Improved SMA-based SCS-CN method incorporating storm duration for runoff prediction on the Loess Plateau, China

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ABSTRACT

In the Soil Conservation Service Curve Number (SCS-CN) method for estimating runoff, three antecedent moisture condition (AMC) levels produce a discrete relation between the curve number (CN) and soil water content, which results in corresponding sudden jumps in estimated runoff. An improved soil moisture accounting (SMA)-based SCS-CN method that incorporates a continuous function for the AMC was developed to obviate sudden jumps in estimated runoff. However, this method ignores the effect of storm duration on surface runoff, yet this is an important component of rainfall-runoff processes. In this study, the SMA-based method for runoff estimation was modified by incorporating storm duration and a revised SMA procedure. Then, the performance of the proposed method was compared to both the original SCS-CN and SMA-based methods by applying them in three experimental watersheds located on the Loess Plateau, China. The results indicate that the SCS-CN method underestimates large runoff events and overestimates small runoff events, yielding an efficiency of 0.626 in calibration and 0.051 in validation; the SMA-based method has improved runoff estimation in both calibration (efficiency = 0.702) and validation (efficiency = 0.481). However, the proposed method performed significantly better than both, yielding model efficiencies of 0.810 and 0.779 in calibration and validation, respectively.

Key words | minimum infiltration rate, runoff, soil moisture accounting (SMA), watershed

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INTRODUCTION

The Soil Conservation Service Curve Number (SCS-CN) method (SCS 1956) is widely used for predicting direct surface runoff from a rain storm. Although developed for the design of agricultural soil conservation structures, the SCS-CN method has also been used for urbanized and forest watersheds. Moreover, it has been integrated into a number of hydrological and water quality models, such as CREAMS (Knisel 1980), EPIC (Sharpley & Williams 1990), SWAT (Arnold *et al.* 1990), PERFECT (Littleboy *et al.* 1992), and AGNPS (Young *et al.* 1989). It is a simple one-parameter (i.e., CN) method which is used for ungauged watersheds (Ponce & Hawkins 1996; Bhuyan *et al.* 2003). The SCS-CN method has been a topic of much discussion in the recent hydrologic literature, especially in the last three decades (Hawkins 1975, 1993; Mishra *et al.* 1999, 2014; Mishra & Singh 2004; Michel *et al.* 2005; Sahu *et al.* 2007, 2010; Shi *et al.* 2009; Singh *et al.* 2015).

The SCS-CN method takes into account the four major runoff producing watershed characteristics, viz., soil type, land use, hydrologic condition, and antecedent moisture condition (AMC). AMC is categorized into three levels: AMC III (wet), AMC II (normal), and AMC I (dry), statistically and, respectively, corresponding to 10, 50, and 90% cumulative probability of exceedance of runoff depth for a given storm (Hjelmfelt *et al.* 1982). The three AMC levels create a discrete relation between the CN and soil moisture, which results in

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corresponding sudden jumps in estimated runoff. Michel *et al.* (2005) unveil several structural inconsistencies in the soil moisture accounting (SMA) that underlies the SCS-CN method, and suggest an improved SMA procedure. They point out that the SCS-CN method should feature initial soil moisture (V_0) conditions instead of an unrealistic parameter in the form of initial abstractions (I_a). However, their improved model includes the existing SCS-CN concept in runoff computation and does not have any formulation for initial soil moisture (V_0). Singh *et al.* (2015) present an improved SMA-based SCS-CN method incorporating a continuous function for antecedent soil moisture, which is more rational and structurally stable than the Michel *et al.* (2005) method. Similar to the original SCS-CN method, storm duration is also not taken into account in model formulation, although it may greatly impact the quantity of runoff (Babu & Mishra 2012).

Storm duration is often an important component of the rainfall–runoff process and may affect the accuracy of runoff estimation. Therefore, an appropriate method incorporating storm duration to improve runoff prediction is essential for the design of soil conservation measures. Thus, the objectives of this study were to (1) modify the Singh *et al.* (2015) method by incorporating storm duration and (2) compare the performance of the proposed method with the original SCS-CN and Singh *et al.* (2015) methods. To this end, rainfall and runoff data which were observed in three experimental watersheds (0.2–71 km^2) on the Loess Plateau of China were used.

METHODS

SCS-CN method

The original SCS-CN method (SCS 1956) is based on the water balance

$$P = I_a + F + Q \tag{1}$$

and two hypotheses

$$\frac{Q}{P - I_a} = \frac{F}{S} \tag{2}$$

$$I_a = \lambda S \tag{3}$$

where P is the observed rainfall (mm), I_a is an initial abstraction (mm), F is the cumulative infiltration (mm), Q is the direct runoff (mm), S is the potential maximum retention (mm), and λ is an initial abstraction ratio (dimensionless). Equation (2) states that the ratio of direct runoff to potential maximum runoff is equal to the ratio of infiltration to potential maximum retention, whereas according to Equation (3), the initial abstraction is some fraction of the potential maximum retention. A combination of Equations (1)–(3) yields

$$Q = \frac{(P - \lambda S)^2}{P + (1 - \lambda)S} \tag{4}$$

For $\lambda = 0.2$, Equation (4) reduces to Equation (5) with S defined by Equation (6)

$$Q = \begin{cases} 0 & (P \leq 0.2S) \\ \dfrac{(P - 0.2S)^2}{(P + 0.8S)} & (P > 0.2S) \end{cases} \tag{5}$$

$$S = \frac{25,400}{CN} - 254 \tag{6}$$

λ is found to range from 0.0 to 0.3 for many geographical locations around the world (Cazier & Hawkins 1984; Bosznay 1989; Huang *et al.* 2007). However, runoff prediction can be improved by considering λ as a fitting parameter. In Equation (6), CN is the curve number (0–100) derivable from NEH-4 tables (SCS 1956) based on the land cover, land management, hydrologic soil group, and the AMC.

