Energetic Relations between the Disappearing Solar Filaments and the Associated Flare Arcades

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Abstract

We present the temporal and statistical relations between the mechanical energies of disappearing solar filaments and the thermal energies of the associated flare arcades in soft X-rays. Measuring the 3-D velocity fields of 10 eruptive filaments, we calculated their mechanical energy gain rate, $mc^2$, per unit volume and compared it to the thermal energy release rate per unit volume, $\epsilon_{\text{th}}$, derived with Yohkoh/SXT data. For the statistical relation, we found a relation that can be approximated as $\epsilon_{\text{th}} \propto \epsilon_{\text{mc}}^{1.9}$. This relation can be explained by interpreting the energy input to an arcade via the Poynting flux in the magnetic reconnection process and the acceleration of a filament by the Lorentz force. This explanation is also supported by the strong dependence of the observed increase rates of both the thermal and mechanical energy densities on the mean magnetic field strength of the source region. We also investigated their temporal variations, and found that the start time of increase in the mechanical energy of a filament preceded that of the thermal energy of the coronal arcade in some cases. These relations imply that the basic mechanisms that accelerate a filament and create a hot plasma are different, and both energy increase rates are determined primarily by the magnetic field strengths.

Key words: Sun: coronal mass ejections (CMEs) — Sun: filaments — Sun: flares

1. Introduction

It is well known that prominence eruptions and filament disappearances are often followed by two ribbon flares in the Hα line, and bright arcade formations in soft X-rays and EUV (e.g., Sheeley et al. 1975; Webb et al. 1976; McAllister et al. 1992, 1996; Hanaoka et al. 1994; Khan et al. 1998; Tripathi et al. 2004). The relation between flares and mass ejections, including filament/prominence eruptions as well as coronal mass ejections (CMEs), has been a major research subject. Every broad study that extensively investigated this relation has also revealed that these two phenomena have strong association (Munro et al. 1979; Webb & Hundhausen 1987; Sheeley et al. 1983; St. Cyr & Webb 1991; Harrison 1995). Owing to detailed studies on solar flares after the launch of Yohkoh satellite, it became commonly recognized that a flare and mass ejection are the processes involved in a magnetic reconnection process (Shibata et al. 1996), based on a model which Hirayama (1974) proposed that a filament eruption triggers magnetic reconnection in the vertical current sheet above the flare loop and other studies on similar magnetic configurations, known as the CSHKP model (Carmichael 1964; Sturrock 1966; Hirayama 1974; Kopp & Pneuman 1976).

It is also known that the origins of filament eruptions and the formations of soft X-ray arcades are magnetic (e.g., Pevtsov 2002). High-intensity flares often occur in active regions where the magnetic field is strong, and low-intensity flares are likely taking place away from active regions. At the same time, a prominence eruption that originates from an active region tends to undergo strong acceleration, and is often called “a spray prominence” (Warwick 1957), while that from a region with a weak magnetic field strength takes much more time (up to several hours) to be accelerated (Valnicek 1964; Tandberg-Hanssen et al. 1980; Rompolt 1990), although it is subject to some exceptions (Sterling et al. 2001; Shakhovskaya et al. 2002).

These facts give us an insight that a filament in its eruption and an arcade or a flare loop formation observed in soft X-rays should have some physical connection to each other. Extensive studies based on observations, however, have failed to figure out certain physical links between them. Hundhausen (1997), analyzing the images of coronal mass ejection (CMEs) taken by the Solar Maximum Mission (SMM) coronagraph, compared the observed signatures of CMEs to those of associated soft X-ray events, and concluded that (i) intense soft X-ray flares are neither a necessary nor a sufficient condition for the occurrence of coronal mass ejections, (ii) the intensity of any soft X-ray flare, which implies heating of the corona, that accompanies a mass ejection is not closely related to the characteristics (such as speed, mass, and energy) of the ejection. Hundhausen (1997) also mentioned that there is no physical reason why the energy available for mass ejection should have any simple relation to the energy available for a soft X-ray flare.

Zhang et al. (2001) carefully investigated the temporal evolutions of the motions of four CMEs observed by Extreme Ultraviolet Imaging Telescope (EIT) (Delaboudiniere et al. 1995) and Large Angle and Spectrometric Coronagraph (LASCO) (Brueckner et al. 1995) instruments aboard the Solar and Heliospheric Observatory (SOHO) spacecraft, and compared it to the Geostationary Operational Environmental Satellite (GOES) soft X-ray flux observations. They demonstrated that the impulsive acceleration phase of CMEs coincides very well with the rising phase of the associated soft X-ray flares. They, however, reported that there is no correlation...
between the energetics of CMEs (in terms of speeds) and flares (in terms of peak flux). Moon et al. (2002) analyzed CMEs with limb flares greater than C1 class, and found a weak correlation between the speed of CMEs and the associated flare X-ray flux.

