Radiation Properties of an Accretion Disk with a Non-Zero Torque on Its Inner Edge

Xinwu CAO
Shanghai Astronomical Observatory, Chinese Academy of Sciences, 80 Nandan Road, Shanghai, 200030, China
cxw@center.shao.ac.cn

and
Ya-Di XU
Physics Department, Shanghai Jiaotong University, Shanghai, 200030, China
ydxu@sjtu.edu.cn

(Received 2002 March 29; accepted 2002 November 8)

Abstract

The structure of the inner edge of the accretion disk around a black hole can be altered, if the matter inside the marginally stable orbit is magnetically connected to the disk. In this case, a non-zero torque is exerted on its inner edge, and the accretion efficiency, \( \epsilon \), can be much higher than that in the standard accretion-disk model. We explored the radiation properties of an accretion disk at its sonic point around a black hole with a time-steady torque exerted on the inner edge of the disk. The local structure of the accretion flow at the sonic point was investigated in the frame of general relativity. It was found that the accretion flow would be optically thin at its sonic point for most cases, if the additional accretion efficiency \( \delta \epsilon \) caused by the torque is as high as \( \sim 10\% \). The results imply that a variable torque may trigger transitions of the flow between different accretion types.

Key words: accretion, accretion disks — black hole physics — galaxies: nuclei — radiation mechanisms: general

1. Introduction

Most works on accretion disks around black holes have been based on the assumption that there is no stress at the inner edge of the disk (Shakura, Sunyaev 1973). The inner edge of the disk should occur very close to the radius of the marginally stable orbit \( r_{\text{ms}} \), since plunging matter in the region of unstable orbits rapidly becomes causally disconnected from the disk. Recently, Krolik (1999) questioned this assumption, and suggested that the matter inside the marginally stable orbit can remain magnetically connected to the disk. If this is the case, an additional torque would be exerted on the inner edge of the disk. Gammie (1999) has shown that the torque enhances the amount of energy released in the disk. The energy of the matter in the plunging region is extracted, and then released in the disk. The extracted energy is transported outwards by the stresses in the disk. The torque alters the disk structure, and then the radiation spectrum of the disk (Agol, Krolik 2000, hereafter AK00). In their calculations, AK00 assumed that the circular motion of the matter in the disk is Keplerian instead of considering the radial force balance; the inner edge of the disk was therefore set at \( r_{\text{ms}} \) manually. They further assumed that the generated energy is dissipated locally in the disk, i.e., the radial energy advection was not considered in their work. General relativistic effects were included in their calculations, as was done by Novikov and Thorne (1973).

For a transonic accretion flow around a black hole, we took the radius of the sonic point \( r_s \) as the inner edge of the disk. We assumed that a time-steady torque is exerted on the inner edge of the disk, as was done by AK00 based on the assumption of a Keplerian disk [and also see Paczynski (2000) and Afshordi and Paczynski (2002)]. We included the angular and radial momentum equations in our calculations, and the circular velocity of the flow was not limited to the Keplerian value. Our calculations were in the frame of general relativity, and the radial energy advection in the flow was included in the energy equation. The local structure of the flow at the sonic point was therefore available. The torque may be variable and extended to a region of the disk outside of the radius of sonic point. For simplicity, we restricted our calculations to the simplest case as AK00, i.e., a time-steady torque is exerted on the inner edge of the disk.

In this work, we investigated the local structure of the accretion disk at the sonic point with a non-zero torque exerted on its inner edge. The radiation properties of the disk at its sonic point were explored. In our calculations, the radial energy advection in the flow was considered, and the circular velocity of the flow was not limited to the Keplerian value.

2. Model

AK00 assumed that the circular motion of the matter in the disk is Keplerian instead of considering the radial force balance; the inner edge of the disk was therefore set at \( r_{\text{ms}} \) manually. They further assumed that the generated energy is dissipated locally in the disk, i.e., the radial energy advection was not considered in their work. General relativistic effects were included in their calculations, as was done by Novikov and Thorne (1973).

