A Shape-and-Density Model of the Putative Binary EKBO 2001 QG\textsubscript{298}

Shigeru TAKAHASHI and Wing-Huen Ip
Institute of Space Science, National Central University, 300 Jungda Road, Chung-Li, Tao-Yuan, 32054, Taiwan
shigeru@astro.ncu.edu.tw, shigeru@nro.nao.ac.jp, wingip@astro.ncu.edu.tw

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Abstract

Recent observations of the Edgeworth–Kuiper belt object (EKBO) 2001 QG\textsubscript{298} (Sheppard, Jewitt 2004) have shown that the lightcurve of this object has a very large amplitude (1.14 ± 0.04 mag), indicating that it is either of an elongated shape or of a binary structure with two components of similar sizes nearly in contact with each other. On the basis of these interesting published data, we employed Roche binary lightcurve simulations to construct a shape model of EKBO 2001 QG\textsubscript{298}. The shape parameters of the best-fitted model were 260 (164) × 205 (130) × 185 (116) km for the primary, and 265 (168) × 160 (102) × 150 (94) km for the secondary in the case of an albedo of 0.04 (0.10). An additional result of this calculation is that the average bulk density of the contact binary system could be estimated to be 630 kg m\textsuperscript{-3}. This value is similar to that of several icy moons of Saturn with a diameter of less than 200 km. We have also used the Jacoby ellipsoidal approximation to compute the shape of one of the largest EKBOs, Varuna. The corresponding shape parameters are \(a : b : c = 1.00 : 0.76 : 0.50\). The lower limit of the bulk density is \(\rho \geq 1000\) kg m\textsuperscript{-3}. These results are in good agreement with the published values of Jewitt and Sheppard (2002), and are consistent with their suggestion that larger icy bodies have higher densities (Sheppard, Jewitt 2002). 

Key words: Kuiper Belt — minor planets, asteroids — planets and satellites: individual (Janus)

1. Introduction

After the first discovery of the moon of (243) Ida, Dactyl, by flyby observations of the Galileo spacecraft (Chapman et al. 1995), binary systems of small bodies in the solar system have been found more and more by ground-based telescopes. So far more than 50 binaries have been found in the main asteroid belt, among the near-Earth objects (NEOs) (Margot et al. 2002) and in the Edgeworth–Kuiper belt (EKB) (Veillet et al. 2002). The increasing number of binary EKBOs found by different observers has led to vigorous investigations of their origin and evolutionary histories (Burns 2004; Funato et al. 2004; Goldreich et al. 2002; Petit, Mousis 2004; Weidenschilling 2002). It has been estimated that about 10% of the total EKBOs are binaries (Burns 2004). The recent finding by Sheppard and Jewitt (2004) that EKBO 2001 QG\textsubscript{298} has a lightcurve amplitude as large as 1.14 ± 0.04 mag is therefore very fascinating. This is because, first, this result indicates that this object could be a close-contact binary, the first ever to be discovered in the Edgeworth–Kuiper belt; second, the actual number of such binaries could be quite substantial (on the order of 10–20%) of the total population at large sizes; Sheppard, Jewitt (2004) based on a statistical argument. The main belt asteroid (216) Kleopatra, with an amplitude of about 1.2 mag for its brightness variation (Zappalà et al. 1983), is a good example of such contact binary systems. Recent radar (Ostro et al. 2000) and interferometric (Tanga et al. 2001) observations have shown (216) Kleopatra to be composed of two equal-sized components in contact. Observations by using adaptive optics have further resolved these two components with a deconvolution technique (Hestroffer et al. 2002a). This experience tells us that EKBO 2001 QG\textsubscript{298} could hence be similar in physical configuration to that of (216) Kleopatra.

