How Do Feynman Diagrams Work?

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1. Introduction

Feynman diagrams (hereafter FDs) are now iconic. Like pictures of the Bohr atom, everyone knows they have something important to do with physics. Those who work in quantum field theory, string theory, and other esoteric fields of physics use them extensively. In spite of this, it is far from clear what they are or how they work. Are they mere calculating tools? Are they somehow pictures of physical reality? Are they models in any interesting sense? Or do they play some other kind of role?

It is safe to say they are linked to some sort of calculation tool, but after that it is far from clear. If you ask me how to get from Toronto to Montreal, I could respond two ways: (1) I could tell you to drive north until you reach the main highway, then turn right and continue on for about five hours, or (2) I could give you a map and tell you where you presently are on it. Both ways provide the information to get you successfully to Montreal. The map in the second method is clearly a model; the instruction in the first method is clearly not. What I'm going to argue is that Feynman diagrams are a lot like (1) in spite of appearing a lot like (2). In other words, they are not pictures or descriptions of reality, nor are they models in any reasonable sense. They play a different kind of role in physics.

I had many pleasant and productive conversations on FDs with Michael Stöltzner and Mauro Dorato while we passed the summer of 2012 in Bielefeld. Michael's student, Jim Talbert wrote a fine thesis that got us started. Thanks also to Letitia Meynell for several helpful conversations and especially for bringing serious aesthetic considerations to the issue, and to Adrian Wüthrich for his excellent recent book, *The Genesis of Feynman Diagrams*, from which I learned much and pinched a lot. Adrian, Letitia, Mauro, Michael, and I had a first go on this topic at a joint session at the European Philosophy of Science Association meeting in Helsinki, August 2013. I'm grateful to the audience there for a valuable discussion.

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It is a truth universally acknowledged, as Jane Austen might say, that anything can stand for anything else. In *Pride and Prejudice* Elizabeth Bennet and Mr. Darcy represent gender relations and class tensions in Regency England. At dinner, someone says a salt shaker stands for Napoleon while the potato salad represents *La Grande Armée*. That same salt and salad is used by another person to model a hydrogen atom; they play the respective roles of electron and nucleus. Add the pepper as a second electron in orbit around the salad and we have a model of helium. If we take a sufficiently liberal view, then there could hardly be a question about Feynman diagrams representing or modeling physical reality. For instance, I could stipulate that Feynman's first diagram stands for Napoleon and his second for Kutuzov. Of course, this is silly and nothing useful is likely to come of it—but it could be done, which is the point. The question is much more interesting when we put a reasonable constraint on it and require some appropriate sort of similarity. I can't say what that similarity is, since there is no consensus on the nature of similarities in models at all. But I will assume that there is a clear difference between the two possibilities that I will eventually discuss: Feynman diagrams might be typical models or representations, on the one hand, or they might be something very different from that, on the other.

I will begin by mentioning a few background assumptions. First, I will somewhat dogmatically adopt a broadly realist outlook. At times I might be forced to be specific about the details of realism, but for the most part my default position is a fairly common view of the matter. Second, let's assume standard quantum theory, as normally understood. Consequently, some of my claims might be incompatible with, say, Bohmian mechanics. The aim is to make sense of FDs in a broad framework that would be generally accepted, a framework that does not stretch normal credence.

There are a number of questions involved in understanding FDs. How did they first arise? How did they evolve so that they were practically useful in QED? How were they extended to quantum chromodynamics and other fields? These are historical questions and have been well discussed in the excellent books by Kaiser (2005), Schweber (1994), and Wüthrich (2010). I am more interested in philosophical issues concerning how FDs are currently used and why they are successful.

When I say "currently used," however, I should note that FDs might be on their way out. New techniques have appeared on the scene that seem more powerful and could replace FDs as the standard method of calculating in the near future.[1] In that case, I would then be discussing something that

1. A good place to start is Bern et al (2012). See also Brito (http://arxiv.org/abs/1012.4493v2) for a review, Brito et al (http://arxiv.org/abs/hep-th/0501052v2) for an important preliminary result, and Arkani-Hamid (arXiv:1012.6030v1 [hep-th]) for important applications.