For a transonic accretion flow around a black hole, we took the radius of the sonic point \( r_s \) as the inner edge of the disk. We assumed that a time-steady torque is exerted on the inner edge of the disk, as was done by AK00 based on the assumption of a Keplerian disk [and also see Paczynski (2000) and Afshordi and Paczynski (2002)]. We included the angular and radial momentum equations in our calculations, and the circular velocity of the flow was not limited to the Keplerian value. Our calculations were in the frame of general relativity, and the radial energy advection in the flow was included in the energy equation. The local structure of the flow at the sonic point was therefore available. The torque may be variable and extended to a region of the disk outside of the radius of sonic point. For simplicity, we restricted our calculations to the simplest case as AK00, i.e., a time-steady torque is exerted on the inner edge of the disk.

The magnetic field plays an important role in the angular-momentum transfer of the matter in the disk. The value of the viscosity \( (\alpha) \) can be determined by the magnetohydrodynamical processes taking place in the accretion flow (Hawley 2000). The value of \( \alpha \) of the flow may vary with the time...
and radius. The angular-momentum transfer in the disk caused by any specific magnetohydrodynamical processes may be approximately described by the parameter $\alpha$, though the precise value of $\alpha$ and its variation is unavailable, unless the detailed magnetohydrodynamical processes in the flow are included in the numerical simulations. The angular-momentum transfer of the matter in the flow caused by the magnetohydrodynamical processes in the flow may correspond to a certain value of $\alpha$, and the torque exerted on the inner edge of the disk is governed by the same magnetohydrodynamical processes. Thus, the values of $\alpha$ and the torque are determined simultaneously by the magnetohydrodynamical processes taking place in the disk. In order to avoid being involved in complicated magnetohydrodynamical processes taking place in the flow, we adopted the $\alpha$-viscosity and another independent parameter ($\delta \nu$) to describe the angular-momentum transfer of the matter in the flow and the torque on the inner edge of the disk, respectively. This may have induced some inconsistency in our calculations, but we could explore the general radiation properties of the disk varying $\alpha$ for a given torque. This would not change our main conclusions on the radiation properties of the disk (see further discussion in section 5).

For a disk with an additional torque exerted on its inner edge, the viscosity $\nu$ at the sonic point is

$$v(r_s) = \nu_0(r_s) + \delta \nu,$$  

(1)

where $\nu_0(r_s)$ is the viscosity at the sonic point without an additional torque; $\delta \nu$ is caused by additional torque exerted on the inner edge of the disk.

The main goal of this work was to explore the local structure and radiation properties of the disk at its sonic point. The radiation properties are mainly governed by the radiative processes and the energy dissipated in the flow. The $\alpha$-description on the angular momentum transfer in the disk was therefore sufficiently good for the present investigation, though the exact value of $\alpha$ is unknown. In this work, we focused on the local structure of the disk and the radiation properties at the sonic point of the flow. We assumed that $\alpha$-viscosity is valid in the flow at the sonic point. The variation of $\alpha$ along the radius may not affect our main results of the radiation properties, since we focused on the local structure of the disk at the sonic point.

3. Equations of Accretion Flows in the Kerr Geometry

Many authors have included general relativistic effects in their models of accretion disks around black holes (e.g., Novikov, Thorne 1973; Lu 1985; Abramowicz et al. 1996; Peitz, Appl 1997; Gammie, Popham 1998; Popham, Gammie 1998; Mannomo 2000). In this work, we mainly adopted the equations presented by Abramowicz et al. (1996, hereafter A96). One may refer to A96 for detail.

The metric of a Kerr black hole on the equatorial plane takes the form given by Novikov and Thorne (1973) ($G = c = 1$),

$$ds^2 = -\frac{r^2 \Delta}{A} + \frac{A}{r^2} (d\varphi - \omega dt)^2 + \frac{r^2}{\Delta} dr^2 + dz^2,$$  

(2)

where

$$\Delta = r^2 - 2Mr + a^2,$$  

(3)

and

$$\omega = \frac{2Ma}{A}.$$  

(5)

Here, $M$ is the black-hole mass and $a$ is the specific angular momentum of the Kerr black hole.

