Turbulence Effects on Bremsstrahlung Emission in the Turbulent Solar Plasma

Young-Dae JUNG\textsuperscript{1,2} and Daiji KATO\textsuperscript{2}

\textsuperscript{1} Department of Applied Physics, Hanyang University, Ansan, Kyunggi-Do 426-791, South Korea
\texttt{ydjung@hanyang.ac.kr}

\textsuperscript{2} National Institute for Fusion Science, 322-6 Oroshi-cho, Toki, Gifu 509-5292

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Abstract

Turbulence effects on bremsstrahlung emission due to the electron–ion interaction were investigated in the turbulent solar plasma. The effective interaction potential, taking into account the correction factor to the nonlinear dielectric function due to plasma turbulence, and the impact-parameter method were employed in order to obtain the bremsstrahlung radiation cross section in a turbulent plasma as a function of the impact parameter, diffusion coefficient, radiation photon energy, projectile energy, and plasma parameters. It has been shown that the turbulence effects considerably enhance the bremsstrahlung radiation cross section. In addition, it was found that the turbulence effects of the solar atmosphere on the bremsstrahlung cross section increase with increasing radiation photon energy, but decrease with increasing thermal energy. It can be expected that the Landau damping effect on the bremsstrahlung spectrum from the solar corona is less important than that from the solar chromospheres due to temperature jumps. However, the effect of the plasma turbulence on the bremsstrahlung spectrum from the solar chromosphere would be smaller than that from the solar corona.

Key words: atomic processes — plasmas — Sun: atmosphere

1. Introduction

The bremsstrahlung process (Spitzer 1964; Bekefi 1966; Melrose & McPhedran 1991; Jung & Yang 1997; Shevelko 1997; Shevelko & Tawara 1998; Padmanabhan 2000; Gould 2006) has received much attention, since bremsstrahlung emissions due to electron–ion interactions in plasmas have provided useful information on various plasma parameters in astrophysical and laboratory plasmas. The electron–ion bremsstrahlung processes in weakly coupled plasmas have been characterized by the Debye shielding model (Bekefi 1966) of the interaction potential. In addition, it has been known that the far-field interaction potential in collisionless plasmas falls off as the inverse cube of the distance between the projectile electron and the target ion (Yu et al. 1972; Shukla et al. 2006). In a turbulent plasma, it would be expected that the projectile electron would be scattered due to random fluctuating electric fields. Moreover, it has been shown that the reaction of the random electric field fluctuations plays an important role in the binary encounters and interaction potentials in turbulent plasmas (Tsytovich 1972, 1995; Shukla & Spatschek 1973; Sitenko & Malnev 1995).

Recently, a detailed and excellent discussion (Landi & Feldman 2003) on the properties of solar plasmas due to spectra detected by the Solar Ultraviolet Measurements of Emitted Radiation instrument on board the Solar and Heliospheric Observatory satellite has been given. Moreover, they (Landi & Feldman 2003) measured the nonthermal mass motions in the plasma for all of the most abundant elements in the solar corona, and also found that a significant radiative excitation occurs for the O VI and Ne VIII resonance lines. Hence, we can expect that the electron bremsstrahlung with these elements would produce a continuous X-ray spectrum based on the results presented in this work. In addition, the bremsstrahlung process due to elastic collisions of electrons and ions has been discussed concerning the solar atmosphere during flares (Somov 2006). It has also been found that the solar-flare atmosphere where the upper hybrid frequency is equal to the low harmonics of the electron cyclotron frequency is in a turbulent state (Barta & Karlicky 2001). Hence, bremsstrahlung emissions caused by electron–ion interactions in turbulent plasmas would be considerably different from those in nonturbulent plasmas due to random fluctuations of the electric field. Thus, in this paper we consider turbulence effects on the soft-photon bremsstrahlung emission spectrum due to electron–ion interaction in a turbulent plasma. The effective screened potential model (Shukla & Spatschek 1973), including the far-field term obtained from the longitudinal dielectric function associated with turbulent plasmas, has been applied to describe screened electron–ion interactions in a turbulent plasma. Impact parameter analysis (Bekefi 1966; Gould 2006) was employed to investigate turbulence effects on the bremsstrahlung radiation cross section as a function of the impact parameter, diffusion coefficient, radiation photon energy, projectile energy, thermal energy, and Debye length of the plasma.

In section 2 we obtain an expression of the bremsstrahlung cross section in a turbulent plasma. In section 3, we discuss the bremsstrahlung radiation cross section in the turbulent solar plasma. In section 4, we discuss the turbulence effects on the bremsstrahlung cross section. Finally, the conclusion is given in section 5.

