Size and Spatial Distributions of Sub-km Main-Belt Asteroids *

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Abstract

This paper presents the result of the first systematic investigation of very small Main-Belt Asteroids (sub-km MBAs) using the Subaru Prime-Focus Camera (Suprime-Cam) with an 8K × 10K mosaic CCD array attached to the 8.2 m Subaru Telescope atop Mauna Kea, Hawaii. We call this survey SMBAS (Sub-km Main-Belt Asteroid Survey). Observations were carried out on 2001 February 22 and 25 (HST) and a ∼ 3.0 deg² sky area near the opposition and near the ecliptic was searched. We detected 1111 moving objects down to R ~ 26 mag (including very slow Trans-Neptunian Objects). In this survey, we could not determine the exact orbits of the moving objects, because of their short observational arc of only 2 hours. Instead, we statistically estimated the semi-major axis (a) and inclination (I) of each moving object from its apparent sky-motion vector, and then obtained the size and spatial distributions of sub-km MBAs. The main results of SMBAS are: (1) The sky number density of MBAs is found to be ~ 290 deg⁻² down to R ~ 24.4 mag (for MBAs) near the opposition and near the ecliptic. (2) The slope of the cumulative size distribution for sub-km MBAs ranging from 0.5 km to 1 km in diameter is fairly shallower (∼ 1.2) than that for large MBAs of more than ∼ 5 km in diameter (∼ 1.8), which was obtained from past asteroid surveys. This means that the number of sub-km MBAs is much more depleted than a result extrapolated from the size distributions for large asteroids. (3) The depletion of sub-km MBAs is clearer in the outer main-belt than in the inner main-belt. (4) It seems that SMBAS asteroids distribute more widely in the I-direction in the outer zone (a = 2.8–3.1 AU) of the main-belt than known large asteroids do. We also discuss the possible causes for the characteristics of the distributions of SMBAS-observed small asteroids.

Key words: asteroids — main-belt — minor planet — size distribution — solar system: general — spatial distribution — surveys

1. Introduction

The current size, spatial, and compositional distributions of Main-Belt Asteroids (MBAs) have been believed to reflect a long-term history of collisional evolution (e.g., Wetherill 1989). Good knowledge of the Cumulative Size Distribution (hereafter CSD) for the main-belt asteroids allows us to gain insight into the collisions of MBAs, the production rate of Near-Earth Asteroids (NEAs) and meteorites, the cratering rate on the surfaces of the inner planets, the impact strengths of asteroids, and so on. It may also provide information about the accretion process in the main-belt region during the initial stage of our solar system. Then, the original mass of the main-belt may be determined (e.g. Kuiper et al. 1958; Anders 1965; Jedicke, Metcalfe 1998).

From such motivations, some systematic survey observations, as summarized in table 1, have so far been made and the CSDs of MBAs have been revealed down to a few km in diameter (D). However, we emphasize here the importance of sub-km MBAs, whose sizes are D < 1 km down to a few hundred meters, from the following two viewpoints: 1) the majority (about 70–80%) of NEAs are sub-km-sized, which are widely supposed to originate from sub-km MBAs, and 2) this size region lies near the border-line separating two typical catastrophic impact mechanisms, namely those in the strength regime and the gravity regime (e.g., Melosh, Ryan 1997; Durda et al. 1998). Concerning the first point, it is generally accepted that NEAs originated from MBAs through collision processes between asteroids in the main-belt and the subsequent gravitational perturbations associated with

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1 See (http://cfa-www.harvard.edu/iau/mpc.html).
the Kirkwood gaps (e.g., Wisdom 1983; Morbidelli, Moons, 1995). However, this dynamical conjecture has never been confirmed observationally, because it is difficult to observe the faint small MBAs corresponding to the NEA’s size. Hence, in this respect, our SMBAS may shed direct light on the physical relations between NEAs and sub-km MBAs. Also, if the second point above is correct, there may be a difference between the CSD slope for sub-km MBAs and that for known large MBAs. Thus, it may possibly be interpreted as a difference in the collisional nature between the strength regime and the gravity regime. For those reasons, we consider that the observational study of sub-km MBAs is very crucial to study the collisional history of the main-belt, and hence conducted the observations described below.

In this paper, section 2 deals with SMBAS observations and data reduction, and section 3 explains our detection technique of moving objects. We describe in section 4 positional and photometric measurements of asteroids, including a determination of the detectable limiting magnitude. In sections 5 and 6, the method of statistical estimations of the semi-major axis and inclination for each asteroid and an observational bias correction method are treated, respectively. Section 7 mentions the main results derived from the observations of this survey, namely the size and the spatial distributions for sub-km MBAs. Finally, in sections 8 and 9, we discuss the physical implications for our obtained results and future prospects.

2. Observations and Data Reduction

2.1. Observations

Observations were performed on 2001 February 22 and 25 (HST) using the 8.2 m Subaru Telescope atop Mauna Kea, Hawaii. We used the 8K × 10K wide-field mosaic camera, abbreviated as Suprime-Cam (Subaru Prime-Focus Camera) (Komiyama et al. 2000), attached at the prime focus of the telescope. Suprime-Cam covers a field of view of 34′ × 27′ at the prime focus (f/2.0) and consists of ten CCD chips (2048 × 4096 pixels for each chip; the pixel scale is 0″2). However, since one CCD chip did not work in our observations, we actually used nine CCD chips. The resulting field of view was 0.23 deg² with nine CCDs. The searched sky included the ecliptic area near opposition at R.A. = 10h22m, DEC. = +10°20′, which was within an ecliptic latitude of ±1°. The seven sky fields were carefully selected so that they would be relatively star-free, and did not include bright stars. The R-band filter used in these observations was most efficient in terms of both the CCD quantum efficiency and the solar spectral distribution. Each exposure time was 7 min. The seeing size was 0″8–1″0 on February 22 and 0″6–0″9 on February 25. The same fields were taken on two nights. The total surveyed area during the two nights amounted to ∼2.97 deg².

Two observational modes were adopted: 1) Wide Field (WF) survey mode and 2) Deep Field (DF) survey mode. In the WF survey, we took three images of the same field with a time interval of about 55 min. In the DF survey, we took eleven images of the same field, which were taken every 11 min in succession. In both modes, the observational arc for each moving object was about two hours. We also observed six Landolt standard stars at different airmasses for photometric calibrations (Landolt 1992).

The observational data described here are actually the same as those for the Wide-Field Survey of Edgeworth–Kuiper Belt Objects (EKBOs) that have been reported by Kinoshita et al. (2002). Thus, the observational modes and exposure times were optimized for the detection of EKBOs. However, since the purpose of the observations and the data-analysis method were quite different between this work and Kinoshita et al., we distinguish our survey from that by Kinoshita et al. for EKBOs by calling this survey the Sub-km Main-belt Asteroids Survey (SMBAS).