Singh *et al.* (2015) method

Michel *et al.* (2005) point out several structural inconsistencies in the original SCS-CN method and amend it in terms of parameterization and a sounder perception of the underlying SMA procedure. In its general form, the Michel *et al.* (2005) method can be expressed as:

$$Q = \begin{cases} 0 & (V_0 \leq S_a - P) \\ \dfrac{(P + V_0 - S_a)^2}{P + S + V_0 - S_a} & (S_a - P < V_0 < S_a) \\ P\left[1 - \dfrac{(S + S_a - V_0)^2}{S^2 + (S + S_a - V_0)P}\right] & (S_a \leq V_0 \leq S + S_a) \end{cases} \tag{7}$$

where S_a is the threshold soil moisture equal to $(V_0 + I_a)$

(mm), V_0 is the initial soil moisture (mm), and S and I_a are the same as in Equation (3).

After analyzing the drawbacks of the Michel *et al.* (2005) method, Singh *et al.* (2015) present a more rational and structurally stable hydrological model based on the concept of C equal to S_r, where C is the runoff coefficient $[Q/(P-I_a)]$ (dimensionless) and S_r is the degree of saturation (dimensionless). Equation (2) is then modified to become:

$$\frac{Q}{P-I_a}=\frac{F+V_0+I_a}{S+V_0+I_a} \tag{8}$$

Coupling Equations (1) and (8), Equation (5) is recast as:

$$Q=\begin{cases}0 & (P\le I_a)\\ \dfrac{(P+V_0)(P-I_a)}{P+S+V_0} & (P>I_a)\end{cases} \tag{9}$$

Thus, Equation (9) is derived after incorporating V_0 in the $C=S_r$ concept, where the runoff could be taken as zero for the condition when $P\le I_a$. This is in contrast to the existing SCS-CN method, which yields a negative runoff.

Substituting $I_a=S_a–V_0$ into Equation (9) results in:

$$Q=\begin{cases}0 & (P+V_0\le S_a)\\ \dfrac{(P+V_0)(P+V_0-S_a)}{P+S+V_0} & (P+V_0>S_a)\end{cases} \tag{10}$$

Similar to the Michel *et al.* (2005) method, the final equations can be obtained by eliminating the mathematical inconsistency for the case when $V_0\ge S_a$, and these can be written as:

$$Q=\begin{cases}0 & (V_0\le S_a-P)\\ \dfrac{(P+V_0-S_a)(P+V_0)}{P+S+V_0} & (S_a-P<V_0<S_a)\\ P\left[1-\dfrac{(S_b-V_0)^2}{SS_b+(S_b-V_0)P}\right] & (S_a\le V_0\le S_b)\end{cases} \tag{11}$$

where S_b is the absolute potential maximum retention equal to $(S+S_a)$ (mm). S and S_a are constants for a given watershed and storm event.

Proposed method

Mishra & Singh (2002) reveal that the cumulative infiltration F consists of the static infiltration (F_c) and dynamic infiltration (F_d). The static infiltration occurs largely due to gravity, while the dynamic infiltration is due to capillarity. So, they suggest that the effect of F_c on Q is identical to that of I_a, and modify the proportionality equation to be

$$\frac{Q}{P-I_a-F_c}=\frac{F_d+M}{S+M} \tag{12}$$

where M is the antecedent moisture (mm). If the static infiltration F_c persists during almost the entire rainfall period and beyond the time to ponding (which can be ignored), then Sahu *et al.* (2012) derive the following equation to compute F_c

$$F_c=f_cT \tag{13}$$

where f_c is the minimum infiltration rate (mm h^{-1}), which is considered to be a constant for a given watershed, and T is the rainfall duration (h).

Based on the above concept, the Singh *et al.* (2015) method can be further improved by distinguishing F from F_d and F_c, so Equation (8) can be rewritten as:

$$\frac{Q}{P-I_a-F_c}=\frac{F_d+V_0+I_a}{S+V_0+I_a} \tag{14}$$

Coupling Equations (1) and (14) results in:

$$Q=\begin{cases}0 & (P+V_0\le S_a+F_c)\\ \dfrac{(P+V_0-F_c)(P+V_0-S_a-F_c)}{P+S+V_0-F_c} & (P+V_0>S_a+F_c)\end{cases} \tag{15}$$

Assuming V_0 and V are the values of soil moisture storage at the beginning of an event and at any time during a storm event, respectively, Q is the corresponding runoff when the accumulated rainfall is equal to P. Then, the expression easily becomes:

$$V=V_0+P-Q \tag{16}$$

where $V - V_0$ corresponds to the amount noted $I_a + F_c + F_d$ in the present study. Taking the derivative of Equation (16) yields

$$\frac{dV}{dt} = p - q \tag{17}$$

where $p = dP/dt$ and $q = dQ/dt$. Replacing Q by its expression from Equation (15) in Equation (16) yields an expression for V

$$V = V_0 + P - \frac{(P + V_0 - F_c)(P + V_0 - S_a - F_c)}{P + S + V_0 - F_c} \tag{18}$$

Taking the derivative of Equation (15) yields

$$q = (p - f_c)\left[\frac{(P + V_0 - F_c)^2 + (2P + 2V_0 - S_a - 2F_c)S}{(P + S + V_0 - F_c)^2}\right] \tag{19}$$

Coupling Equations (18) and (19) results in

$$q = (p - f_c)\left[1 - \frac{(S_b + F_c - V)^2}{SS_b}\right] \tag{20}$$

Separating F_c from V, Equation (17) can be rewritten as:

$$\frac{d(V - F_c)}{dt} = p - q - f_c \tag{21}$$

According to reasonable prediction, if $V_0 = S_b$ (which means the soil is fully saturated), then Q should be equal to $P - F_c$ or P while $f_c = 0$. However, putting $V_0 = S_b$ in Equation (15) results in:

$$Q = (P - F_c) + \frac{SS_b}{(P - F_c) + S + S_b} \tag{22}$$

Equation (22) gives a value of Q greater than $P - F_c$, which suggests that Equation (15) needs further refinement under the condition of $V_0 = S_b$. This inconsistency can be eliminated by coupling Equations (20) and (21), as follows:

$$\frac{d(V - F_c)}{dt} = (p - f_c)\frac{(S_b + F_c - V)^2}{SS_b} \tag{23}$$

The final expression of Q can be obtained by integrating Equation (23) and substituting V with its expression from Equation (16)

$$Q = (P - F_c)\left[1 - \frac{(S_b - V_0)^2}{(P - F_c)(S_b - V_0) + SS_b}\right] \tag{24}$$

