement with the measured runoff rate. The performances of the TGAM, BGAM, MLGAM, and MGAM in the simulations of the runoff rate were reflected in their $MAE$, $ARE$, $RSME$, and $NSE$ values, as shown in Table 13. The majority of the $MAE$ for the BGAM were found to be in the range of 0.006 to 0.083 cm/min; the TGAM range was from 0.007 to 0.048 cm/min; and the MLGAM ranged from 0.036 to 0.152 cm/min, whereas the $MAE$ for the MGAM was less than 0.021 cm/min. The majority of the $ARE$ for the BGAM range from −25.5% to 34.3%; the TGAM were found to be less than −28.12%; and that of the MLGAM ranged from −14.00% to 107.23%, whereas the range for the MGAM was from −10.1% to 9.4%. The majority of the $RSME$ values for the BGAM were in the range of 0.009 to 0.24 cm/min; the TGAM were larger than 0.014 cm/min; and the MLGAM were found to be larger than 0.04 cm/min, whereas the MGAM were less than 0.028 cm/min. Also, the majority of the $NSE$ values for the MGAM were found to be larger than 0.75; the BGAM were less than 0.77; the TGAM less than 0.73; and the MLGAM were less than −0.54.

The underestimation of the runoff rate by the TGAM was due to the fact that it ignored the air entrapment and air compression. The TGAM assumed that the soil water content profile above the wetting front was fully saturated...
with water (Green & Ampt 1911). The water content and hydraulic conductivity of the soil water content profile above the wetting front were referred to as the saturated soil water content, and hydraulic conductivity, respectively.
Figure 12 | Continued.
In reality, during the rainfall-infiltration-runoff process, the air in the soil was displaced and compressed ahead of the wetting front. The compression of the air in the soil led to a substantial decrease in the infiltration rate. In addition, having been affected by the entrapped air, the water content profile of the soil above the wetting front was unsaturated. The hydraulic conductivity of the soil water content profile above the wetting front was smaller than the saturated
one. Therefore, the TGAM overestimated the infiltration rate, and underestimated the runoff rate.

The overestimation of the BGAM for the loading pattern of the clay-loam-sand and loam-sand-clay was due to the fact that the hydraulic conductivity of the soil water content profile above the wetting front was assumed to be only half of the saturated hydraulic conductivity. In reality, the actual hydraulic conductivity of the first layer in soil water content profile above the wetting front was larger than half of the saturated hydraulic conductivity. In regard to the loading pattern of the sand-clay-loam, the BGAM overestimated the actual hydraulic conductivity of the first layer in the soil water content profile above the wetting front. Therefore, for the loading pattern of the clay-loam-sand and loam-sand-clay, the BGAM underestimated the infiltration rate, and overestimated the runoff rate, and vice versa for the loading pattern of the sand-clay-loam.

In this study, the MLGAM was found to be suitable for the simulations of homogeneous soil with uniform initial moisture content under steady rainfall. For all of the loading patterns, the underestimations or overestimations of runoff rates by the MLGAM were due to the fact that all of the MLGAM parameters experienced no changes during the infiltration process. In fact, when the wetting front entered the interface between two soil layers, the changes in the soil physics resulted in changes in the infiltration and runoff rates, due to the different water movement parameters.

### SUMMARY AND CONCLUSIONS

This study presented a modified Green–Ampt model, and then evaluated the model’s performance by the laboratory observed soil water content, runoff rate, and soil cumulative infiltration under different soil loading patterns (different combinations of sand, loam, and clay) during rainfall events. In particular, the model was compared with the BGAM (Bouwer 1966), TGAM (Jia & Tamai 1997), and MLGAM (Mein & Larson 1973). The influence of the mechanism of the soil properties, and the order of the soil layers during the infiltration-runoff process, were very complicated. MGAM was able to achieve a better match with the observation than the TGAM, BGAM, and MLGAM, by considering the air resistance in the layered soil. In addition, all of the parameters in MGAM had physical meaning, and could be determined from the physical and hydraulic properties of the soil. This indicated that the modified Green–Ampt model had the ability to effectively depict the soil type and water content of the layered soil infiltration–runoff process during unsteady rainfall events.

Although MLGAM (Mein & Larson 1973), GGAM (Grismer 2016) did not consider the soil stratification, the model parameters’ correction based on field rainfall simulation results may compensate for the lack of consideration of stratification. These results may be mainly on soils with high water permeability. The soil stratification may not be obvious in these watersheds (such as Tahoe Lake soil, volcanic ash soil, etc.), so their parameters’ modification results are still relatively good. However, for clearly stratified soils (including natural geological stratification, and human-affected strata, such as farmland and urban soils, etc.), it is very significant to consider soils’ stratification.

MGAM considered many influencing factors, such as the soil layers, air resistance, unsteady rainfall, and so on, which potentially could have great effects on the infiltration-runoff process in a field or real catchment scale, to easily eliminate interference occurring in a field or a real catchment scale. These may include effects as such unbalanced amounts of water, large observational errors, and so on. Therefore, in order to better validate the MGAM, corresponding laboratory experiments were designed, and then the model was evaluated by the laboratory data.

<table>
<thead>
<tr>
<th>Model</th>
<th>Treatments</th>
<th>MAE/ cm/min</th>
<th>ARE/ %</th>
<th>RSME/ cm/min</th>
<th>NSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>MGAM</td>
<td>Clay-loam-sand</td>
<td>0.006</td>
<td>-10.1</td>
<td>0.013</td>
<td>0.78</td>
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<tr>
<td></td>
<td>Sand-clay-loam</td>
<td>0.011</td>
<td>9.4</td>
<td>0.014</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>Loam-sand-clay</td>
<td>0.021</td>
<td>9.1</td>
<td>0.028</td>
<td>0.75</td>
</tr>
<tr>
<td>BGAM</td>
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<td>0.006</td>
<td>34.3</td>
<td>0.009</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>Sand-clay-loam</td>
<td>0.083</td>
<td>-25.5</td>
<td>0.24</td>
<td>0.77</td>
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<tr>
<td></td>
<td>Loam-sand-clay</td>
<td>0.046</td>
<td>30.5</td>
<td>0.038</td>
<td>0.69</td>
</tr>
<tr>
<td>TGAM</td>
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<td>0.007</td>
<td>-28.12</td>
<td>0.014</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>Sand-clay-loam</td>
<td>0.026</td>
<td>-32.8</td>
<td>0.078</td>
<td>0.73</td>
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<tr>
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<td>Loam-sand-clay</td>
<td>0.048</td>
<td>-38.7</td>
<td>0.048</td>
<td>0.45</td>
</tr>
<tr>
<td>MLGAM</td>
<td>Clay-loam-sand</td>
<td>0.036</td>
<td>103.32</td>
<td>0.04</td>
<td>-1.35</td>
</tr>
<tr>
<td></td>
<td>Sand-clay-loam</td>
<td>0.152</td>
<td>-14.00</td>
<td>0.27</td>
<td>-0.78</td>
</tr>
<tr>
<td></td>
<td>Loam-sand-clay</td>
<td>0.048</td>
<td>107.23</td>
<td>0.05</td>
<td>-0.54</td>
</tr>
</tbody>
</table>
When the MGAM model is applied to real catchment scale, in order to obtain model parameters, based on the principle of geostatistics, the soil types and stratified features in the real catchment scale were investigated, stratified sampling was carried out, and then brought back into the room to measure the water characteristic parameters, including the water characteristic curve, saturated hydraulic conductivity, and field water capacity, etc.

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**REFERENCES**


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