In these studies, however, they only focused on the temporal relation, or compared the peak intensities of soft X-ray events with the velocities of the mass ejections. We therefore, in order to seek physically more meaningful temporal and statistical relations, estimated both the kinetic and potential energies of eruptive filaments, and compared them directly to the released thermal energies of the associated arcades or flare loops. Furthermore, we investigated the relation between the measured increase rates of the mechanical energy density and the thermal energy density with the mean photospheric magnetic field strength of the source region, because both processes are magnetic, and it may be appropriate to relate them in terms of the magnetic field strength.

In our previous paper (Morimoto & Kurokawa 2003a, hereafter Paper I), we studied the 3-D velocity fields of disappearing filaments (disparitions brusques: DBs) on the solar disk, applying Beckers’ cloud model to 5 DB events observed in Hα line center and Hα ± 0.8 Å with the Flare Monitoring Telescope (FMT) at Hida Observatory. By measuring the 3-D velocity fields of 35 DBs on the solar disk, we studied the causal relation between the motions of Hα DBs and the associated coronal phenomena (Morimoto & Kurokawa 2003b, hereafter Paper II). In this paper, we present the temporal and statistical relations between the mechanical energies of eruptive filaments and the thermal energies of the associated flare arcades in soft X-rays.

In section 2, we describe the data used in this study and the selection of events. A brief explanation of the methods used to derive the line-of-sight and transversal velocities of a disappearing filament is given in section 3. We then show the results of two events to see the temporal relation of the energy of a disappearing filament and the thermal energy of the associated arcades in the first half of section 4. We also present their statistical relation using all of the selected events. Finally, summarizing the results of all events, we give a conclusion in section 5.

2. Observations and Data

2.1. Hα Observations

We used Hα images obtained by the Flare Monitoring Telescope (FMT: Kurokawa et al. 1995) at Hida Observatory, Kyoto University to investigate the velocities and energetics of disappearing filaments. This telescope provides full-disk solar Hα images not only in its line center, but also in blue and red wings, whose central wavelength are Hα − 0.8 Å and Hα + 0.8 Å, respectively. The FMT observations in the 3 bands enable us to measure the line-of-sight and transversal velocities of the filaments, and hence their complete 3-D velocity fields.

One example of FMT observations of a filament disappearance is displayed in figure 1. This event took place on 2000-May-08 near to the central meridian in the southern hemisphere. Before the onset of disappearance, the filament is visible in only the Hα line center image (left panels). From 04:17 UT, the southern part of the filament starts to ascend from its southern leg, and appears as a dark feature in the blue wing (middle panels), and then fades in the Hα line center images. At 05:36 UT, when the southern part vanishes completely, a two-ribbon flare appears at both sides of the disappeared filament, as shown in the bottom-left panel of figure 1. At this moment, the middle and northern parts of the filament still continue to move towards the north-west, and it becomes prominent in the red wing (right panels), which indicates that the receding motion is dominant in the later phase.

2.2. Yohkoh Observations

Yohkoh Soft X-ray Telescope (SXT: Ogawara et al. 1991; Tsuneta et al. 1991) provides full and partial frame images (FFIs and PFIs, respectively) of the solar corona in soft X-rays from 1991. An FFI is obtained, usually, per 50 seconds with a pixel size of 4.9’’ (half resolution) or 9.8’’ (quarter resolution). A PFI is obtained with a pixel size of 2.5’’ (full resolution), and the range of time of the cadence is from a few seconds to 30 seconds. This telescope is equipped with five X-ray analysis filters, which enables us to derive the temperatures and emission measures of arcades with filter ratio method (Harra et al. 1992).

The Yohkoh/SXT images of the 2000-May-08 event are illustrated in figure 2. At 04:36:37 UT, before the onset of this event (top panel), the Hα filaments are indicated by the white contour. The soft X-ray corona was found to become bright at the southern part of a large faint sigmoid-like structure at 05:57:27 UT, or just after the data gap due to Yohkoh night. An arcade of bright cusp-shaped loops is seen in the image at 07:46:13 UT. These locations of the cusp-shaped loops correspond to the southern part of the Hα filament, which disappeared in the FMT Hα center images (figure 1). The rest part of the soft X-ray sigmoid-like structure, which corresponds to the middle and northern parts of the Hα filament, is also seen to slightly brighten. In an analysis of 10 events, we used FFIs only in the cases in which we did not have appropriate PFIs, because the spatial and temporal resolutions of PFIs were better than those of FFIs.

2.3. Other Data

The soft X-ray flux data obtained by Geostationary Operational Environmental Satellite (GOES) are used to see the peak fluxes associated with the soft X-ray events. Since the acceleration of a filament and heating of coronal plasma appear to be the consequences of the magnetic field changes, we examined the photospheric magnetic field strength, $B_\tau$, of the region as an indicator of the coronal magnetic field strength, $B$. As the photospheric magnetogram, we used data from the National Solar Observatory/Kitt Peak Vacuum Telescope. The values of $B_\tau$ are listed in the sixth column of table 1, by taking the unsigned average over the arcade region. We also used white-light coronagraph images from the LASCO instrument to see the velocity evolution of the CME accompanied with the event on 2000-May-08, which will be discussed in some detail later.