It is apparent that if $V_0 = S_b$, then $Q = P - F_c$ according to Equation (24). Similarly, in the lowest case where $V_0 \leq S_a + F_c - P$, then $Q = 0$; where $S_a + F_c - P < V_0 < S_a + F_c$, then Q can be computed using Equation (15) as an intermediate case. Moreover, P should be greater than F_c in intermediate cases and the highest case.

In summary, the final equations under these three conditions can be written as follows:

$$Q = \begin{cases} 0 & (V_0 \leq S_a + F_c - P \; or \; P \leq F_c) \\ \dfrac{(P + V_0 - S_a - F_c)(P + V_0 - F_c)}{P + S + V_0 - F_c} & (S_a + F_c - P < V_0 < S_a + F_c \; \& \; P > F_c) \\ (P - F_c)\left[1 - \dfrac{(S_b - V_0)^2}{SS_b + (S_b - V_0)(P - F_c)}\right] & (S_a + F_c < V_0 < S_\mathrm{b} \; \& \; P > F_c) \end{cases} \tag{25}$$

where S_a and V_0 can be computed using the equations suggested by Mishra *et al.* (2006)

$$V_0 = \alpha\sqrt{P_5 S} \tag{26}$$

$$S_a = \beta S \tag{27}$$

where P_5 denotes the antecedent 5-day rainfall amount (mm), and α and β are coefficients (dimensionless).

STUDY AREA AND DATA

Study area

This study was conducted with data from three experimental watersheds: Jiuyuangou (JYG), Peijiamaogou (PJMG), and

Yangdaogou (YDG), which are tributaries of the Yellow River (Figure 1).

The YDG watershed (latitude: 37°31′N; longitude: 111°15′E; elevation: 1,000–1,320 AMSL; area: 0.21 km^2) is located in Lishi county, the gully region of the Loess Plateau. It has a mean annual temperature of 9 °C and a mean annual precipitation of 510 mm, 81% of which falls between May and September. Soil texture in the YDG watershed is classified as sandy loam (FAO-UNESCO 1988).

The JYG watershed (latitude: 37°33′N; longitude: 110°16′E; elevation: 820–1,180 AMSL) is located in Suide county, the hilly region of the Loess Plateau with an area of 70.7 km^2. The climate is semi-arid with a mean annual temperature of 10.2 °C and a mean annual precipitation of 524 mm, most of which falls between June and September. The main soil type is silty loam soil.

The PJMG watershed (latitude: 37°33′N; longitude: 110°16′E; elevation: 790–1,140 AMSL) is also located in Suide county, 6 km from the JYG watershed with an area of 41.5 km^2. The climate is similar to that of the JYG watershed.

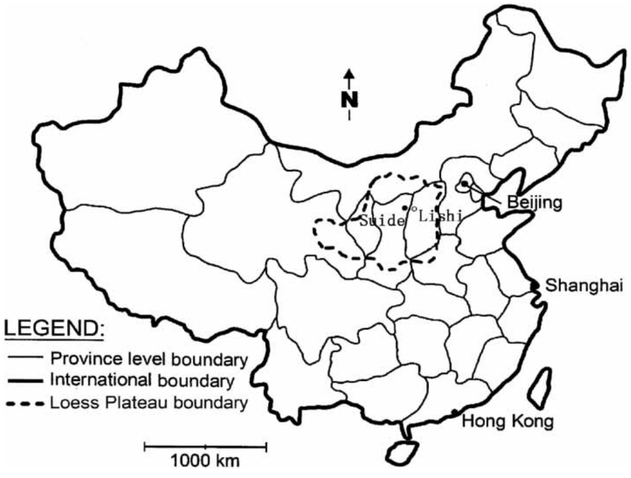

Figure 1 | Location of the experimental sites.

Topographies of the three watersheds are very similar, and are characterized by upland, gently sloping ridges, steep hillslopes, and well-defined alluvial valleys with incised channels. Slopes vary from 0 to 5° on the upland, 5 to 15° on the ridges and valleys, and above 15° on hillslopes. Soils of these watersheds (depths varying from 20 to 50 m) are developed on a deep loessal mantle. Their physical characteristics are described in Table 1.

Data collection

Flows in the above three watersheds are ephemeral, with most runoff occurring between May and October, and were measured during this period using either a rectangular weir (at the outlets of the JYG and PJMG watersheds) or a 60° V-notch sharp-crested weir (the YDG watershed). The total event runoff volume was calculated according to:

$$R = \sum_{i=1}^{n} Q(t_i)\Delta t_i \tag{28}$$

where t_i is the measurement time (s), $Q(t_i)$ is the stream flow (m^3 s^{-1}), Δt_i is the time interval between two successive measurements (s), R is the total runoff volume for an event (m^3), i is an index for the number of measurements, and n is the total number of measurements per storm event. The frequency of discharge measurement was once every 1–5 min during peak discharge and once every 5–10 min otherwise.

Daily rainfall characteristics, including (spatially averaged) rainfall depth, rainfall duration, and average rainfall intensity, which were measured using eight self-recording rain gauges in the JYG and PJMG watersheds and three rain gauges in the YDG watershed, were determined by the arithmetic mean value of self-recording rain gauges. This information was compiled on a storm basis by the

Table 1 | Summary of experimental watershed characteristics

Watershed	Area (km^2)	Average slope (%)	Channel length (km)	Channel density (km/km^2)	Observation period	Vegetative cover	Canopy cover (%)
JYG	70.7	48.8	18.0	5.34	1964–1969 1974–1979	Grass	<50
PJMG	41.5	53.2	11.0	2.69	1960–1969	Grass	<50
YDG	0.21	60.1	0.75	3.82	1956–1970	Grass	<50

Yellow River Administration Committee of the Ministry of Water Resources (1983) and the Shanxi Institute of Soil & Water Conservation (1984), and used in the analyses herein.

