Abstract

We derive the pairwise peculiar velocity distribution function of dark matter particles by applying a dark-matter halo approach. Unlike previous work, we do not assume a Gaussian velocity distribution function of dark matter in a single halo, but compute it self-consistently with the assumed density profile for the dark-matter halo. The resulting distribution function is well approximated by an exponential distribution which is consistent with the previous observational, numerical, and theoretical results. We also compute the pairwise peculiar velocity dispersion for different density profiles, and provide a practical fitting formula. We apply an empirical biasing scheme into our model and present a prediction for a pairwise peculiar velocity dispersion of galaxies, and reproduce the previous results of simulations using our semi-analytical method.

Key words: cosmology: theory — dark matter — large-scale structure of universe

1. Introduction

Since Davis and Peebles (1983) first analyzed the anisotropy in the galaxy distribution from the Center for Astrophysics (CfA) redshift catalog, it has been recognized that the peculiar velocity field of galaxies induces a significant systematic effect in the statistics of the observed galaxy distributions in redshift space. In particular, the virialized random motion of galaxies produces an elongated pattern of the galaxy distribution along the line of sight, called finger-of-God. This effect significantly suppresses the amplitude of the two-point correlation function of galaxies in redshift space, especially on scales below \( \sim 1 \) Mpc.

A proper account of this redshift-space distortion requires a detailed model for the pairwise velocity distribution function (hereafter, PVDF) of galaxies. Davis and Peebles (1983) discovered that the PVDF of the CfA galaxy sample is approximately described by an exponential distribution, instead of a Gaussian:

\[
f_{12}(v_{12}; r_{12}) = \frac{1}{\sqrt{2} \sigma_{12}(r_{12})} \exp \left[ -\frac{\sqrt{2} v_{12}}{\sigma_{12}(r_{12})} \right],
\]

where \( r_{12} \) is the separation length and \( v_{12} \) denotes the pairwise peculiar velocity between a pair of galaxy along the line-of-sight direction. The quantity \( \sigma_{12}(r_{12}) \) is the peculiar velocity dispersion (PVD). The exponential form of PVDF was also confirmed later by analyses of \( N \)-body simulations of dark-matter particles and of other samples of galaxies (e.g., Efstathiou et al. 1988; Fisher et al. 1994; Marzke et al. 1995).

Theoretical models for the origin of the exponential PVDF of dark matter were put forward by Sheth (1996) and Diaferio and Geller (1996), and more recently by Sheth and Diaferio (2001). They phenomenologically introduced a nonlinear model of PVDF using the Press–Schechter formalism (Press, Schechter 1974). To be more specific, they assume that any dark-matter particle belongs to one of the virialized clumps (dark halos) with the 1-point velocity distribution function being a Maxwellian form. If one considers sufficiently small scales, the particle pairs of those separations are likely to be in the same halo, and thus their PVDF is approximately given by

\[
f_{12}(v_{12}; r_{12}) = \frac{\int dM n(M) N_{\text{pair}}(r_{12}|M) f_{12,1h}(v_{12}|M) f_{12,1h}(v_{12}|M) dx}{\int dM n(M) N_{\text{pair}}(r_{12}|M)},
\]

where \( n(M) \) is the mass function of dark halos, and \( f_{12,1h}(v_{12}|M) \) is the PVDF of dark-matter particles within a halo of mass \( M \). The quantity \( N_{\text{pair}}(r_{12}|M) \) represents the statistical weight proportional to the number of particle pairs with separation \( r_{12} \) in the halo,

\[
N_{\text{pair}}(r_{12}|M) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \rho(r_1|M) \rho(r_2|M) \delta_D(r_1 - r_2 - |r_1 - r_2|),
\]
Fig. 1. Schematic illustration of our procedure to compute the pairwise velocity distribution function.

where \( \rho(r|M) \) is the density profile of the halo of mass \( M \), and \( \delta_D \) is the Dirac delta function. Adopting the singular isothermal distribution as a particular choice of the dark halo profile, Sheth (1996) showed that the scale-free model of \( P(k) \propto k^n \) with \( n = -1 \) exactly reproduces the exponential PVDF (1).