2.4. Event Selection

In Paper II, we extensively analyzed 3-D velocity fields of 35 DBs, and classified them into eruptive and quasi-eruptive
types. “Eruptive” means that all or, at least a part of the disappearing filament is ejected into interplanetary space. The “quasi-eruptive” filament ascends, breaks into fragments, and then flows down into the chromosphere at several places, or remains in the solar atmosphere. We selected 10 events out of these 35 events with the following criteria: (i) The filament is judged to be eruptive, since we are concerned with the problem of how a filament is accelerated. (ii) We have sufficient SXT images taken with different filters and good temporal cadence with a reasonable exposure time to obtain the temporal variations of coronal temperatures and emission measures, and to determine the thermal energy release rate in the rise phase of soft X-ray events. We focused on the rise phase because the energetics of disappearing filaments in their acceleration phase is considered to be related to the rise phase of the soft X-ray events. The selected 10 events are listed in table 1 along with their occurrence time and locations.

3. Analysis

3.1. Energetics of the Hα Filaments

3.1.1. The velocities of disappearing filaments

The detailed calculation procedures of the velocity in the disappearing Hα filaments are presented in Paper I, so we only...
briefly explain them in this paper. The 3-D velocity field of a disappearing filament is obtained by calculating the line-of-sight and transversal velocities of the filament. In this new method, the time variation of the line-of-sight velocity is derived using the three filtergrams obtained in Hα center, Hα − 0.8 Å, and Hα + 0.8 Å on the basis of Beckers’ cloud model (Beckers 1964). We also make corrections for the effective band widths of the FMT filters, the scattered light and the Doppler brightening effect (DBE) in order to accurately determine the line-of-sight velocity. The transversal speeds and directions are measured by tracing the internal structures (blobs) of a filament on successive FMT images. This transversal velocity enables us to determine the trajectory of each element inside the filament. With the line-of-sight and transversal velocities of each trajectory, we calculate the corresponding upward velocity. The transversal velocity is next used to derive the trajectory of each pixel on the filament. This enables us to obtain the time-dependent evolution of the 3-D velocity for each pixel. This procedure is performed for all the pixels on the filament individually to reconstruct the complete 3-D velocity field.

In order to see an example of the 3-D velocity evolution, we define a sample point (point A in figure 1) on the filament of the 2000-May-08 event at 04:17 UT before its onset. The defined and subsequent positions of point A are shown on the FMT images with “+” marks in figure 1. The top-left panel of figure 3 shows the temporal evolution of the line-of-sight velocity of the point. The point A is accelerated from the onset of the disappearance, and attains −48 km s−1 at 05:18 UT just before its disappearance. The top-right panel in figure 3 shows the upward velocity of point A. Since the motion of point A is almost along the line-of-sight, the upward velocity is quite similar to the line-of-sight velocity.

3.1.2. The mechanical energy gain rate

The mechanical energy gained by an element x with unit volume inside a filament \( e_{\text{mc}}(x, t) \) at an arbitrary time \( t \) is the sum of its kinetic and potential energies; it can be written as

\[
e_{\text{mc}}(x, t) = \frac{1}{2} \rho_p v(x, t)^2 + \int_{R_0}^{H(x, t) + R_0} G \frac{M_{\odot} \rho_p}{r^2} dr,
\]

where \( R_0, M_{\odot}, \) and \( G \) are the solar radius, solar mass, and gravitational constant, respectively. Since the density of a filament, \( \rho_p \), can not be obtained with FMT observations alone, we adopted an average value of an ordinary prominence, \( \rho_p = 1.7 \times 10^{-13} \text{ g cm}^{-3} \), which corresponds to a number density of \( 10^{11} \text{ cm}^{-3} \) (Jefferies & Orrall 1963) with a filling factor of 0.1 (Engvold 1976; Simon et al. 1986). The total velocity, \( v(x, t) \), is obtained by the velocity diagnostics described above and in Paper I, and the height \( H(x, t) \) is given by the upward velocity, \( v_u(x, t) \), via an equation,

\[
H(x, t) = \int_{t_0}^t v_u(x, t') dt',
\]

where \( t_0 \) is the onset time of the acceleration of the element. Using equation (1) and equation (2), we obtain the final mechanical energy, \( e_{\text{mc}}(x, t_f) \), of the eruptive element on the trajectory at time \( t_f \) when the element disappeared from the FMT field of view. We then divide it by the time difference between \( t_0 \) and \( t_f \) to obtain the mechanical energy gain rate, \( e_{\text{mc}}(x) = e_{\text{mc}}(x, t_f)/(t_f - t_0) \). Finally, we average the derived mechanical energy gain rates over all the eruptive elements and obtain the mean mechanical energy gain rate per unit volume \( e_{\text{mc}} = \langle e_{\text{mc}}(x) \rangle \).