Parameter estimation

For the application of the three methods described above, the available dataset for each watershed was split into two parts, with one half used for calibration and the other for validation. The model parameters for all three methods were optimized using a Marquardt (1963) algorithm for solving constrained nonlinear least-squares problems and the observed data in the calibration group. For the original SCS-CN method, the initial estimate of CN was taken as 50 and it was allowed to vary from 1 to 100. For the Singh *et al.* (2015) and proposed methods, α was allowed to vary from 0.01 to 2 with an initial estimate of 0.1. The initial estimate of β was taken as 0.1, and it was allowed to vary within a range of 0.001 to 1. S was assumed to vary from 0 to 1,000 with an initial value of 100 mm. In the proposed method, f_c was allowed to vary from 0 to 25 with an initial estimate of 1 mm h^{-1}.

Data analyses

The Nash–Sutcliffe efficiency (NE) (Nash & Sutcliffe 1970; Risse *et al.* 1994) and coefficient of determination (r^2) were used to evaluate model performance

$$NE = 1 - \frac{\sum_{j=1}^{N} (Q_j - Q_j^*)^2}{\sum_{j=1}^{N} (Q_j - \overline{Q})^2} \tag{29}$$

$$r^2 = \frac{\left(\sum_{j=1}^{N} (Q_i - \overline{Q})(Q_j^* - \overline{Q}^*)\right)^2}{\sum_{j=1}^{N} (Q_j - \overline{Q})^2 \sum_{j=1}^{N} (Q_j^* - \overline{Q}^*)^2} \tag{30}$$

where Q_j is the jth measured runoff (mm), $\overline{Q}$ is an average of measured runoff (mm), Q_j^* is the jth calculated runoff (mm), $\overline{Q}^*$ is an average of model calculated runoff (mm), and N is the total number of storm events. The NE was employed to indicate the agreement between observed and computed runoff values; it varies from 0 to 1, with lower values indicating larger differences between predicted and observed values and therefore poorer model performance, and vice versa. The coefficient of determination (r^2) between the measured and predicted values was used to describe the proportion of the variance in the observed data that can be explained by the model; it ranges from 0 to 1, with lower values indicating poorer agreement, and vice versa.

RESULTS AND DISCUSSION

The optimized parameters resulting from the application of all three methods to the three watersheds are presented in Table 2, while Table 3 presents a comparison of the overall performance of the three methods using the datasets from the three watersheds based on statistical indexes.

Calibration and validation

SCS-CN method

The total number of observed rainfall–runoff events for calibration, validation, and full (combined) datasets was 152, 151, and 303, respectively, as shown in Table 3. Figure 2 shows the estimated runoff values with the corresponding observations used in the calibration, validation, and full datasets. In this figure, the 1:1 line is indicative of perfect fit. The SCS-CN method underestimates large (runoff depths of 40–80 mm) as well as some small (runoff depths of 5–37.5 mm) rainfall–runoff events. The data in Table 3 also show that the SCS-CN method, with a regression line

Table 2 | Optimized parameters for the three methods for the three watersheds

		Watershed		
Method	**Parameter**	**JYG**	**PJMG**	**YDG**
SCS-CN method	S (mm) (for AMCII)	117.35	38.82	105.40
Singh *et al.* (2015)	α	0.061	0.010	0.014
	B	0.544	0.108	0.001
	S	206.32	153.10	861.19
Proposed method	α	0.181	0.055	0.672
	β	0.028	0.001	0.015
	f_c (mm/h)	2.201	1.034	4.610
	S (mm)	174.18	157.28	236.41

Table 3 | Comparative overall model performance on three datasets

		Linear regression			
Method	**Events**	**Interception**	**Slope**	**r^2**	***NE***
Calibration					
SCS-CN method	152	−0.77	0.83	0.71	0.63
Singh *et al.* (2015)	152	0.19	0.78	0.71	0.70
Proposed method	152	−0.09	0.87	0.82	0.81
Validation					
SCS-CN method	151	0.95	0.36	0.20	0.05
Singh *et al.* (2015)	151	1.37	0.61	0.50	0.48
Proposed method	151	0.76	0.66	0.81	0.78
Full data					
SCS-CN method	303	0.41	0.47	0.30	0.18
Singh *et al.* (2015)	303	0.97	0.62	0.56	0.53
Proposed method	303	0.37	0.74	0.82	0.79

slope of 0.465 and an intercept of 0.405 for all (combined) rainfall–runoff events (Figure 2), underpredicts large runoff events and overpredicts most small runoff events. The inference is consistent with Van Mullen (1991) for rangeland and cropland in Montana and Wyoming and King *et al.* (1999) for Mississippi.

The SCS-CN method underestimates runoff in both calibration and validation, with regression line slopes of 0.83 and 0.36 and *NE* values of 0.63 and 0.05, respectively. For the full dataset, this method underpredicts runoff depths for 266 out of 303 events ($NE = 0.35$). The resulting low *NE* values are indicative of the poor performance of the SCS-CN method on all three datasets.

Singh *et al.* (2015) method

Table 2 presents the values of parameters (α, β, and S) optimized in calibration. The α values range from 0.010 to 0.061, in contrast to the range of 0.010–0.450 obtained by Singh *et al.* (2015) in their application to 35 US watersheds. However, the upper bounds of β and S, which range from 0.001 to 0.544 and 153.10 to 861.19, respectively, are beyond the corresponding ranges of 0.001–0.386 and 49.06–826.45 derived from Singh *et al.* (2015) in their application to 35 US watersheds.