2.2. Data Reduction

Image reduction was carried out on a chip-by-chip basis using the standard method with NOAO IRAF. First, the averaged output value for the overscan region of each CCD was subtracted from each CCD image data. Second, the overscan region was trimmed, and then an image consisting of only the effective area was made. Third, in order to correct the two-dimensional bias pattern of each CCD, the bias image was subtracted from each CCD image. The bias image was produced by averaging a few raw bias frames which were taken every night. Next, we made corrections for the difference in the pixel-sensitivity over a CCD-chip, namely a traditional
flat-field calibration. For that purpose, we took several images of the twilight sky with the field-centers offset slightly from each other. Then, a median flat-field image was constructed from them, by which each CCD image was divided to obtain uniform sensitivity.

3. Detection of Moving Objects

There are two approaches to detect moving objects in observed images, that is, by visual inspection and that by computer software. We adopted here the former approach, whereas most large-scale survey programs conducted in the last decade for NEAs and EKBOs relied on the latter one (e.g., SDSS, LINEAR). However, both approaches have their own merits and demerits. Software detection is believed to be objective, free from careless mistakes made by the human interface and appropriate for handling a large amount of data. However, a properly designed procedure of visual detection can also be as objective as the software approach.

On the other hand, there seems to be a tendency that the use of only software detection gives a limiting magnitude of roughly at least 1.0–1.5 mag shallower than that for visual detection; this is reasonable because at critical signal levels and/or in blended images, even sophisticated detection algorithms can never surpass the overall judging ability of the human eye and brain. As a result, software detection generally needs some help of a more or less visual confirmation. We therefore believe, as shown later, that our technique of visual detection gives reliable results compatible with the software approach, especially for medium-sized data of less than a few thousand objects. Regarding this, it is worthwhile to cite a recent survey observation of EKBOs by Millis et al. (2002), who eventually adopted a visual detection method after comparing with software detection. They report that their technique works well even with only two-exposure pseudo-colored images.

After the basic reduction mentioned in section 2 was applied to all object frames, we made combined images to easily recognize and count moving objects. Concretely, for all object frames in the WF survey which consisted of three exposures, we subtracted the second image from the first one, and added the resulting image to the third image. An example of such new images made by the above operation is shown in figure 1a. One can see moving objects as trains of black-and-white dots. Figure 1b shows an image to which the above operations were similarly applied using a series of 11 images taken in the DF survey. One can recognize moving objects as a sequence of black-and-white striped bars. In composite images (see figure 1a), stars are generally seen as groupings of black-and-white dots slightly shifted each other; this was caused by a telescopic pointing error during the 2-hour exposure interval. This also helps us distinguish stars from moving objects.

This technique is basically the same as that proposed in Yoshida et al. (2001). We then counted the number of moving objects by a careful eye-inspection. When multiple images were combined, the absolute value of the mean sky-level for positive and negative images was equated to within a certain error level. This operation inevitably increased the dispersion of the sky level by some amount compared to that for single-exposure images (as for quantitative aspect of sky-level fluctuation, see subsection 4.2). However, the easy detectability of asteroids shown as trains of black-and-white dots was much more advantageous than some degradation of the S/N ratio in composite images.

This black-and-white image technique also enables us to easily identify moving objects as a time sequence. For example, in the WF survey, we can surely confirm that the white dot corresponds to the image taken at the second exposure. It may also help us judge whether one elongated object is either a moving object or a galaxy, because all galaxies are always seen as black images, and we can confidently distinguish all white images as moving objects. Our technique is also useful to confirm that a moving object is the same one when it appears on the neighboring CCD chips as a result of its motion.

In an actual examination of moving objects, we divided all of the processed images into small partially overlapped sub-frames whose size was about $100'' \times 100''$ (about 500 $\times$ 500 pixels); its size could sufficiently cover the range of motion of the asteroids in the inner main-belt during the two-hour exposure time. We then magnified all sub-frames on a PC screen and checked moving objects by careful eye-inspections, twice separated by a few days. As a result, we detected 1194 moving objects. Then, after removing the same moving objects that strode over neighboring CCDs, we eventually recognized 1111 moving objects.

4. Photometry and Measurement of Positions

4.1. Photometry

We carried out aperture photometric measurements of detected moving objects using the `apphoto` task of IRAF. We then applied the following two corrections to the measured brightness of each moving object. First, we made a correction of the difference in the sensitivity between the CCD chips. The relative response for each CCD to the incoming radiation on Suprime-Cam was calibrated by comparing the mean count of the sky-background brightness between the CCDs. Second, a correction of the atmospheric extinction arising from the variation of the airmass was made using the extinction coefficients obtained from several Landolt photometric standard stars observed at some different airmasses on each night.

From measurements of the brightness of moving objects at each exposure time, we found that the mean amplitude of the intrinsic light variations of the moving objects (caused by their rotation) was $\sim 0.25$ mag. This value is about ten times larger than the mean measuring photometric error ($\sim 0.03$ mag) of each object. Therefore, the absolute magnitudes of all detected moving objects may include an error of $\sim 0.25$ mag. However, we emphasize that an error of this kind can accordingly be averaged out when we construct size or spatial distributions from many objects.

4.2. Determination of the Limiting Magnitude

According to Kinoshita et al. (2002), the $R$-band limiting magnitude of point sources for our observing run was 26.1 mag. However, it is necessary for us to independently examine the limiting magnitude of moving objects by our own method, because our moving objects were much more trailed
Fig. 1. Moving objects detected in each one of the CCD images in the WF and DF surveys. (a: left) Only some trains of black-and-white dots identified as asteroids are indicated by shaded lines for clarity. Actually fifteen moving objects were detected in this 2K × 4K chip. Black-white-black dots appear fairly separated because of long exposure intervals (≈55 min). Field stars and galaxies appear as slightly shifted groupings of black-white-black dots, due to the telescopic guiding error during the three exposures. (b: right) Twenty-three detected moving objects were included altogether in this image. They appear as black-and-white straight bars because of short exposure intervals (≈11 min). Field stars and galaxies appear as black images. Up is north and left is east in these images. All moving objects moved from left to right (due to retrograde motions near the opposition).

than EKBOs and we used a special technique, that is, the black-and-white image composite method. Since we combined multiple images to detect moving objects, the mean sky fluctuation was increased to 1.8–2.0σ (σ: standard deviation of the sky brightness variation for a single exposure image) for 3-exposure composites and to 2.9–3.3σ for 11-exposure ones. Considering that the variation of the sky brightness follows photon statistics (namely, Poisson or Gaussian statistics), the detection probability of a star with the peak intensity of 1σ was calculated to be 68.3%, and 95.5% for the 2σ-peak, 99.7% for the 3σ-peak, respectively (Meyer 1975) for single-exposure images. Hence, this can be interpreted as meaning that the objects barely seen in 3-exposure composite images with a sky of 1.8–2.0σ have 92.8–95.5% detection probability, and 99.6–99.9% probability for those in 11-exposure composites with 2.9–3.3σ. In other words, we may safely say that all of the detected asteroids in our composite images can be found in single exposure images with probabilities higher than 90%.
This is a quantitative basis for detecting moving objects in SMBAS.