Moreover, there are actually two kinds of asteroidal binaries. The most common type is characterized by small size ratios (i.e., 0.2–0.3), and the second type with binary components of similar size has only three members: (99) Antiope (Michałowski et al. 2002), (216) Kleopatra, and probably also the Trojan asteroid (624) Hector (Weidenschilling 1980). Why should the EKBO binaries so far detected all have components of similar sizes? Is this simply because of a selection-type constraint in observing these faint objects? Or, is it because of the formative history of the EKBOs? It is expected that the physical structures (namely, shapes and bulk densities, and hence the internal properties) of these objects might provide clues to their evolutions. And this is the reason why EKBO 2001 QG\textsubscript{298} gives us the first opportunity to study such a unique system in detail.

One technique developed to study close binaries is the Roche binary approximation (Leone et al. 1984; Cellino et al. 1985). A Roche binary is described by a pair of homogenous bodies in hydrostatic equilibrium and in circular orbital motion around each other. One advantage of such a method is that the bulk density of the objects could be deduced if their shapes and the rotation period are known. Under the condition that the internal structures of the binary components are not of monolithic nature, but rather made up of many small fragmental pieces (i.e., the rubble pile) held together by mutual gravitational attraction, the fluid approximation can be applied to asteroids and EKBOs. In fact, this method was tested on (216) Kleopatra, and the numerical results demonstrated that the shape model so derived fit the lightcurves from previous observations extremely well; at the same time, the density estimate is very similar to the value given by radar observations (Cellino et al. 1985; Ostro et al. 2000; Hestroffer et al. 2002b; Takahashi et al. 2004). It is for this reason that we proceed with numerical
simulations of EKBO 2001 QG298 as a Roche binary.

2. Simulations

The equations of the Roche ellipsoids are given in the manner of Leone et al. (1984) as follows:

\[
\begin{align*}
(3 + q)a^2 + c^2 & = 2(A_1a^2 - A_3c^2), \\
qb^2 + c^2 & = 2(A_2b^2 - A_1c^2),
\end{align*}
\]

where

\[
A_1 = abc \int_0^{\infty} (a^2 + u)^{-1/2} (b^2 + u)^{-1/2} (c^2 + u)^{-1/2} du,
\]

\[
A_2 = abc \int_0^{\infty} (a^2 + u)^{-1/2} (b^2 + u)^{-1/2} (c^2 + u)^{-1/2} du,
\]

\[
A_3 = abc \int_0^{\infty} (a^2 + u)^{-1/2} (b^2 + u)^{-1/2} (c^2 + u)^{-1/2} du.
\]

In the equations, \(a, b, c\) are axes of the primary or secondary, and \(q, \omega, \rho, \) and \(G\) are mass ratio, angular velocity, density, and gravity constant, respectively. We solved the equations with changing axial parameters, the primary axes \(b (b/a)\) and \(c (c/a)\) (see figure 1), and obtained the mass ratio, \(q\), and the density, \(\rho\). The ranges of the axial parameters are \(b \sim 0.0–1.0, c \sim 0.0–1.0, (b \geq c)\) with a step of 0.001. Then, pairs of axial parameters \((a', b', c')\) were calculated self-consistently by setting the mass ratio as \(1/q\). If two components were contacted \((l \leq a + a')\), the solutions were omitted. After solving the equations, we randomly sampled 500 sets of the Roche solutions for simulations.

The lightcurve simulations were carried out by taking into consideration the illuminated areas that are visible from the Earth as well as the shadow and occultation effects of the primary and secondary components. In addition, the empirical scattering model on an asteroidal surface, proposed by Kaasalainen, Torppa, and Muinonen (2001), was used. The flux of an asteroid is composed of both single and multiple scatterings on the surface. To describe the light-scattering properties (e.g., reflectance \(r\)), formulae of Lumme–Bowell and Hapke (Bowell et al. 1989; Hapke 1993) are widely used. These formulae contain several parameters, so that a certain number of observations at different phase angles are needed to determine proper values. For 2001 QG298, we cannot apply these formulae because the data were measured only in 2002 September and 2003 August–September. Instead, we took the idea of Kaasalainen, Torppa, and Muinonen (2001) to denote the scattering properties; that is, the contribution of single scattering attributed to the Lommel–Seeliger law and that of multiple scattering to the Lambert law. These fundamental scattering laws were linearly combined with one parameter (weight factor) \(k\). The reflectance, \(r\), is written as

\[
r \sim (1 - k) \frac{\mu_0}{\mu_0 + \mu} + k \cdot \mu_0
\]

or

\[
r \sim \frac{\mu_0}{\mu_0 + \mu} + c \cdot \mu_0 \left( c \equiv \frac{k}{1 - k} \right).
\]