The Lorentz factor $\gamma$ is defined by

$$\gamma = \frac{1}{\sqrt{1 - (v^{(r)})^2}},$$  

(6)

where

$$v^{(r)} = \dot{R} \Omega.$$  

(7)

The quantities $\dot{R}$ and $\Omega$ are defined by

$$\dot{R}^2 = \frac{A^2}{r^4 \Delta}, \quad \dot{\Omega} = \Omega - \omega.$$  

(8)

The radial velocity component $V$ is given by

$$V = \frac{\nu^{(r)}}{\sqrt{1 - \Omega^2 r^2}}.$$  

(9)

Combining equations (6)–(9), we have

$$\gamma^2 = \left( \frac{1}{1 - \Omega^2 r^2} \right) \left( \frac{1}{1 - V^2} \right)$$  

(10)

and

$$V = \frac{\nu^{(r)}}{\sqrt{1 - \Omega^2 r^2}}.$$  

(11)

The angular momentum conservation is

$$\dot{M} \frac{dl}{2\pi r} dr + \frac{1}{r} \frac{d}{dr} \left( \Sigma \nu A^{3/2} \Delta \gamma^3 \frac{d\Omega}{r^4} \right) = 0,$$  

(12)

where

$$l = \gamma \left( \frac{A^{3/2}}{\Delta^{1/2}} \right) \tilde{\Omega}$$  

(13)

is the specific angular momentum per unit mass, and the term of the angular momentum carried by the vertical flux of radiation has been neglected (A96).

Integrating equation (12), we obtain

$$\frac{\dot{M}}{2\pi \Omega} (l - l_0) = -\frac{\Sigma \nu A^{3/2} \Delta \gamma^3}{r^4} \frac{d\Omega}{dr},$$  

(14)

where $l_0$ is the integral constant. We assume that an additional torque is exerted on the inner edge of the disk, $r = r_s$; here, $r_s$ is the radius of the sonic point. At the sonic point, the viscosity $v(r_s)$ is described by equation (1). The additional accretion efficiency caused by the torque is $\delta \epsilon$. We can obtain the following relation at $r = r_s$: 

$$\delta \epsilon \dot{M} = \frac{-\Sigma \nu A^{3/2} \Delta \gamma^3}{r^4} \left. \frac{d\Omega}{dr} \right|_{r=r_s}.$$  

(15)

In the absence of an additional torque, the viscosity $\nu_0(r_s)$ at the sonic point is very low, namely a zero-torque approximation. In this work, we considered the cases with significant additional torques on the inner edges of the disks, so $\delta \nu \gg \nu_0$
is usually satisfied at the sonic point. For simplicity, we used an approximation, \( v(r_s) \approx \beta v \), in our calculations. We further assumed that \( \alpha \)-viscosity is valid even at the sonic point. The value of \( \alpha \) at the sonic point may be different from that in the region outside the radius of the sonic point.

The radial component of the momentum conservation is

\[
\frac{V}{1-V^2} \frac{dV}{dr} = A \frac{1}{r} \frac{dP}{dr},
\]

where

\[
A = \frac{M A}{r^3 \Delta \Omega_{K} \Omega_{K}} \frac{(\Omega - \Omega_{K}^{+})(\Omega - \Omega_{K}^{-})}{1 - \Omega^2 R^2},
\]

and \( P = 2H\rho \) is the vertical integrated pressure.

The angular velocities of the co-rotating and counter-rotating Keplerian orbits are

\[
\Omega_{K}^{\pm} = \pm \sqrt{\frac{M}{r^{3/2}} \pm aM^{1/2}}.
\]

The surface energy generation rate \( F^+ \) and the cooling rate \( F^- \) are

\[
F^+ = \nu \sum \frac{A^2}{r^4} \gamma^4 \left( \frac{d\Omega}{dr} \right)^2,
\]

and

\[
F^- = \frac{16\alpha T^4}{3\pi},
\]

respectively. Here, we only consider the optically-thick case.