While a perturbation theory (Seto, Yokoyama 1998; Juszkiewicz et al. 1998) also qualitatively explained why the Gaussian initial models approach the exponential PVDF, the above model is much more successful quantitatively. Further, a significant influence of the finger-of-God effect appears at small scale, where the perturbative approach cannot be applied. Therefore, in the present paper, we attempt to improve the Sheth (1996) model (2) for the PVDF in several aspects: First, we consider more popular CDM models instead of the scale-free power-spectra. Second, we adopt a series of more realistic density profiles for dark halos (Hernquist 1990; Navarro et al. 1997; Fukushige, Makino 1997, 2001a; Moore et al. 1998; Jing, Suto 2000). Third, we derive the one-point PVDF of dark matter particles in a halo directly from the Abel integral of the above density profiles, instead of assuming the Maxwellian form \textit{a priori}. This approach is important, since we can incorporate the scale- and mass-dependence of the PVD in a consistent fashion, unlike in the previous modeling. Finally, we also apply the selection function following Jing, Börner, and Suto (2002) so as to phenomenologically attempt to predict the PVD of galaxies out of that of dark-matter particles.

This paper is organized as follows: Section 2 describes our improved modeling for the PVDF based on the dark-matter halo approach. In section 3, we present the resultant PVD in various cosmological models and discuss how the underlying halo profiles are sensitive to those results. We also provide a simple fitting formula of the PVD in the currently popular spatially-flat CDM model, which is useful in modeling the redshift-distortion effect. We then attempt to consider the effect of the spatial biasing of galaxies relative to the dark-matter particles on the PVD by applying a phenomenological biasing scheme. Finally, section 4 is devoted to a summary and conclusions.

2. Dark-Matter Halo Approach to Compute the Pairwise Velocity Distribution Function

Our present method to compute the PVDF is schematically shown in figure 1. We describe the details of the procedure below. Throughout the paper, we consider the three representative CDM models parameterized by the density parameter, \( \Omega_0 \), the dimensionless cosmological parameter, \( \lambda_0 \), the amplitude of the mass fluctuation smoothed over the top-hat radius of \( 8h^{-1}\text{Mpc} \), \( \sigma_8 \), and the Hubble constant in units of \( 100\text{km s}^{-1}\text{Mpc}^{-1} \), \( h \); standard CDM (\( \Omega_0 = 1.0, \lambda_0 = 0, \sigma_8 = 0.6, h = 0.5 \); SCDM), lambda CDM (\( \Omega_0 = 0.3, \lambda_0 = 0.7, \sigma_8 = 1.0, h = 0.7 \); LCDM), and open CDM (\( \Omega_0 = 0.45, \lambda_0 = 0, \sigma_8 = 0.83, h = 0.7 \); OCDM). Those models are normalized to satisfy the X-ray cluster abundances (Kitayama, Suto 1997).

2.1. Density Profile

The density profile of dark-matter halos plays a key role in our method. Following recent suggestions from high-resolution numerical simulations, we adopt the following specific form:
\[ \rho(r|M) = \begin{cases} \frac{\bar{\rho}(z)\delta_c}{(r/r_s)^\nu(1 + r/r_s)^\nu} & (r < r_{\text{vir}}), \\ 0 & (r > r_{\text{vir}}). \end{cases} \] (4)

In the above, \( M \) is the mass of the halo, \( \bar{\rho}(z) = \Omega_0\rho_0(1 + z)^3 \) is the mean density of the universe at \( z \), \( \rho_0 \) is the present critical density, \( \delta_c(M) \) is the characteristic density excess, and \( r_{\text{vir}}(M) \) and \( r_s(M) \) indicate the virial radius and the scale radius of the halo, respectively.