In the above equations, \(\mu_0\) and \(\mu\) are the cosines of the angles between the surface normal and the incidence and emission directions, respectively. As far as calculations of the amplitudes and lightcurves are concerned, both equations (6) and (7) give the same results. Although equations (6) and (7) are empirical formulae, asteroidal shapes have been well reproduced via precise lightcurve inversions (Kaasalainen et al. 2001). Kaasalainen, Torppa, and Muinonen (2001) have also reported that their scattering model is in good agreement with results using the Hapke or Lumme–Bowell method. Note that even though equation (7) is the original expression derived by Kaasalainen, Torppa, and Muinonen (2001), we employed equation (6) in our following study because we needed to consider the case of Lambert scattering with \(k = 1.0\). It is expected that a low-albedo body, such as an EKBO, has a small \(k\) or \(c\) value because the single scattering process is dominant on the surface. On the other hand, a high-albedo body, on which the multiple scattering process is dominant, is expected to have a large value. Kaasalainen, Torppa, and Muinonen (2001) obtained \(c = 0.1 \ (k = 0.091)\) for an S-type (high albedo) asteroid (433) Eros (geometric albedo: \(A_p = 0.29 \pm 0.02\); Domingue et al. 2002), so that for a low-albedo body, such as EKBO (geometric albedo \(A_p \leq 0.1\)), the \(k\) or \(c\) value is expected to be smaller (\(c, k \leq 0.1\)).

The best-fitted shape model was obtained by comparing the simulated lightcurves with the observations of EKBO 2001 QG298 by Sheppard and Jewitt (2004). The averaged differences between simulations and observations were taken as criterion for selecting the models. The criterion \(\Theta\) is denoted as

\[
\Theta (\text{mag}) = \sqrt{\frac{\sum_{i=1}^{n} (\text{obs}_i - \text{cal}_i)^2}{n}}.
\]

where \(n\) is a data number, and \(\text{obs}_i\) and \(\text{cal}_i\) are observed and calculated magnitudes. We selected the models that satisfied \(\Theta < 0.1\) mag, which corresponds to 2.5–3 times the data uncertainties (\(\pm 0.03–0.04\)).
3. Results and Discussions

3.1. Model Shapes

Figure 2 shows a lightcurve constructed by using the best-fitted model (table 1 and figure 3) with \( \theta = 0.066 \). This model is able to reproduce very well the main facets of the observations, namely, the amplitude, the differences between the first and second minima and the maxima. There existed some small discrepancy in fitting the maxima (\( \sim 0.05 \) mag). However this could have been caused by both albedo variations and local topographical variations of 2001 QG\(_{298}\). We believe that this model should represent the real shapes of the binary components as a whole. The aspect angle is defined as the angle between the rotation axis and the line of sight. As would be expected, when the aspect angle approaches 0\( \circ \) (i.e., a pole-on configuration), the variation amplitudes of the simulated light curves become smaller because of a decrease in the changes of the light-scattering cross sections. We examined different solutions over a wide range of aspect angles, and found that only for a nearly equatorial view with an aspect angle of about 90\( \circ \) could the observations be reproduced. If the albedo was assumed to be 0.04 (0.10), the best solution had average sizes of 260 (164) \( \times \) 205 (130) \( \times \) 185 (116) km for the primary and 265 (168) \( \times \) 160 (102) \( \times \) 150 (94) km for the secondary. The separation distance between the two mass centers is 273 (172) km, indicating that the two bodies are nearly in contact with each surface, separated by only 10 (6) km at the closest distance.