2. Bremsstrahlung Cross Section in a Turbulent Plasma

The expression for the low-energy electron–ion
bremsstrahlung cross section (Gould 2006), $d\sigma_{\text{b}}$, is given by integration over $db$,

$$d\sigma_{\text{b}} = 2\pi \int db \, dw_{\omega}(b),$$

where $b$ is the impact parameter and $dw_{\omega}(b)$ is the differential probability of emitting a photon of frequency $\omega$ within $d\omega$. For all impact parameters, $dw_{\omega}(b)$ can be calculated by the Larmor formula (Melrose & McPhedran 1991) for the instantaneous far-field term due to the influence of the random electric field.

The absolute square, $|F_{\omega}(b)|^2$, is then represented by the Fourier coefficients of the components of force parallel, $F_{||\omega}$, and perpendicular, $F_{\perp\omega}$, to the projectile velocity, $v_0$:

$$|F_{\omega}(b)|^2 = |F_{||\omega}(b)|^2 + |F_{\perp\omega}(b)|^2 = |v_0(v_0 \cdot F_{\omega})/v_0^2|^2 + |v_0(v_0 \times F_{\omega}) \times v_0/v_0^2|^2.$$

The remarkably useful analytic form of the effective screened potential (Shukla & Spatschek 1973) of a moving test charge in a turbulent plasma has been obtained by the longitudinal nonlinear dielectric function, including a modification factor (Birmingham & Bornatici 1972), $\exp(-k^2 D t^3/3)$, due to plasma turbulence, where $k$ is the wave number and $D$ the diffusion coefficient. Using the effective screened potential model (Shukla & Spatschek 1973), the interaction potential, $V(r, \gamma)$, between a slowly moving projectile electron, i.e., $v_0 < v_t$, and a target ion with nuclear charge $Ze$, including the far-field term due to the influence of the random electric field fluctuations in a turbulent plasma in the limit $r > \lambda_D$, is then obtained by

$$V(r, \gamma) = \frac{Ze^2}{r} \exp(-r/\lambda_D)$$

$$- \frac{Ze^2}{r} \frac{2 \sqrt{2}}{\sqrt{\pi}} \cos(\lambda_D r) \left( \frac{v_0}{v_t} \right)^2 \left( 1 - \frac{9}{4} \frac{\sqrt{\pi} D r}{v_t} \right),$$

where $r = b + v_0 t$ is the position vector of the projectile electron with respect to the target ion, $\gamma$ the angle between $v_0$ and $r$, $v_t(=\lambda_D \alpha_0)$ the thermal velocity, $\lambda_D$ the Debye length, and $\omega_0$ the plasma frequency. A detailed discussion on the calculation of the diffusion coefficient by a new perturbation theory was given by an excellent paper by Dupree (1966). In this effective interaction potential the effect of the random fluctuation of the electric field is considered based on the plasma dielectric function, which contains resonance bonding (Dupree 1966; Birmingham & Bornatici 1972; Shukla & Spatschek 1973). In addition, nonlinear damping or growing is neglected in this potential (Shukla & Spatschek 1973), since the test-particle velocity is small compared to the thermal velocity in plasmas. The Landau damping frequency width, $\gamma_L$, is assumed to be much smaller than the plasma frequency, $\omega_0$. In weak turbulent plasmas, it has been shown that the diffusion coefficient, $D$, is independent of $t$ (Dupree 1966).

In a magnetized turbulent plasma, an excellent discussion on the test-charge potential has been obtained by Tegeback and Stenflo (1975). A detailed discussion on the Landau damping and plasma turbulence apart from the usual Debye shielding and $r^{-3}$ terms. From equations (3), (4), and (5) with the impact-parameter method, we then obtained the following closed forms of the scaled Fourier coefficients $\tilde{F}_{||\omega} = - [Ze^2/(\pi aZ_0 v_0)]^{-1} (v_0 \cdot F_{||\omega})/v_0$ and $\tilde{F}_{\perp\omega} = - [Ze^2/(\pi aZ_0 v_0)]^{-1} (b \cdot F_{\perp\omega})/b$:

$$\tilde{F}_{||\omega} = \int_0^\infty d\tau \cos \xi \tau \left[ -2 \sqrt{2} \frac{\tau}{\lambda_D} \left( \frac{v_0}{v_t} \right) - \frac{4 D r^2}{v_t^2} + \frac{1}{f^2} \right]$$