The virial radius is defined according to the spherical collapse model as

\[ r_{\text{vir}}(M) = \left( \frac{3M}{4\pi \bar{\rho} \Delta_{\text{coll}}} \right)^{1/3}. \] (5)

We use the following expressions (Kitayama, Suto 1996) for the critical over-density \( \Delta_{\text{coll}} \):

\[ \Delta_{\text{coll}}(\Omega_0, \lambda_0) = \begin{cases} 18\pi^2(1 + 0.4093\omega_{\text{vir}}^{0.9052}) & (\Omega_0 < 1, \Omega_0 + \lambda_0 = 1), \\ 4\pi^2 \left( \frac{c_\text{vir}}{c_\text{0}} - 1 \right)^2 \left( 1 - \frac{\sinh \eta_{\text{vir}}}{\eta_{\text{vir}}} \right)^2 & (\Omega_0 < 1, \lambda_0 = 0), \end{cases} \] (6)

where \( \omega_{\text{vir}} \) and \( \eta_{\text{vir}} \) are, respectively, given as \( \omega_{\text{vir}} = 1/\Omega_0 - 1 \) and \( \eta_{\text{vir}} = \cosh^{-1}(2/\Omega_0 - 1) \), in terms of the density parameter at the collapse time, \( \Omega_0 \).

In practice, we focus on three specific profiles: (i) the original NFW profile with \( \alpha = 1 \) and \( \nu = 2 \) (Navarro et al. 1997), (ii) the modified NFW profile with \( \alpha = \nu = 3/2 \), indicated by higher-resolution simulations (Fukushige, Makino 1997, 2001a,b; Moore et al. 1998; Jing, Suto 2000), and (iii) the Hernquist profile with \( \alpha = 1 \) and \( \nu = 3 \) (Hernquist 1990) for which the analytic expression of the phase-space distribution function is known.

The two parameters \( r_s \) and \( r_{\text{vir}} \) are not independent, and are related in terms of the concentration parameter,

\[ c(M, z) \equiv \frac{r_{\text{vir}}(M, z)}{r_s(M, z)}. \] (7)

The condition that the total mass inside \( r_{\text{vir}} \) is equal to \( M \) relates \( \delta_c \) to \( c \). Therefore, the halo mass-dependence of the above profiles is entirely specified by \( c = c(M) \). In the case of the original NFW profile, we use the approximate fitting function from the simulation data of Bullock et al. (2001),

\[ c_0(M) = \frac{8.0}{1 + z} \left( \frac{M}{10^{14} M_\odot} \right)^{-0.13}. \] (8)

For the other profiles, we first compute the amplitude of the two-point correlation functions of dark matter following the procedure of Seljak (2000) and Ma and Fry (2000), and then find the amplitude of the concentration parameter which reproduces the Peacock and Dodds (1996) fitting formula. This calibration yields \( c(M) = c_0(M)/2 \) for the modified NFW profile (Oguri et al. 2001), and \( c(M) = c_0(M)/3 \) for the Hernquist profile, which we adopt throughout the analysis below.

2.2. Phase-Space Distribution Function in a Halo

Our next task is to compute the phase-space distribution function in a single halo from the given density profile (4). While Sheth (1996) and Sheth et al. (2001) simply adopt the Gaussian velocity distribution function, we eliminate this assumption and derive the velocity distribution function in a fully consistent manner.

For this purpose, we make use of the Jeans theorem, which states that for a spherically symmetric and stationary system the solution of the collisionless Boltzmann equation can be expressed as a function of the specific binding energy, \( E = \psi(r) - \nu^2/2 \), alone. Here, we define \( \psi \) as the minus of the gravitational potential satisfying the boundary condition \( \psi(r \to \infty) \to 0 \).

One may wonder whether halos in hierarchical universes that should experience repeated merger and destruction continually are well approximated as stationary. Nevertheless, Natarajan et al. (1997) and Hanyu and Habe (2001) found that the phase-space distribution function directly estimated from their particle simulations agrees well with that derived from the Jeans theorem. Thus, the above assumption is justified, at least empirically.