Varuna is one of the largest EKBOs with a size of \( 900^{+125}_{-145} \) km (Jewitt et al. 2001). It is also a fast rotator with a rotation period of 6.3442 \( \pm \) 0.0002 hr (Jewitt, Sheppard 2002). For the sake of a comparison, we used the table of Jacobi solutions listed in a book by Chandrasekhar (1969), assuming that Varuna is made up of a rubble pile and rotates with the perpendicular axis towards the Earth. By applying the same criterion as given in equation (3), we found that the best-fitted ellipsoidal model in terms of the three axes can be described by \( a : b : c = 1.00 : 0.76 : 0.50 \) with \( a = 691^{+111}_{-76} \) (see figure 4). Jewitt and Sheppard (2002) previously gave very similar values (\( a : b : c = 1.00 : 0.68 : 0.47 \)). We confirm that each values also reproduce the observations within \( \theta < 0.1 \).

3.2. Weight Factor \( k \)

The best-fitted \( k \) value of the scattering parameter is \( k \sim 0.6 \sim 0.8 \) for 2001 QG\(_{298}\) and \( k = 0.6 \) for Varuna. The lightcurves of 2001 QG\(_{298}\) for different \( k \) values are shown in figure 5. It can be seen that the amplitudes become larger with increasing \( k \) values (i.e., becoming more Lambert-like). Small \( k \) values (\( k \sim 0.2 \sim 0.4 \)) cannot fit the observed light curves and amplitudes too well. Because a dark surface would be dominated by single scattering (the Lommel Seeliger law), the \( k \) values of EKBOs are thus expected to be smaller than those of the S-type asteroids, such as (433) Eros with \( k = 0.09 \) (Kaasalainen et al. 2003).
The presently derived value of $k \sim 0.6–0.8$ might hence appear to be inappropriate. However, we should note that the $k$ value of an M-type asteroid, that is (216) Kleopatra, itself, has almost the same $k$ value ($0.6–0.8$), although its surface is darker than those of the S-type asteroids (Takahashi et al. 2004). To a certain extent, this effect might be related to the unique mineralogical property of the M-type asteroids. On the other hand, since no other $k$ values of EKBOs have been precisely estimated, a large $k$ value for 2001 QG298 could therefore not be ruled out.

3.3. Density

We show the best-fitted shapes of 2001 QG298 lightcurves for different density values in figure 6. The density of the best model is $\rho = 630 \text{kg m}^{-3}$. The other solutions have nearly the same values (average is $\rho = 640 \text{kg m}^{-3}$). As the density of model becomes higher, kinks and broader maxima appear in the lightcurves. This is due to large separations of the high-density models; i.e., a large separation causes long time lags of the starting mutual occultations during a rotation. We found that the densities up to $\rho \sim 700 \text{kg m}^{-3}$ reproduce the lightcurves with $\Theta < 0.1 \text{mag}$, but we could not find good solutions for densities of $\rho > 700 \text{kg m}^{-3}$. We conclude that the density of 2001 QG298 is lower than $\rho \sim 700 \text{kg m}^{-3}$ if the shape obeys the Roche binary. We obtained a density of $\rho = 1000 \text{kg m}^{-3}$ for Varuna, which is the equal to the value obtained by Sheppard and Jewitt (2002). This value is the lower limit density for Varuna, because we assumed that the axis is perpendicular to being viewed from the Earth. If the rotation axis were more pole-on view like, the density would become higher.