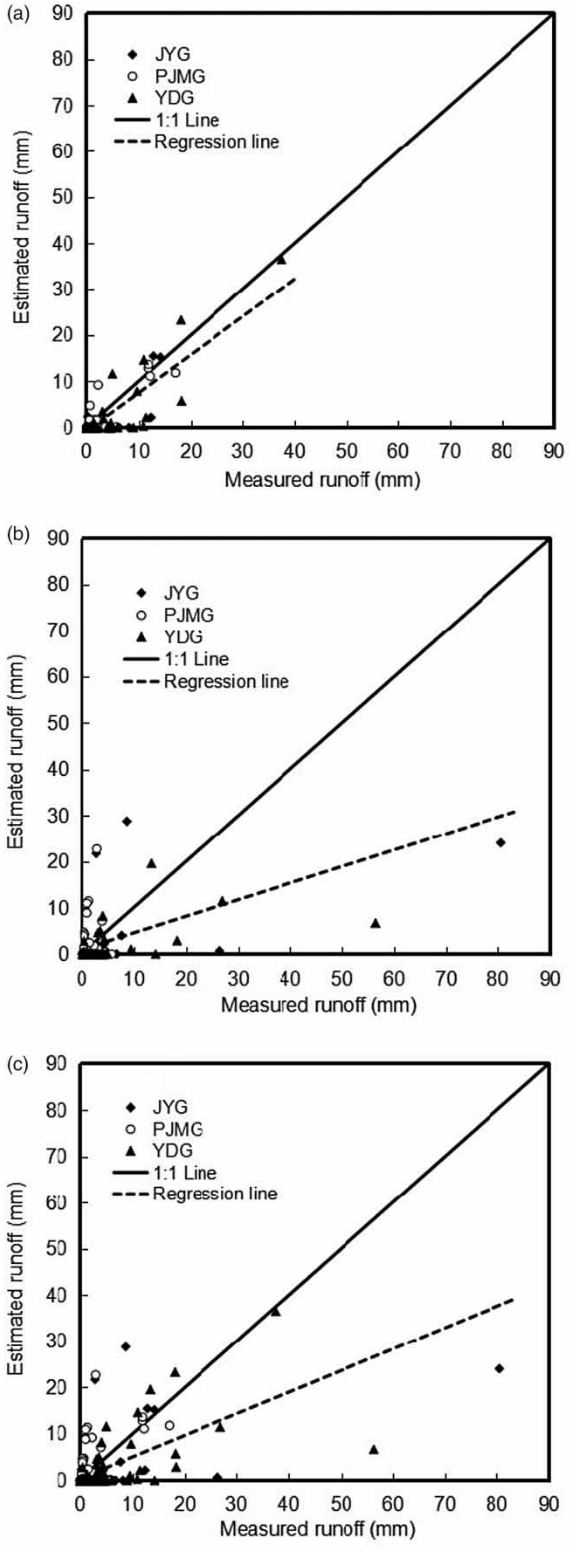

Figure 2 | Measured versus estimated runoff depth (SCS-CN method) for (a) calibration, (b) validation, and (c) full dataset.

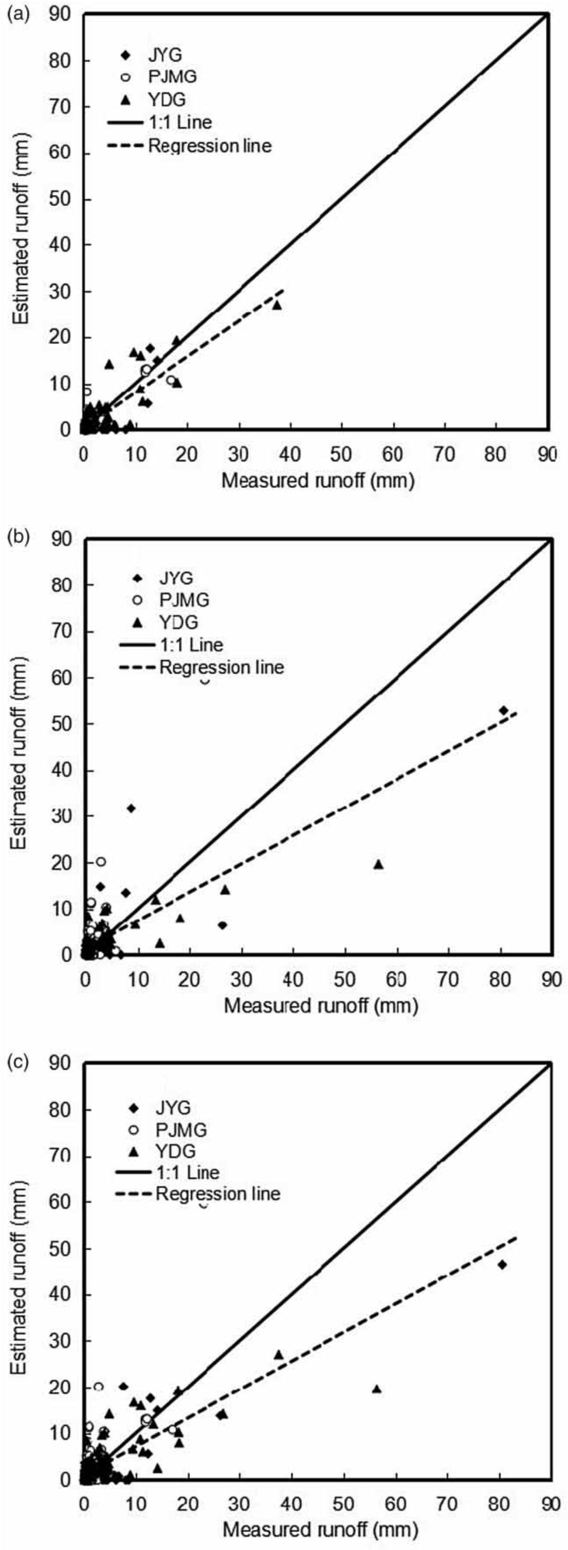

Figure 3 | Measured versus estimated runoff depth (Singh *et al*. (2015) method) for (a) calibration, (b) validation, and (c) full dataset.