For example, the height of the element that corresponds to point A of the 2000-May-08 event at 05:18 UT just before its disappearance was \( 4.9 \times 10^3 \text{ cm} \), and its potential energy is therefore \( 2.1 \text{ erg cm}^{-3} \). The kinetic energy of the same element is \( 2.1 \times 10^{-1} \text{ erg cm}^{-3} \) which, combined with the
Table 1. Complete list of the events analyzed in this study.*

<table>
<thead>
<tr>
<th>No</th>
<th>Time</th>
<th>Location/NOAA</th>
<th>$\epsilon_{\text{mc}}$</th>
<th>$\epsilon_{\text{th}}$</th>
<th>$B_t$</th>
<th>GOES</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>1993-Apr-20</td>
<td>S22E70/7480</td>
<td>3.8</td>
<td>8.3</td>
<td>6.7</td>
<td>B7.4</td>
</tr>
<tr>
<td>2</td>
<td>1993-May-15</td>
<td>S28E30</td>
<td>2.7</td>
<td>3.8</td>
<td>5.2</td>
<td>B3.6</td>
</tr>
<tr>
<td>3</td>
<td>1993-Oct-21</td>
<td>S15W08/(7605)</td>
<td>8.1</td>
<td>4.4</td>
<td>6.0</td>
<td>B2.2</td>
</tr>
<tr>
<td>4</td>
<td>1994-Jan-05</td>
<td>S10W22/7647</td>
<td>25</td>
<td>250</td>
<td>46</td>
<td>M1.0</td>
</tr>
<tr>
<td>5</td>
<td>1994-Feb-20</td>
<td>N01W00/7671</td>
<td>14</td>
<td>370</td>
<td>22</td>
<td>M4.0</td>
</tr>
<tr>
<td>6</td>
<td>1994-Sep-05</td>
<td>S05W03/7773</td>
<td>8.7</td>
<td>19</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1997-Oct-20</td>
<td>S30E43</td>
<td>5.0</td>
<td>19</td>
<td>12</td>
<td>B1.2</td>
</tr>
<tr>
<td>8</td>
<td>1998-Sep-20</td>
<td>N20E70/8340</td>
<td>11</td>
<td>570</td>
<td>41</td>
<td>M1.8</td>
</tr>
<tr>
<td>9</td>
<td>2000-Jan-19</td>
<td>N08W18/8829</td>
<td>2.8</td>
<td>4.2</td>
<td>8.1</td>
<td>C1.4</td>
</tr>
<tr>
<td>10</td>
<td>2000-May-08</td>
<td>S21W03</td>
<td>4.0</td>
<td>9.0</td>
<td>9.8</td>
<td>B6.8</td>
</tr>
</tbody>
</table>

* From the left column, event number, time of H$\alpha$ filament disappearance, the approximate location of the filament with the NOAA number for a case in which the filament is belonging to an active region, the mechanical energy and thermal energy release rates per unit volume ($10^{-4}$ erg cm$^{-3}$ s$^{-1}$), the mean photospheric magnetic field strength below the soft X-ray events (G), the peak soft X-ray flux of the associated arcade formation obtained by GOES satellite, respectively. We left a blank for the soft X-ray flux of the 1994-Sep-05 event because we have no GOES data for this event.

Fig. 3. Line-of-sight velocity of the point A of the 2000-May-08 event plotted as a function of time at the top-left panel in which negative values correspond to approaching motion. The upward and the total velocities of the same point are given at the top-right and bottom-left panels, respectively.
potential energy, gives a total mechanical energy of $e_{mc}(A, t_1) = 2.3 \text{ erg cm}^{-3}$. Since the start time of acceleration and the time of its disappearance of the element are $t_0 = 04:17$ UT and $t_1 = 05:18$ UT, respectively, we obtained its mechanical energy gain rate to be $\dot{e}_{mc}(A) = 6.3 \times 10^{-4} \text{ erg cm}^{-3} \text{ s}^{-1}$. This value is used together with those of the other trajectories which are judged to be ejected into the interplanetary space to calculate the mean value of the mechanical energy gain rate, $\dot{e}_{mc}$, of all eruptive elements. The calculated results for all events are listed in the fourth column of table 1.

