Dirac's Prediction of the Positron: A Case Study for the Current Realism Debate

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Much debate has ensued regarding the challenge to scientific realism provided by consideration of certain problematic episodes of theory change in the history of science. This paper contends that there is an interesting case which has been overlooked in this debate, namely the prediction of the positron by Dirac from his 'hole' theory, and its subsequent replacement by a theory which failed to contain a central, and essential, theoretical posit: the 'Dirac sea' of negative energy electrons. Accounting for this case with the 'divide and conquer' strategy of contemporary scientific realism proves particularly troublesome, even for the structural realist.

1. Introduction

Scientific realism is often described as the attitude that we should have towards our most successful scientific theories, namely that they are, in an appropriate sense, *true.* That is, they get things basically right about the world. Currently, local relativistic quantum field theories comprise a large class of our best theories. We take them to provide the best description of reality we have in circumstances where energies are large enough to require relativistic treatment but small enough such that gravitational effects can be neglected. To a high degree of accuracy, The Standard Model (a renormalizable local Yang-Mills gauge theory with the internal symmetry group $SU(3) \times SU(2) \times U(1)$) provides a description of all the forces

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and phenomena encountered so far in terrestrial physics, save gravity. One of the central characteristics of these theories is the ubiquity of antimatter: each particle (or better, quantum matter field) possesses a partner with, roughly, the same mass and opposite charge.[1]

The existence of antimatter was predicted by Dirac in 1931 from formal properties of the relativistic electron equation he had discovered in 1928, on the basis of arguments that were almost entirely theoretical. Since the Dirac Equation was the first successful application of the requirement of relativistic invariance to quantum mechanics, *prima facie* it seems Dirac's successful prediction of antimatter embodies just the characteristics that the realist finds most compelling: a unification of two theories with disparate empirical support resulting in the prediction of entirely novel phenomena. However, the 'hole' theory he based his arguments upon was later supplanted, along with its essential posit: a completely filled 'sea' of negative energy electrons.

The realism debate has been characterized as a pull between two competing arguments: the No Miracles Argument (attributed to Putnam 1975), which argues from the success of scientific theories to their truth, and the Pessimistic Meta-Induction (attributed to Laudan 1981), which points to instances where theories which the realist would have taken as true have been replaced by theories failing to contain their central theoretical posits. The task for the contemporary realist in making sense of the historical record is, therefore, to construct a philosophical account of science that respects the realist intuition of the NMA, while avoiding falling foul of the cases adduced in support of the PMI.

This paper contends that philosophers of science engaged in this project have failed to consider a rich historical episode that deserves their attention: Dirac's prediction of antimatter, and the subsequent replacement of hole theory by local relativistic quantum field theory (the Quantum ElectroDynamics of Feynman, Schwinger and Tomonaga). It is my contention that Dirac's negative energy electron sea deserves a place in Laudan's famous 'laundry list' of discarded theoretical posits alongside caloric and the ether, and is a case worthy of serious consideration by the current generation of scientific realists due to its close relation (genetically and theoretically) to our current best physical theories.

One reason which might explain why this example of theory replacement has been overlooked is the lack of a suitable historical resource. Although there are many which cover some of the requisite territory (Moyer

1. For a clear account of the "naïve" view of antimatter, and a mathematically sophisticated replacement see Baker and Halvorson (2010). On this topic see also Wallace (2009).

1981a, 1981b; Pais 1986; Darrigol 1988; Kragh 1990; Miller 1994; Schweber 1994; Roque 1997; Mehra and Rechenberg 2001), the role of hole theory in the transition from non-relativistic quantum mechanics to the quantum electrodynamics of the post-war years has not yet been subject to a comprehensive study. The first aim of this paper, therefore, is to present the historical details in a readily digestible form, which demonstrates to the philosopher of science the relevance of this episode for the scientific realism debate.

The second aim is to present a philosophically motivated account of the challenge that the success of hole theory, closely followed by its replacement, poses to scientific realism. One of the guiding assumptions is that the historical and theoretical details matter. In other words, it is not enough to consider theories in the abstract, as set-theoretic constructions or otherwise, since doing so would obscure or ignore distinctions and developments which are relevant to the questions scientific realism seeks to address. However, another reason why this episode might have been overlooked is the continued controversy surrounding the interpretation of relativistic quantum field theory, and quantum mechanics is general. Insofar as it is possible, these general concerns will be bracketed in what follows, but it is assumed that it is incumbent on the realist to account for the success of these theories in a way that also respects their historical development.

The philosophical work in which Dirac's prediction has featured most prominently—namely as a case study for Hanson (1963) in his work on the 'logic of discovery'—could be taken to question whether Dirac's prediction of his *anti-electron* deserves to be considered as a prediction of the *positron* soon to be observed by C. D. Anderson. Although it will become apparent that there is a degree of fortuity about the chain of reasoning which led to Dirac's prediction, I will maintain that there is no reason not to accord to it the status of a genuine prediction of antimatter. However, as will become apparent, the details of the logic of discovery and its replacement by a different 'logic of justification' are important for the sorts of considerations that feature in asking the central question, namely: is scientific realism a plausible explanation of the empirical and predictive success of the Dirac equation, and its hole theoretic interpretation?

The paper is organized as follows. In Section 2, I present the relevant history pertaining to Dirac's prediction and the subsequent replacement of hole theory by QED. Although much of the story is well known, Weyl's role in the prediction of antimatter is often underappreciated and proves to be crucial in understanding Dirac's chain of reasoning. In Section 3, I use this case to mount a Laudan-style challenge to the scientific realist. I then proceed to assess the prospects for success of two well known responses of the contemporary scientific realist to this kind of argument. Section 4 discusses what I term *restrictive realism,* exemplified by Psillos (1999), while Section 5 considers how structural realism might fare in this case. I argue that each faces difficulties in accounting for the success of hole theory, but it seems that structural realist has more room to maneuver. The reader who wishes to skip the historical detail and go straight to the philosophical discussion may proceed directly to Section 3, aided by this schematic account of the history.

Chronology of Developments Leading to Dirac's Prediction

1. Relativistic invariance of the wave equation (Klein 1926; Gordon 1926).
2. Consistency with Dirac's quantum mechanics (his transformation theory of Dirac 1927b).
   → Linearity of the wave equation in time and momentum.
   → The Dirac Equation (Dirac 1928).
3. Klein paradox (Klein 1929).
4. Pauli Exclusion Principle (phenomenologically justified).
   → Filled negative energy states (the 'Dirac sea').
   → 'Holes' in the sea are positive energy particles (hole theory). The holes behave as positively charged particles with positive energy.
   → Holes are protons. Pair production/annihilation. (Dirac 1930).
5. Completely filled vacuum state (Oppenheimer 1930).
6. Symmetrical masses in the presence of interactions (Weyl 1931).
   → Prediction of 'anti-electron' (Dirac 1931).

2. Historical Case Study: The 'Hole' Story

1. The Magic: The Dirac Equation

With the completion of his transformation theory (1927b), Dirac regarded the interpretation of quantum mechanics—both kinematically and dynamically—as essentially fixed. By extending and modifying classical mechanics, he had established a systematic means for giving unambiguous answers to all experimental questions that could be posed of an ensemble of identical quantum systems (the probabilistic nature of the answers provided by the application of this recipe to a single system never really troubled Dirac). With this work completed, he devoted his energies to the task of finding a satisfactory relativistic treatment of the electron.[2]

Relativity was Dirac's early passion, and he had been concerned with

2. This section draws on Dirac (1977), Kragh (1990), and Mehra and Rechenberg (2001).

the relativistic treatment of problems in quantum theory since he first became a graduate student at St. John's, Cambridge in 1925. This concern led to his relativistic treatment of the Compton effect (1926, 1927a), in which radiation is emitted by an electron in electromagnetic field. A note added in proof to the later paper indicates his awareness of Gordon's (1926) work, which utilized the Klein-Gordon equation to calculate the field produced by a moving electron.