$$+ \int_0^\infty d\tau \sin \xi \tau \left( \frac{\tau}{f^2} + \frac{\pi}{\lambda_D} \right) \exp(-\tau/\lambda_D).$$

$$\tilde{F}_{\perp\omega} = \int_0^\infty d\tau \cos \xi \tau \left( \frac{\hat{b}}{f^2} + \frac{\hat{b}}{\pi \lambda_D} \right) \exp(-\tau/\lambda_D)$$

$$+ \int_0^\infty d\tau \sin \xi \tau \left[ -2 \sqrt{2} \frac{\tau}{\lambda_D} \left( \frac{v_0}{v_t} \right) - \frac{4 D r^2}{v_t^2} \right]$$

$$+ \int_0^\infty d\tau \sin \xi \tau \left( \frac{\tau}{f^2} + \frac{\pi}{\lambda_D} \right) \exp(-\tau/\lambda_D).$$

where $\tau(= v_0 t/a_Z)$ is the scaled time, $a_Z(= a_0/Z)$ the Bohr radius of the hydrogenic ion with nuclear charge $Ze$, and $\alpha_0(= (h^2/m_e^2))$ the Bohr radius of the hydrogen atom; $\xi(= \omega a_Z/v_0, t(= r/a_Z)$ is the scaled distance, $\lambda_D(= \lambda_D/a_Z)$ the scaled Debye length, $b(= b/a_Z)$ the scaled impact parameter, and $D(= D a_Z/v_0^2)$ the scaled diffusion coefficient. The low-energy electron–ion bremsstrahlung cross section is then represented by the scaled Fourier coefficients of the force,

$$d\sigma_{\text{b}} = \frac{16 \alpha^2 a^3 d\omega}{3 E^2} \int db \, b \left( |\tilde{F}_{||\omega}|^2 + |\tilde{F}_{\perp\omega}|^2 \right),$$

where $\alpha(= e^2/(4\pi \epsilon_0 \hbar c))$ is the fine-structure constant, $E \equiv E/(Z^2 R_y)$, $E(= m v_0^2/2)$ the kinetic energy of the projectile electron, and $R_y(= e^2/(2\hbar^2) \approx 13.6 \text{eV})$ the Rydberg constant.

3. Bremsstrahlung Radiation Cross Section in the Turbulent Solar Plasma

Since we are interested in the soft-photon bremsstrahlung spectrum from the solar plasma, we consider the condition $v_0 < v_t$ in an effective interaction potential. It would be expected that the Landau damping and plasma turbulent effects would increase with increasing the projectile...
velocity. It has been shown that the continuous bremsstrahlung spectrum could be investigated through the bremsstrahlung radiation cross section (Jackson 1999), defined as \( d\chi_b / d\xi \equiv (d\sigma_b / h\omega)h\omega \). After some mathematical manipulations, the scaled differential bremsstrahlung radiation cross section, \( \partial_{\xi,\tilde{b}} \equiv d^2\chi_b / (d\tilde{e} d\tilde{b})/\pi\alpha_0^2 \), in units of \( \pi\alpha_0^2 \) due to the electron–ion interaction in a turbulent plasma including the influence of the random electric field fluctuations is then found to be

\[
\partial_{\xi,\tilde{b}} \tilde{\chi}_b = \frac{16 \alpha_0^3}{3\pi \tilde{b}} \left( \int_0^\infty \right. d\tau \cos \left( \frac{\tilde{e}\tau}{2\sqrt{E}} \right) \left. \tilde{\lambda}_{\tilde{D}}^2 \right) \\
\times \sqrt{\frac{E}{E_t} \left( \frac{4\tau^2}{(b^2 + \tau^2)^3} - \frac{1}{(b^2 + \tau^2)^2} \right)} \\
-9\tilde{D} \left[ \frac{3\tau^2}{(b^2 + \tau^2)^{3/2}} - \frac{1}{(b^2 + \tau^2)^{3/2}} \right] \right)^2 \\
+ \left( \int_0^\infty \right. d\tau \sin \left( \frac{\tilde{e}\tau}{2\sqrt{E}} \right) \left. \tilde{\tau} \right) \left[ \frac{1}{(b^2 + \tau^2)^{3/2}} \right] \\
+ \left( \int_0^\infty \right. d\tau \cos \left( \frac{\tilde{e}\tau}{2\sqrt{E}} \right) \left. \tilde{b} \right) \left[ \frac{1}{(b^2 + \tau^2)^{3/2}} \right] \\
+ \left( \int_0^\infty \right. d\tau \sin \left( \frac{\tilde{e}\tau}{2\sqrt{E}} \right) \left. \tilde{D} \right) \left[ \frac{4\tilde{b}\tau}{\sqrt{\pi} (b^2 + \tau^2)^{5/2}} \right] \\
-9\tilde{D} \left[ \frac{3\tilde{b}\tau}{(b^2 + \tau^2)^{5/2}} \right]^2 \right) \end{align}