### 3.2. Thermal Energy Release Rate

The mean increasing rate of thermal energy per unit time ($\dot{e}_{th}$) is calculated in the following procedure based on Isobe et al. (2002). (i) We select an SXT image (obtained at $t = t_{S0}$) without saturated pixels. (ii) We select two SXT images taken with a different filter and without saturated pixels before ($t = t_{S1} < t_{S0}$) and after the first image ($t = t_{S2} > t_{S0}$). (iii) We average the intensity of the arcade, and (iv) we derived two temperatures ($T_{S1}, T_{S2}$) and emission measures ($EM_{S1}, EM_{S2}$) from the averaged intensities of the two pairs of images. Finally, we obtain $T$ and $EM$ at $t = t_{S0}$ by a linear interpolation, i.e.,

$$T = \frac{T_{S2} - T_{S1}}{t_{S2} - t_{S1}}(t_{S0} - t_{S1}) + T_{S1}. \tag{3}$$

Then, we calculate the electron number density, $n$, and the mean thermal energy per unit volume, $e_{th}(t)$, with the following equations:

$$n(t) = \sqrt{\frac{EM(t)}{V(t)}}, \tag{4}$$

$$e_{th}(t) = 3n(t)k_bT(t), \tag{5}$$

where the volume $V$ is the area of one pixel on the SXT images multiplied by the line-of-sight length, $l$, which we assume to be equal to the height or the width of the arcade. The calculated temporal variation of the thermal energy density $e_{th}(t)$ of the arcade of the 2000-May-08 event is plotted in figure 4 with diamonds. Since we have PFIs for this event, the time cadence of the images is less than 30 seconds, and we can see the increase in the thermal energy density. We also take the radiative and conductive coolings into account for a more accurate measurement of the released energy. The thermal energy release rate per unit volume, $\dot{e}_{th}$, is therefore given by

$$\dot{e}_{th} = \frac{de_{th}}{dt} + l_r + l_c, \tag{6}$$

where $l_r$ and $l_c$ are the mean radiative and conductive loss rates per unit volume, respectively. They are written as

$$l_r = n^2 Q(T) \simeq 10^{-17.73} T^{-2/3} n^2 \, \text{ (erg s}^{-1} \text{ cm}^{-3})}, \tag{7}$$

$$l_c = \frac{d}{ds} \left( \kappa \frac{d}{ds} T \right) \simeq 9.0 \times 10^{-7} \frac{T^{7/2}}{s^2} \, \text{ (erg s}^{-1} \text{ cm}^{-3}). \tag{8}$$

In the above equations, $Q(T)$ is the radiative loss function for the temperature range of $10^6 < T < 10^7$ (Rosner et al. 1978), $\kappa = 9.0 \times 10^7 \text{ erg s}^{-1}$ the Spitzer thermal conductivity (Spitzer 1956), and $s$ is the half-length of an arcade loop. The time derivative of the observed thermal energy density, $de_{th}/dt$, is obtained by applying the least-squares fitting method to the temporal evolution of the observed thermal energy density, $e_{th}$. Using equations (6)–(8), we then calculated the thermal energy release rate, $\dot{e}_{th}$, per unit volume.

In analyzing an event for which we only have PFIs, we make another assumption to obtain a more appropriate energy release rate. In this case, to derive the energy release rate, we assume that the start time of the soft X-ray event is equal to the time when an Hα ribbon flare first appears on FMT images, and that all of the surplus thermal energy is supplied after onset of the Hα flare. The thermal energy release rates per unit volume are listed in the fifth column of table 1.

### 4. Results and Discussions

In this section, we first present the temporal relation between the evolution of the mechanical energy density, $e_{mc}$, of disappearing filaments and that of the thermal energy density, $e_{th}$, of the associated soft X-ray arcades, using two events that occurred on 2000-May-08 and 1998-Sep-20. In these events, we fortunately have SXT PFIs of the associated arcades with a high time cadence, which is suitable for a detailed study of the temporal variation of the thermal energy densities of the arcades. We then discuss the statistical relation between these two energy increase rates, using the results of all 10 events.

#### 4.1. The Temporal Relation

##### 4.1.1. The 2000-May-08 event

The temporal variation of the mean mechanical energy density, $e_{mc}$, of elements inside the filament is plotted in figure 4. The crosses show the mean kinetic and the squares the mechanical energy density, which is the sum of the kinetic and...
potential energy densities. It is clear that the filament possesses much more potential energy than the kinetic energy. The mean increase rate of $\epsilon_{mc}$ is $\epsilon_{mc} = 6.4 \times 10^{-4} \text{erg cm}^{-3} \text{s}^{-1}$. Since the increase of the mechanical energy density, $\epsilon_{mc}$, can be approximated not by a linear function, we measured the upper and lower limits by dividing the event into several time ranges in which the increase of the total velocity is more than $15 \text{km s}^{-1}$. This is because, as we mentioned in Paper I, the magnitude of the measured velocities is less than $15 \text{km s}^{-1}$. The upper and lower limits of $\epsilon_{mc}$ of this event are $1.8 \times 10^{-3}$ and $3.1 \times 10^{-4} \text{erg cm}^{-3} \text{s}^{-1}$, respectively.