The next step is to determine the limiting magnitude for trailed asteroids in the black-and-white composite images that we made. For this purpose, we conducted simulation experiments using the \texttt{mkobjects} task of IRAF. We first produced a stellar image with the FWHM of an average seeing-size for the observed night; we then made its slightly shifted image, added the two, and continued the process to give a train of superimposed stellar images; this was done to mimic the trailed images of asteroids. We assumed a mean apparent motion for mid-belt asteroids (14′ d$^{-1}$) with a trail length corresponding to a 7-min exposure time. Then, series of trails of different brightness with a 0.2-mag step (covering about 4 mag span) and with a fixed separation were output randomly in location on the black-and-white composite images (see figure 2). The left-most trail in each series was the brightest one. By a careful eye-inspection, we measured the magnitude of the discernible faintest trail with 0.1 mag accuracy, relative to the brightest one.

In practice, because the overlapping of some parts of the trails with background stars and galaxies often occurred, we had to attempt many series of trails. Among them, we picked up 87 cases in which the brightest and faintest discernible trails could be safely measured, and plotted a percentage detection frequency of the faintest trails as a function of the magnitude (in figure 3). The origin of the magnitude in the abscissa is arbitrary. By measuring the magnitude differences between the brightest trail and some nearby photometric standard stars using asteroid trails as a mediator, we connected the abscissa to the standard magnitude system. One can see that the detection probability in figure 3 changes from 100% to 0% over a magnitude range of 0.8–0.9. This is in good agreement with the trend in figure 4 of Millis et al. (2002).

Here, we adopted a 90%-perfect detection level in figure 3 as the limiting magnitude in SMBAS. This magnitude corresponds to 24.4 mag in the $R$-band. Note that the limiting magnitude is for mid-belt MBAs with their typical motions, not for stars (namely point sources).

4.3. Positional Measurements of Moving Objects

The position for each moving object was measured again with IRAF-\texttt{apphot} relative to about ten USNO-A2 stars\textsuperscript{2} that we picked up on the same frame. The apparent velocity for each object was calculated from its position using all of the exposure images. Figure 4 shows the apparent daily motions along the ecliptic longitude and the ecliptic latitude for moving objects detected in SMBAS. From figure 4, we can easily distinguish between the MBAs and other groups of moving objects by their motions. We discuss only MBAs in the next section, because our interest focuses on small asteroids in the main-belt in this paper.

\textsuperscript{2} See \url{http://tdc-www.harvard.edu/software/catalogs/ua2.html}.
5. Estimates of the Semi-Major Axis and Inclination for Asteroids

Since the observational arc for each asteroid detected in SMBAS is only two hours, we cannot determine its exact orbital elements. Thus, instead we adopted a method to derive the approximate semi-major axis \( a \) and inclination \( I \) from the sky-motion vector of each asteroid under the assumption that its orbital eccentricity \( e \) is zero. This method is based on geometrical and kinematical relations in the two-body problem, which was initially proposed by Bowell et al. (1990). We call it Bowell’s method in this paper. Since, however, the \( e \)-values of the typical MBAs actually lie in the range from 0 to \( \sim 0.2 \), we had to estimate in a statistical sense by Monte Carlo simulations the possible errors between the \( a \) and \( I \) obtained by Bowell’s method and the true orbital elements. We hereafter denote the former elements as \( a' \) and \( I' \) and the latter as \( a \) and \( I \). The following is an outline of the results from Nakamura and Yoshida (2002).

First, we generated many hypothetical asteroids with various orbital elements in a computer and picked up a few thousand asteroids that entered the observational window in SMBAS. We selected the ranges of the orbital elements for the generated asteroids to be slightly wider than the ranges of known MBAs. The observational window area was set to be nearly the same as the actual observational window in SMBAS. The orbital-element ranges of the generated asteroids and the observational windows are listed in table 2. Next, we calculated their daily motions using a two-body ephemeris generator. We then compared the \( a \) and \( I \) for each ephemeris with the \( a' \) and \( I' \) calculated from its motion vector.

Figure 5a is a reproduction from Nakamura and Yoshida (2002), which shows the \( a \) vs. \( a' \) plot calculated by the above simulation. It seems that there is little systematic difference between the \( a \) and \( a' \), though the scattering attains to \( \sim 0.1 \) AU. Figure 5b shows the \( I \) vs. \( I' \) relation calculated by the same simulation. The difference between the \( I \) and \( I' \) is seen to be considerably large, especially for \( I > 10^\circ \). We summarize those quantitative results in tables 3 and 4.

After we calculated the \( a' \) and \( I' \) of asteroids detected in SMBAS with Bowell’s method, we corrected the systematic errors in their \( a' \) and \( I' \) using the mean values of \( (a-a') \) and \( (I-I') \) given in tables 3 and 4. In order to give random components of errors in the estimated \( a' \) and \( I' \), we also calculated the standard deviations, namely SD\( (a-a') \) and SD\( (I-I') \), which are given in the fourth column of tables 3 and 4. From table 3, we can see that random errors, namely SD\( (a-a') \), exceed the systematic errors, namely the mean\( (a-a') \), for each zone, so that a correction of the systematic error may bring about slight improvement to the \( a' \)-estimate. On the other hand, table 4 shows that the mean\( (I-I') \) for the high-inclination MBA zone is larger than SD\( (I-I') \), and hence a correction of the systematic error is essential, though random errors in \( I \) are fairly large for three zones. We comment that these errors are for individual asteroids, and that their effects are much more averaged out in the overall size and spatial distributions of the small MBAs obtained in SMBAS.

### Table 2. Orbital-element ranges of simulation-generated asteroids and the assumed observational window.

<table>
<thead>
<tr>
<th>Element</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a ) (AU)</td>
<td>2.75 ± 0.75</td>
</tr>
<tr>
<td>( I ) (deg)</td>
<td>15 ± 15</td>
</tr>
<tr>
<td>( e )</td>
<td>0.2 ± 0.2</td>
</tr>
<tr>
<td>Other angular elements</td>
<td>uniform over ( 0^\circ )–( 360^\circ )</td>
</tr>
<tr>
<td>Observational window</td>
<td>( 2^\circ \times 2^\circ )</td>
</tr>
</tbody>
</table>
Fig. 6. (a: left) Relative bias as a function of $a$. The relative bias is the number ratio between near-ecliptic asteroids in the distant orbits ($r \sim 6$ AU) and those at $r = a$ (AU). The relative bias curves were calculated for circular, near-circular, and elliptic orbits. (b: right) Relative bias as a function of $I$. The relative bias is the number ratio between ecliptic asteroids ($I \sim 0$) and those with $I$. Three relative bias curves were calculated for the inner, middle, and outer MBAs. These figures were taken from Nakamura and Yoshida (2002).

Table 3. Errors of the semi-major axis obtained from Bowell’s orbit.

