Effect of a Magneto-Rotational Instability on Jets from Accretion Disks

Takahiro KUDOH
National Astronomical Observatory, Mitaka, Tokyo 181-8588
kudoh.takahiro@nao.ac.jp

Ryoji MATSUMOTO
Department of Physics, Chiba University, Inage-ku, Chiba 263-8522
matumoto@c.chiba-u.ac.jp

and

Kazunari SHIBATA
Kwasan Observatory, Kyoto University, Yamashina, Kyoto 607-8471
shibata@kwasan.kyoto-u.ac.jp

(Received 2001 August 9; accepted 2001 December 21)

Abstract

We present the results of 2.5-dimensional MHD simulations of jet formation from accretion disks that are unstable for a magneto-rotational instability. Numerical simulations show that magnetically driven jets are ejected from magneto-rotationally-unstable disks. The velocities of the jets are of the order of the Keplerian velocities of the disks. In this paper, we mainly focus on the effect of the magneto-rotational instability on magnetically driven jets from thick disks. For that purpose, we initially impose a sinusoidal perturbation with finite amplitude on a rotating disk that is threaded by a vertical uniform magnetic field. The perturbation grows by the magneto-rotational instability, and the nonlinear development of the instability leads to a channel flow which causes violent accretion. As the accretion continues, the accretion flow is partially turned outward to the outflow that is accelerated by the magnetic force along the poloidal magnetic field line, i.e., a magnetically driven jet is produced. Models with initial finite amplitude perturbation (i.e., $|\delta v|/V_s = 0.1$ where $\delta v$ is the velocity perturbation and $V_s$ is the sound velocity) are compared with those without any perturbation that we previously studied. The mass-accretion rate, mass-ejection rate, and jet velocity are larger when the perturbation is imposed on in the disk. However, the jet velocity is of the order of the Keplerian velocity of the disk, almost independent of the perturbation.

Key words: accretion disks — instabilities — jets — methods: numerical — MHD

1. Introduction

Magnetically driven jets from accretion disks have been considered as models for astrophysical jets in young stars, active galactic nuclei, and X-ray binaries. Magnetohydrodynamic (MHD) jet formation from accretion disks was first proposed by Blandford and Payne (1982) and has been developed by many authors (e.g., Pudritz, Norman 1986; Sakurai 1987; Lovelace et al. 1991; Pelletier, Pudritz 1992; Contopoulos, Lovelace 1994; Najita, Shu 1995; Li 1995; Fendt, Camenzind 1996, etc.). In most theoretical models of jets from accretion disks, however, accretion disks are treated as boundary conditions. Accretion disks only play a role of supplying energy and mass to the jets, and neither accretion flow nor internal structure of disks are considered in these models.

Jet models including dynamic accretion of disks were first studied by Uchida and Shibata (1985) and Shibata and Uchida (1986) by performing time-dependent MHD numerical simulations. Initially, they assumed a rotating disk threaded by a vertical uniform magnetic field. During the evolution, since the disk angular momentum is rapidly extracted by the magnetic field, it begins to contract toward the central region even when its initial rotation is Keplerian. They showed that the magnetic field is highly twisted by the rotation of the disk, and a bipolar jet is accelerated from the disk by the $J \times B$ force in a relaxing magnetic twist. These numerical simulations were followed by Stone and Norman (1994), Matsumoto et al. (1996), and Kudoh, Matsumoto, and Shibata (1998).

On the other hand, in a recent development of accretion-disk theory, Balbus and Hawley (1991) as well as Hawley and Balbus (1991) showed that perturbations in a differentially rotating magnetized disk grow when the magnetic pressure is smaller than the gas pressure in the disk (i.e., magneto-rotational instability). The nonlinear evolution of this instability has been studied by three-dimensional local MHD numerical simulations by adopting a shearing box approximation (Hawley et al. 1995; Matsumoto, Tajima 1995; Brandenburg et al. 1995; Stone et al. 1996; Miller, Stone 2000). These simulations showed that the magnetic turbulence driven by this instability can be the origin of the viscosity in the accretion disk. Recent global three dimensional MHD simulations (Matsumoto 1999; Hawley 2000; Machida et al. 2000) confirmed the result of local simulations. If the magnetic field is not strong in the disk, the disk is unstable for the magneto-rotational instability and it may affect the jet emanating from the disk. Stone and Norman (1994) and Matsumoto et al. (1996) found that the surface layers of disks accrete faster than the equatorial part when a weak magnetic field is assumed in Uchida and Shibata’s (1985) situation. They argued that the surface accretion is a result of...
the magneto-rotational instability.