Sheppard and Jewitt (2002) have reported the tendency that larger icy bodies have higher densities. Our results support this trend. The $D \sim 200\text{km}$ sizes of two components of 2001 QG298 have a density of $\rho \sim 640 \text{kg m}^{-3}$, while the larger body Varuna ($D = 900^{+125}_{-145} \text{km}$) has a lower limited value of $\rho \geq 1000 \text{kg m}^{-3}$. We plotted these results on a size-density plot, which had been made by Sheppard and Jewitt (2002) (figure 7). The position of 2001 QG298 is almost equal to those of several $D < 200\text{km}$ satellites of Saturn (e.g., Janus: $\rho = 670 \pm 10 \text{kg m}^{-3}$, $D = 177.6 \pm 8.0\text{km}$; Yoder et al. 1989), on the other hand Varuna is the same as $D \sim 1000\text{km}$ sized.
satellites (e.g., Tethys: \( \rho = 991 \pm 9 \) kg m\(^{-3} \), \( D = 1060 \pm 20 \) km; Morrison et al. 1984). It should be noted that the density of 2001 QG\(_{298} \) is almost equal to those of smaller bodies and short period comets; Halley: \( \rho = 600_{-400}^{+900} \) kg m\(^{-3} \) and Shoemaker–Levy 9: \( \rho = 500 \) kg m\(^{-3} \) (Asphaug, Benz 1994; Solem 1994; Sagdeev et al. 1988). This result indicates that \( D \approx 200 \) km sized EKBOs and short-period comets have similar porosities, and may imply a relation between them.

So far, almost all theoretical simulations seem to treat the densities of EKBOs as \( \rho = 1000 \) kg m\(^{-3} \) or higher. This study suggests that theoretical simulations take into account the size-density distribution of EKBOs. We propose that for at least diameter \( D \approx 200 \) km sized EKBOs the densities are \( \rho \sim 640 \) kg m\(^{-3} \).

4. Conclusions

1. The Roche binary shape can well reproduce the observations of EKBO 2001 QG\(_{298} \) by Sheppard and Jewitt (2004). The shape parameters are: the primary axis ratio \( a : b : c = 1.0 : 0.79 : 0.72 \), the secondary axis \( a' : b' : c' = 1.02: 0.62: 0.57 \), and distance \( l = 2.10 \). Those values correspond to 260 (164) \( \times 205 (130) \times 185 (116) \) km for the primary and 265 (168) \( \times 160 (102) \times 150 (94) \) km for the secondary if the albedo is 0.04 (0.10). The separation distance between the two mass centers is 273 (172) km, indicating that the two bodies are nearly in contact with each surface, separated by only 10 (6) km at the closest distance.

2. The EKBO Varuna was simulated with the Jacobi ellipsoid assumption. The best-fitted shape parameters of Varuna, which are based on the Jacobi ellipsoid assumption, are \( a : b : c = 1.00 : 0.76 : 0.50 \), whose values are almost equal to those by Jewitt and Sheppard (2002).

3. The best-fitted weight factor, \( k \), values of the scattering model by Kaasalainen, Torppa, and Muinonen (2001) are \( k \approx 0.6–0.8 \) for 2001 QG\(_{298} \) and \( k = 0.6 \) for Varuna. The derived values are large for dark objects, such as EKBOs. However, since no other \( k \) values of EKBOs have been precisely evaluated, large \( k \) values for 2001 QG\(_{298} \) cannot be ruled out.

4. The obtained density is \( \rho = 630 \) kg m\(^{-3} \) for EKBO 2001 QG\(_{298} \). The lower limit of the density for Varuna is \( \rho \geq 1000 \) kg m\(^{-3} \). These values support the suggestion that larger icy bodies have higher densities (Sheppard, Jewitt 2002).

5. Interestingly, the density value \( \rho = 630 \) kg m\(^{-3} \) for 2001 QG\(_{298} \) is almost the same as those of several \( D < 200 \) km satellites of Saturn, such as Janus: \( \rho = 670 \) kg m\(^{-3} \), smaller objects, and short period comets, such as Halley: \( \rho = 600_{-400}^{+900} \) kg m\(^{-3} \) and Shoemaker–Levy 9: \( \rho = 500 \) kg m\(^{-3} \) (Asphaug, Benz 1994; Solem 1994; Sagdeev et al. 1988).

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