Then, the phase-space distribution function \( F(E|M) \) in a single halo is directly computed from its given density profile, \( \rho(r|M) \), as follows (e.g., Binney, Tremaine 1987):

\[ F(E|M) = \frac{1}{\sqrt{2\pi}} \int_0^E d\psi \frac{d}{d\psi} \frac{d\rho}{E - \psi}. \] (9)

Figure 2 plots the dimensionless phase-space distribution function \( f(\epsilon) \equiv F(E|M)(Gm_s/r_s)^{3/2}/(\delta_c \bar{\rho}) \), evaluated numerically from equation (9), where \( m_s = 4\pi r_s^3 \delta_c \bar{\rho} \) is the characteristic mass of the halo and \( \epsilon = Er_s/(Gm_s) \) is the dimensionless specific energy. The Hernquist model has an analytical solution for \( F(E|M) \) which is reproduced by our numerical result almost within an accuracy of 2\%, except for the tiny region \( \epsilon \sim 0 \), where the error reaches at 7\%, but the effect is safely negligible for a
Fig. 2. Dimensionless phase-space distribution function $f(\varepsilon) \equiv F(E|M)(Gm_s/r_s)^{3/2}/(k,\rho)$ as a function of the dimensionless binding energy, $\varepsilon = E_r/(Gm_s)$; NFW ($\alpha = 1$) profile (solid), NFW ($\alpha = 1.5$) profile (dotted), and the Hernquist profile (dashed).

Later analysis. Since the three halo profiles that we adopt have a central cusp, $f(\varepsilon)$ diverges at a corresponding value of $\varepsilon$. The modified NFW profile has $f(\varepsilon)$, which extends more broadly up to $\varepsilon \sim 2$, reflecting the stronger central concentration than that of the original NFW case.

2.3. Single-Particle Velocity Distribution Function in a Halo

Once the phase-space distribution function is given, one can also compute the one-dimensional single-particle velocity distribution function along a particular direction by integrating over the other two components. Assuming the isotropic velocity distribution, one has

$$f_1(v_1|M;r) = \frac{1}{\sqrt{2\pi} \sigma(r|M)} \exp \left[ -\frac{v_1^2}{2\sigma^2(r|M)} \right],$$

(11)

where we project along the direction of $v_1$ and the quantity $E_1 \equiv \psi - v_1^2/2$ is the corresponding binding energy. Figure 3 shows the dimensionless velocity distribution function, $v_s f_1(v|M;r)$, at $r/r_s = 1$ (thin lines) and $r/r_s = 10$ (thick lines), where $v_s \equiv (Gm_s/r_s)^{1/2}$ is the scaling velocity. The figure indicates that the one-dimensional velocity distribution function in a halo can be reasonably approximated by a Gaussian,

$$f_1(v|M;r) = \frac{1}{\sqrt{2\pi} \sigma(r|M)} \int dv_2 \int dv_3 F(E|M)$$

$$= \frac{1}{\sqrt{2\pi} \rho(r|M)} \int_0^{E_1} dE \int_0^E d\psi \frac{d\rho}{d\psi} \sqrt{E - \psi},$$

(10)

where we project along the direction of $v_1$ and the quantity $E_1 \equiv \psi - v_1^2/2$ is the corresponding binding energy. Figure 3 shows the dimensionless velocity distribution function, $v_s f_1(v|M;r)$, at $r/r_s = 1$ (thin lines) and $r/r_s = 10$ (thick lines), where $v_s \equiv (Gm_s/r_s)^{1/2}$ is the scaling velocity. The figure indicates that the one-dimensional velocity distribution function in a halo can be reasonably approximated by a Gaussian,

$$f_1(v|M;r) = \frac{1}{\sqrt{2\pi} \sigma(r|M)} \exp \left[ -\frac{v^2}{2\sigma^2(r|M)} \right].$$

(11)

although it has a sharp cutoff around the escape velocity of the halo, $v_{esc} = (2\psi)^{1/2}$.