In our previous paper, Kudoh et al. (1998) found that the jet from the surface accretion can be understood using steady state theory even when nonsteady accretion occurs. In this paper, we consider how the magnetically driven jet is affected by the magneto-rotational instability in the disk. The initial condition of this problem is the same as those of Matsumoto et al. (1996) and Kudoh et al. (1998), except that a sinusoidal perturbation is imposed in the disk. Some of the preliminary results have already been discussed by Kudoh, Matsumoto, and Shibata (1999a). In section 2, we discuss the model used in this paper. In section 3, we present our results. Section 4 is devoted to discussion.

2. Numerical Model

2.1. Assumptions and Basic Equations

Our simulations made the following assumptions: (1) axial symmetry around the rotational axis, including the azimuthal components of a velocity and a magnetic field (i.e., 2.5 dimensional approximation); (2) ideal MHD; (3) a specific heat ratio of \( \gamma = 5/3 \); (4) a point-mass gravitational potential, with the disk self-gravity being neglected.

The basic equations we used were the ideal MHD equations:

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \rho \nabla \cdot \mathbf{v} &= -\rho \nabla \cdot \mathbf{v}, \\
\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} &= -\frac{1}{\rho} \nabla p + \frac{1}{4\pi \rho} (\nabla \times \mathbf{B}) \times \mathbf{B} - \nabla \Phi, \\
\frac{\partial \Phi}{\partial t} + \mathbf{v} \cdot \nabla \Phi &= -\left( \frac{B}{\rho} \right) \nabla \cdot \mathbf{v}, \\
\frac{\partial \mathbf{B}}{\partial t} &= \nabla \times (\mathbf{v} \times \mathbf{B}),
\end{align*}
\]

where \( \Psi = -GM/\sqrt{r^2 + z^2}^{3/2} \) is the gravitational potential, \( G \) is the gravitational constant, and \( M \) is the mass of the central object. Other variables are summarized in table 1. The equations were solved in cylindrical coordinates \((r, \phi, z)\).

In addition to the MHD equations, the time evolution of the magnetic stream function \( \Phi_m \) was calculated by the equation,

\[
\frac{d\Phi_m}{dt} = \frac{\partial \Phi_m}{\partial t} + v_r \frac{\partial \Phi_m}{\partial r} + v_z \frac{\partial \Phi_m}{\partial z} = 0,
\]

by using the velocity field obtained by the MHD equations. The magnetic stream function \( \Phi_m \) was used to draw the poloidal magnetic field lines that are shown by the contour of \( \Phi_m \) in \((r, z)\) plane.

2.2. Initial Conditions

As an initial condition, we assumed an equilibrium disk rotating in a central point mass gravitational potential. Exact solutions for these conditions are obtained under simplifying assumptions for the distribution of angular momentum \((L)\) and pressure:

\[
\begin{align*}
L &= L_0 r^a, \\
p &= K \rho^{1 + 1/n},
\end{align*}
\]

where \( L_0 \) is a parameter which shows the angular momentum of the disk when \( a = 0 \). Then the distribution of material in the disk is given by

\[
\psi = \frac{GM}{(r^2 + z^2)^{3/2}} + \frac{1}{2(1-a)} L_0^2 r^{2a-2} + (n+1) \frac{p}{\rho} = \text{constant}.
\]

We used \( a = 0 \) and \( n = 3 \) throughout this work. (Though the choice of \( a = 0 \) leads to a torus rather than a disk, we call it a disk in this paper.) These values of parameters are the same as those in Matsumoto et al. (1996) and Kudoh et al. (1998).