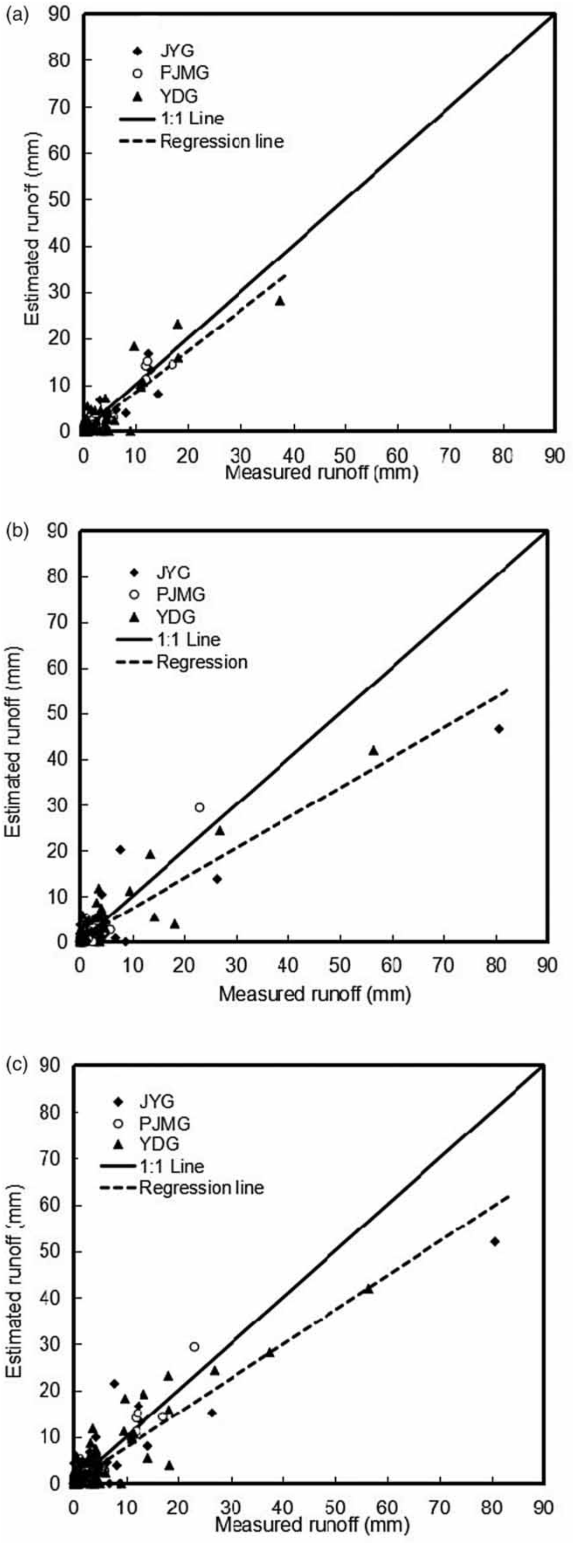

Figure 4 | Measured versus estimated runoff depth (proposed method) for (a) calibration, (b) validation, and (c) full dataset.

Figure 3(a) demonstrates the improvement in the calibration using the Singh *et al.* (2015) method vs. the original SCS-CN method (Figure 2(a)). However, the intercept of the regression line at 0.190 implies that the Singh *et al.* (2015) method overestimates small runoff events in calibration. A comparison of Figure 3(b) vs. Figure 2(b) shows the improvement, with the Singh *et al.* (2015) method producing less scatter and a significantly improved value of *NE* (0.48 vs. 0.05) (Table 3). From Table 3, the slope (0.62) and intercept (0.97) of the Singh *et al.* (2015) method imply an underestimation for large runoff events and an overestimation for small runoff events in validation. For the full dataset, the Singh *et al.* (2015) method underpredicted runoff depths for 167 (out of 303) events. Overall, more values are near the perfect line than for the SCS-CN method, and the value of *NE* (0.53) was much improved compared to the SCS-CN method (0.18).

Proposed method

Figure 4(a) compares estimated runoff depths from the calibration dataset vs. observations. The figure shows that the proposed method performs satisfactorily ($NE = 0.81$) as most data points lie close to the 1:1 line, indicating a close match with the observed runoff. The performance is better than both the Singh *et al.* (2015) ($NE = 0.70$) and SCS-CN ($NE = 0.62$) methods.

Figure 4(b) shows that the runoff estimation is much improved over the SCS-CN method and the Singh *et al.* (2015) method in validation because the data points lie quite close to the 1:1 line, even for large runoff events, exhibiting a good match between the estimated and observed runoff. As per Table 3, the slope of the regression line increases to 0.66 from 0.36 (SCS-CN method) whereas the intercept decreases to 0.76 from 1.37 (Singh *et al.* (2015) method), indicating that the runoff predictions by the proposed model are closer to observed values. The proposed method performed better ($NE = 0.78$) than the Singh *et al.* (2015) method ($NE = 0.48$) and the SCS-CN method ($NE = 0.05$) in validation.