<table>
<thead>
<tr>
<th>Zone</th>
<th>Range (AU)</th>
<th>Mean($a - a'$)</th>
<th>SD(*($a - a'$))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner-belt</td>
<td>$2.0 &lt; a &lt; 2.6$</td>
<td>0.075</td>
<td>0.14</td>
</tr>
<tr>
<td>Middle-belt</td>
<td>$2.6 &lt; a &lt; 3.0$</td>
<td>0.070</td>
<td>0.13</td>
</tr>
<tr>
<td>Outer-belt</td>
<td>$3.0 &lt; a &lt; 3.5$</td>
<td>0.083</td>
<td>0.15</td>
</tr>
</tbody>
</table>

* SD: the standard deviation.

Table 4. Errors of the inclination obtained from Bowell’s orbit.

<table>
<thead>
<tr>
<th>Zone</th>
<th>Range (deg)</th>
<th>Mean($I - I'$)</th>
<th>SD(*($I - I'$))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low incl.</td>
<td>$0 &lt; I &lt; 10$</td>
<td>0.27</td>
<td>1.5</td>
</tr>
<tr>
<td>Medium incl.</td>
<td>$10 &lt; I &lt; 20$</td>
<td>2.35</td>
<td>4.4</td>
</tr>
<tr>
<td>High incl.</td>
<td>$20 &lt; I &lt; 30$</td>
<td>6.37</td>
<td>5.8</td>
</tr>
</tbody>
</table>

* SD: the standard deviation.

6. Observational Bias Corrections

SMBAS was conducted in a very small sky area only near opposition and near the ecliptic. For such observational conditions, we must consider some specific observational biases. Nakamura and Yoshida (2002) have already estimated observational biases for a small area near the opposition and near the ecliptic (see figures 6a,b). They calculated the observational biases as functions of $a$ and $I$ for an assumed observational field of view ($5^\circ \times 4^\circ$, slightly wider than in actual SMBAS), centered at the opposition and on the ecliptic. Figure 6a shows the relative bias as a function of $a$. The relative bias is defined here as the number ratio between the near-ecliptic distant asteroids with, say, $r \sim 6$ AU ($r$: heliocentric distance) and those with $r = a$ AU. Three relative bias curves were calculated for circular, near-circular, and elliptic orbits. Figure 6b shows the relative bias as a function of $I$. The relative bias is defined as the number ratio between ecliptic asteroids ($I \sim 0$) and those with $I$. The relative bias curves were calculated for the inner ($a = 2.3$ AU), middle ($a = 2.7$ AU), and outer MBAs ($a = 3.1$ AU), respectively.

The $a$-distribution and $I$-distribution for MBAs detected in SMBAS, which we describe in section 7, were corrected by using the bias corrections shown in figures 6a and b. Since we are mainly interested in profiles of the size and spatial distributions, those relative bias corrections are sufficient for our purpose and absolute bias corrections are unnecessary.

7. Results

7.1. Overview of Results

In the present work, we defined the main-belt zone as $a = 2–3.5$ AU in accordance with past surveys. After applying the corrections for the $a$ and $I$ described in section 5, we regarded all moving objects whose $a'$ values fell between 2 and 3.5 AU as being MBAs. We found 861 MBAs in SMBAS. From this, we estimated that the sky number density of MBAs down to $R = 24.4$ mag is $\sim 290$ deg$^{-2}$ near the opposition and near the ecliptic.

Next, we calculated the absolute magnitude ($H_R$) of each MBA (e.g., see Ephemerides of Minor Planets for 2001) by

$$H_R = V - 5\log(\Delta \cdot r) - P(\alpha) - \delta V,$$

where $V$ is the apparent magnitude in the R-band of the asteroid in question, $\Delta$ and $r$ stand for the geocentric and heliocentric distances (in AU), respectively, $P(\alpha)$ is the phase function ($\alpha$: phase angle, namely Earth–asteroid–Sun angle),
and $\delta V$ is the light variation due to the asteroid’s rotation. Since SMBAS observation was done near the opposition, the value of $P(\alpha \sim 0^\circ)$ is negligibly small. Though $r = a(1 - e^2)/(1 + e\cos (f + \omega))$ near the ecliptic, where $\omega$ is the argument of the perihelion and $f$ is the true anomaly, we can regard $r \sim a$ for each asteroid, because of our assumption that $e = 0$. Although most asteroids show $\delta V$ of less than 0.5 mag for individual objects, they are random in phase, and therefore their effect will be averaged out when we construct the size distributions from their observations (Nakamura, Yoshida 2002). We thus give $\delta V = 0$ here.

If the albedo ($A$) of an asteroid is known or assumed, its diameter ($D$) can be obtained approximately from its $HR$ (except for color effects) by equation (2) (Bowell et al. 1989), which is a modified version of a formula by Bowell and Lumme (1979),

$$\log D = 3.130 - 0.5 \log A - 0.2 HR$$

Here, we used an empirical formula, $\log D = 3.65 - 0.2 H$, corresponding to the averaged albedo for C- and S-type asteroids, because we could not measure the albedos of asteroids in SMBAS.

Figure 7 shows a comparison between the $HR$-distribution of MBAs detected in SMBAS and the $H$-distribution of 85150 known MBAs as of 2000 September. Since the ($V - R$) color for asteroids with typical taxonomic types is known to fall in the range between $-0.05$ and $+0.25$, we need not distinguish the $HR$- and $H$-magnitudes within the accuracy of our diameter estimate; the comparison given in figure 7 is therefore justified. An approximate $D$-value corresponding to $HR$ is also indicated by the arrows in figure 7. From figure 7, we can see that the peak of the $H$-distribution for known MBAs is $\sim 15$ mag ($D \sim 4.5$ km), whereas the peak for MBAs obtained with SMBAS is $\sim 20$ mag ($D \sim 450$ m). Hence, one can understand that our SMBAS could observe substantially sub-km MBAs, whose size region is one order of magnitude smaller than that of known asteroids; this is really an unknown world. The minimum and maximum sizes of MBAs in SMBAS range from $\sim 0.1$ to $10$ km.

### 7.2. Distributions of $a$ and $I$

Figure 8a shows the $a$-distribution of 861 MBAs detected in SMBAS. The black, white, and gray boxes, respectively, indicate the raw $a$-distribution of 861 MBAs, the $a$-distribution of MBAs corrected by the relative bias calculated for elliptic orbits, and that of MBAs corrected by the relative bias calculated for circular orbits given in figure 6a. In section 5, we stated that the $a$ of each asteroid in our estimation has a mean error of 0.13–0.15 AU. However, the $a$’s error in the histogram of figure 8a should generally be smaller than 0.1 AU, because the error in each $a$-bin is statistically improved by the amount of $\Delta a/\sqrt{n}$ ($\Delta a$: the $a$-error for a single asteroid; $n$: the data number in that $a$-bin), unless the data number is smaller than several. Hence, we took 0.1 AU as the bin-step in figure 8a.