The mass distribution outside the disk was assumed to be that of a high-temperature isothermal corona in hydrostatic equilibrium without rotation. The density distribution in hydrostatic equilibrium is

\[
\rho = \rho_c \exp \left\{ \alpha \left[ \frac{r_0}{(r^2 + z^2)^{1/2}} - 1 \right] \right\},
\]

where \( r_0 \) is a normalized unit of length defined by

\[
r_0 = (L_0^2 / GM)^{1/(1-2a)},
\]

\[
\alpha = (\gamma V_{K0}^2 / V_{sc}^2),
\]

\( V_{K0} = (GM/r_0)^{1/2} \) is the Keplerian velocity at radius \( r_0 \), and \( \rho_c \) is the coronal density at radius \( r_0 \). We used \( a = 1 \) throughout this work. The parameter \( \rho_c \) was taken to be \( \rho_c/\rho_0 = 10^{-3} \) for all models, where \( \rho_0 \) is the initial disk density at \((r, z) = (r_0, 0)\).

To distinguish the material that are initially in the disk from that in the corona, we introduced a scalar variable, \( \Theta \), initially defined as \( \Theta = 1 \) interior to the disk and \( \Theta = 0 \) outside of the disk. The interior of the disk is defined as the region where the density obtained by equation (9) is positive. The time evolution of \( \Theta \) is followed by the equation,

\[
\frac{d\Theta}{dt} = \frac{\partial \Theta}{\partial t} + v_r \frac{\partial \Theta}{\partial r} + v_z \frac{\partial \Theta}{\partial z} = 0,
\]

by using the velocity field obtained by solving the MHD equations.

2.3. Initial Magnetic Fields

The initial magnetic field was assumed to be uniform and parallel to the axis of rotation for simplicity:

<table>
<thead>
<tr>
<th>Physical quantities</th>
<th>Normalization unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>( r_0 / V_{K0} )</td>
</tr>
<tr>
<td>( r, z )</td>
<td>( r_0 )</td>
</tr>
<tr>
<td>( \rho )</td>
<td>( \rho_0 )</td>
</tr>
<tr>
<td>( p )</td>
<td>( p_0 V_{K0}^2 )</td>
</tr>
<tr>
<td>( \mathbf{v} )</td>
<td>( V_{K0} )</td>
</tr>
<tr>
<td>( \mathbf{B} )</td>
<td>( \sqrt{\rho_0 V_{K0}^2} )</td>
</tr>
<tr>
<td>( e )</td>
<td>( V_{K0}^2 )</td>
</tr>
</tbody>
</table>

Note: The unit length, \( r_0 = (L_0^2 / GM)^{1/(1-2a)} \), is the radius of the density maximum in the initial disk. Throughout this paper, it is assumed \( a = 0 \). The unit velocity \( V_{K0} \) is the Keplerian velocity at \((r, z) = (r_0, 0)\). The unit density \( \rho_0 \) is the initial density at \((r, z) = (r_0, 0)\).
\[ B_z = B_0 = \text{constant}, \quad B_r = B_\theta = 0, \]  
and \[ \Phi_m = \frac{1}{2} B_\theta r^2. \]

### 2.4. Perturbations in the Disk

In order to initiate the growth of the magneto-rotational instability in the disk, radial velocity perturbations were imposed at the first stage of the calculation (at \( t = 0 \)):
\[ \delta v_r = \delta V_{z0} V_{z0} \cos(k_z z), \]
where \( \delta V_{z0} \) is the amplitude of the perturbation, \( V_{z0} \) is sound velocity in the disk at \((r, z) = (r_0, 0)\), and \( k_z \) is the wave number of the perturbation. The perturbation was applied only in a disk where \( \Theta = 1 \).

#### 2.5. Boundary Conditions

On the axis \((r = 0)\), \( p, \rho, v_z, \) and \( B_z \) are symmetric, while \( v_r, v_\theta, B_r, \) and \( B_\theta \) are anti-symmetric. The side, top, and bottom surfaces are free boundaries. In contrast to the previous paper (Kudoh et al. 1998), we did not use the mirror symmetric boundary on the equatorial plane. In order to avoid a singularity at the origin, the region around \( r = z = 0 \) was treated by softening the gravitational potential with the softening radius, \( 0.2r_0 \) (see Kudoh et al. 1998).