The energy equation can be written as

\[
F^{adv} = F^+ - F^-.
\]

The equation of state is

\[
p = \frac{1}{3} \frac{aT^4}{\mu_{H}},
\]

where the ratio of the specific heats of the gas, \( \gamma_g = 5/3 \), has been adopted. The magnetic pressure has been neglected, since we restrict our calculations to the optically-thick case. For the optically-thick case, the radiation processes are not relevant to the magnetic pressure.

Using equation (22), the entropy gradient can be calculated as

\[
T dS = \frac{P}{\rho} \left( 12 \frac{21}{2} \beta \right) d\ln T - (4 - 3\beta) d\ln \rho,
\]

where \( \beta = \rho_g / \rho \).

The advection cooling rate due to the radial motion of the gas is

\[
F^{adv} = -\frac{M}{2\pi r} \frac{dS}{dr} = -\frac{M}{2\pi r} \rho \left( 12 \frac{21}{2} \beta \right) d\ln T - (4 - 3\beta) \frac{d\ln \rho}{dr}.
\]

The vertical force balance gives

\[
\frac{p}{\rho H^2} = \gamma^2 \frac{M}{r^3} \left[ \frac{(r^2 + a^2)^2}{(r^2 + a^2)^2 - \Delta a^2} \right].
\]

Substituting equations (22), (24), and (25) into equation (16), we can rewrite the radial motion equation as

\[
\frac{dV}{dr} = \frac{N}{D} (1 - V^2),
\]

where

\[
D = V - \frac{72 - 51\beta - 9\beta^2 c_s^2}{56 - 45\beta - 3\beta^2 V},
\]

and

\[
N = \frac{A}{r} + \frac{72 - 51\beta - 9\beta^2 c_s^2}{56 - 45\beta - 3\beta^2 V} \frac{d\ln \Delta^{1/2}}{dr} + \frac{-8 + 3\beta + 9\beta^2 c_s^2}{56 - 45\beta - 3\beta^2 V} \times \frac{d}{dr} \left\{ y^2 M \left[ \frac{(r^2 + a^2)^2 + 2\Delta a^2}{(r^2 + a^2)^2} \right] \right\}.
\]

The sonic point is defined by the condition

\[
D = N = 0,
\]

and we have the radial velocity of the flow at the sonic point,

\[
V_s = \left( \frac{72 - 51\beta - 9\beta^2 c_s}{56 - 45\beta - 3\beta^2} \right)^{1/2} c_s.
\]

The standard \( \alpha \)-viscosity,

\[
\nu = \frac{2}{3} \alpha c_s H,
\]

was adopted in this work.

Substituting equations (15), (19), (20), and (24) into equations (28) and (29), the physical quantities \( \rho_s \) and \( T_s \) of the flow at the sonic point are available, if the parameters \( M, M, a, r_s, \alpha, \delta \epsilon, \) and \( \Omega_s \) are specified.

The effective optical depth in the vertical direction is

\[
\tau_{eff} = 0.5 \sum (\kappa_{es} + \kappa_{H}) \delta \epsilon,
\]

where \( \kappa_{es} \) and \( \kappa_{H} \) are the electron-scattering opacity and free–free opacity, respectively.

4. Results

In principle, the set of equations describing the accretion flow around a black hole can be integrated with suitable outer boundary conditions. The solution is required to satisfy the regularity condition (29) at the sonic point, \( r = r_s \), for a given \( \delta \epsilon \). The specific angular momentum \( l_s \) at the sonic point and the radius of sonic point \( r_s \) were simultaneously given by the numerical solution. In the case of \( \delta \epsilon = 0 \), we have \( l_s \approx l_0 \). The main numerical difficulties in solving the set of equations were the particular value of \( l_s \), which should satisfy the regularity condition (29) (see A96 for the cases of \( l_s \approx l_0 \)).