Figure 8b shows the $a$-distribution of 85150 known MBAs discussed in subsection 7.1 and figure 7. In order to fairly compare the result of known MBAs with that of the MBAs from SMBAS, we drew figure 8b by intentionally degrading the resolution for the $a$-distribution down to 0.1 AU. In the $a$-distribution of known MBAs, it is well known that the diminution of asteroids near 2.1, 2.5, and 2.9 AU is reflected by the existence of the Kirkwood resonant gaps. One can see in figure 8a the vague depression near $a = 2.4$ AU for SMBAS asteroids, which corresponds to the depression seen at $a = 2.5$ AU in the $a$-distribution of the known MBAs (figure 8b). This might imply that, even for sub-km MBAs, the 3:1 Kirkwood gap ($a = 2.5$ AU) still gives detectable dynamical effects. On the other hand, the depression of the known MBAs at $a \sim 2.9$ AU (figure 8b) seemingly corresponds to an enhancement at the same $a$-value for asteroids in figure 8a, and a shallow depression exists at $a \sim 2.8$ AU. This situation, if real, will be clearer along with the accumulation of small asteroid data.

Figure 9a shows the $I$-distribution of MBAs detected in SMBAS. The black and white boxes represent the raw $I$-distribution of 861 MBAs and that of MBAs corrected by the relative bias calculated for middle-belt asteroids in figure 6b, respectively. According to the statistical consideration already mentioned above, a mean random-error of $1\!.5$–$5\!.8$ in the $I$ for individual asteroids in our estimation (see section 5) should also be improved by the amount of $\Delta I/\sqrt{n}$ ($\Delta I$: the $I$-error for a single asteroid), unless the data number is too small. Therefore, in this case, a value of two degrees as the $I$-error in the histogram is much more realistic than the $0.1–0.15$ AU stated for elliptic orbits given in figure 6a. In section 5, we stated that the $I$-distribution of 85150 known MBAs, the $I$-distribution down to 0.1 AU. In the $I$-distribution of known MBAs, it is well known that the diminution of asteroids near 2.1, 2.5, and 2.9 AU is reflected by the existence of the Kirkwood resonant gaps. One can see in figure 8a the vague depression near $a = 2.4$ AU for SMBAS asteroids, which corresponds to the depression seen at $a = 2.5$ AU in the $a$-distribution of the known MBAs (figure 8b). This might imply that, even for sub-km MBAs, the 3:1 Kirkwood gap ($a = 2.5$ AU) still gives detectable dynamical effects. On the other hand, the depression of the known MBAs at $a \sim 2.9$ AU (figure 8b) seemingly corresponds to an enhancement at the same $a$-value for asteroids in figure 8a, and a shallow depression exists at $a \sim 2.8$ AU. This situation, if real, will be clearer along with the accumulation of small asteroid data.

Figure 9b shows the $I$-distribution of the 85150 known MBAs. The black, white, and gray boxes, respectively, indicate the relative-bias-corrected $I$-distribution of the 85150 known MBAs, the $I$-distribution of the relative-bias-corrected middle-belt asteroids, and that of known MBAs at $a \sim 2.9$ AU (figure 8b) seemingly corresponds to an enhancement at the same $a$-value for asteroids in figure 8a, and a shallow depression exists at $a \sim 2.8$ AU. This situation, if real, will be clearer along with the accumulation of small asteroid data.

First, the $I$-distribution of the relative-bias-corrected SMBAS asteroids is seen to be comparatively uniform over the $I$-range, as shown in figure 9a, though that of known MBAs steeply decreases along with an increase of $I$ (figure 9b). Second, the lack of asteroids near $I \sim 12^\circ$ is much more conspicuous for small MBAs compared with that for the known MBAs (figure 9b). It is not clear now whether the difference between figures 9a and b is real or an artifact caused by the statistics of a small sample number; it will be a target for future exploration.
7.3. Size Distribution for the Whole Main-Belt

Next we discuss the Cumulative Size Distribution (hereafter CSD) of sub-km MBAs detected in SMBAS. It is well known that a cumulative number distribution for MBAs brighter than a magnitude $H_R$ is approximately expressed as

$$\log N(<H_R) = C + \alpha H_R,$$

where $\alpha$ and $C$ are constants. The value of $\alpha$ is often referred to as the slope for the log $N$ vs. $H_R$ plot. If we rewrite equation (3) with the help of equation (2), the result is equal to

$$N(>D) \propto D^{-b}.$$  \hspace{1cm} (4)

The power-law index $(b)$ in equation (4), which corresponds to the slope for the log $N$ vs. log $D$ plot, is connected to $\alpha$ by $b = 5\alpha$. In this paper, we use $b$ to express the slope of the CSD of asteroids.

Figure 10 shows the differential (white-box histogram) and cumulative (filled dots) $H_R$-magnitude distributions for 861 MBAs detected in SMBAS. The $D$ corresponding to a $H_R$ is also shown on the upper horizontal axis. The $H_R$ of each asteroid was estimated from its $a$ with the help of equation (1) (see subsection 7.1). Therefore, the $H_R$ of each asteroid includes the $a$-error caused by the assumption that $e = 0$. This $H_R$-error amounts to 0.25–0.38 mag. Again statistically, however, the $H_R$-error in a $H_R$-bin is reduced to $\Delta H_R/\sqrt{n}$ ($\Delta H_R$: the mean error for individual asteroids). For clarity, the error bars are shown only for the cumulative $H_R$-magnitude in figure 10.

The solid line was drawn for a comparison to show the slope ($b \sim 1.75$) for the CSD from PLS and Spacewatch surveys. It seems that the slope of the CSD for asteroids brighter than $H_R \sim 17$ is steeper than $\sim 1.75$, and that of asteroids fainter than $H_R \sim 18$ is much more gentle. Recent
Spacewatch (Jedicke, Mec-Falce 1998) and SDSS (Ivezić et al. 2001) projects ascertained that the slope of the CSD for small MBAs is shallower than previous estimates and cannot be represented by a simple power-law. We can easily confirm their conclusions in figure 10. For a quantitative comparison with the past results, we attempted to obtain the \( b \) for our data with a least-squares method.

By assuming a mean albedo for known C- and S-type MBAs for equation (2), the best-fit value of the \( b \) for the asteroids with \( 0.5 \text{ km} < D < 1 \text{ km} \) (corresponding to \( 18.3 < H_R < 19.8 \)) was found to be \( 1.18 \pm 0.03 \). Remember, however, that we derived this value for the 90%-complete data at the limiting magnitude. We therefore re-calculated the cumulative number corresponding to the 100%-completeness based on figure 3, by multiplying a factor of 100/90, and found a slope of \( 1.19 \pm 0.02 \) for the 100%-detection through a least-squares fitting. All of the slopes described below represent those for 100%-detection. Hence, even for 100%-detection statistics, we conclude that our \( b \)-value (1.19) is much shallower than those for PLS and Spacewatch surveys, and even shallower than that for SDSS (Ivezić et al. 2001), implying that the past number estimates of km- to sub-km MBAs extrapolated based on the number of large MBAs (\( D > 5 \text{ km} \)) was definitely overestimated.