The soft X-ray event associated with this event is a long duration event (LDE), very gradually starting at 05:01 UT and reaching its peak at 06:23 UT according to GOES soft X-ray flux data, shown by the solid line in figure 4. The sharp peak around 07:00 UT is not related to the event in the problem. The start time of the soft X-ray event is more than a half hour after the onset time of the acceleration of the filament, which is consistent with the results by Hundhausen (1997), who found that the onset times of X-ray events commonly lag behind the start times of the acceleration of mass ejections. Since the Yohkoh/SXT observations of the arcade formation (see figure 2) associated with the Hα filament eruption start at 05:55 UT after the Yohkoh night, we do not have enough SXT data that covers the acceleration phase of the Hα filament. The thermal energy density, $\epsilon_{th}$, is plotted in figure 4 with diamonds. It increases within the specified time range, and their mean increase rate is found to be $9.1 \times 10^{-4} \text{erg cm}^{-3} \text{s}^{-1}$.

According to the CME observation by SOHO/LASCO, a CME with a ragged front appeared at 06:50 UT in the LASCO C2 field of view. This time was one and a half hours after disappearance of the Hα filament in FMT images (5:20 UT). The CME velocity obtained by measuring the position of its leading edge on the time-sequenced LASCO images is also shown in the same panel with asterisk marks. This CME is continuously accelerated within the LASCO C2 field of view until 08:20 UT. The acceleration of a CME in LASCO/C2 FOV does not necessarily mean that the CME or the associated prominence has followed the same profile since its onset (Tripathi et al. 2006). Thus, we cannot distinguish whether the acceleration of the filament from its onset of eruption at 04:17 UT continued or not. The increase in the thermal energy release rate during SXT observations from 05:55 UT to 06:26 UT, may be related to the accelerations of both the filament and the CME.

4.1.2. The 1998-Sep-20 event

A filament eruption occurred near the solar east limb on 1998-Sep-20. A series of partial FMT images are displayed in figure 5. The dark filament is initially located to the east of an active region, NOAA 8340, and is visible only in the Hα line center image at 02:00 UT before its disappearance. From 02:40 UT, a two-ribbon flare takes place at the west side of the main body of the filament. This flare is clearly seen on FMT images at 03:09 UT, and we can see that the filament is activated, but still visible, in all images. The filament was lifted up to a height of 30–40 Mm with a maximum upward velocity of $27 \text{km s}^{-1}$ when the flare occurred.

This filament eruption was followed by soft X-ray arcade formation from 02:33 UT with a peak integrated flux of M1.8 class, observed by GOES, which is shown in figure 6 by the solid line. The Yohkoh observation started from 02:40 UT, and one of the PFIIs of this flare is shown in the bottom-right panel of figure 5. The temporal variations of the mean mechanical energy density, $\epsilon_{mc}$, of the filament and the mean thermal energy density, $\epsilon_{th}$, of the arcade are also shown in figure 6. The thermal energy density, $\epsilon_{th}$ (diamonds), rises and then falls almost consistently with the soft X-ray flux (the solid line). The mean release rate of the thermal energy per unit volume, $\epsilon_{mc}$, during its rise phase (from 02:40 UT to 02:56 UT) was calculated to be $\epsilon_{th} = 0.57 \text{erg cm}^{-3} \text{s}^{-1}$. Similar to the thermal energy density, the mean mechanical energy of the filament, $\epsilon_{mc}$, shows a notable increase with a maximum rate of $\epsilon_{mc} = 2.7 \times 10^{-4} \text{erg cm}^{-3} \text{s}^{-1}$ during the rise phase of the soft X-ray event from 02:33 UT.

The growth of both thermal and mechanical energy densities, however, are not in coincidence after 02:42 UT. Although the thermal energy density, $\epsilon_{th}$, increases until its peak at 02:56 UT, the mean mechanical energy density of the filament, $\epsilon_{mc}$, does not show any significant enhancements until 03:08 UT. This corresponds to a temporary suspension of filament motion from 02:42 UT. Then, from 03:08 UT when the thermal energy density was slowly decaying, the “eruptive” part of the filament started to be accelerated upward again and disappeared, thus causing an increase in the mechanical energy density, $\epsilon_{mc}$. One should notice that the increase rate of the mechanical energy density in the later phase is up to $\epsilon_{mc} = 1.9 \times 10^{-3} \text{erg cm}^{-3} \text{s}^{-1}$, and this is roughly an order of magnitude larger than the maximum increase rate in the initial phase of this event. The contribution of kinetic energy of the filament to the mechanical energy is very large in the initial acceleration phase until 02:42 UT, though it then becomes negligible compared to the potential energy, since the filament ceased its motion at 02:42 UT. This result agrees with the result of the 2000-May-08 event, and indicates that the potential energy is very important during the acceleration phase of a filament eruption.