These boundary conditions are different from those in the recent MHD simulations of jets from accretion disks performed by Ouyed, Pudritz, and Stone (1997), Romanova et al. (1997), Meier et al. (1997) and Ustyugova et al. (1999). The main difference is the condition on the equatorial plane. They assumed an inflowing fixed boundary condition (i.e., the angular momentum is continually injected from the boundary). Therefore, their numerical simulations did not allow disk accretion due to magnetic braking nor growth of the magneto-rotational instability.

#### 2.6. Normalized Units and Parameters

We normalized the physical quantities with their initial value at \((r, z) = (r_0, 0)\), taking \( r_0 = (L_0^2/GM)^{1/(1-\alpha)} \) so that the initial density of the disk would have the maximum value, \( \rho_0 \), at \((r, z) = (r_0, 0)\). The normalization unit for each variable is summarized in table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Normalized Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_{\text{th}} )</td>
<td>( \frac{V_{z0}^2}{\gamma k_0^2} )</td>
</tr>
<tr>
<td>( E_{\text{mg}} )</td>
<td>( \frac{V_{A0}^2}{\gamma k_0} )</td>
</tr>
</tbody>
</table>

where \( V_{z0} = (\gamma p_0/\rho_0)^{1/2} \), \( V_{A0} = B_0/(4\pi \rho_0)^{1/2} \), and \( p_0 \) is the initial pressure at \((r, z) = (r_0, 0)\). We used \( E_{\text{th}} = 0.05 \) throughout this work, the same as those in Matsumoto et al. (1996) and Kudoh et al. (1998). We also used \( E_{\text{mg}} = 5.0 \times 10^{-4} \) for all models.

#### 2.7. Numerical Method

The numerical schemes we used were the CIP method (Yabe, Aoki 1991; Yabe et al. 1991) and the MOC–CT method (Evans, Hawley 1988; Stone, Norman 1992). The magnetic induction equation was solved by MOC–CT and the others by CIP. The CIP has an advantage of tracing the contact surface sharply, so that it is less diffusive when we solve an advection equation, such as equation (6) or (12). This CIP–MOC–CT scheme is summarized in Kudoh, Matsumoto, and Shibata (1999b). The minimum grid size was 0.01\( r_0 \) both for the \( r \) and \( z \) directions. The grid spacing was uniform (\( = 0.01r_0 \)) for \( r/r_0 < 1 \) and \( z/z_0 < 1 \), and stretched in \( r \) and \( z \) for \( r/r_0 > 1 \) or \( z/z_0 > 1 \). The maximum grid spacing is 0.2\( r_0 \). We used \( N_r = 168 \) and \( N_z = 448 \) or \( N_r = 280 \) and \( N_z = 584 \), where \( N_r \) and \( N_z \) are the radial and vertical grid number, respectively. The maximum scales are \( r_{\text{max}} \sim 5.7 \) and \( z_{\text{max}} \sim 17.3 \) for \( N_r = 168 \) and \( N_z = 448 \) and \( r_{\text{max}} \sim 28.1 \) and \( z_{\text{max}} \sim 30.9 \) for \( N_r = 280 \) and \( N_z = 584 \).

#### 3. Results

Figure 1 shows the time evolution of the density, the toroidal magnetic field (\( B_z \)), and the plasma \( \beta (\beta = 8\pi p/B^2) \), the ratio of gas pressure to the total magnetic pressure) for the case of a uniform magnetic field without any perturbation (i.e. \( \delta V_{z0} = 0 \)). The white lines show the poloidal magnetic field lines, and the arrows show the poloidal velocity. The initial parameter is \( E_{\text{mg}} = 5.0 \times 10^{-4} \). This is the same as the parameter of a typical case in Kudoh et al. (1998). The surface layer of the disk falls faster than the equatorial part. This surface accretion (surface avalanche) was also observed in Stone and Norman (1994) as well as Matsumoto et al. (1996).