### 7.4. Size Distributions for Three Zones of the Main-Belt

In this subsection, we consider how CSDs change depending upon their locations in the main-belt. For the purpose, we partitioned the main-belt into the inner, middle, and outer zones, defined by \( 2.0 < a < 2.6, 2.6 < a < 3.0, \) and \( 3.0 < a < 3.5 \text{ AU} \), respectively. This division is conformable to that for the previous surveys: Yerkes–McDonald survey (YMS), PLS, and Spacewatch survey. Since the limit of the detectable magnitude becomes brighter along with the increase of an asteroid’s heliocentric distance, it is important to take such a distance effect into account when we examine the CSD of MBAs for each zone of the main-belt. The limiting \( H_R \)-magnitudes for each zone were calculated here at the farthest position of each zone, namely \( 2.6, 3.0, \) and \( 3.5 \text{ AU} \), with equation (1) and the detectable limiting magnitude discussed in subsection 4.2; they are listed in table 5. Note that the values of \( H_R \) or \( D \) given in table 5 are those corresponding to 90%-complete detection for SMBAS.

The \( H_R \)-differential distribution (histograms) and the \( H_R \)-cumulative distribution for each zone (symbols with error bars) are shown in figure 11. The inner-, mid-, and outer-MBAs are respectively represented by blue, black, and red colors. The vertical dashed-lines show the \( H_R \)-limiting magnitude for each zone (see table 5). It seems that, for asteroids with \( H_R \gtrsim 15–16 \), the slope of the CSD for the outer region (red open circles) is apparently gentler than that for the inner region (blue crosses). However, when seen more in detail, we recognize that the slope of the CSD changes continuously with the \( H_R \)-range for any of three zones. Thus, we calculated the local slope values as a function of \( H_R \), and then drew them in figure 12. In figure 12, the connected crosses, triangles, and open circles, respectively, show the changing nature of the CSD slopes for the inner-, mid-, and outer-MBAs. For asteroids with \( H_R \gtrsim 17.5 \), it is obviously seen that the slopes of the CSDs become systematically gentler in the order of the inner-, middle-, and outer-belt. Hence, one can easily expect that such a trend should have some physical implications. On the other hand, for asteroids with \( H_R \lesssim 17.5 \), the slopes change largely, particularly for the middle-belt. We consider that this behavior may be an artifact caused by small sample statistics.

Now we focus on the CSD slopes of sub-km MBAs ranging from \( 0.5 \text{ km} \) to \( 1 \text{ km} \) in diameter in each zone; we are especially interested in this size region, since it has never been observed systematically before. The discussion of those MBAs is adequately meaningful, because the number of MBAs detected in SMBAS is large enough to deal with it statistically, and SMBAS detection of MBAs is nearly complete down to \( D \sim 0.5 \text{ km} \) in the whole main-belt (see table 5). To grasp the slope change within this narrow size-range quantitatively, we examined the \( b \) of the CSD for the MBAs with \( 0.5 \text{ km} < D < 1 \text{ km} \) by a least-squares fitting. The thus-calculated slopes of the CSDs for the three main-belt zones were found, respectively, to be \( 1.37 \pm 0.03, 1.15 \pm 0.03, \) and \( 0.98 \pm 0.03 \) (see table 6). From these results, one can see that even for a sub-km size region, there is a clear zone-dependence of the CSD slopes.

<table>
<thead>
<tr>
<th>Belt zone</th>
<th>Inner</th>
<th>Middle</th>
<th>Outer</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a ) (AU)</td>
<td>2.6</td>
<td>3.0</td>
<td>3.5</td>
</tr>
<tr>
<td>( H_R ) (mag)</td>
<td>21.4</td>
<td>20.6</td>
<td>19.8</td>
</tr>
<tr>
<td>( D ) (km)</td>
<td>0.23</td>
<td>0.34</td>
<td>0.49</td>
</tr>
</tbody>
</table>

Table 5. Limiting \( H_R \)-magnitudes of asteroids for three zones in the main-belt.
7.5. Spatial Distribution in the Whole Main-Belt

Figure 13a shows the spatial distribution (a vs. I) plot of MBAs detected in SMBAS. Note that each data point includes the errors for the a and I mentioned in section 5, namely an error of \( \sim 0.1 \) AU in a and an error of 2° \( (\sim 6^\circ) \) for high inclinations) in the I for each asteroid (also see tables 3 and 4). Probably due to these errors, one cannot recognize the well-known Kirkwood gaps in figure 13a.

Apparently, we notice the following four trends: (1) there seem to be several vague gaps near \( I \sim 12^\circ \) and \( \sim 23^\circ \) over the whole main-belt in figure 13a, (2) the a’s of the high-inclination asteroids seem to be located near the heliocentric distances corresponding to the mean motion resonances with Jupiter, namely 2.5, 2.8, and 3.0 AU, (3) there seems to be a trend that asteroids with larger a’s show higher I’s, and (4) a comparison of the spatial distribution of SMBAS asteroids with that of 85150 known MBAs (figure 13b) seems to indicate that, between about 2.8 and 3.1 AU, a larger fraction of MBAs for SMBAS distribute in a higher I-range than does the distribution of known MBAs.

However, we also know that, since the orbital errors of each SMBAS asteroid in figure 13a are fairly large, it is premature for us to take all of the above four trends to be certain. Thus, we only discuss in section 8 possible interpretations for the causes of the spatial distribution of the sub-km MBAs shown in figure 13a.

8. Discussion

8.1. Interpretations for Size Distributions of Sub-km MBAs

8.1.1. Overall nature

First we comment on the sky number density of small MBAs, \( \sim 290 \text{ deg}^{-2} \) \( (R < 24.4 \text{ mag}) \), which is given in subsection 7.1. Although Poisson statistics teaches us that the formal error of this sky density is about 10%, we suspect that the actual error will probably be much higher, perhaps by several ten percent. In fact, such a high variation of the number density can be seen depending on time, if we plot on the sky the positions of known asteroids of about a hundred thousand.\(^4\) Hence, we consider that the sky number density of asteroids is a less-stable quantity to characterize the size distribution of small MBAs than their slope, meaning that the latter is more important in a statistical sense.

We have already seen in section 7 that the CSD slope for sub-km MBAs with \( D \sim 0.5–1 \text{ km} \) detected in SMBAS is obviously shallower than that for multi-km asteroids estimated with YMS, PLS, and Spacewatch surveys. Yoshida et al. (2001) also found that the CSD slope of sub-km MBAs is \( \sim 1.0 \) from their preliminary survey, which was similar in principle to SMBAS, but adopted a more simplified approach than SMBAS. Ivezić et al. (2001) recently found that the slope is 1.3 for the MBAs with 0.4 km < \( D < 5 \) km in the SDSS project, though their data were weighted toward larger asteroids than ours. Considering the above three results, it

seems to be highly certain that very small asteroids are not so plentiful as had been expected from past observations of larger ones.

When seen in more detail, however, our CSD slopes of sub-km MBAs with \( D \sim 0.5–1 \text{km} \) are shallower than that obtained in SDSS. This implies that the number of very small sub-km MBAs detectable only in SMBAS is much more depleted compared with the prediction by Ivezić et al. (2001). We thus infer that there is a size-dependence in the mechanism to remove asteroids from the main-belt. In relation to this, for example, Nakamura (1994) found that smaller asteroids exist more abundantly toward the centers of some of the Kirkwood gaps, from which NEAs are believed to be supplied.