(9)

Here, the characteristic bremsstrahlung parameter, \( \xi \), has been replaced by \( \tilde{\xi} = \tilde{\xi} / \sqrt{2E} \) in the nonrelativistic case; \( \tilde{\xi} \equiv \frac{\tilde{e}}{(Z^2 R)} \), \( \tilde{\xi} \equiv \frac{\tilde{E}}{(Z^2 R)} \), \( \tilde{\xi} \equiv (k_B T) \) is the thermal energy, \( k_B \) the Boltzmann constant, and \( T \) the plasma temperature. This bremsstrahlung radiation cross section [equation (9)] is anticipated to be quite trustworthy since the classical bremsstrahlung cross section, \( d\sigma_b / h\omega \), and effective interaction potential, \( V(r, \gamma) \), are both dependable for low projectile velocities. If we neglect the reaction of the turbulent fluctuations on the electron–ion bremsstrahlung process, the scaled differential bremsstrahlung radiation cross section, \( \partial_{\xi,\tilde{b}} \tilde{\chi}_b \), becomes

\[
\partial_{\xi,\tilde{b}} \tilde{\chi}_b = \frac{16 \alpha_0^3}{3\pi \tilde{b}} \left( \int_0^\infty \right. d\tau \cos \left( \frac{\tilde{e}\tau}{2\sqrt{E}} \right) \left. \tilde{\lambda}_{\tilde{D}}^2 \right) \\
\times \sqrt{\frac{E}{E_t} \left( \frac{4\tau^2}{(b^2 + \tau^2)^3} - \frac{1}{(b^2 + \tau^2)^2} \right)} \\
+ \left( \int_0^\infty \right. d\tau \sin \left( \frac{\tilde{e}\tau}{2\sqrt{E}} \right) \left. \tilde{\tau} \right) \left[ \frac{1}{(b^2 + \tau^2)^{3/2}} \right] \\
+ \left( \int_0^\infty \right. d\tau \cos \left( \frac{\tilde{e}\tau}{2\sqrt{E}} \right) \left. \tilde{b} \right) \left[ \frac{1}{(b^2 + \tau^2)^{3/2}} \right] \\
+ \left( \int_0^\infty \right. d\tau \sin \left( \frac{\tilde{e}\tau}{2\sqrt{E}} \right) \left. \tilde{D} \right) \left[ \frac{4\tilde{b}\tau}{\sqrt{\pi} (b^2 + \tau^2)^{5/2}} \right] \\
-9\tilde{D} \left[ \frac{3\tilde{b}\tau}{(b^2 + \tau^2)^{5/2}} \right]^2 \right) \end{align}

\[
+ \left| \int_0^\infty \right. d\tau \cos \left( \frac{\tilde{e}\tau}{2\sqrt{E}} \right) \left. \tilde{b} \right| \left[ \frac{1}{(b^2 + \tau^2)^{1/2}} \right] \\
+ \left( \int_0^\infty \right. d\tau \sin \left( \frac{\tilde{e}\tau}{2\sqrt{E}} \right) \left. \tilde{\lambda}_{\tilde{D}}^2 \right) \left[ \frac{E}{E_t} \sqrt{\pi} \left( \frac{4\tilde{b}\tau}{(b^2 + \tau^2)^{1/2}} \right)^2 \right] \\
+ \left( \int_0^\infty \right. d\tau \sin \left( \frac{\tilde{e}\tau}{2\sqrt{E}} \right) \left. \tilde{D} \right) \left[ \frac{4\tilde{b}\tau}{\sqrt{\pi} (b^2 + \tau^2)^{1/2}} \right] \right) \end{align}

(10)

4. Turbulent Effects

In order to investigate the turbulence effects on the bremsstrahlung emission spectrum, we chose \( \tilde{E} < 1 \), i.e., \( \nu_0 < Z\alpha c \), since the integral form of the bremsstrahlung cross section [equation (8)] and the impact-parameter method are known to be reliable for \( \epsilon < E \leq Z^2 R \). In addition, we chose the thermal energy as \( \tilde{E}_t > \tilde{E} \), since the effective interaction potential, \( V(r, \gamma) \), is dependable for \( \nu_i > \nu_t \). The radiation photon energy due to the bremsstrahlung process in solar plasmas, such as the solar chromosphere and corona, would be mainly soft-X-ray and UV radiations, since the temperature range is about \( 10^4 - 10^6 \) K (Cox 2000).