The Klein-Gordon equation was the result of a straightforward relativistic generalization of the Schrödinger equation, obtained by applying the quantum substitutions for the momentum and energy to the relativistic mass-energy equation $E^2 = p^2c^2 + m^2c^4$. In fact, this equation had been written down by Schrödinger himself as early as 1925, but was discarded due to its failure to reproduce the correct hydrogen spectrum. The Schrödinger equation for a free massive particle (in one dimension) is obtained from the classical expression for the Hamiltonian as follows,

$$H = \frac{p^2}{2m};\; p \rightarrow \hat{p} = i\hbar\frac{\partial}{\partial x}$$

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = \hat{H}\psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t)\,.$$

The (free) Klein-Gordon equation results from taking the relativistic energy and performing the same substitutions of momentum and energy operators for their classical counterparts. Since classically the Hamiltonian represents the energy of the particle, this suggests that the operator on the left hand side above is a suitable substitution for the energy *E.* So, in three dimensions this time, we have,

$$p^2 \rightarrow \hat{p}^2 = -\hbar\nabla^2 = -\hbar\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right);\; E \rightarrow \hat{E} = i\hbar\frac{\partial}{\partial t}\,;$$

$$E^2 = p^2c^2 + m^2c^4 \rightarrow \hbar^2\frac{\partial^2}{\partial t^2}\psi(x,t) = (\hbar^2c^2\nabla^2 - m^2c^4)\psi(x,t)\,.$$

While this wave equation retains the Lorentz invariance of the original expression, from the perspective of Dirac's transformation theory it has a major flaw: whereas the Schrödinger equation is first-order in the time derivative, the Klein-Gordon equation is second-order. This meant that the interpretation of transformation theory that Dirac had developed could not be applied in full generality to the Klein-Gordon equation since the evolution of the system was no longer determined by the specification of a state at a single time.

Convinced of the validity of transformation theory, Dirac set about finding a relativistic wave equation for the electron which would satisfy this requirement while agreeing with the Klein-Gordon equation in an appropriate way. Since the equation he sought would be first-order in time, i.e. linear in the zeroth component of the 4-vector momentum p_0, he took relativistic symmetry to imply it must also be linear in the vector momenta p_r, $r = 1,2,3$. This meant that a satisfactory equation would be a linearized version of the Klein-Gordon equation, which would then result from taking the (Minkowski) product of the linear equation with itself. Speaking loosely, such an equation would be the square-root of the Klein-Gordon equation. So, absorbing factors of c into p_0, the wave equation he sought is of the form,

$$(p_0 + \alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p_3 + \alpha_4 mc)\psi = 0.$$

Although the four coefficients α_μ were initially unknown, the requirement of consistency with the Klein-Gordon equation placed significant constraints upon them since all cross-terms involving more that one momentum p_r must vanish, while squaring to unity. That is,

$$\alpha_\mu^2 = 1;\ \alpha_\mu\alpha_\nu + \alpha_\nu\alpha_\mu = 0, \text{ for } \mu \neq \nu.$$

The failure of commutativity displayed by these relations meant that the coefficients α_μ couldn't be ordinary numbers (c-numbers in Dirac's language). However, Dirac noticed that the 2×2 Pauli spin matrices had exactly these properties, although they would only serve to linearize the massless wave equation. The problem was solved when Dirac realized that 4×4 matrices would afford a representation of the algebraic relations imposed by these conditions on the α_μ. In doing so Dirac had found an equation of just the form he desired, which could be written in terms of a Hamiltonian operator, and could be shown to retain the Lorentz invariance of the Klein-Gordon equation. That is, the Dirac Equation (here without an electromagnetic potential) which could be written as

$$i\hbar\frac{\partial}{\partial t}\psi = \hat{H}_d\psi;\ \hat{H}_d = c\boldsymbol{\alpha}\cdot\nabla + \alpha_4 mc^2\ .$$

However, the interpretation in terms of transformation theory was not entirely straightforward. Whereas the Schrödinger equation applies to a single wave function, the 4×4 matrices appearing in the Dirac Equation meant that ψ represented not a single wavefunction but a vector of 4 wave functions, $\psi_n(\mathrm{x},t)$. Two of these were enough to account for the known properties of electrons with opposite spin—a triumph—but this left two more solutions (also with opposite spin), and *negative* energy.

This property is inherited from the classical expression for relativistic energy $E = \pm\sqrt{p^2c^2 - m^2c^4}$, which provides both positive and negative solutions for a given mass and momentum (and shared by the Klein-Gordon equation). As Dirac (1928) notes, his linear wave equation had only managed to solve the first of the interpretative difficulties associated with the Klein-Gordon equation: the problematic negative energy solutions remained, only now there were two of them. It was convenient to ignore these negative energy solutions for the initial purposes of calculation, but it soon became apparent that they could not be discounted.

However, the success that Dirac's breakthrough brought was remarkable. At a stroke, Dirac had explained not only the spin of the electron, but also derived its gyromagnetic ratio and (by considering the Dirac equation with a central Coulomb potential) calculated the fine structure of the hydrogen atom spectrum, seemingly just from the requirements of Lorentz invariance and conformity with quantum mechanics. These two results were not to be improved upon for 20 years.

2. The Sickness: Negative Energies and the Advent of Hole Theory

Whereas the free Dirac equation seemed to allow positive and negative energy solutions to be considered independently, including interactions into the Hamiltonian led to transitions from positive to negative energy and *vice versa.* In what became known as the 'Klein paradox,' Klein (1929) demonstrated conclusively that the negative energy solutions could not be ignored, and furthermore their presence led to behavior that initially seemed pathological. In the situation he considered, an electron with energy E approaches a step potential from the left (see Figure 1). The electron is represented by an incident plane wave $\psi = e^{(i(px \quad Et))}$, but as such ψ cannot be restricted to a positive energy part alone. As Darwin (1928) had observed, the system of linear equations encapsulated by the Dirac Equation introduced a dependency between the coefficients of the positive and negative energy solutions which entailed the inclusion of a small negative energy term in a plane wave solution. Continuity requirements at the boundary between Region I and II put conditions on the transmitted and reflected waves so that the current density is properly normalized. Now, Klein asked, what percentage of the wave is transmitted or reflected, according to the Dirac Equation?

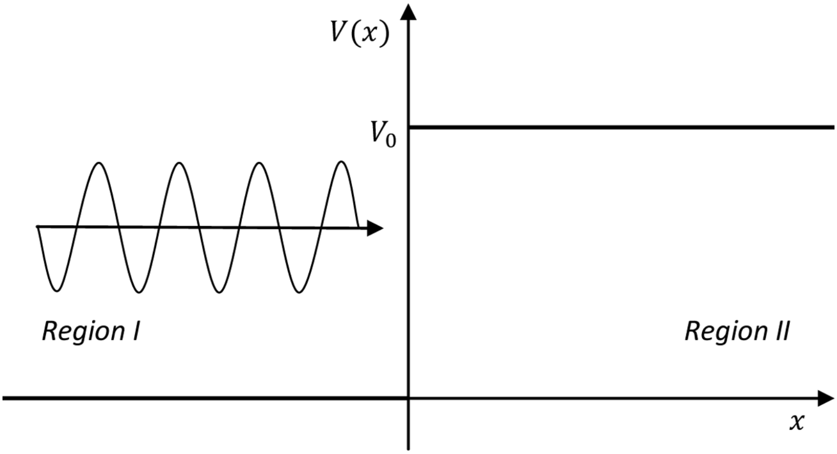

Figure 1. The Klein Paradox (Setup). An electron approaches a step potential from the left.

He found that if $V_0 < E + mc^2$ then the transmitted wave decays exponentially in Region II, with some proportion reflected back,—just as one would expect if it obeyed the Schrodinger equation. However, if $V_0 > E + mc^2$ then the transmitted wave is a plane wave in Region II which, normalized to respect the continuity condition, is directed *toward* Region I, so that the total reflected current appeared to be greater than the total incident current. This result seemed to be pure nonsense, and was deeply troubling from the point of view of contemporary atomic theory which (prior to the discovery of the neutron) explained the neutral charge of the nucleus by the hypothesis of electronic nuclear confinement.

Bohr wrote to Dirac in late 1929 posing this difficulty and expressed his concerns that a wide scale conceptual revolution would be required to resolve it. Dirac confronted the problem in his reply by proposing his 'hole' theory, which interprets the transmitted wave as the current of a *positive* particle moving right. His reasoning appeared as follows:[3]

- Electrons may transition to negative energy states by spontaneous emission of radiation. However, low (negative) energy states will be stable against further descent since to jump back up requires incident high energy radiation.
- Since electrons are fermions they obey the Pauli exclusion principle so a state can be occupied by at most one electron. Suppose

3. See Moyer (1981a) for details of this exchange.

that (nearly) all of the stable low (negative) energy states are occupied. This forces positive energy electrons to remain in positive energy states.

- The negative energy electrons will have uniform (infinite) density so the net electromagnetic field is zero. Only deviations from uniformity will be observable.
- 'Holes' in the negative energy state distribution will act like they have positive energy, but with opposite (positive) charge. These are protons, which are annihilated when an electron drops into the corresponding negative energy state.