Figure 2 shows the case when a perturbation of \( k_z = 8\pi \) and \( \delta V_{z0} = 0.1 \) is imposed on the disk. The initial conditions are the same as those of figure 1, except for the perturbation. Since the initial magnetic pressure is smaller than the gas pressure in the disk (\( \beta > 1 \)), the perturbation grows due to the magneto-rotational instability. The magnetic field lines with the perturbation are compared with those without any perturbation in figure 3. The magneto-rotational instability is caused by angular-momentum transfer through the magnetic fields in the disk. The nonlinear growth of this instability in an axially symmetric case generates a channel solution on an orbital time scale (Hawley, Balbus 1992; Goodman, Xu 1994). This leads to accretion of the disk material initially in the region near the equator, and attains a larger accretion rate than that of the surface accretion. The plasma \( \beta \) distribution in the final stage shows that the low-\( \beta \) regions (\( \beta < 1 \)) appear even near the equator of the disk because of the generation of a toroidal magnetic field. We can see that the magnetic islands are created by magnetic reconnections in the disk. Since we assumed ideal MHD, the magnetic reconnection is caused by numerical diffusion. Numerical diffusion is inevitable for finite differential numerical schemes, even though we do not include the magnetic diffusivity explicitly. The numerical magnetic diffusivity \( (\eta_{\text{num}}) \) in this problem is of order of \( \eta_{\text{num}}/(V_{K0}r_0) < 10^{-3} \), which is estimated from a comparison with a numerical simulation including uniform diffusivity, though it is difficult to determine it (cf. Sano et al. 1998; Fleming et al. 2000).

Figure 4 shows the large scale structure of the later stage for the model shown in figure 2. The color shows the temperature. It shows that the jet is propagating along the poloidal
Fig. 1. Time evolution of the density (top), the toroidal magnetic field (middle), and the plasma $\beta$ (bottom) for the case of the initial uniform magnetic field without any perturbation (i.e., $\delta V_r = 0$). The plasma $\beta$ is the ratio of gas pressure ($P_{\text{gas}}$) to the total magnetic pressure ($P_{\text{mag}} = B^2/8\pi$). The white lines show the poloidal magnetic field lines, and the arrows show the poloidal velocity.
Fig. 2. Time evolution of the density (top), the toroidal magnetic field (middle), and the plasma $\beta$ (bottom) for the case of an initial uniform magnetic field with a perturbation ($k = 8\pi$ and $\delta V_{r0} = 0.1$). The plasma $\beta$ is the ratio of the gas pressure ($P_{\text{gas}}$) to the total magnetic pressure ($P_{\text{mag}} = B^2/8\pi$). The white lines show the poloidal magnetic field lines, and the arrows show the poloidal velocity.
magnetic field lines, that is, a cylindrical jet appears. The dotted line shows the Alfvén surface where the poloidal velocity equals the poloidal Alfvén velocity \( (v_{\text{Ap}}) \), defined by 
\[ v_{\text{Ap}} = \sqrt{\left( B_r^2 + B_z^2 \right) / (4\pi \rho)} \]. We found that a bipolar super-Alfvénic jet is ejected from the accretion disk even when the magneto-rotational instability is taking place.

Figure 5 shows the velocity along a stream line at \( t = 10 \) (the thick red line in figure 4). The poloidal velocity increases as the rotational velocity increases. Since the sound velocity is much smaller than the Alfvén velocity, the gas pressure is negligible for the acceleration of jet. The jet is accelerated by the magnetic Lorentz force, i.e., the magneto-centrifugal force and magnetic pressure. After the velocity exceeds the poloidal Alfvén velocity \( (v_{\text{Ap}}) \), the acceleration finishes and the velocity tends to the terminal velocity. The terminal poloidal velocity is comparable to the fast magnetosonic velocity, \( v_f \), for a cold plasma, defined as 
\[ v_f = \sqrt{\left( B_r^2 + B_z^2 + B_\phi^2 \right) / (4\pi \rho)} \].

The rotational velocity still remains at a certain value after the poloidal velocity tends to the terminal poloidal velocity, though it is smaller than the poloidal velocity. These results are similar to the characteristic of a steady cylindrical jet in which the cylindrical radius is nearly constant along the jet (cf. Heyvaerts, Norman 1989; Ostriker 1995). In these simulations, however, the jet is not in the steady state. The fluctuation of \( v_{\text{Ap}} \) and \( v_{\text{Ap}} \) around \( z = 3.5 \) are the result of the non-steady mass ejection that is seen in figure 6.