In figure 3, the dashed lines indicate a Gaussian fit which has the same velocity dispersion, as that evaluated from equation (10). It seems that the empirical Gaussian approximation can reasonably reproduce the PVDF, and thus we use the approximation in the numerical integrations below so as to reduce the computational time.

Figure 4 plots the velocity dispersion, $\sigma(r|M)$, computed from the best-fit Gaussian, which clearly shows the scale-dependence that was neglected in the previous analysis (Sheth 1996; Sheth et al. 2001). Note also the different scale-dependence from the circular velocity $V_c \equiv Gm(r)/r$, where $m(r)$ is the mass inside radius $r$ (thin lines in figure 4). In subsequent modeling of the PVDF, we use the Gaussian approximation with the fitted $\sigma(r|M)$ rather than repeating the full numerical integration.
Fig. 3. Single-particle velocity distribution function in a single halo at \( r/r_s = 1 \) (thin-solid) and \( r/r_s = 10 \) (thick-solid). The dashed lines show the corresponding Gaussian fits [see equation (11)]. Left: NFW (\( \alpha = 1 \)); middle: NFW (\( \alpha = 1.5 \)); right: Hernquist profile.

Fig. 4. Peculiar velocity dispersion \( \sigma(r|M) \) of dark-matter particles in a single halo resulting from a Gaussian fit as a function of position \( r/r_s \) (thick lines). For a comparison, the thin lines show the circular velocity, \( V_c \), evaluated from the relation \( V_c(r) = Gm(r)/r \), where \( m(r) \) represents the mass inside radius \( r \). The solid lines, dotted lines, and dashed lines represent the results for NFW (\( \alpha = 1 \)), NFW (\( \alpha = 1.5 \)), and Hernquist profiles, respectively.

2.4. Pairwise Relative Velocity Distribution Function

Finally, we are in a position to estimate the PVDF by combining all of the above results. Since we are interested in small scales, the particle pairs with the corresponding separations are approximated to reside in the common halo. Then Sheth (1996) derived the following expression for the PVDF:
The relation between the quantities used in expressions (12) and (13) are schematically summarized in figure 5.

\[ f(v_{12}; r_{12}) = N^{-1} \int dM n(M) \int d^3r_1 d^3r_2 \frac{\rho(r_1|M)\rho(r_2|M)}{2\pi \sigma(r_1|M)\sigma(r_2|M)} \times \int dv_1 dv_2 f_1(v_1|M;r_1) f_1(v_2|M;r_2) \delta_D(r_{12} - |r_1 - r_2|) \delta_D(v_{12} - v_1 + v_2), \]

where \( r_{12} \) is the pair-separation and \( N \) is the normalization factor given by

\[ N = \int dM n(M) \int d^3r_1 d^3r_2 \rho(r_1|M)\rho(r_2|M)\delta_D(r_{12} - |r_1 - r_2|). \]

The integrals over the two velocity components, \( v_1 \) and \( v_2 \), and also over the directions of the position vectors can be performed analytically, and equation (14) reduces to

\[ f(v_{12}; r_{12}) = \frac{N^{-1}}{4\pi r_{12}^3} \int_{M_{\text{min}}(r_{12})} dM n(M) r_{12} \int^{r_{12}}_{r_{\text{max}}(0,r_{12},v_{12})} dr_2 \int_{r_{12}}^{\min(r_{12},r_{12} + r_2)} dr_1 \frac{\rho(r_1|M)\rho(r_2|M)}{2\pi \sigma^2(r_1|M) + \sigma^2(r_2|M)} \exp \left\{ -\frac{v_{12}^2}{2\sigma^2(r_1|M) + \sigma^2(r_2|M)} \right\}, \]

where \( M_{\text{min}}(r_{12}) \) is the minimum mass of the halo including the pair with separation \( r_{12} \) [i.e., \( v_{\text{vir}}(M_{\text{min}}) > r_{12}/2 \)]; the normalization factor \( N \) is now given by

\[ N = \frac{1}{4\pi r_{12}^3} \int_{M_{\text{min}}(r_{12})} dM n(M) r_{12} \int^{r_{12}}_{r_{\text{max}}(0,r_{12},v_{12})} dr_2 \int_{r_{12}}^{\min(r_{12},r_{12} + r_2)} dr_1 r_1 r_2 \rho(r_1|M)\rho(r_2|M). \]

Actually, it turns out that this term corresponds to the one-halo contribution of the two-point correlation function, \( \xi_{1h}(r_{12}) \), in the dark halo approach (Seljak 2000; Ma, Fry 2000).