To explain how Dirac had resolved Klein's paradox will require a further diagram (see Figure 2). On the left in Region I, electrons may have energies greater than $+mc^2$ (the rest mass) or less than $-mc^2$, and the electron current of a solution with energy $+E$ is directed right. However, since in this situation we have $V_0 > E + mc^2$, this energy level corresponds to a *negative* energy solution in Region II, where the potential V_0 acts to raise the energy of the "vacuum" state.[4] In the hole theory interpretation, the negative energy states are everywhere filled, but the states on the right are above the "mass gap" and so, in Region I, are allowed positive energy electron states. Thus the leftward directed current which seemed paradoxical due to the absence of electrons in Region II now had a ready explanation: in Dirac's hole theory there are plenty of electrons there to travel left, in fact, an uncountable infinity of them. While the current directed left is indeed an electron current, electrons moving to the left leave a 'hole' moving right, which is to be interpreted as a proton (later to become the antielectron). If an electron drops into such a hole, then both particles, each having positive energy, are annihilated. This ingenious solution explained how negative energy solutions could be interpreted as particles with positive charge and positive energy, since a hole in an otherwise filled negative energy electron sea corresponds to the *absence* of a negative energy particle.

The direct interpretation of negative energy solutions as protons had been proposed by Weyl (1929), but only by adopting Dirac's interpretation of protons as holes could they be included in the theory as particles with physically reasonable properties.[5] However, as Dirac was immediately aware, the mass discrepancy between the electron and proton (a factor of roughly 1800) was a significant problem for his new theory, since the dynamics appeared to be entirely symmetric with respect to the negative and positive energy solutions. This meant that there could apparently

4. Scare quotes indicate the use of anachronistic language.

5. For example, Dirac (1930) pointed out that on Weyl's hypothesis a proton radiating energy would *accelerate,* and would have to absorb energy to come to rest.

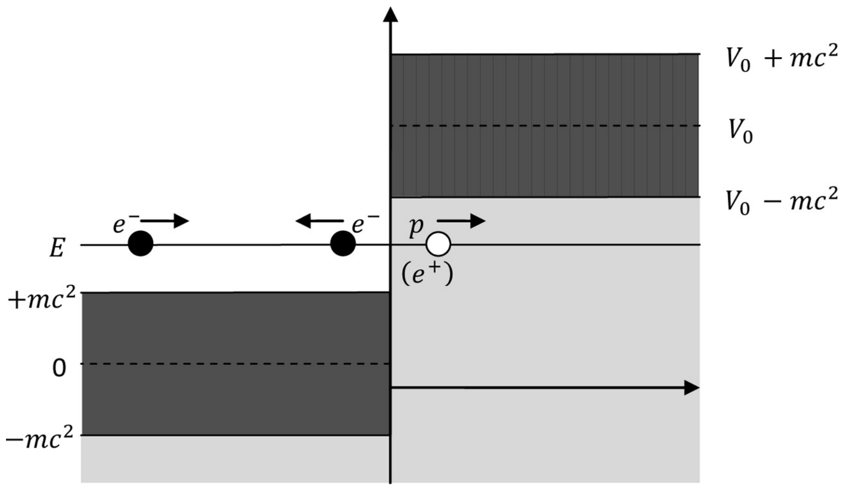

Figure 2. The Klein Paradox (Resolution). The electron current moving to the left is now accompanied by a 'hole' moving to the right, which is interpreted as a proton (later positron).

be no dynamically relevant difference between a situation in which the vacuum state was filled by protons, with the holes being positive energy *electrons,* and the situation in which the holes were protons, which made it hard to see how they could not have the same mass.

In his letter to Bohr, he expressed his hope that this asymmetry might emerge from taking into account interactions between the vacuum electrons, but he had not managed to formulate the problem in a relativistically invariant way. Dirac (1930) gave a sketch of a formal argument in the non-relativistic context, in which he claimed that the *infinite* number of electrons in the vacuum would serve to break the symmetry if the Hamiltonian includes the energy due to pair-wise interactions. Since only the term containing the sum over the vacuum electrons would contain an infinite sum, the situation could not fail to be asymmetric under an exchange of holes and electrons. This would serve to break the dynamical symmetry and so might explain the observed difference in mass.

When Dirac later said that only "pure cowardice" prevented him from immediately positing the existence of the anti-electron,[6] he ignored the plausibility of his expectation that the interaction of the holes with the surrounding infinity of vacuum electrons might serve to break the mass symmetry. However, as he freely admitted, this asymmetry must be dem-

6. This phrase appears in Dirac's 1963 interview with Thomas Kuhn.

onstrated to hold in the relativistic setting. Weyl took up this challenge by considering the symmetry properties of a fully interacting relativistic theory of the electron and proton, and demonstrated that the symmetry of the masses remained unbroken.[7]

3. Interactions and Symmetries: The Role of Weyl

Although Weyl is often credited for providing Dirac with the crucial symmetry argument that convinced him that the holes could not be protons, the significance and sophistication of Weyl's argument is perhaps not always fully appreciated.[8] Weyl's background in pure mathematics made him ideally placed to provide a general framework in which he could explore the consequences of the nascent relativistic quantum treatment of matter. Key to this progress was his understanding of the geometrical properties of the Dirac equation, which allowed him to express the equation in manifestly relativistic form and so generalize it beyond Dirac's (1928) single particle equation.

It had soon been realized (again by Darwin 1928) that Dirac's electron equation was not a tensor equation, and so its solutions were not the relativistic objects familiar to physicists at the time. These new relativistic covariants were christened *spinors* (apparently by Ehrenfest) and were initially regarded with puzzlement by the physics community. However, Weyl's geometrical knowledge provided him with the means to connect spinors to the general Lorentz transformation since, as he was well aware, spinors had in fact already been discovered by the French mathematician Elie Cartan as a means to represent rotations in three-dimensional space by a pair of complex numbers.

In the second edition of *Gruppentheorie und Quantenmechanik* (1931) Weyl showed that while spinor representations of the restricted Lorentz Group (of boosts and rotations) exist, representations of the full Lorentz Group (including spatial and temporal reflections) required bispinors (with 4 complex numbers) corresponding to the two pairs of solutions to the Dirac Equation. The Dirac matrices could be understood as forming a basis in this bispinor representation, analogous to the role of the quaternions (known to physicists as the Pauli matrices) in the representation of rotations in three-dimensional space. This geometrical connection must have been of great interest to Dirac given his fondness for projective

7. Pauli also claimed to have demonstrated a similar result, which Dirac was informed of in a letter from Tamm in 1930. However, it seems that Dirac considered Weyl's demonstration to be superior since he was never to refer to Pauli's proof. Lacking the details of Pauli's argument, it is hard to know whether or not they were equivalent.

8. See Darrigol (1986, p. 243) for a notable exception.

geometry, which, as Weyl had explained in the first edition (1928), also provided a means to understand Lorentz transformations geometrically.[9]

With this understanding in place, Weyl was able to derive the Dirac equation from an action principle written in manifestly relativistically covariant form, which enabled him to easily investigate the symmetries of the equations of motion. He went on to address the invariance properties of the Dirac equation in two different contexts, first without an electromagnetic-matter interaction (1932, pp. 225–27), and then in fully interacting form, called by Weyl "The Maxwell-Dirac Field Equations." This second treatment apparently provided the rigorous treatment that Dirac required, but rather than confirm his hypothesis that interactions would explain the mass asymmetry of electrons and protons, Weyl instead displayed the insensitivity of hole theory to the exchange of electrons and holes.

Weyl's initial treatment of the (first-quantized) Dirac Equation is essentially as a classical theory, applied to a single electron under the influence of an external classical field (without back reaction). He made it clear that a full quantum treatment of the interaction would require relativistic quantum field theory, and thus second-quantization.[10] That is, rather than treating electrons as quantized point particles, Weyl was to use Jordan's formalism of matter fields, which had been extended to the relativistic context by Heisenberg and Pauli.[11] Having done so the crucial argument which establishes the equality of masses of the electrons and holes appears in the context of Weyl's relativistic quantum field theoretic treatment of the electromagnetic field, its interaction with matter, and the matter *field.*

Whereas the variables of the electromagnetic field obey Canonical Commutation Relations (CCR's), the wave functions featuring in the matter part of the action obey canonical *anti*-commutation relations (CAR's), which serve to enforce the Pauli exclusion principle. Following Dirac's (1927c) approach to quantum electrodynamics, Weyl (1932, pp. 256–60) constructs creation and annihilation operators for the electromagnetic

9. The role of projective geometry in Dirac's approach to relativistic quantum theory and his derivation of the Dirac equation is the subject of Pashby (unpublished).

10. Second quantization refers to the procedure of describing a classical field system by field configuration variables and their conjugate momenta, and then applying the quantization procedure to promote *these* to quantum operators, rather than the position and momentum variables of individual particles, which became known as *first*-quantization. Thus a second-quantized system is not "quantized twice," but rather quantized according to a procedure which was developed subsequently to the original.