As possible causes of the depletion of small asteroids in the main-belt, researchers have so far proposed the following three physical processes:

(1) Small asteroids may become a part of large asteroids, consisting of strengthless “rubble-piles”. The likely existence of rubble-pile asteroids, which consist of re-accumulated impact fragments was theoretically predicted for the first time by Weidenschilling (1981). In fact, the asteroid (253) Mathilde observed by NEAR spacecraft (Veverka et al. 1999) has been regarded as having a rubble-pile structure, because of the observed low bulk density. Recent collisional theories and experiments suggest that the impact energy needed to disperse an asteroid is greater than that to thoroughly shatter it, for asteroids larger than a few km to sub-km in size. This means that it is more difficult to disperse collisional fragments for asteroids of such sizes. If this is the case, the number of small MBAs should be depleted, because they will more likely be incorporated into part of those large rubble-piled asteroids (Melosh, Ryan 1997).

(2) Small asteroids would have been thrown into the Kirkwood gaps by the Yarkovsky effect (namely a repulsion force due to anisotropic thermal emission), and then removed from the main-belt (e.g. Farinella, Vokrouhlický 1999). According to their calculations, the \( a \) of asteroids with \( \sim 1–10 \text{km} \) in radius can be moved by a few hundredths of AU by the Yarkovsky effect during their collisional lifetimes (\( \sim 10–1000 \text{ million years} \)). In particular, the \( a \) of small asteroids with \( \sim 10–100 \text{m} \) in radius will change more effectively. Since this size region spans a part of the sizes of asteroids observed in SMBAS, the Yarkovsky effect could be another candidate for the depletion of small MBAs.

(3) Small asteroids would have been ejected out of the main-belt with high speeds acquired in a collision beyond the escape velocity. Seemingly, this is the simplest process to remove asteroids from the asteroid belt, though the existence of such high-speed fragments has not yet established in laboratory experiments.

Presently we cannot say, from only our SMBAS, which of the above three candidate processes is more plausible. To solve the problem, we may need detailed observations for the CSD of NEAs and/or for the CSDs of craters on the surfaces of the inner planets and satellites, in addition to the CSD of small MBAs. In any case, we can say that our investigation of the CSD for sub-km MBAs (namely NEA-sized asteroids) provides an important step in estimating both the supply rate of NEAs and the formation rate of rubble-pile asteroids.

8.1.2. Belt-zone dependence

We next discuss the CSDs of sub-km MBAs investigated for three zones of the main-belt. As we have already shown in subsection 7.4, it is fairly certain that there is a difference in the slopes between the inner and other zones for asteroids with \( \sim 0.5–1 \text{km} \) in diameter. The slope in the inner zone is relatively steep (\( \sim 1.4 \)), while that in the outer zone is shallow (\( \sim 1.0 \)) (see table 6). However, we must remember here that transformation from \( H_R \)-magnitude of asteroids to the size considerably depends on their albedo. For well-observed MBAs, we know that S-type asteroids with a high albedo are abundant in the inner main-belt, while C-type ones with a low albedo are dominant in the outer region of the main-belt; namely, the number ratio of the S-type and C-type asteroids varies with the
heliocentric distances$^5$ (Gradie et al. 1989). Note that the size of a C-type asteroid is about twice larger than that of a S-type one for the same absolute magnitude, due to the difference in the albedo. On the other hand, Xu et al. (1995) showed, in the Small Main-belt Asteroid Spectroscopic Survey (SMASS), that the majority of small MBAs ($D < 20$km) are C- and S-type asteroids, and that their distributions are similar to that of large asteroids.

Therefore, we assumed that the inner-belt, the middle-belt, and the outer-belt consist of asteroids with the albedo of S-type, with the mean albedo being between S-type and C-type, and with the albedo of C-type, respectively. We then re-estimated the CSD slopes for the three main-belt zones by taking this albedo effect into account, and the results are shown in table 7. In table 7, the mean slope 1 indicates the slope of the CSD for the asteroids in the size range estimated from the mean albedo of well-known MBAs, which is the same as those given in table 6. The mean slope 2 represents the slope estimated by considering the ratio mentioned above of the S-type and C-type asteroids in the main-belt. In table 7, except for the mean slope 2 for the outer-belt asteroids, all of the slopes were calculated for asteroids with $0.5 \text{ km} < D < 1 \text{ km}$. The mean slope 2 of the outer-belt was estimated for asteroids with $0.7 \text{ km} < D < 1 \text{ km}$, because we defined the limiting magnitude of asteroid detection to be $H_\text{lim} = 19.8 \text{ mag (} D = 0.7 \text{ km)},$ assuming the albedo of C-type asteroids) in this region (refer to table 5). The above result indicates that a consideration of the S/C number ratio for MBAs clarifies the difference in the slopes of CSD between the inner and outer MBAs.

We therefore insist here that there really exists a difference in the CSD depending upon the location of sub-km MBAs; namely, it is comparatively steep in the inner-belt, and shallow in the outer-belt. Then, what is the cause of this difference? Here are some considerations. We may infer that some specific mechanisms to remove a large number of small asteroids had worked in the distant past, or rubble-pile asteroids have been produced more effectively in the outer-belt rather than in the inner-belt. For larger asteroids, in this respect, Anders (1965) had suggested that the frequency of collisions is different between the inner-belt and the outer-belt: in the inner-belt, the impact frequency is only a few times throughout its history, while in the outer-belt collisions were more severe and asteroids were highly fragmented due to proximity to Jupiter. We may also consider that the zone-dependence of the slopes is the result of a difference in the distributions between S- and C-type asteroids. It has generally been believed that C-type and S-type asteroids are, respectively, like carbonaceous chondrites and silicate rocks. This belief was strengthened by the results from recent space-probe explorations that the bulk densities of (243) Ida (S-type asteroid) and (253) Mathilda (C-type asteroid) are respectively $\sim 2.6 \text{ g cm}^{-3}$ and $\sim 1.3 \text{ g cm}^{-3}$. It is therefore likely that different outcomes would occur in collisions of bodies that have different materials and densities.

In fact, the following recently conducted investigation indicates that S-type and C-type materials are likely to behave differently in impact events, resulting in their different size distributions. Housen et al. (1999) performed laboratory experiments to explain huge craters on the asteroid Mathilde, which were discovered by the NEAR spacecraft (Veverka et al. 1999). They conducted crater-formation experiments using a certain material simulating Mathilde, with a porosity of more than 50%, corresponding to the unusually low bulk density of $\sim 1.3 \text{ g cm}^{-3}$ for the asteroid. The experiment showed that collisional fragments produced after an impact were much more “absorbed” into Mathilde, rather than expelled from around the impact site. This unexpected outcome is interpreted as being due to the very high porosity of the asteroid. If such a behavior is common for impacts of projectiles onto sizable C-type asteroids, it will bring about more efficient depletion of small collisional products than in the case of S-type asteroids, which are likely to have a lower porosity because of their higher bulk density. Then, this process, if effective, may predict a possible difference in the size distributions for C-type and S-type asteroids.