In this work, we searched all possible solutions in the parameter plane \( (r_s, l_s) \) instead of integrating the equations with outer boundary conditions numerically. Using this method, we could study the local properties of the disk at the sonic point.
Fig. 1. Parameter plane \((r_s, l_s)\) for all possible solutions with different values of \(\delta\) (labeled near the curves). The parameter \(M = 10^9 M_\odot\), \(a = 0.95\), and \(M/M_{\text{Edd}} = 0.001\) were adopted. The parameters in the left side of the curves are for optically-thick solutions, i.e., \(\tau_{\text{eff}} > 1\). The radius of the black hole horizon \(r_h\), radius of the minimal bound circular orbit \(r_{\text{mb}}\), and radius of the marginally stable orbits \(r_{\text{ms}}\) are indicated in the figure. The upper panel is for \(\alpha = 1\), while the lower panel is for \(\alpha = 0.1\).

Fig. 2. Same as figure 1, but for the case of a larger accretion rate \((a = 0.95\) and \(M/M_{\text{Edd}} = 0.1\)).

Fig. 3. Same as figure 1, but for the case of a Schwarzschild black hole \((a = 0\) and \(M/M_{\text{Edd}} = 0.001\)).

Fig. 4. Same as figure 1, but for the case of a Schwarzschild black hole along with a larger accretion rate \((a = 0\) and \(M/M_{\text{Edd}} = 0.1\)).
presented in figures 1–4 for different values of $\dot{M}/\dot{M}_{\text{Edd}}, \alpha$, and $a$. We found that the $r_s-l_s$ plane for all possible optically-thick solutions is sensitive to these parameters. It was interesting to find that there is an upper limit on the radius of the sonic point in the solution of the $r_s-l_s$ plane for given parameters. The upper limits on the radius of sonic point $r_{s,max}$ for possible solutions are plotted in figures 5 and 6.

### 5. Discussion of the Results

The physical, global solutions for optically-thick flows are available by integrating a set of equations with suitable inner and outer boundary conditions. The problem is that the search for such a parameter plane for all physical global solutions with all possible different inner and outer boundary conditions would be very difficult. The $r_s-l_s$ plane for physical, global solutions would be included in the present $r_s-l_s$ plane, but the global solution would set a more strict constraint on the existence of optically-thick flows. Thus, the conclusion is tight, that no physical, optically-thick disk solution at the sonic point would be present outside of the $r_s-l_s$ plane obtained here, though the possibility cannot be ruled out that the flow is optically thin at the sonic point, while it becomes optically thick at a large radius outside of the sonic point.

It was found that the location of the sonic point for an optically-thick accretion flow moves towards the horizon of the hole with an increase of $\delta\varepsilon$. For high $\delta\varepsilon$, no optically-thick accretion flow at the sonic point is present. A higher $\delta\varepsilon$ means that much energy of the plunging matter inside the marginally stable orbit is extracted to the disk, and the disk is heated to a higher temperature. The free–free opacity $\kappa_{ff}$ decreases with an increase of the temperature. The disk then becomes optically thin.

Comparing the results for different values of $\alpha$, we find that the accretion flow with a low $\alpha$ will be optically thin, even for a low $\delta\varepsilon$, i.e., optically-thick flows at the sonic point with low $\alpha$ are present only for low $\delta\varepsilon$ cases. A high $\alpha$ makes the energy extracted from the plunging matter inside the inner edge of the disk transported outwards efficiently. Therefore, only a small fraction of the energy is released locally at the inner edge of the disk, and the flow can still remain optically thick for a relatively high $\delta\varepsilon$. For the lower $\alpha$ case, much energy is released locally, the disk is heated to a higher temperature, and the disk becomes optically thin at the sonic point. For a high accretion rate of $\dot{m} = \dot{M}/\dot{M}_{\text{Edd}}$, the surface density of the disk is so high that the disk is optically thick even for a relatively high $\delta\varepsilon$ (compare the results in figures 1–4 for different values of $\dot{m}$).