11. See Darrigol (1986) for an admirably lucid account of the development of quantum matter fields.

field and derives a dynamical equation which takes into account the interaction of an electron with the electromagnetic field by emission and absorption of photons. Although Weyl doesn't go on to carry it out in any detail, he sketches the outlines of an analogous treatment of the Dirac Equation with many of the features we recognize from the modern account.

> There is, of course, nothing to prevent us from quantizing the matter waves in a manner analogous to that applied to electro-magnetic waves. . . . [The energy] will then depend on the quantum number n_μ which corresponds to the characteristic values μ which may take only the values 0 and 1, and in addition on the numbers N_ν of photons. . . . The dynamical law allows only those quantum jumps of the particles in which one n_μ falls from 1 to 0 and another $n_{\mu'}$ jumps from 0 to 1. (1932, p. 262)

Although this is just a sketch of a fully interacting field theoretic hole theory, the continuity with the later quantum field theory program and the prescience Weyl shows is striking. Indeed, as Darrigol (1986, p. 243) observes, this section contains what must be the first statement of the CPT invariance of a relativistic quantum field theory. The argument for the mass equality of electrons and holes follows. First Weyl notes that:

> [T]here is nothing to prevent us from replacing the numbers $n_{-\mu}$ for negative $-\mu$ by $n_\mu^- = 1 - n_\mu$, keeping n_μ^+ for positive μ. The theorem of conservation of charge is then $\Sigma n_\mu^+ - \Sigma n_\mu^- = const.$ $(\mu > 0)$. (1932, p. 263)

Here we see the appearance of the idea that while charge is conserved, the dynamics is indifferent to the number of positive and negatively charged pairs of particles. This leads Weyl to a discussion of hole theory, and the resulting interpretation of pair production and annihilation, about which he says:

> However attractive this idea may seem at first, it is impossible to hold without introducing other profound modifications to square our theory with the observed facts. Indeed, according to it the mass of a proton should be the same as the mass of an electron; furthermore, no matter how the action is chosen (so long as it is invariant under interchange of right and left), this hypothesis leads to the essential equivalence of positive and negative energy under all circumstances—even on taking the interaction between matter and radiation rigorously into account. (1932, p. 263)

The first thing to note is that Weyl takes the failure of hole theory to explain the divergent masses of the known particles to be evidence *against* hole theory. In contrast, Dirac will retain his faith in the essential correctness of his theory, and so turn this inference around to make his prediction of antimatter. It is also clear that Weyl sees his argument as answering directly to Dirac's hypothesis about the role of interaction in the mass asymmetry, asserting that the details of the interaction are unimportant so long as it preserves the symmetries of the action. It is remarkable how close Weyl might have come to identifying instead the holes with the "positive electrons" he had discussed earlier in the context of the first-quantized equation (1932, p. 225). But he did not! It was left to Dirac to complete the final step of the argument.

4. Interpreting Holes: The Prediction of Antimatter

The manifest relativistic invariance of Weyl's treatment of the interaction of the electron with the electromagnetic field had made it clear to Dirac that holes could not be protons. While Weyl seems to have thought, like Bohr, that another conceptual revolution would be required to explain the difference between "positive and negative electricity" (1932, p. 264), Dirac was instead led to straightforwardly identify holes with 'anti-electrons'—positively charged particles with positive energy and the same mass as the electron. While this may seem today an almost trivial step to have taken, the conceptual inertia generated under the sway of what Kragh (1990) terms the "two particle paradigm" must have been enormous. For 30 years, there had been two forms of matter: electrons and protons.

No experimental data compelled Dirac's (1931) prediction of the existence of an entirely novel period. His prediction was made solely on the basis of consideration of theoretical arguments, and the expectation that since relativity and quantum mechanics were separately valid a theory combining them must be also. Dirac's prediction of the existence of antimatter on the basis of hole theory deserves quotation in full.[12]

> A hole, if there was one, would be a new kind of particle, unknown to experimental physics, having the same mass and opposite charge to an electron. We may call such a particle an anti-electron. We

12. As Kragh (1990) notes, the prediction is not even the boldest made by Dirac in the paper in which it appears! Dirac's prediction occurs in the introduction of 'Quantised singularities in the electromagnetic field (1931),' which predicts the existence of magnetic monopoles. The prediction of the anti-electron (yet to be observed, remember) is offered *in support* of the "method of theoretical advance" which leads Dirac towards that prediction

> should not expect to find any of them in nature, on account of their rapid rate of recombination with electrons, but if they could be produced experimentally in high vacuum they would be quite stable and amenable to observation. An encounter between two hard γ-rays (of energy at least half a million volts) could lead to the creation simultaneously of an electron and anti-electron. . . . Presumably the protons will have their own negative-energy states, all of which normally are occupied, an unoccupied one appearing as an anti-proton. (1932, pp. 60–1)

So the prediction concerns quite specific conditions under which an 'anti-electron' might be observed experimentally through the phenomenon of pair production, and an explanation of why they have yet to be observed. Note also that antimatter is expected by Dirac to be a quite general phenomenon since by the same lights protons are predicted to have their own anti-particles. Although, as he later said, Dirac would have reason at the time to know of the possibility of observing such particles as tracks in bubble chamber cosmic-ray experiments (Dirac 1977, p. 145), it would be quite uncharitable to deny to this bold yet cautiously expressed statement the status of a genuine prediction of the positron, soon to be observed by Anderson. The details of the development of Dirac's reasoning given above serve to demonstrate that was a *bone fide* theoretical prediction of hole theory, depending essentially on Dirac's hypothesis of the filled 'sea' of negative energy electrons, and the subsequent interpretation of holes as anti-electrons (positrons).

As it happened, just months later C. D. Anderson at Caltech first reporting the observation of cloud chamber tracks which had the signature of a positively charged electron, which were observed independently in Cambridge by Blackett and Occhialini. Surprisingly, even the Cambridge experimentalists were initially unaware of Dirac's prediction of anti-electrons, and their explanation in terms of hole theory. While the identification of these particles with the anti-electrons of Dirac's hole theory was initially resisted, the prediction soon came to be seen as a vindication of hole theory, which provided essential tools for understanding their behavior.

In 1933, when Dirac received his Nobel prize for "the discovery of new productive forms of atomic theory," hole theory stood triumphant: the existence of the 'anti-electron' had been posited and independently confirmed, and Blackett and Occhialini had begun to explore the properties of this exotic new form of matter under controlled conditions in the labora-

(which despite being revived several times in the intervening years, has yet to be confirmed today).

tory. Roque (1997) argues that it was confirmation afforded to hole theory by these latter developments which were essential to its acceptance. Not only did hole theory show positrons to be a consequence of Dirac's relativistic electron equation, but it also provided a theoretical and explanatory framework for the prediction of qualitatively new processes, and gave quantitative predictions which were in agreement with experiment.

While some physicists remained skeptical of Dirac's hole theory despite its apparent success, there were notable converts, including Blackett, whose enthusiasm regarding the identification of positrons with Dirac's anti-electron had initially been lukewarm at best. In late 1933 he expressed his faith in the theory thus: "That Dirac's theory of the electron predicts the existence of particles with just these properties, gives strong reason to believe in the essential correctness of his theory" (in Roque 1997, p. 110). This attitude was typical of the physics community at that time, and was a reasonable view to take given the prevalence of evidence in favor of hole theory and the absence of a credible competitor.

5. Replacement: The Fate of Hole Theory

Despite his acceptance of Weyl's argument for mass symmetry, Dirac remained of the opinion that while the electromagnetic field should be subject to second-quantization, electrons should continue to be treated as particles undergoing Schrödinger-style evolution. He continued to regard the use of Jordan's quantized matter fields as a formal device which could lead to conceptual confusion, and proposed an alternative.[13] In his 'many-time' theory (Dirac 1932), which in the interests of relativistic invariance assigned each particle its own time variable, particles and the fields through which they interacted were kept separate. It was soon realized (Dirac, Fock, and Podolosky 1933) that this was equivalent to the field-theoretic description, and with the benefit of hindsight we can see that Dirac's attachment to the description of an electron as a quantized point particle was misguided.[14]

The best indication of this at the time was provided by the papers of Fock (1932), and Pauli and Weisskopf ([1934] 1994) who provided a reinterpretation of the Klein-Gordon equation in field theoretic terms without the use of negative energies. Since this equation did not apply to spin-half particles, Pauli's exclusion principle couldn't apply and so negative

13. Darrigol (1986, pp. 199–200) attributes this to their differing presentations of transformation theory: Jordan's was axiomatic and abstract, while Dirac's was closely tied to the classical transformation theory.

14. However, although soon abandoned by Dirac, these formal and conceptual tools proved to be essential to the development of Tomonaga's 'super-many-time' version of QED.

energy states could not have been "filled" by any number of particles. As Pauli and Weisskopf made clear (Pauli called it his "anti-Dirac paper"), the existence of this interpretation of the Klein-Gordon equation made Dirac's original dissatisfaction with it unjustified, and made hole theory's essential use of the exclusion principle seem somewhat unprincipled.