Figure 6 shows the time evolution of the mass-accretion rate, \( \dot{M}_a(t) \), the mass-ejection rate, \( \dot{M}_e(t) \), and the maximum vertical velocity, \( V_z(t) \), for both cases without and with a perturbation \( (k_x = 8\pi) \). The mass-flow rates and the velocity are defined by

\[
\dot{M}_a(t) = 2\pi \left[ \int_{r=0.3}^{0.5} \rho(-v_r) \Theta r \, dz \right], \tag{18}
\]

\[
\dot{M}_e(t) = 2\pi \left\{ \left[ \int_0^1 \rho v_r \Theta r \, dr \right]_{z=1} + \left[ \int_0^1 \rho(-v_r) \Theta r \, dr \right]_{z=-1} \right\}, \tag{19}
\]

and

\[
V_z(t) = \text{Max}[v_z(r, z, t)\Theta(r, z, t)]. \tag{20}
\]

The mass-accretion rate, \( \dot{M}_a(t) \), is defined at \( r = 0.3 \) and the mass-ejection rate, \( \dot{M}_e(t) \), is obtained by summing the mass flux at \( z = 1 \) and \( z = -1 \). The maximum vertical velocity, \( V_z(t) \), is the maximum value of \( v_z \) at each time. These values are defined for the material that is initially in the disk by multiplying by \( \Theta \), in order to exclude the transient ejection of the coronal material. For the case with a perturbation, the mass-ejection rate and velocity increase after \( t = 6 \), which is caused by an increase in the accretion rate just before \( t = 6 \). The increase in the accretion rate is caused by inflow of the disk material, which is initially in a deep region near the equator. The nonsteady mass-accretion rate and mass-ejection rate cause the fluctuation of the jet along the stream line in figure 5.

The parameter-dependencies of the time averages of \( \dot{M}_a(t) \), \( \dot{M}_e(t) \), and \( V_z(t) \) are summarized in table 2 as \( \bar{\dot{M}}_a \), \( \bar{\dot{M}}_e \), and \( \bar{V}_z \). The time averages were made within \( 4 < t < 10 \) for \( \dot{M}_a(t) \) and \( 6 < t < 10 \) for \( \dot{M}_e(t) \) and \( V_z(t) \). When \( k_z = 32\pi \), these values are almost the same as those without any perturbation, because the disk is stable for a magneto-rotational instability for such large wave number. The instability occurs for the cases of \( k_z = 4\pi \), \( k_z = 8\pi \), \( k_z = 16\pi \). A nonlinear growth of this instability leads to a larger mass-accretion rate and a larger mass-ejection rate than those of the surface accretion. However, the ratio of the mass-accretion rate to that of the mass-ejection rate is the
order of 0.1 and is roughly independent of \( k_z \). The velocities of the jets are slightly larger than those in the case without a magneto-rotational instability, but are of the order of the Keplerian velocity of the disk (cf. Kudoh, Shibata 1997).

4. Discussion

The ejection and acceleration mechanisms of the outflow are the same as those studied by Kudoh et al. (1998), and most of the properties can be understood by using steady state theory (e.g., Blandford, Payne 1982), though the jet outflow and accretion did not reach a steady state in our simulations. In this problem, a large-scale structure of the jet shows a cylindrical structure (e.g., Ostriker 1995). In this kind of time-dependent numerical simulation, the collimated structure may depend on the initial magnetic field. The cylindrical structure often appears when the vertical uniform magnetic field is initially assumed. If the strength of the initial magnetic field was assumed to decrease with the distance from the disk, the jet would show a different type of collimation, such as the paraboloidal collimation which appeared in Sakurai (1985, 1987). The collimation of the jet will be investigated in a forthcoming paper. [The preliminary result is discussed in Kudoh et al. (2000)].