Though the evolutions of $\epsilon_{mc}$ and $\epsilon_{th}$ in the rise phase of the soft X-ray event is temporally in coincidence, the temporal variations of the thermal energy density decreases during an abrupt increase of the mechanical energy density, $\epsilon_{mc}$, from 03:08 UT. According to the GOES X-ray flux, however, the soft X-ray flux increases again from 03:45 UT, and reaches a second peak of C8.0 at 03:57 UT. The FMT observations also show another Hα two ribbon flare at the same site associated with an enhancement in the soft X-ray flux just after the filament disappeared from the FMT field of view. Increases in its mechanical energy from 02:32 UT and 03:08 UT may correspond to not only the soft X-ray events from 02:33 UT, but also to events from 03:45 UT.

4.2. The Statistical Relation

The relation between the derived mean mechanical energy gain rates of the Hα filaments per unit volume, $\epsilon_{mc}$, and the thermal energy release rate per unit volume, $\epsilon_{th}$, obtained with SXT data for all 10 events are displayed in figure 7. We also give rough values of the mean magnitude of photospheric magnetic field beneath the arcade $B_f$ in the figure. This relation clarifies that (i) the thermal energy release rates and the mechanical energy gain rates have a positive correlation and
Fig. 5. FMT and SXT observations of the event on 1998-Sep-20. This event occurred near the east limb, followed by an intense Hz (FL) and soft X-ray flares. The bottom-right panel is one of the SXT PFIs which are used to derive the thermal energy densities shown in figure 6. The filament is located between the arrows shown in the top-left panel with an abbreviation of “FIL”. The SXT field of view of the bottom-middle panel is indicated by the white box in the left-bottom panel and the white box in the bottom-middle panel stands for the SXT PFI field of view.

(ii) more energy is supplied for events with stronger photospheric magnetic field strengths. To calculate the mean value of the mechanical energy gain rate of all the eruptive elements, we used the value of one representative element. From observations it appears that different parts of a filament move with different speeds during an eruption. The velocity differences of each element would affect the kinetic energy of the filament, and in turn the mean mechanical energy, but we believe that such effects are included in the errors of the measured velocities (15 km s\(^{-1}\), mentioned in Paper I).

The most probable correlation between the increase rates of the thermal energy of the coronal arcades and the mechanical energy of filaments derived by the least-squares fitting method is shown by the dashed line in figure 7, which represents the relation \( \epsilon_{\text{th}} \propto \epsilon_{\text{mech}}^{1.9} \). In order to interpret this relation, we compile an order-of-magnitude analysis of both the energy increase rates by assuming that the stored magnetic energy is carried by the Poynting flux to the magnetic reconnection region, and is converted into the observed thermal energy, and a filament is accelerated by the Lorentz force. Then, for the increase rate of thermal energy, equation (6) can be rewritten as

\[
\epsilon_{\text{th}} = 2 \frac{B^2}{4\pi L} v_{\text{in}},
\]

where \( B \), \( v_{\text{in}} \), and \( L \) are the coronal magnetic field strength, the inflow speed, and the length scale of the reconnection region.
Fig. 6. Thermal energy density ($e_{th}$: diamond) of the arcade and the mean mechanical energy of the disappearing filament per unit volume ($e_{mc}$: square) with its kinetic energy density (cross) are plotted as functions of time. The solid line represents the soft X-ray flux observed by GOES. The magnitudes of mechanical and kinetic energy densities are indicated by the righthand axis and the level of class (B, C, M) of soft X-ray flux is given beside the righthand axis.

Fig. 7. Relation between the mechanical energy gain rate per unit volume, $e_{mc}$, and the thermal energy release rate per unit volume, $e_{th}$, for the selected 10 events. The dashed line represents the relation $e_{th} \propto e_{mc}^{1.9}$ obtained by the least-squares fitting method. The rough magnitudes of the mean photospheric magnetic field strength beneath the arcade, $B_I$, are indicated by the circles.

respectively (Isobe et al. 2002). The inflow speed is expressed as $v_{in} = M_A V_A$ with the Alfvén velocity, $V_A$, and the reconnection rate, $M_A$, which characterizes the rate at which the field lines move through the X-type neutral point. The dependence of the reconnection rate, $M_A$, on the magnetic field strength, $B$, differs from slow to fast reconnection mechanisms. For slow and fast reconnections, we consider the Sweet–Parker (Sweet 1958; Parker 1957) and the Petschek (Petschek 1964) mechanisms in which the reconnection rates, $M_A$, are given by the magnetic Reynolds number, $R_m$, as $M_A \approx 1/R_m^{1/2}$ and $M_A \approx (\pi/8) \log R_m$, respectively, by an order-of-magnitude analysis while ignoring the effect of pressure gradients (Priest & Forbes 2000). Since $\log R_m$ is slowly varying, this yields the dependence of the thermal-energy release rate per unit volume, $e_{th}$, on the magnetic field strength, $B$, as $e_{th} \propto B^{2.5}$ and $e_{th} \propto B^2$ for the Sweet–Parker and the Petschek mechanisms, respectively.