In the Fock space treatment (Fock 1932) the Hilbert space of the field configurations is written as a direct sum of *n*-particle Hilbert spaces. In this context creation and annihilation operators move between these subspaces, and the number operators defined from them return the number of particles or antiparticles in a given state. The vacuum state was defined as the state that results from the application of annihilation operators to remove all particles (i.e., the state 'with no particles in it') and adding particles to this state by application of creation operators returns a state with *positive* energy. The key feature of the new quantum field theory was the symmetrical treatment of particles and anti-particles by creation and annihilation operators.[15]

In Pauli and Weisskopf's second-quantized Klein-Gordon theory the canonical commutation relations apply not to position and momentum operators for particles, but *field* operators and their conjugate momenta, defined by the usual recipe from the Lagrangian. Solving Hamilton's equations allows one to write down an equation of motion for the *operators* that is second-order in time (in the case of the Klein-Gordon equation), while the dynamical evolution of the *state* of the system (still represented by a ray in Hilbert space) is determined by a first-order Schrodinger equation. Thus Dirac's initial dissatisfaction with the first-quantized Klein-Gordon equation no longer applied: Pauli and Weisskopf's theory was entirely consistent with Dirac's transformation theory.

In this theory, there was no negative energy sea, no special role for the Pauli exclusion principle, and a completely symmetrical treatment of matter and antimatter in which particle number is not necessarily conserved. Although the merits of this approach seem obvious to us now, it is significant that Weisskopf continued to use the conceptual and theoretical framework of hole theory and negative energies even when working with the second-quantized Dirac equation.[16] In retrospect, once Dirac had ac-

15. Introduced, ironically, by Dirac (1927c) to deal with photons. However, his use of the formalism had more in common with hole theory than Fock's conception of the vacuum.

16. Weisskopf (1939) considered the theory that he had constructed with Pauli as a competitor to Dirac's rather than providing the means for a reconstruction of Dirac's hole theory. He continued to use the language of hole theory in his (1949) explanation of pair production and vacuum polarization even though his theoretical treatment could have been expressed without making use of negative energies.

cepted Weyl's argument that the interacting theory would be entirely symmetric between particles and anti-particles, it amounted to a mere choice of labeling to decide which were which in the theory and so the failure of hole theory to respect this symmetry of the dynamics was an indication that a more parsimonious description was possible.

In fact, Fock (1933) and Furry and Oppenheimer (1934) had already applied the formalism of creation and annihilation operators to the Dirac equation with the aim of doing away with the postulation of negative energy electrons, apparently without knowledge of Pauli and Weisskopf's work. They showed that in the second-quantized theory the Hamiltonian operator could be constructed from annihilation and creation operators with *positive* energy. However, due to difficulties introducing interaction without breaking gauge invariance, their presentation of the theory was limited to the non-interacting case. This became an apparently insurmountable problem for their program during the 1930's,[17] and as a result hole theory, in either first- or second-quantized incarnation, remained essentially the only game in town for understanding the electron and positron.

In various ways, the problem was solved after the Second World War by Feynmann, Tomonaga and Schwinger, whose formulations of Quantum ElectroDynamics were shown by Dyson to be essentially equivalent.[18] Note that throughout this development there was little conceptual revolution of the kind anticipated by Bohr in 1929: it turned out the formalism required only an extension of the relativistic quantum electrodynamics proposed by Heisenberg and Pauli in 1929, and was ultimately consistent with Dirac's transformation theory. Crucial to the acceptance of the theory was the realization that the empirically relevant content of the theory (given by the S-matrix) could be extracted without infection by the divergences (arising from the electron self-interaction and vacuum polarization) which had been thought to doom the program to failure.

3. The Challenge to the Scientific Realist

Laudan's (1981) critique of realism on the basis of the historical record marked the beginning of a "historical turn" in the realism debate. Recent defenses of scientific realism have sought to make sense of the historical record in ways that favor a realist interpretation, introducing criteria which allow the cursory dismissal of most of Laudan's list of abandoned theories while subjecting a few problematic cases to detailed analysis. This

17. Mehra and Rechenberg (2001, chap. 4) provide a useful account of these developments.

18. See Schweber (1994) for an account of this history.

characteristic response of the contemporary realist to this predicament has been dubbed the "divide and conquer" strategy. However, they have failed to address the historical episode I have detailed here, which fits exactly the logical form of Laudan's argument against the realist's use of success as an indicator of the truth.

I claim that in Dirac's hole theory we have a theory with strong realist credentials which was subsequently replaced by a theory failing to contain its central theoretical posit, the 'Dirac sea' of negative energy electrons. The argument against the scientific realist arising from consideration of this historical episode is therefore as follows:

1. Hole theory was an empirically successful and scientifically serious theory that enjoyed considerable predictive and explanatory success, including novel predictive success.
2. Therefore, the scientific realist would have maintained that hole theory was deserving of a realist attitude, including the existence of the negative energy electron 'sea.'
3. However, hole theory was replaced by a successor in which its central theoretical posit failed to appear.
4. Therefore, even novel predictive success does not provide compelling evidence for the adoption of a realist attitude towards our current best theories.

The first move of the divide and conquer strategy is to attempt to exclude such problematic cases from the set of theories to which a realist would have been committed. Leplin (1997) suggests that novel predictive success is the gold standard against which empirical success is to be judged, since this alone provides convincing evidence for the validity of the abductive reasoning characteristic of science. He claims that no theories in Laudan's list meet this criterion, including theories of the ether (1997, p. 146). Yet hole theory did enjoy remarkable novel predictive success, comfortably meeting Leplin's criteria of independence and uniqueness (1997, p. 77).

Still, the realist could argue that there was something about the theory that gave it an essentially preliminary character. In support she might offer the views of contemporary scientists such as Pauli who held the theory to be deeply flawed. While it is certainly the case that that there was widespread skepticism in the relevant community in the period 1931–3, remarks of Pauli's such as "I do not believe in your perception of 'holes', even if the existence of the 'anti-electron' is proved!" (in Moyer 1981b) should not be taken to characterize the attitude of the community as a whole once the relevance of hole theory to the observed phenomena had been established.

Bohr had reportedly made remarks almost identical to that of Pauli's, but, impressed by the continued empirical success of hole theory, his closing lecture of the 1934 Solvay conference spoke of "the marvelous confirmation of Dirac's theory of the electron brought about by the discovery of the positron" (in Roque 1997, p. 108). As Roque emphasizes, hole theory at this time was not just an abstract theoretical framework without experimental application. It had been put to good use deriving quantitative predictions concerning experimental phenomena such as *bremsstrahlung* and pair creation by a group of theorists including Peierls, Bethe, and Oppenheimer.

Furthermore, hole theory enjoyed considerably fecundity as a framework from which to address the foundational problems that had been opened up by the consideration of relativistic quantum electrodynamics, most pressing of which were the seemingly unavoidable divergences that arose in calculating electron self energy and vacuum polarization. The former of these was known to be a problem that arose in the classical field theory, but the latter was unique to the quantum realm. Hole theory provided the basis of the first partial successes of renormalization techniques (Dirac 1934; Weisskopf ([1936] 1994) and provided a vital explanatory framework for understanding the properties of antimatter, even with the use of second-quantized methods.

But what counts most strongly against excluding hole theory from the set of theories to which the realist would have been committed is the absence of a credible competitor: the realist recommends belief in the best theory available, provided that the theory has enjoyed the requisite success. It would be unprincipled to deny that status to hole theory on the basis of anachronistic considerations. In order to argue that the theory doesn't deserve to be included under the remit of the No Miracles Argument, the realist needs to supply a non-question begging criterion operating at the level of whole theories that clearly separates the successful but false from the successful but true. In the absence of such a criterion which works for Dirac's hole theory, which not only enjoyed considerable success but was formulated in a mature science in mathematical language, let us assume that the realist takes hole theory to be a mature scientific theory. Therefore, its success counts for its truth, by application of the NMA.

Divide and Conquer?

Having failed to exclude hole theory from the list of theories to which she would have been committed, the realist retreats to defend instead the claim that, despite its replacement, the empirical and explanatory success of hole theory would not have entailed an ontological commitment to any problematic theoretical entities. The thought behind this strategy is the following: the PMI relies on the failure of reference to theoretical entities to be preserved through theory change, but theory change tends to be progressive, in that it results in a theory which not only retains and expands upon the empirical successes of its predecessor, but also explains its success. This thesis is often called *preservative* realism. It involves an identification of something that is a matter of historical contingency, namely facts about what is or is not retained in successive theories, with something that is the basis of the realist argument—the parts of theories that "get things right."