Figure 6 plots the resulting PVDF (in the LCDM model) at \( r_{12} = 1h^{-1}, 0.3h^{-1} \) and \( 0.1h^{-1} \) Mpc against the pairwise velocity, \( v_{12} \), normalized by the PVD at each separation. We adopt the Press–Schechter mass function for definiteness (Press, Schechter 1974). Note that the quantitatively similar behavior is obtained for other cosmological models, but with different PVDs. The PVDF for small separation pairs \( (r_{12} < 1h^{-1}\text{Mpc}) \) is well described by the exponential distribution. As \( r_{12} \) increases, the central region resembles the Gaussian distribution, while the exponential tail is still clear at large velocities. Neither the inner nor outer slope of the density profile produces any systematic difference in the non-Gaussian tails of PVDF, in contrast to the single-particle velocity distribution in figure 3. This qualitative behavior is in complete agreement with the result of Sheth (1996), assuming the scale-free model and the singular isothermal sphere. Therefore, we conclude that the exponential distribution of the PVDF is a rather general consequence in the gravitational instability picture fairly independent of the underlying cosmological model.
3. Pairwise Relative Peculiar Velocity Dispersions

3.1. Dark Matter Particles

We have shown that the shape of the PVDF is well approximated by the exponential in a fairly insensitive manner to either the cosmological model or the dark halo density profile. Note, however, that this does not imply that the PVD is determined independently of the cosmology, but, rather, it is the basic source for the cosmological model-dependence, through the mass function of halos. Since the PVDF is already obtained, one may evaluate the PVD $\sigma_{12}(r_{12})$ directly as

$$\sigma_{12}^2(r_{12}) = \int_{-\infty}^{\infty} dv_{12} f_{12}(v_{12};r_{12}) v_{12}^2.$$  \hspace{1cm} (17)

We note, however, that the above expression is valid only when the one-halo term, $\xi_{1h}(r_{12})$, is sufficiently larger than unity. If one takes account of particle pairs residing in two different halos that we neglect in the present modeling, one should rather replace the normalization factor, $N$, by $1 + \xi(r_{12})$, since the factor physically corresponds to the relative probability of finding a pair at separation $r_{12}$. This consideration implies the normalization of the PVD from the one-halo contribution should be

$$\sigma_{12}^2(r_{12}) = \frac{\xi_{1h}(r_{12})}{1 + \xi(r_{12})} \int_{-\infty}^{\infty} dv_{12} f_{12}(v_{12};r_{12}) v_{12}^2.$$  \hspace{1cm} (18)

In fact, this agrees with equation (21) of Sheth et al. (2001). We compute the two-point correlation function, $\xi(r_{12}) = \xi_{1h}(r_{12}) + \xi_{2h}(r_{12})$, on the basis of the dark halo approach (Seljak 2000), and we also confirm that the resulting $\xi(r_{12})$ is in good agreement with the fitting formula of Peacock and Dodds (1996) by an appropriate choice of the concentration parameter $c(M)$, as discussed in subsection 2.1.

Figure 7 shows the PVD calculated according to equation (18). The comparison among the different model predictions indicates that the amplitude of PVD depends sensitively on the cosmological parameters through the mass function, $n(M)$, but is almost insensitive to the density profile of the dark halo. Note, however, that this is partly because we have chosen the value of $c(M)$ so as to reproduce the same $\xi(r)$, irrespectively of the density profile. We also show the result with neglecting the scale-dependence of the velocity distribution in the case of the LCDM model (long-dashed lines). This indicates that the isothermal approximation (Sheth et al. 2001) is quite acceptable in predicting the PVD.