The surface accretion, which has the same origin as the magneto-rotational instability, is triggered by an initial kick at the interface between the rotating disk, and the non-rotating corona. This kick is due to the rotation of the disk, and is inevitable for the initial magnetic fields that connect the interior of the disk to the initially non-rotation ambient medium (\( \partial B/\partial t = \nabla \times (\mathbf{v} \times \mathbf{B}) \neq 0 \) at the interface). Since this kick
is larger than an ordinary linear perturbation with a velocity amplitude of 1% of the sound velocity, the surface accretion dominates when the perturbation in the disk is small. This is the reason why we need a large perturbation in the disk (i.e. \( \delta V_r = 0.1 \)) to investigate the magneto-rotational instability for this kind of simulation. The case of \( \delta V_r = 0.01 \) in Table 2 shows that the kick near the disk surface grows faster than the perturbation in the disk, so that the result is nearly the same as that without a perturbation. The normalized critical wave number \( k_{zc} \) of the linear magneto-rotational instability in an uniform unperturbed state is about

\[
k_{zc} = \frac{V_A}{\Delta} \left| \frac{d\Omega_\phi}{dr} \right|^{1/2} r_0 \sim \frac{2V_K}{\Delta} \approx \frac{2E_{mg}^{-1/2}}{\Delta}
\]

(Balbus, Hawley 1991), which results in \( k_{zc} \approx 2.85 \pi \) for \( E_{mg} = 5.0 \times 10^{-4} \). This is consistent with the claim that the disk is stable for \( k_{zc} = 32 \pi \).

Goodman and Xu (1994) showed that the magneto-rotational instability modes are exact solutions of nonlinear fluid equations, even if the perturbed magnetic field is much larger than the unperturbed field. This means that the nonlinear growth of this instability approaches a channel solution on the orbital time scale within the assumption of axial symmetry. Our results are consistent with the channel solution found by Goodman and Xu (1994). When we consider non-axisymmetric perturbations, the channel flow breaks up and a chaotic structure appears in the disk (e.g., Hawley et al. 1995). Therefore, a 3-dimensional calculation is needed for the long-term evolution of the disk. However, the growth time of the non-axisymmetric perturbation is larger than that of an axially symmetric perturbation (Matsumoto, Tajima 1995). Therefore, the channel solution may appear in the initial stage of this kind of 3-dimensional calculations where a poloidal magnetic field is initially assumed. The jet may be ejected within the orbital time scale as the axisymmetric perturbation grows, and large mass accretion may be caused by the channel solution. Some results of 3D MHD simulations are given in Matsumoto (1999). The results for longer time scale 3-dimensional simulations will be presented in forthcoming papers.

We consider some cases in which the simulations run longer, up to \( t = 20-30 \). Even in such longer time-scale simulations, jets do not reach steady states. As discussed in Kuwabara et al. (2000), we also found that mass accretion and mass ejection take place intermittently in the case without a perturbation. The first event is caused by surface accretion, and the second one is caused by accretion of the inner dense region of the disk where the magneto-rotational instability develops later. For the case with a perturbation \( (k = 8 \pi) \), however, because the surface accretion and the accretion of the inner region of the disk occur almost at the same time, the second event overlaps the first one. (The increase of mass ejection near \( t \approx 6.5 \) in figure 6 is the second event.) The mass accretion and mass ejection decrease after \( t = 10 \), since most of the disk mass falls into the central region within about \( t = 10 \). This depends on our initial condition that the initial disk is not sufficient to supply mass through a longer time scale; the outer edge of the initial disk is at \( r \approx 2.5 \). Longer time-scale simulations are important, and should be done in the future, assuming that the outer edge of the initial disk is located at a long distance from the central region.

In Figure 7, we plot the time evolutions of the toroidal magnetic field and the plasma \( \beta \) for a reference point at \((r, z) = (1.0, 0.1)\) for longer time scale simulations \((k = 8 \pi)\). The toroidal field is plotted as the toroidal Alfvén velocity, defined as \( v_{A\phi} = B_\phi/\sqrt{4\pi \rho} \). The increase in the toroidal magnetic field and the decrease in the plasma \( \beta \) stop when \( v_{A\phi} \) become the same order of the Keplerian speed of the disk \((v_{A\phi} \sim 1)\). After that, the toroidal field is saturated and spiky oscillations can be seen. The spiky oscillations are caused by magnetic islands that are created by the numerical diffusivity. The final value of the plasma \( \beta \) is about \( \beta \sim 3 \) in this case. Curry and Pudritz (1995) found the global instability related to a strong toroidal magnetic field whose Alfvén speed is of the order of the rotational