Whereas all parties (including the anti-realist) agree that there is progress at an empirical level, in order to support a realist conclusion it must be argued that there is an accumulation of a suitable kind at the theoretical level. So the tendency of successive theories over time to preserve and expand upon the empirical applications of their predecessors is consistent with this thesis, but alone it is not sufficient. Preservative realism requires something stronger: it requires the retention through theory change of *theoretical* content which suffices to explain the success of the replaced theory in light of its replacement. It should be clear, therefore, that this claim is very much hostage to the particularities of episodes of theory change.

The challenge for the preservative realist lies in providing a non-question begging criterion that serves to delimit the parts of the theory that were responsible for their success from those which were not, and so can be safely discarded by the successor theory without doing damage to the NMA. Here is an example of a criterion which will *not* do: the parts of a theory that were responsible for its success are just those which are preserved through theory change. Clearly this begs the question since there is no reason why what is preserved would be what was responsible for previous successes unless one presupposes preservative realism.[19]

Of course, the preservative realist thinks that there is cumulative progress at the theoretical level, and so aims to provide just such a criterion. I will distinguish between two brands, *restrictive* realism which retains a mostly traditional account of the reference of theoretical terms (Psillos, 1999), and *structural* realism, which involves a novel semantics (yet to be fully specified) (e.g., Worrall 1989, 2007; French and Ladyman 2003; Ladyman and Ross 2007). They each make significant claims about the nature of the historical record.

19. Note that the techniques of renormalization critical to dealing with these divergences only received theoretical justification with the later Wilsonian program, which provided a means to display the insensitivity of the dynamics of the theories (at energies relevant for experimental particle physics) to contributions from smaller length scales.

Restrictive Realism Evidential support accrues differentially to different parts of the theory. Taking into account the differing empirical support of various components of a theory at the time it was replaced and the attitudes of contemporary scientists, we can separate those parts which were well-supported by the evidence and those which were not. It turns out that only parts with little or no empirical support were replaced.

Structural Realism The success of a theory is explained by its ability to capture the structure of the physical world. These structural (but physical) relations are described by mathematically expressed theories in terms of abstract structures and relations. Whereas there is little or no preservation of ontology (in terms of objects and their properties) structure *is* preserved through theory change.[20] The question the remainder of this paper will address is this: assuming hole theory to be a proper object of scientific realism, does its eventual replacement offer conclusive evidence for or against either of these expressions of preservative realism?

Against Restrictive Realism

The restrictive realist argues that we should only be committed to the parts of the theory that were empirically supported at the time. In order to avoid the challenge posed above she must argue that the theoretical content that was retained was responsible for the success of the theory, so wouldn't have committed her to the existence of the negative energy sea. The problem in this case is making the cut in such a way that includes more than mere empirical predictions, but avoids the negative energy sea. Since she uses standard referential semantics, I claim the restrictive realist is faced with a dilemma concerning the successful use of hole theory by Dirac and other physicists in the 1930's:

Either the empirical support for hole theory is confined merely to quantitative predictions, in which case there is nothing to offer as an explanation of the success of hole theory which differs from what an instrumentalist might say, *or* the successful use of the theory entails ontological commitment to vacuum electrons.

In support of the second horn, note that the existence of the anti-electron and the possibility of pair creation involve completely novel phe-

20. Structural realism may be further classified according to the stance taken on the ontological status of objects: Worrall's *epistemic* structural realism maintains that all we can *know* about objects are their structural relations, while French and Ladyman's *ontic* structural realism maintains that there *are* no objects but only structural relations (or at least that objects are not metaphysically primitive). This difference will be largely irrelevant for what follows since either involves a commitment to the historical thesis above.

nomena, which are only implied by the Dirac equation if the existence of negative energy electrons is taken seriously. This is worth emphasizing. The *presence* of negative energy solutions was not enough to predict antimatter: each of the negative energy states had to be occupied by an electron for 'anti-electrons' (holes) to have physically reasonable properties, and moreover properties which in detail turned out to be instantiated by the positron. In the conceptual framework of hole theory, there could be no antimatter without the Dirac sea. What is more, since the stability of matter required that *all* the negative energy states be filled (pointed out by Oppenheimer in 1930), the empirical adequacy of the Dirac equation relied on the hole theory hypothesis.

Hence the defense offered by Psillos (1999, p. 113) to explain away the replacement of caloric will not work here. Although it is feasible that the hypothesis that heat is a material substance failed to feature essentially in the derivation of any thermodynamic law, the assumption that all the negative energy states of hole theory were filled by electrons was essential to predict the existence of positrons, and to make physical sense of the Dirac equation. In the case of the disappearing ether, the empirical support accrued by electrodynamics was based on the unification of theories of well-known electric and magnetic phenomena, and led to the discovery of new phenomena such as electromagnetic radiation. Compare also Psillos' description of Maxwell's methodology: "The new theory of electromagnetism was to be built up slowly, in response to the evidence available and background knowledge of the physical world" (1999, p. 136). Instead, Dirac's methodology was driven by concerns of consistency with background theory, accompanied by occasional logical leaps made in response to conceptual difficulties and new theoretical results. Since Dirac proceeded from the initial success of the Dirac equation to his prediction of antimatter with little empirical input, the restrictive realist can't use the criterion of differing empirical support to partition off successful parts of the theory.

However, ultimately Psillos argues that the mechanical ether featured essentially only in heuristic models which were regarded as such by the scientists who made use of them. He claims that although Maxwellian field theory appeared to replace the mechanical ether with a new kind of entity which allowed the propagation of electromagnetic phenomena through free space (the field), the role the ether had played as a mechanical bearer of electromagnetic properties was merely heuristic. Could a similar move account for the success of hole theory? This is certainly the most promising avenue since as I already mentioned one could certainly argue that hole theoretic models were regarded in the same skeptical light as mechanical ether models by certain scientists. However, the difference here is that while mechanical ether models were (arguably) either demonstrations of known phenomena or played a merely heuristic role in theory construction, the hole theoretical account of pair production, say, played an ineliminable role in providing quantitative predictions and qualitative understanding of genuinely novel phenomena.

This brings into play the first horn of the dilemma: if hole theory was not to be interpreted realistically, how does one explain its success in realist terms? The difficulty for the realist here is again that the empirical success enjoyed by hole theory involved novel predictive success. Of course, she may decide that she could do without this episode of novel predictive success, but that move would be ill advised since then she would have broken the association of realism with genuine novel predictive success, supposedly the most compelling evidence in its favor (Musgrave 1988). To do so would be to throw out the baby with the bathwater.

Moreover, it was not just the ontology of hole theory that was discarded: the explanations offered by hole theory became invalid in light of its replacement. For example, the explanation of pair production in terms of transitions of electrons from negative to positive energy states was completely undermined. As explanatory success plays a key role for the scientific realist in the confirmation of a theory, the disappearance of an entire explanatory framework should be just as worrying as the accompanying ontological discontinuity. A last gasp attempt to retain something of the traditional realist account of reference would be to claim that by "the Dirac sea" scientists had meant "the vacuum state" all along, in analogy to Hardin and Rosenberg's claim that by "luminiferous ether" scientists had meant "electromagnetic field" (defended in Psillos 1999, chap. 12). However, it is unclear what set of causal properties would ground such a continuity of reference here.

In sum, having been ostensibly committed to hole theory in, say, 1934, there appears to be little that the restrictive realist could have hung on to through the vicissitudes of theory change which would suffice to explain her initial commitment to hole theory while doing justice to her realist credentials. On the other hand, the structural realist has a way out of this dilemma since she rejects the standard story of ontological commitment entailed by the successful use of a theory. According to Worrall, episodes of theory change exemplified by Maxwell's electrodynamics can be characterized as "cumulative growth at a structural level combined with radical replacement of the previous ontological ideas" (1989, p. 160). The correct level of theoretical commitment, says the structural realist, is at the level of *structure.*

4. The Case for Structural Preservation: Symmetries

In the progression towards Dirac's prediction of antimatter, Weyl's argument was crucial in convincing him of the inherent symmetry between matter and antimatter. While it is tempting to attribute to Weyl (1931) a prescient understanding of the second-quantized theories of electrons that were to follow, the account he gives of his theory is only a rudimentary sketch. Hence there is a puzzle here about Dirac's unquestioning acceptance of Weyl's argument. Firstly, Weyl had provided no proof that Dirac's theory was equivalent to his treatment, and the account he gave of this alternative "theory" scarcely warrants the name. Secondly, Dirac's misgivings about the second-quantization of matter were well known, and soon led him to propose his alternative 'multi-time' formalism. So why was Dirac persuaded?