We attempted an empirical fitting to our numerical results at different redshifts (figure 8) by adopting a power-law form,

$$\sigma_{12}(x_{12}) = A \left( \frac{x_{12}}{1h^{-1}\text{Mpc}} \right)^\nu.$$  \hspace{1cm} (19)
in terms of the comoving pair separation \( x_{12} \equiv r_{12}(1 + z) \). The values of the amplitude \( A \) and the power-law index \( \rho \) fitted for \( 0.01 h^{-1} \text{Mpc} < x_{12} < x_{\text{max}} \) are listed in table 1 for the LCDM model, where \( x_{\text{max}} \) is the comoving separation at which the PVD becomes maximum. Since the two-halo term \( \xi_{2h}(r_{12}) \) becomes the dominant contribution to \( \xi(r_{12}) \) for larger separations, equation (19) becomes inaccurate for \( x > x_{\text{max}} \). The fit is accurate within 5%, which is comparable to the other systematic errors including the numerical integration or the Gaussian approximation. This result may be compared with an independent prediction based on the cosmic virial theorem (Peebles 1976; Suto 1993; Suto, Jing 1997). For a reference, if the correlation function of dark matter is given by \( (r/5.4 h^{-1} \text{Mpc})^{-1.8} \), the cosmic virial theorem implies that

\[
\nu_{12}(r_{12})_{\text{CVT}} = 990 \left( \frac{\Omega_0}{0.3} \right)^{1/2} \left( \frac{Q}{2.0} \right)^{1/2} \left( \frac{r_{12}}{1 h^{-1} \text{Mpc}} \right)^{0.1} \text{km s}^{-1},
\]

where \( Q \) is the normalized amplitude of the three-point correlation function. The value of \( Q \) for dark-matter clustering is somewhat uncertain, but may be close to 2 in LCDM (Suto 1993), and thus this estimate is fairly consistent with our prediction presented here.

3.2. Effect of Biasing

It is well known that the PVD of the observed galaxies is generally smaller than the value predicted in the current popular models (e.g., Davis, Peebles 1983; Mo et al. 1993; Suto 1993; Suto, Jing 1997). This may be interpreted as a manifestation of the spatial biasing of galaxies relative to the dark matter.

Jing, Mo, and Börner (1998) analyzed the Las Campanas Redshift Survey of galaxies, and developed a phenomenological biasing model, CLuster underWeight bias (CLW, hereafter), which successfully accounts for the amplitudes of the two-point correlation function and the PVD simultaneously. More recently, Jing, Börner, and Suto (2002) performed a similar analysis of galaxies in the PSCz catalog (Saunders et al. 2000), which are selected from the InfraRed Astronomical Satellite (IRAS) Point Source Catalog (PSC; Beichman et al. 1988). They found that the IRAS-selected galaxies, which are likely to be dominated by late-types, have significantly smaller PVD than those in other catalogues. In addition, they applied the CLW bias scheme to mock samples from N-body simulations, and concluded that the PVD of the PSCz galaxies is significantly smaller than those predicted from the CLW bias in the popular CDM models. In this subsection, we revisit this issue by combining the biasing effect with our analytical model of the PVD in a complementary manner to the direct method using the N-body data.

The CLW scheme can be applied to our analytical model easily. According to Jing et al. (2002), we adopt that the selection function of galaxies in a halo of mass \( M \) is proportional to \( (M/10^{14} M_\odot)^{-\beta} \). This is simply equivalent to replacing the mass function, \( n(M) \), in equation (15) by \( n(M)(M/10^{14} M_\odot)^{-\beta} \). Because this biasing model puts a lower weight on the massive...
clusters where the velocity dispersion is large, increasing $\beta$ suppresses the mean PVD. In fact, the Las Campanas redshift survey data are consistent with $\beta = 0.08$, and the PSCz data prefer a much larger value $\beta = 0.25$. Physically speaking, this phenomenological prescription should be understood as the dependence of the efficiency of galaxy formation on the mass of the hosting halo.