On the other hand, Nolan et al. (2001) found by their numerical simulations that since a shock wave after a collision fractures an asteroid in advance of crater-excavation flow, impact results are controlled by gravity; the tensile strength is unimportant whether asteroids are initially intact or rubble-piles. If this is the case, it means that the dispersion of fragments after a collision is independent of the tensile strength of the parent bodies. Hence, in short, whether a correlation exists or not between the size distribution of the collisional fragments and the tensile strength of the parent bodies is not yet known. Therefore, in order to pursue the causes of the difference in the slopes of the CSDs for three zones of the main-belt that we found, it is necessary to investigate the size distributions of C-type and S-type asteroids separately. This point is discussed in a little more detail in subsection 9.2.

8.2. Spatial Distribution of Sub-km MBAs

Finally, we discuss the spatial distribution of sub-km MBAs. First we noticed unclear “gaps” near $I \sim 12^\circ$ and $\sim 23^\circ$ in figure 13a. The cause of the depletion of asteroids with $I \sim 23^\circ$ is known theoretically (Scholl 1987). It is due to secular perturbations by Jupiter. On the other hand, the causes of the asteroid depletion near $I \sim 12^\circ$ seen in figure 9a have not been clear so far. Although a weak depletion of asteroids with $I \sim 10^\circ$–$12^\circ$ was pointed out in PLS (Van Houten et al. 1970), it was interpreted as meaning that the existence of the new Io family probably caused the depletion. However, since the size

\begin{table}
\centering
\caption{Two kinds of mean slopes of CSDs for MBAs with $0.5 < D (\text{km}) < 1$.
\begin{tabular}{lccc}
\hline
Belt zone & Inner belt & Middle belt & Outer belt \\
& $2.0 < a < 2.6$ & $2.6 < a < 3.0$ & $3.0 < a < 3.5$ \\
\hline
Mean slope 1 & $1.37 \pm 0.03$ & $1.15 \pm 0.03$ & $0.98 \pm 0.03$ \\
Mean slope 2 & $1.55 \pm 0.05$ & $1.15 \pm 0.03$ & $0.97 \pm 0.08$ \\
\hline
\end{tabular}
\end{table}
ranges covered by PLS and our SMBAS are largely different, there are no a priori reasons to reject the apparent gap near \( I \sim 12^\circ \) seen in SMBAS; it is also possible that the distributions are actually different for PLS and SMBAS. Considering large errors for the \( a (\sim 0.1\text{AU}) \) and \( I (\sim 2^\circ - 6^\circ) \) in SMBAS, therefore, we would say that this problem should be further investigated in the near future with a larger sample of small asteroids and an improved orbital estimate.

Next, we discuss the issue pointed out above that a small number of asteroids with a high inclination were seen in the neighborhood of the distinct Kirkwood gaps at 2.5, 2.8, and 3.0 AU. Dynamical considerations generally show that, once an asteroid gets trapped in a gap, it undergoes a chaotic orbital transition, its \( e \) grows unexpectedly to a very high value, and finally it is ejected from the main-belt to the near-Earth region or to other regions in the solar system. In some cases where a mean-motion resonance is coupled with a secular resonance or the Kozai resonance, it is shown that the \( I \) of an asteroid also pumps up to a high level (Morbidelli, Moons 1995). It is thus possible that the high-inclination asteroids near the Kirkwood gaps in figures 13a and b might correspond to such chaotic asteroids. This may imply that those asteroids are under a transport process in which the asteroids are exported through the resonances from the main-belt to different places in the solar system.

Figures 13a and b also indicate that there seems to be a trend that asteroids with a larger \( a \) show a higher \( I \). This situation was similar for both MBAs in SMBAS and known MBAs (see figures 14a and b), though it seems that the high-inclination fraction in the outer-belt is more pronounced in SMBAS than in known MBAs. Since it is likely that there is a weak inverse correlation between the size and the velocity for fragments produced in collisions among asteroids (e.g., Nakamura, Fujiwara 1991), smaller fragments should have a faster release velocity. If so, it is reasonable that the \( I \)-distribution of smaller MBAs has a wider range than that of large MBAs.

However, we again encounter a problem here that because the orbital errors of each asteroid in SMBAS were considerably large (see section 5), we could not confidently state that small asteroids have a different orbital-element distribution from that for larger asteroids in figures 13a and b. Clearly, we need many more samples. If we can divide a sufficient number of asteroids into smaller \( a \)-zones and discuss the corresponding \( I \)-distribution for each zone, we will probably obtain more meaningful results. For that reason, we hope to widen the survey area in near-future observations.

9. Conclusions and Future Prospects

9.1. Summary and Conclusions

We detected 1111 moving objects down to \( R = 24.4\text{mag} \) in a sky area of \( 2.97\text{deg}^2 \) near the opposition and near the ecliptic in SMBAS. We then identified 861 MBAs by estimating the \( a \) of each moving object from its sky motion vector. The sky number density of MBAs was \( \sim 290\text{deg}^{-2} \) down to \( R = 24.4\text{mag} \) near the opposition and near the ecliptic. We found that the slope of the CSD for small MBAs ranging from a few km to sub-km is fairly shallower (\( \sim 1.0 - 1.4 \)) than that for large MBAs (\( \sim 1.8 \)) obtained from past asteroid surveys (see table 1). This means that the number of sub-km MBAs is much more depleted than that indicated based on a result extrapolated from the size distribution for large asteroids. The CSD slope of the inner sub-km MBAs was somewhat steeper than that of the outer sub-km MBAs.

On the other hand, we could not detect any clear difference between the spatial distributions of sub-km MBAs and that of known large MBAs. This is perhaps because of the insufficiency of the number of asteroids detected in SMBAS, rather than the inaccuracy of the orbital elements of each asteroid. Hence, we plan to widen the survey area in near-future observations, in order to make our conclusions more reliable.

9.2. Future Prospects

As a next step, we must clarify and better interpret the differences in the slopes of the CSD for the three regions of the main-belt. For this purpose, it is necessary for us to investigate each size distribution for C-type and S-type asteroids, because these two taxonomic types are major components of MBAs. Those
two types can be discriminated from \((B - V)\) or \((V - R)\) color observations. We performed such observations about an year ago, and data reductions are now in progress. As mentioned above, since it is believed that there is some correspondence between the asteroid material and taxonomic types, such a survey observation as those conducted by us will also allow us to argue interrelations between the collisional processes, orbital evolution, and material distribution of MBAs.

We thank Dr. Yutaka Komiyama (support astronomer), Mr. Gray Fujiwara, Mr. Bob Potter, and Ms. Sumiko Harasawa (night operators), Suprime-Cam team, and the Subaru telescope supporting staff for their kind support in our observations. We would also like to express our thanks to Dr. Takashi Ito and staff of the Astronomical Data Analysis Center, NAOJ, whose advice in data reduction was very useful. Finally, our thanks are due to Dr. M. V. Sykes, whose comments and suggestions as a referee of this paper were very useful to improve the original manuscript.

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