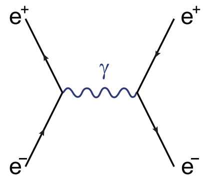


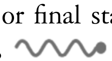
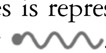
Figure 1. Simple Feynman diagram: electron-positron interaction. (Time runs from left to right.)

after 70 years of fruitful service is about to become passé. No matter; the nature of FDs remains philosophically interesting even if we are only trying to make sense of old techniques.[2]

2. Basic Properties

There are a number of rules for constructing FDs. For the most part I will stick to QED, where the relevant particles are fermions (electrons e^-, positrons e^+) and bosons (photons γ) that can interact with one another. Diagrams represent the different ways interactions could happen. For example, an electron and a positron annihilate each other and in so doing they create a photon, which in turn creates another electron and positron (Figure 1).

The phrase "Feynman diagram" is somewhat generic. It often covers both the diagrams and the process of constructing the mathematical expression associated with the diagrams. There are, however, distinct rules for each. The process of constructing the mathematical expression to go with a diagram is, unsurprisingly, known as "the Feynman rules." I will briefly explain both, starting with the diagrams.

Three symbols are used to depict QED processes: solid straight lines for the fermions, wavy lines for photons, and dots or vertices for the interaction. An electron in the initial state is represented by a solid line with an arrow pointing toward the vertex, , but in the final state it is represented by a line with an arrow pointing away from the vertex, . A positron in the initial (final) state reverses the direction of the arrows. A photon in the initial or final states is represented, respectively, by a wavy line meeting a vertex,  or . A vertex always has three lines attached to

2. I should mention that I had a brief go at this topic several years ago as part of a more general account of diagrams in science (Brown 1996). My account of FDs in that article was criticized by Meynell (2008) and Wüthrich (2010). The present paper is an attempt to go deeper into the issue. My beliefs about how FDs work remains largely intact, somewhat to my surprise, though I hope they have become a bit more sophisticated and better defended.

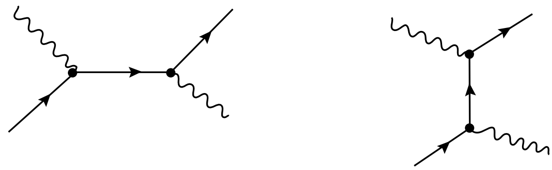


Figure 2. Two second order diagrams of the Compton effect. (Time runs from left to right.)

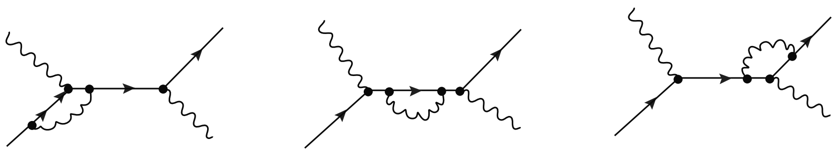


Figure 3. Some 4th-order diagrams of the Compton effect.

it: one boson, one fermion pointing in, and another fermion pointing out. The order of a FD is the number of vertices in it. Figure 1 above, for instance, is second order. The higher the order, the greater the complexity and the more terms in the corresponding series.[3] For a given process draw an FD of lowest order. There might be more than one at lowest order, so it is important to draw all possible. The reason for this is Feynman's particular way of doing quantum mechanics, which is to sum over all possible processes. Thus, for instance, there are two second order diagrams for the Compton effect (Figure 2).

Both of these have the same input (a photon and an electron) and the same output (a photon and an electron). The electron between the two vertices is a virtual electron. Its existence is very short-lived, so it does not violate the energy-time uncertainty principle. On the left, a photon and electron are annihilated; a virtual electron is created; it in turn is annihilated in favor of an electron and photon. On the right, a photon and electron exchange a virtual electron and create an electron and a photon.