<table>
<thead>
<tr>
<th>( k )</th>
<th>( \delta V_r )</th>
<th>( M_a )</th>
<th>( M_j )</th>
<th>( V_z )</th>
<th>( M_j/M_a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0</td>
<td>0.023</td>
<td>0.0024</td>
<td>1.0</td>
<td>0.10</td>
</tr>
<tr>
<td>4\pi</td>
<td>0.1</td>
<td>0.056</td>
<td>0.022</td>
<td>1.4</td>
<td>0.39</td>
</tr>
<tr>
<td>8\pi</td>
<td>0.1</td>
<td>0.061</td>
<td>0.0085</td>
<td>1.9</td>
<td>0.14</td>
</tr>
<tr>
<td>16\pi</td>
<td>0.1</td>
<td>0.040</td>
<td>0.010</td>
<td>2.0</td>
<td>0.25</td>
</tr>
<tr>
<td>32\pi</td>
<td>0.1</td>
<td>0.023</td>
<td>0.0022</td>
<td>1.1</td>
<td>0.096</td>
</tr>
<tr>
<td>8\pi</td>
<td>-0.1</td>
<td>0.057</td>
<td>0.0088</td>
<td>1.9</td>
<td>0.15</td>
</tr>
<tr>
<td>8\pi</td>
<td>0.01</td>
<td>0.038</td>
<td>0.0024</td>
<td>0.93</td>
<td>0.063</td>
</tr>
</tbody>
</table>

In addition to the standard accretion-disk model (Shakura, Sunyaev 1973), advection-dominated accretion flow (ADAF) is now widely recognized (Ichimaru 1977; Narayan, Yi 1995; Abramowicz et al. 1995). Since we neglect radiative cooling, we simulate non-radiative accretion flows (Stone, Pringle 2001; Hawley et al. 2001). Our simulation shows that the low-β region (β < 1) appears interior to the disk because of the generation of the toroidal magnetic field (see figure 2). This should be related to the low-β disk proposed by Mineshige, Kusnose, and Matsumoto (1995). They argued that a standard accretion disk is a high-β disk, and that an optically-thin geometrically-thick accretion disk tends to be a low-β disk. The magneto-rotational instability causes channel-like accretion flow in the disk, and a low-β region (β < 1) is produced interior to the disk. The channel-like accretion flow causes a large amount of mass accretion, and finally the jet is ejected along the large-scale poloidal magnetic field. The fact that the jet is ejected just after an increase of the accretion rate may explain the observation of GRO J1655-40, which shows that the radio burst appeared just after an increase of X-rays (Tingay et al. 1995).

5. Conclusions

In this paper, we focus on the effect of the magneto-rotational instability on magnetically driven jets from thick disks. For that purpose, we initially impose a sinusoidal perturbation with finite amplitude (i.e., |δv|/V_s = 0.1 where δv is the velocity perturbation and V_s is the sound velocity) on a rotating disk that is threaded by vertical uniform magnetic fields. These results are compared with those without a perturbation, which we previously studied.

- We found that a bipolar super-Alfvénic jet is ejected from the accretion disk where a magneto-rotational instability takes place.
- The mass-accretion rate, mass-ejection rate, and jet velocity are larger when a perturbation is imposed on the disk.
- The jet velocity is of the order of the Keplerian velocity of the disk, almost independent of a perturbation.

In the future, 3-dimensional simulations including a magneto-rotational instability in the disk should be made to confirm the result found in this paper.

Numerical computations were carried out on VPP300 and VPP5000 at the Astronomical Data Analysis Center of the National Astronomical Observatory. This work was partially supported by Japan Science and Technology Corporation (ACT-JST) and a Grant in Aid of the Ministry of Education, Culture, Sports, Science and Technology (10147105).

References

Sakurai, T. 1987, PASJ, 39, 821
Shibata, K., & Uchida, Y. 1986, PASJ, 38, 631
Uchida, Y., & Shibata, K. 1985, PASJ, 37, 515