The sonic point of the accretion flow usually locates at a radius close to the marginally stable orbit, since the matter will plunge to the hole rapidly inside $r_{\text{ms}}$. The radius of the sonic point is in principle available by integrating a set of the equations with suitable outer and inner boundary conditions. In the present work, we only considered the inner boundary condition at the sonic point, and only the range of the radius of the sonic point is given. From figures 5 and 6, we found that $\delta\varepsilon \leq 0.07$ should be satisfied for an optically-thick flow at the sonic point around a Kerr black hole ($\alpha = 0.95$), if $r_s \geq r_{\text{ms}}$ is assumed (see figure 5). For a Schwarzschild black hole, $\delta\varepsilon \leq 0.02$ should be satisfied (see figure 6). In our calculations, we found that the

---

**Fig. 5.** Maximal radius of the sonic point $r_{s,max}$ as a function of $\delta\varepsilon$ for $\alpha = 0.95$. Different values of the parameters were adopted: $\dot{M}/\dot{M}_{\text{Edd}} = 0.001, \alpha = 1$ (solid line); $\dot{M}/\dot{M}_{\text{Edd}} = 0.001, \alpha = 0.1$ (dashed); $\dot{M}/\dot{M}_{\text{Edd}} = 0.1, \alpha = 1$ (dotted); and $\dot{M}/\dot{M}_{\text{Edd}} = 0.1, \alpha = 0.1$ (dot-dashed).

**Fig. 6.** Same as figure 5, but for a Schwarzschild black hole ($\alpha = 0$).
results are insensitive to the black-hole mass.

Recently, the XMM-Newton observation of the Seyfert 1 galaxy MCG−6-30-15 reveals an extremely broad and red-shifted Fe Kα line (Wilms et al. 2001). An explanation for the observed spectrum requires the disk to have a very steep emissivity profile with an index of around 4.3–5.0. A possible explanation is provided by the magnetic connection between the inner region of the disk and the plunging matter inside the inner edge of the disk (Wilms et al. 2001; Krolik 1999; AK00). Another possible explanation is based on the magnetic connection between a rotating black hole and a disk (Wilms et al. 2001; Li 2002a,b). However, it is also suggested that the origin of the broad Fe Kα line can be explained within the frame of an illuminated relativistic accretion disk (Martocchia et al. 2002). AK00 suggested that the locally generated surface energy flux scales as $r^{-7/2}$ at large $r$ in the limit of infinite efficiency or zero accretion rate in their model, rather than that scaling as $r^{-3}$ in the standard thin-disk model. This is helpful to explain the required steep emissivity profile. In this work, we investigated the local structure and radiation properties of the disk at the sonic point. The global solution to this problem could be tested against the observed broad fluorescent Fe Kα line in MCG−6-30-15, which will be given in our future work.

In this work, we found that the radiation properties of the disk at the sonic point are sensitive to the torque exerted on the inner edge of the disk. This torque is probably variable (e.g., Armitage et al. 2001; Hawley 2001; Hawley, Krolik 2001, 2002; Reynolds, Armitage 2001). Our results imply that a variable torque may trigger transitions of the flow between optically-thick and optically-thin accretion types. Global solutions are necessary to attack this problem, which is beyond the scope of the present work, and it will be reported in our future work.

In the present calculations, we assumed that the torque is exerted on the inner edge of the disk, as done by AK00. If the torque is exerted on the extended region outside the inner edge of the disk, the disk can be optically thick at the sonic point, even for a relatively higher $\delta\epsilon$ than that reported here.

XC is grateful to J. F. Lu for helpful discussions. We thank an anonymous referee for his/her helpful comments. This work was supported by the NSFC (No. 10173016) and NKBRSF (No. G1999075403).

References


Downloaded from https://academic.oup.com/pasj/article-abstract/55/1/149/1493245 by guest on 29 December 2018