In addition we consider another biasing to take into account the observed density–morphology relation of galaxies. Since spiral galaxies preferentially avoid the central region of massive clusters (i.e., halos in the present context), the PVD of spirals is generally suppressed, especially in the case of the modified NFW profile that has a stronger central concentration (cf. figure 4). We attempt to incorporate this effect by introducing a selection probability that depends on the distance from the center of the halo,

$$p(r|M) = a + \frac{r}{r_{\text{vir}}(M)} b.$$  \hfill (21)

We set $a = 0.2$ and $b = 0.6$ so as to reproduce the observed ratio of spirals to ellipticals; 2:8 in the inner part and 8:2 in the outer part. In practice, we calculate the PVD by replacing $\rho(r|M)$ by $\rho(r|M)p(r|M)$ in equation (18).

The results are shown in figure 9 for the modified NFW profile in the LCDM model. The dotted and short-dashed lines represent the results taking account the CLW bias effect, while the long-dashed lines consider the density–morphology relation [equation (21)] in addition to the CLW bias. The degree of suppression of the PVD is in agreement with the simulation results of
Fig. 9. Pairwise peculiar velocity dispersion for galaxies with empirical biasing scheme in LCDM. Solid: dark matter particles; dotted: CLW with \( \beta = 0.08 \); short-dashed: CLW with \( \beta = 0.25 \); long-dashed: CLW with \( \beta = 0.25 \) and density–morphology relation. The filled squares indicate the values for the PSCz galaxies estimated by Jing et al. (2002).

Jing et al. (2002). Even with the density–morphology relation, the PVD of the IRAS PSCz galaxies is too small to be reconciled in the current model, as Jing et al. (2002) claimed.

4. Summary

We have presented a detailed prediction for the pairwise peculiar velocity distribution function (PVDF), applying the dark-matter halo approach. In particular, we have derived the PVDF in a direct and self-consistent manner with the assumed density profile for dark-matter halo for the first time. On the other hand, we neglected the halo–halo contribution by assuming that any pair of particles resides in a common halo. Thus, our predictions are quantitatively valid only on small scales of \( \lesssim 1h^{-1}\text{Mpc} \), but our result turns out to be fairly close to the previous one by Sheth (1996) and Sheth et al. (2001), who assumed an isothermal velocity distribution in a single halo. In this sense, our independent approach may be regarded as providing an empirical justification for their simplifying assumptions; also, our predictions are fairly accurate on those small scales.

We have shown that the shape of the PVDF is well approximated by an exponential function in a fairly insensitive manner to either the cosmological model or the dark halo density profile. The dependence of the PVD on the halo density profiles that we employed is also fairly small, yielding a difference of less than about 10 percent.

We have also obtained a practical fitting formula for the PVD of dark-matter particles [equation (19)] at different cosmological models as a function of the pair separation. This may be useful in modeling the redshift-space distortion of clustering. The result is in reasonable agreement with an estimate based on the cosmic virial theorem. Furthermore, we applied an empirical biasing scheme into our model and attempted to predict the PVD of galaxies. We could reproduce the previous simulation results based on our analytical method, and also confirmed that the very small PVD estimated for the PSCz galaxies (Jing et al. 2002) is difficult to be reconciled with a simplistic biasing model and/or the underlying CDM model.

The discrepancy between the prediction and the observation shown in subsection 3.2 indicates the presence of a velocity bias, in addition to the other selection effects. In fact, each luminous galaxy is a clump composed of baryons as well as dark-matter particles, whose bulk motion might not trace the random motion of the individual dark-matter particles. In this case, the galaxy–galaxy interaction through tidal field or gas pressure of baryons might be an important source for a velocity bias. At least, our present treatment using a dark-matter halo approach can provide a quantitative prediction for the PVD of the dark-matter particles. Therefore, a next step, the effect of velocity bias including these interactions, should be incorporated into our scheme to account for the PVD of the galaxies.

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