Figure 1 shows the scaled differential bremsstrahlung radiation cross section, \( \partial_{\xi,\tilde{b}} \tilde{\chi}_b \), as a function of the scaled impact parameter, \( \tilde{b} \), for various values of the diffusion coefficient, \( \tilde{D} \). As can be seen in this figure, it is found that the turbulence effects of the solar atmosphere significantly enhance the bremsstrahlung radiation cross section. Figure 2 represents a surface plot of the function \( F_{TE}(\tilde{\xi}) \equiv \partial_{\xi,\tilde{b}} \tilde{\chi}_b / \partial_{\xi,\tilde{b}} \tilde{\chi}_b \) of the turbulence effects on the bremsstrahlung radiation cross section as a function of the scaled radiation photon energy, \( \tilde{\xi} \), and scaled impact parameter, \( \tilde{b} \). In addition, figure 3 represents the function \( F_{TE} \) of the turbulence effects as a function of the scaled impact parameter, \( \tilde{b} \), for various values of the radiation photon energy, \( \tilde{\xi} \). From these figures, the turbulence effects of the solar atmosphere on the electron–ion bremsstrahlung process are found to increase with increasing radiation photon energy. In addition, it has been shown that the turbulence effects on the bremsstrahlung process are less important near the target ion and, however, become significant with increasing the impact parameter. Thus, the turbulence effects would be important for bremsstrahlung emissions due to distant encounters of electrons and ions in a turbulent plasma. Figure 4 shows the function of turbulence effects, \( F_{TE} \), as a function of the scaled impact parameter, \( \tilde{b} \), for various values of the thermal energy, \( \tilde{E}_t \). For example, it has been found that the turbulence effect enhances the bremsstrahlung radiation cross sections by more than 10% in solar plasmas in soft-bremsstrahlung radiation for \( \tilde{b} = 60 \). In addition, it is found that the turbulence effects on the bremsstrahlung cross section decrease with an increase of the thermal energy of the plasma. It has also been found that the Landau damping effect on the electron–ion Coulomb bremsstrahlung process in dense Maxwellian plasmas caused by the imaginary part of the plasma dielectric function \( \Im \varepsilon_p \) (Jung & Kim 2001) reduces the bremsstrahlung cross section in plasmas. In addition, it was shown that the Landau damping effect in the soft-photonic case is more important than that in the hard-photonic...
plasma frequency, less than 5%, since photons whose frequency is lower than the plasma frequency cannot be propagated in the plasma. It is less important than that from the solar chromosphere due to the temperature dependence of the effective interaction potential. In addition, the quantum degeneracy effects can be neglected in weakly coupled solar plasmas, since $\hbar \omega_p < E_t$. According to the plasma shielding effects, the X-ray bremsstrahlung spectrum would be suppressed due to the finite range of the electron–ion binary interaction, i.e., the Debye length. Hence, the bremsstrahlung radiation due to electron–ion encounters in strongly coupled plasmas (Jung & Yoon 2000) is found to be weaker than that in weakly coupled plasmas. In addition, we have found that the turbulence effects enhance the bremsstrahlung radiation cross section, since the plasma turbulence produces additional acceleration and the electric field of the projectile electron in the bremsstrahlung process.

5. Conclusion

In the present work, we investigated the turbulence effects on the soft-photon bremsstrahlung emission due to the electron–ion interaction in the turbulent solar plasma. We employed the effective interaction potential, while taking into account the correction factor to the nonlinear dielectric function due to plasma turbulence, and the impact-parameter method in order to obtain the bremsstrahlung radiation cross section in a turbulent plasma as a function of the impact parameter, diffusion coefficient, radiation photon energy, projectile energy, and...
plasma parameters. From this work, it has been shown that the turbulence effects considerably enhance the bremsstrahlung-radiation cross section. In addition, we found that the turbulence effects of the solar atmosphere on the bremsstrahlung cross section increase with increasing radiation photon energy, but decrease with increasing thermal energy. Thus, we can understand that the turbulence effects caused by the random fluctuations of the electric field play important roles in the electron–ion bremsstrahlung process in the turbulent solar plasma. These results would provide useful information about the turbulence effects on the radiation and collision processes in the turbulent solar plasma.

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