On the full dataset, Figure 4(c) shows that the runoff data based on the proposed method are closer to the perfect line (slope = 0.74) than those generated from the Singh *et al.* (2015) method (0.62) or SCS-CN method (0.47). The proposed method also yielded a higher value of *NE* (0.79)

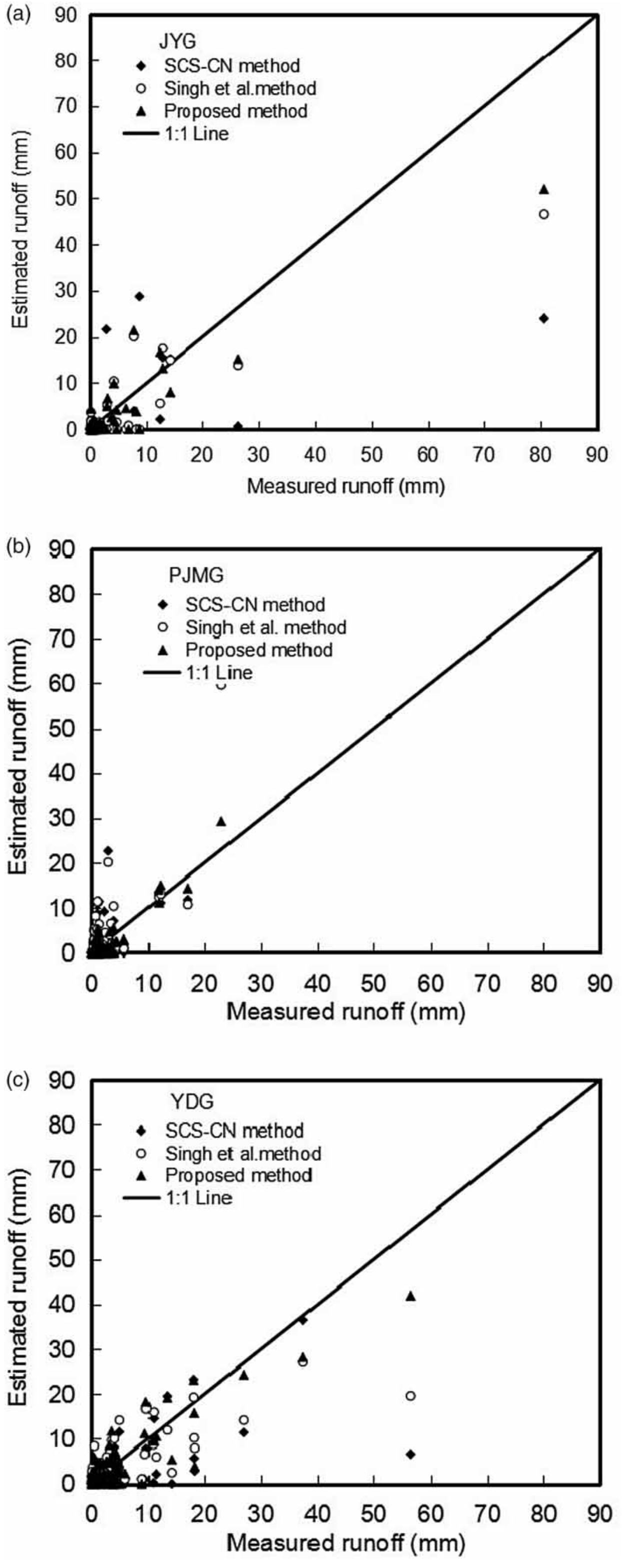

Figure 5 | Measured versus estimated runoff depth for the three methods for (a) JYG, (b) PJMG, and (c) YDG watersheds.

Table 4 | Coefficient of determination (r^2) and efficiency (*NE*) results by the application of the three models in the three watersheds

			SCS-CN method		Singh *et al.* (2015)		Proposed method	
Watershed		**Events**	r^2	*NE*	r^2	*NE*	r^2	*NE*
JYG	Calibration	46	0.58	0.41	0.64	0.50	0.81	0.78
	Validation	44	0.34	0.40	0.72	0.75	0.86	0.79
	Full data	90	0.35	0.32	0.82	0.74	0.85	0.80
PJMG	Calibration	45	0.72	0.66	0.72	0.71	0.90	0.87
	Validation	54	0.79	−3.24	0.79	−2.10	0.87	0.79
	Full data	99	0.58	−1.58	0.59	−0.91	0.87	0.82
YDG	Calibration	61	0.74	0.66	0.74	0.74	0.80	0.79
	Validation	53	0.26	0.13	0.66	0.50	0.83	0.81
	Full data	114	0.41	0.32	0.62	0.58	0.81	0.80

than the Singh *et al.* (2015) (0.53) or SCS-CN (0.180) methods (Table 3). Overall, the proposed model clearly performs the best of the three methods compared.

Measured and estimated data using the three methods are plotted in Figure 5. For each watershed, the runoff data for the proposed method are always closer to the line of perfect fit than those produced by either the SCS-CN or Singh *et al.* (2015) methods. Table 4 presents the performance of the investigated methods for each watershed. The proposed method always yields higher values of r^2 and *NE* compared to the other two methods for each of the three watersheds. In particular, the SCS-CN and Singh *et al.* (2015) methods result in negative values of model efficiency (*NE*) in the PJMG watershed, while the proposed method results in a positive efficiency during validation (0.79) and with the full dataset (0.82). Clearly, the proposed method performs better than the other two methods considered.

Sensitivity analysis

The above results indicate that the proposed method presents a greater accuracy than the other two methods. The sensitivity analysis can distinguish parameters which are more sensitive for their employment and further explore the robustness of the proposed method. Therefore, in this study, the calibrated parameters of the proposed method (α, β, f_c, and S) were varied for observing the impact of variation on the calculated runoff values in terms of *NE* with the full datasets of JYG watersheds.

Figure 6 depicts the sensitivity analysis of the proposed model parameters, in which the efficiency varies with the parameter, while it is either sharply increased or decreased from the calibrated one. It can be seen that parameter S is apparently the most sensitive to variation, which may be because the parameter S not only represents the characteristics of the study watershed but also has impact on the antecedent soil moisture condition (S_a) (Shi *et al.* 2017). The parameter β appears to be the least sensitive, while the parameters α and f_c are seen to be less sensitive than S but more than β. In general, the sensitivity of model parameters decreased in the following order: $S > f_c > \alpha > \beta$.