The answer lies in the generality of Weyl's argument, which followed from the symmetry properties of the relativistic action. The elegance and power of Weyl's group theoretic approach must have been quite clear to Dirac, and it would have been immediately apparent that this afforded an invaluable geometrical understanding of the Dirac equation in terms of relativistic covariants: spinors. Since the CPT invariance of the action followed directly from its construction from certain kinds of Lorentz covariant quantities (scalars, tensors, and spinors), it followed that any theory constructed within these constraints would have this symmetry. Hence, irrespective of the precise details, any theory representing electromagnetism, matter, and their interaction must inherit the symmetrical description of matter and antimatter. Dirac's conviction that this was a general property of such theories is evinced by his prediction that protons would have to have anti-particles too, despite a complete lack of any plausible theory of protons at the time.

Thus rather than following from the detailed description of matter according to a precisely expressed theory, Dirac's prediction of antimatter relied upon the formal characterization of symmetry properties that such a theory must possess. This is just the sort of continuity that the structural realist claims explains the success of the replaced theories in terms of their replacements. In fact, the foundational role of group theory in modern quantum field theory can be traced directly back to Weyl's work on unitary representations, which led eventually to proofs that *any* Lorentz invariant local quantum field theory must be CPT invariant, and Wigner's characterization of the allowed matter fields according to the irreducible unitary representations of the Poincaré Group.

Clearly then, there is an important historical structural continuity here. But just what level of structural continuity is sufficient to explain the success of a replaced theory in terms of its replacement? Are these symmetry properties alone enough to provide an explanation of the success of hole theory in realist terms? One could protest that symmetries are properly understood as properties of the phenomena. Certainly this would be closer to Weyl's philosophy of transcendental idealism, where the objectivity of our description of nature arises from a common phenomenology (see Ryckman 2003).

The structural realist takes a more ontological view of symmetries as (somehow) structural properties of the world, but should admit that that there is more to a theory "getting things right" than just capturing the correct symmetries. Writing in another context, here is Steven French:

> the lesson [of metaphysical underdetermination] for the structuralist is that this essential structure must be expanded beyond the group-theoretical 'object' structure to include the dynamical. Following Bain . . . we can informally represent this essential structure in the following terms: [state space, dynamics, symmetries]. The structural realist will insist this is what we should be realists about. (French 2011, p. 220)

So it seems that we are justified in demanding the preservation of more than just these general dynamical symmetries. Turning first to the continuity of state space, note that in Fock space the state of the system is represented by a one-dimensional ray in an infinite-dimensional separable Hilbert space, and any two such Hilbert spaces are isomorphic. Similarly, there is continuity in terms of the representation of the evolution of the state of the system by a family of unitary operators parameterized by t, generated by the Hamiltonian operator.

But the suspicion arises that the display of shared structure is perhaps too opportunistic: modern physical theories are expressed in rich mathematical language and so it is not at all surprising to find *some* common structures. Is the fact that many successful theories happen to be expressed within the setting of an infinite-dimensional separable Hilbert space miraculous on an instrumentalist account of science? It seems to me that this kind of fact receives a perfectly satisfactory explanation in terms of the utility of generalized function spaces. The question to ask whether or not the structural continuities displayed by the structural realist are sufficient to warrant a *realist* interpretation of the historical record.[21]

21. Bain and Norton (2001) argue that the most suitable display of continuity in the history of the electron is at the level of the Lagrangian. While I agree that this provides a useful heuristic for understanding of the way in which theories of the electron are related, it is blind to the structural discontinuities articulated here (which I take to be problematic for the structural realist).

5. The Case Against Structural Preservation: The Dirac Equation

At first sight, the Dirac Equation seems to be a most plausible candidate for structural preservation since a casual comparison of Dirac (1928) and the modern equation of a spin-half quantum field found in a standard quantum field theory textbook reveals a striking similarity of form. Compare the first quantized Dirac Equation of Section 2.1 with Peskin and Schroeder (1995, p. 52), where the Hamiltonian appears as,

$$\hat{H}_D = \int d^3x \hat{\psi}^\dagger(x)(-i\alpha \cdot \nabla + m\beta)\hat{\psi}(x) .$$

However, in the replacement of the first-quantized Dirac equation by its second-quantized field theoretic analog a significant structural change has occurred that is disguised by the formal similarity of the equations. Namely, for Dirac the equation concerned the evolution of a four-component wave function (living in Hilbert space) which described the probability distribution associated with a definite number of electrons, whereas for the modern physicist the Dirac equation applies to field *operators* $\hat{\psi}(x)$, which are expressed as sums of creation and annihilation operators.

That said, there are important structural continuities here introduced by the construction of Fock space from the single particle Hilbert space. Roughly, Fock space is just an (infinite) direct sum of n-particle Hilbert spaces each formed by taking the (anti-symmetrized) n-fold tensor product of the space of solutions to the single particle Dirac equation. This leads to a natural extension from operators defined on the single particle space to operators on the Fock space.[22] However, note that while a complete specification of field operators determines an operator-valued distribution over space-time points, it does not serve to determine a unique state of the system. The field theoretic Dirac equation determines the evolution of the field operators, not the system state.

A more acute discontinuity arises from the failure of particle number to be conserved (in general) in the second-quantized case. For the first-quantized Dirac equation charge conservation amounts to the statement that the number of electrons is conserved by the dynamics. We can see this requirement as leading to hole theory: if electrons cannot be created or destroyed they must be waiting in the vacuum, waiting to become manifest. However, in the second-quantized field theory conservation of particle number is the exception rather that the rule since the dynamics is not confined to any one n-particle subspace (unless the field is free). As Saunders (1991, pp. 93–103) demonstrates, this fact leads to considerable

22. See Bratelli and Robinson (1997, p. 8), Saunders (1991), and Baez, Segal and Zhou (1992, pp. 163–65) for accounts of this procedure.

difficulties in setting up a correspondence between the first- and second-quantized dynamics.

I contend that these structural differences correspond to accompanying shifts in metaphysical or ontological assumptions. A central assumption of Dirac's hole theory is the use of spatial wave functions $\psi(x)$, which can be thought of as complex valued fields encoding the probability for finding the particle within a particular spatial region. Despite the quantum description employed, which provides a definite probability amplitude *for* position rather than a definite value *of* position, Dirac (1928) made clear that an electron is a point particle in this picture. Since a particle with negative energy is described by exactly the same spatial properties, a negative energy solution is just as "particle-like" as a positive energy solution.

When Dirac (1930) fills these negative energy solutions with electrons, each one of them corresponds to a particle with definite spatial properties. In the first-quantized expression of hole theory, charge conservation amounts to the statement that the number of electrons is conserved (insofar as it makes sense for infinity to be the conserved value). So while positrons (corresponding to the absence of an electron) are in a sense created from the vacuum, the number of electrons remains unchanged. However, in the second-quantized theory particle number is *not* conserved by the dynamics, only charge is.

That said, there may be an easy way out for the structural realist here since she may plausibly deny that hole theory ever received a coherent first-quantized expression, i.e., it could be argued that there *is* no hole theory without second quantization, on the basis that the first-quantized Dirac Equation only has the structural resources to describe finitely many electrons, whereas hole theory had to involve infinitely many. In so doing the structural realist neatly sidesteps any problems relating to the structural discontinuities I described above, but it remains for her to explain away the apparently discontinuous change resulting from the later reinterpretation of antimatter in terms of presence rather than absence.[23]

6. The Structure of Vacuum

One reason to think the change in vacuum might involve another structural discontinuity is the change that occurred in the spectrum of the Hamiltonian operator, which represents the energies that a system de-

23. A potential problem this move brings is the need to then account for the success of the first-quantized Dirac equation in accounting for situations involving interactions. This must be addressed on a case by case basis but I suspect that to do so is unproblematic. For example, note that the Bethe-Salpeter equation recovers in field-theoretic form Dirac's initial treatment of the hydrogen atom as a Coulomb potential (Bethe and Salpeter 1951).

scribed by the theory may take: in hole theory it is unbounded, whereas in modern quantum field theories it is bounded from below, reflecting the requirement that energies must be positive. Writing out the Hamiltonian in terms of creation and annihilation operators, we have,

$$\hat{H}_D = \int d^3\mathbf{k}E(\mathbf{k})\Big(a^\dagger(\mathbf{k},\sigma)a(\mathbf{k},\sigma) + b^\dagger(\mathbf{k},\sigma)b(\mathbf{k},\sigma)\Big),$$

where $a^\dagger(\mathbf{k},\sigma)$ creates an electron with momentum $\mathbf{k}$ and spin σ, $b^\dagger(\mathbf{k},\sigma)$ creates a positron with momentum $\mathbf{k}$ and spin σ, while their respective conjugates act to destroy them. But why does this expression result in *positive* energies? The answer lies in the freedom of choice provided by the Canonical Anti-commutation Relations (CAR's): since $b^\dagger$ and b appear symmetrically in the relation $\{b^\dagger(\mathbf{k},\sigma),b(\mathbf{k},\sigma)\} = 1$ it is essentially a matter of convention as to whether $b^\dagger$ is chosen to be the operator that *lowers* the energy or *raises* it.