For the increase rate of the mechanical energy of a filament, we find that the work done by the Lorentz force per unit time for an element with unit volume inside a filament is $|\mathbf{j} \times \mathbf{B}|/c (v + 1/2 \alpha_p)$, where $v$ is the velocity of the element and $\alpha_p$ is its acceleration value. Since $\alpha_p$ is usually much smaller than the velocity, $v$, except for the very beginning of activation, the dependence of the mechanical energy gain rate of the element on the magnetic field strength, $B$, is approximated as $e_{mc} \propto B^2$. This then yields the relations between two energy release rates as $e_{th} \propto e_{mc}^{1.9}$, where $\gamma = 1.3$ for the Sweet–Parker type reconnection and $\gamma = 1.5$ for the Petschek type. These two order-of-magnitude analyses based on an assumption of the Poynting flux as a source of thermal energy and the Lorentz force as a driving force for the filament, can explain the relation $e_{th} \propto e_{mc}$ as well as the dependence of the two energy increase rates on the magnetic field strengths, $B_I$, which is roughly indicated in figure 7.

5. Summary and Conclusions

Using the Hζ line center, Hζ $\pm$ 0.8 Å images obtained by the Flare Monitoring Telescope (FMT) at Hida Observatory, we derived the complete 3-D velocity field of 10 solar disappearing filaments by measuring both the line-of-sight and transversal velocities. The line-of-sight velocity is obtained by interpreting the temporal variation of contrasts that are created by the intensity difference between the filament and the surrounding chromosphere in the Hζ line center and Hζ $\pm$ 0.8 Å at every pixel on the filament due to the Doppler shift of the Hζ line profile of the filament. The transversal velocity is derived by tracing the internal structures of the filament. These two components of the velocity are combined to yield the complete 3-D velocity field of the filament.

This new method is described in more detail in our previous study (Paper I), and enables us to measure the temporal variations of the kinetic and potential energies, which are difficult to obtain with coronagraph observations or ordinary Hζ observations in the Hζ line center alone. We also calculated the released thermal energy density of the associated flare arcades by applying the filter ratio method (Hara et al. 1992) to Yohkoh/SXT data. In order to make the more accurate measurement of the thermal energy release rate, we took the radiative and conductive coolings into account. Since previous studies on the energetics of mass ejections and flare/arcade formations have been done in terms of the speed of mass ejections and the intensity of the associated soft X-ray events, this study is the first one that presents the real energetic relation between them.

The temporal variations of the mechanical energy density, $e_{mc}$, and the thermal energy density, $e_{th}$, were compared for two events (the 2000-May-08 and the 1998-Sep-20 events) in which we had SXT PFIs. In both events, an increase of the mechanical energy density, $e_{mc}$, seems to always correspond...
to an increase of the thermal energy density, $e_{th}$, though the increase of the mechanical energy density, $e_{mc}$, often temporally precedes the corresponding thermal energy density, $e_{th}$. This temporal disagreement is consistent with our previous study in which the onset times of X-ray events commonly lag behind the start times of the acceleration of mass ejections (Hundhausen 1997). In our 10 data sets, this was also confirmed in 8 events, while in the remaining 2 events, we could not identify the start time of their soft X-ray events.

We next calculated the mean increase rate of the mechanical energy density, $e_{mc}$, and that of the thermal energy, $e_{th}$, by taking the average of the amounts of increase at both energies over the time during the acceleration and impulsive phases, respectively. We then found a strong correlation between these two energy increase rates, which is shown in figure 7. Since previous studies on the energetic relation between the CME kinematics and the flare strength found only a weak correlation (e.g., Moon et al. 2002), our result is the first evidence of a strong correlation between the mass ejection energy and the flare energy. This also indicates that it is very important to accurately measure the velocity fields, which are free from any projection effect of mass ejections as well as the released thermal energy to seek the actual relation between them.

The correlation between these two energy increase rates shown in figure 7 is best represented by the relation $e_{th} \propto e_{mc}^{1.9}$. This relation can be explained by a simple model in which the stored magnetic energy is carried by the Poynting flux to be converted into thermal energy via magnetic reconnection; also, the Lorentz force plays the primary role in accelerating a filament. The strong dependence of both the energy increase rates on the mean photospheric magnetic field strength, $B_l$, of the arcade region also supports this interpretation.

These above results mean that the energy increase rates of the filaments and the associated flare arcades are related to each other, though they are temporally not coincident. There may be no direct connection between the mechanisms of the accelerating mass ejections and heating the coronal plasma; they merely have a connection in terms of the magnetic field strength. This supports the idea that the filament is accelerated by the outward Lorentz force, while the arcade plasma is heated by some mechanisms closely involved in magnetic reconnection; also, they are two different manifestations of the process that dissipates the stored magnetic energy.

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