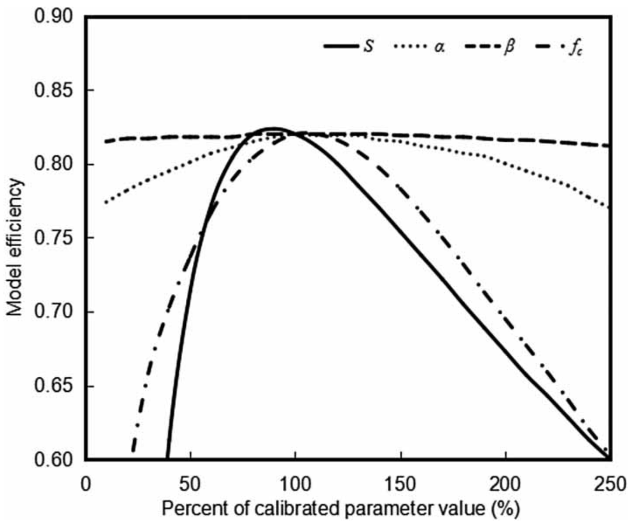

Figure 6 | Sensitivity analysis of the four proposed model parameters: S is the maximum water retention; f_c is the minimum infiltration rate; α is the empirical parameter of Equation (26); and β is the empirical parameter between the threshold soil moisture (S_a) and S.

Effect of rainfall duration on runoff estimation

Based upon rainfall depth and duration, 303 rainfall events of the three watersheds were divided into three groups to test the performance of the SCS-CN method for different types of rainfall regimes using K-means clustering (Hong 2003) (Table 5). The classification of the three rainfall regimes reaches the ANOVA criterion for a significant level (***$P < 0.001$). Rainfall Regime 1 is the group of rainfall events with lower precipitation, short duration, and the most frequent occurrence (74.91%); Rainfall Regime 3 consists of rainfall events with large rainfall and long duration which had the least occurring frequency occupying 4.29% of the total events; while Rainfall Regime 2 (20.79%) is composed of rainfall events which have moderate rainfall eigenvalues.

Figure 7 presents the predicted versus the corresponding measurement runoff for different rainfall regimes. The original SCS-CN method underpredicted most storm-runoff events of the Rainfall Regime 2 and 3 (Figure 7(a)). This is because the SCS-CN method ignores the storm duration which only accounts for the rainfall amount. However, the underprediction of Rainfall Regimes 2 and 3 was reduced when incorporating the storm duration in the proposed method as compared with the traditional SCS-CN method (Figure 7(b)). The better performance indicated that storm duration plays a vital role in rainfall–runoff generation and prediction (Mishra *et al.* 2008; Reaney *et al.* 2010), and the proposed method can accurately predict the runoff generated by a different type of rainfall regime with varying duration.

Table 5 | Statistical features of the rainfall regimes in the study area

Rainfall regime	Eigenvalue	Mean	Standard deviation	Variation coefficient	Frequency (%)
Regime 1	P (mm)	13.07	6.64	0.51	74.91
	D (h)	3.96	4.48	1.13	
Regime 2	P (mm)	40.53	30.47	0.75	20.79
	D (h)	12.76	9.60	0.75	
Regime 3	P (mm)	97.37	11.00	0.11	4.29
	D (h)	25.28	12.42	0.49	

P: precipitation depth; *D*: storm duration.

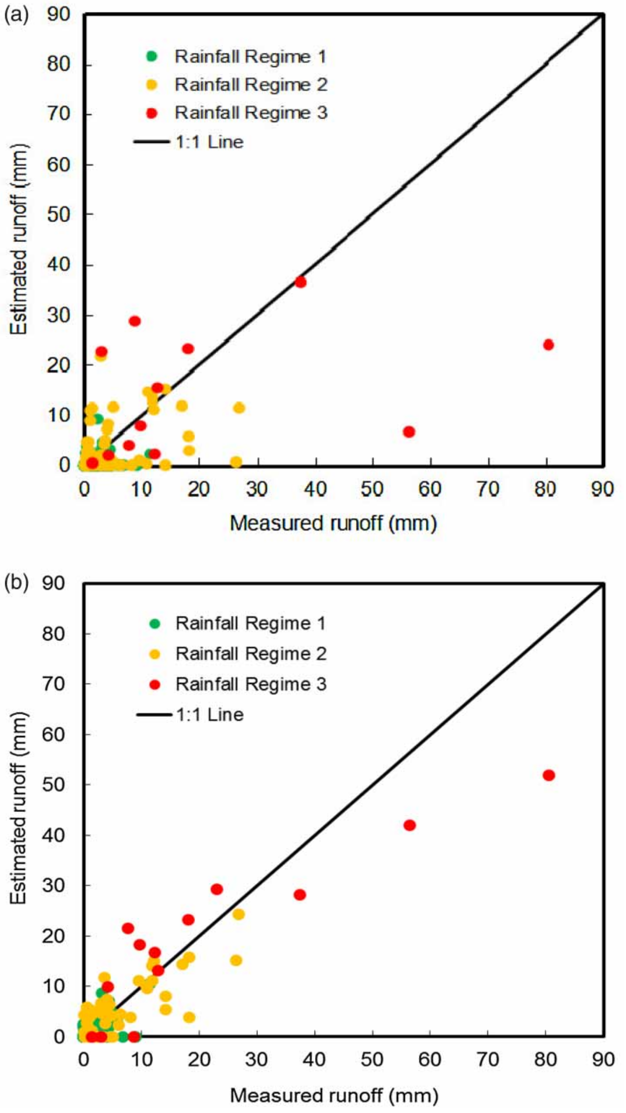

Figure 7 | Measured versus estimated runoff depths for (a) the original SCS-CN method and (b) the proposed model of the three rainfall regimes.

CONCLUSIONS

In this paper, the Singh *et al.* (2015) method based on the revised SMA procedure for runoff estimation was modified by incorporating storm duration. A dataset of 303 rainfall–runoff events from three experimental watersheds (JYG, PJMG, and YDG) on the Loess Plateau of China was employed to test the applicability of the original SCS-CN, Singh *et al.* (2015) and proposed methods. The proposed method incorporating rainfall intensity could accurately predict runoff and had greater reliability than the SCS-CN method and the Singh *et al.* (2015) method in the region of Loess Plateau.

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DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available on request from the corresponding author.

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