Let's see how this works. One way of defining creation and annihilation operators would be to have pairs of operators $(\hat{a}_+^\dagger, \hat{a}_+)$ which create and destroy positive energy electrons, and a pair $(\hat{a}_-^\dagger, \hat{a}_-)$ which create and destroy negative energy electrons. In order to get a workable theory, we define the vacuum as the state where every possible negative energy electron state is filled but every positive energy electron state is empty, that is $|1,1,1, \ldots; 0,0,0, \ldots.\rangle$. Then in order to create a positive energy positron, we use $\hat{a}_+$ to destroy a negative energy state, creating a positive energy *hole.* This is the hole theory interpretation.

To obtain the modern vacuum interpretation, we exploit the symmetry of the anti-commutation relation $\{\hat{a}_-^\dagger, \hat{a}_-\} = 1$ to define operators $\hat{b} \equiv \hat{a}_-^\dagger, \hat{b}^\dagger \equiv \hat{a}_-$ so that $\hat{b}^\dagger$ *raises* the energy and creates a positive energy positron, which is then destroyed by the corresponding $\hat{b}$. Accordingly, the vacuum is defined as the state $|0,0,0, \ldots; 0,0,0, \ldots.\rangle$ and so the theory no longer allows negative energy particles.[24] This is maneuver is known within the standard account of quantum field theory as a Bogoliubov transformation.

So the structural realist who maintains that only second-quantized hole theory was worthy of a realist attitude could argue that this equivalence allows us to see the structural continuity of the completely filled negative energy vacuum state with the "empty" vacuum that replaced it. Unfortunately, Baez, Segal and Zhou (1992, 166) demonstrate that the inter-

24. Note that in so doing we replace the $\hat{a}_-^\dagger\hat{a}_-$'s appearing in the Hamiltonian operator by $\hat{b}_+^\dagger\hat{b}_+$'s, conveniently forgetting about the infinite constant that results from the CAR equality $a_-^\dagger\hat{a}_- + \hat{a}_-\hat{a}_-^\dagger = 1$. Saunders (2002) discusses in depth the resulting reinterpretation of vacuum energy.

change of creation and annihilation operators described above is *not* unitarily implementable, which implies that there is a significant structural discontinuity corresponding to the replacement of the hole theoretic vacuum state since they cannot be accommodated within a single separable Hilbert space.

On the other hand, this may be no reason for the structural realist to despair, since it turns out that even the Klein-Gordon vacuum state admits continuously many unitarily inequivalent Hilbert space representations. This arises since, although in quantum mechanics (involving finitely many canonical variables) the Stone-von Neumann theorem guarantees the unitary equivalence of irreducible representations of the CCR's, the CCR's of quantum field theory involve infinitely many variables, so allowing for irreducible representations that are unitarily inequivalent. This fact, therefore, could be taken to indicate that unitary equivalence is a poor criterion for physical equivalence in the setting of quantum field theory anyway.[25] If the structural realist chose instead to invest only the *algebraic* structure of the CAR's with physical significance then she could point to the fact that there is a unique CAR C^*-algebra (up to *-isomorphism), of which there may be many unitarily inequivalent Hilbert space representations.[26]

However, it will not do to invest *only* the algebraic structure with physical significance since in certain contexts the difference between unitarily inequivalent representations of an algebra of observables *does* seem to have physical significance, for example in explaining spontaneous symmetry breaking in quantum statistical mechanics (Ruetsche, 2003). French (forthcoming) suggests that, although each case requires consideration on its own merits, the algebraic structure can be understood by the structural realist as encoding physical possibilities at a suitably abstract level, modalities which may or may not be realized in our world.

On this understanding, although the hole theoretic vacuum and the "empty" vacuum disagree about the structure of the actual world, they can be seen as incompatible realizations of the way the world *could* have been according to the abstract algebraic description of the theory. However, given the centrality of the spectral condition in formulations of quantum field theory today (which amounts to the assumption that energy must be

25. See Reutsche (2011) for a comprehensive (and comprehensible) introduction to these issues.

26. See Theorem 5.2.5 of Bratelli and Robinson (15, 1997), which demonstrates that any two C^*-algebras generated by bounded operators obeying the CAR's (on a separable Hilbert space) are related by a unique *-isomorphism. Furthermore, in this setting a Bogoluibov transformation is represented by a unique *-automorphism on the algebra of observables.

positive), there is also a clear sense in which hole theory is *not* a possible world of contemporary quantum field theory.

The point I wish to make is this: theory replacement involves structural discontinuity as well as structural preservation, and since these discontinuities may reflect the problematic ontological or metaphysical shifts that the structural realist had hoped to avoid, she had better account for them too. So rather than pointing to structural preservation to explain the success of discarded theories in terms of their replacements, I would ask the structural realist for an account of the historical record that addresses instead structural *discontinuities* that arise. In taking account of cases such as the replacement of hole theory, the danger is that the common thread of shared structure becomes so thin as to fail to support an expressly realist interpretation of the success of the replaced theory.

7. Conclusion

The purpose of this paper was to suggest a particular episode in the history of modern physics as being particularly pertinent for the contemporary scientific realism debate. The history I have tried to uncover reveals a surprising interpenetration of theoretical methods and an undeniable logic to the chain of reasoning that led to Dirac's prediction of antimatter. The disappearance from the succeeding theories of the central posit of his hole theory interpretation—the Dirac sea—is therefore all the more interesting. It is hoped that the present discussion has served to make accessible the details of this episode to philosophers of science, while also demonstrating the relevance of the details of these events for the preservative realist, who has already sought to sophisticate her account in response to challenges posed by the history of physics.

The main conclusion of this case study is a negative one: the position I termed restrictive realism, exemplified by Psillos (1999), either fails to accommodate the disappearance of the Dirac sea, or fails to account for the empirical and predictive successes of hole theory. Therefore, the argument of Laudan (1981) against the necessary connection of the reference of central theoretical terms with empirical success and (in this case) novel predictive success goes through despite the potential accommodation of other cases of theory change by the restrictive realist.

The realist might be inclined to say, "Well, so much the worse for novel predictive success!" But, aside from the inherent problem of reducing the force of one of realisms key motivations, I have argued that hole theory also enjoyed considerable empirical and explanatory success. Perhaps there is some other criterion that could be provided for the acceptance of a theory which would exclude this case, or perhaps we should adopt a more cautious, yet hopeful, picture where it is only the ultimate, "fundamental" theory which is deserving of a realist attitude, and our commitment to present theories is to be cashed out in relation to this future theory. But these would be quite different positions to the one considered here.

It seems that the structural realist fares considerable better in accommodating the peculiarities of this case of theory change, which indicates that the accumulation of theoretical content is best understood in terms of continuity between the mathematical frameworks in which the theories are expressed rather than the commitments entailed by their current degree of empirical support. In particular, the perspective on the Dirac equation offered by Weyl's group theoretic understanding of Lorentz transformations proved not only to be vital to Dirac's prediction of antimatter, but also to the future development of the Standard Model. It would be hard to deny that this deserves recognition as a compelling instance of structural preservation.

However, despite this striking continuity of dynamical symmetries, the structural realist faces problems accounting for the shift from the first-quantized Dirac equation to second-quantized hole theory, and thereby to the modern notion of an "empty" vacuum. I have suggested that she might avoid concerns relating to the first of these shifts by arguing that only the second-quantized theory was deserving of a realist attitude. However, there remains the problem of accounting for the disappearance of the Dirac sea, since this difference is ontology is reflected in a structural distinction that can be understood as reflecting the asymmetry between matter and antimatter displayed by the hole theoretic vacuum state.

Perhaps this is the lesson to be drawn for structural realism: theory change also involves discontinuity at the structural level. Thus even restricting her theoretical commitments to structural components does not suffice to insulate the structural realist from problems of theoretical change. What makes the case considered here particularly interesting is that it demonstrates how shifts in the metaphysical commitments of a theory may also be displayed in terms of changes in theoretical structure. It remains to be argued that a structuralist understanding of theory change suffices to support a compellingly realist interpretation of scientific progress.[27]

27. French and Ladyman (2011, p. 32) have recently claimed that only "a case where the structure of a theory that had enjoyed novel predictive success was completely abandoned" would provide a legitimate challenge to structural realism. However, in so doing they forget the positive role that the historical record is supposed to play in establishing the plausibility of realism in the first place via the NMA. In other words, structural realism is the best explanation of the success of science only to the extent that its competitors are not.

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