The next step is to draw higher-order diagrams, starting with all possible 4^{th}-order diagrams (four interaction vertices). An electron, for instance, could emit and absorb a virtual photon. The next figure (Figure 3) shows some of the possibilities. In principle we continue drawing higher-order diagrams; there is no limit. In practice, of course, we stop after a while, since there will be no measurable consequences of going beyond a certain point.

3. The terminology is not uniform. Some authors use "1st-order" for diagrams with two vertices, "2nd-order" for diagrams with four vertices, and so on. Consequently, some caution is called for when encountering phrases such as "2nd-order effect."

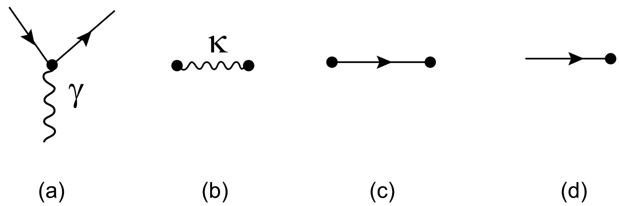


Figure 4. Diagram elements corresponding to Feynman rules.

The next step is to construct the series of mathematical terms that are associated with the elements of each of the diagrams. Freeman Dyson played a huge role in the development of QED and FDs in particular. Dyson (1949) proved a crucial result that links the diagrams and the mathematical terms: There is a one-to-one correspondence between them. Here are some of the explicit rules for setting up the mathematical expressions.

1. For each vertex (as in Figure 4(a)) write $ie\ \gamma^{\alpha}$
2. For each internal photon with momentum k (as in Figure 4(b)) write $i - g_{\alpha\beta}/k^2 + i\varepsilon$
3. For each internal electron with momentum p (as in Figure 4(c)) write $i/\not{p} - m + i\varepsilon$ (where $\not{p}$ is an instance of the slash notation, an abbreviation for a long formula that I won't include here)
4. For each external electron (as in Figure 4(d)) write $u_r(\mathbf{p})$

Add the terms together. Do this for each diagram. Finally, we add all the terms from all the diagrams and calculate the end result, M, which (when squared) gives us the probability of the event.

These are just a few of the many rules. Some of the ones I am skipping are quite important; for instance, they involve considerations of symmetry and the requirement that momentum be conserved at each vertex. What I have included, however, should be sufficient for my purposes, which is to provide an account of three things: an FD, the perturbation series associated with the FD, and the physical process associated with the perturbation series and hence to the FD. This process of drawing diagrams and assigning mathematical terms is terminated at some stage. The contribution coming from higher-order diagrams is rapidly diminishing, so typically the series is cut off rather soon.

3. Feynman's Understanding

What did Feynman think of FDs? Did he take them to be pictures of physical process or were they devices for keeping track of computations, or perhaps a combination of the two? It is far from clear.

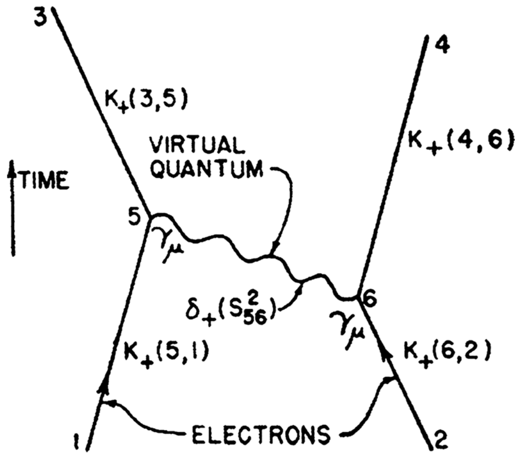


Figure 5. Feynman's first published diagram (Feynman, 1949a, 1949b).

Freeman Dyson knew Feynman well and spent many hours with him in the late 1940s discussing his approach to QED. Of course, this does not guarantee that he understood Feynman properly, but he was in a good position to get this right. It seemed to Dyson at this time that Feynman took his diagrams to provide a picture of physical processes.

> In Feynman's theory the graph corresponding to a particular matrix element is regarded, not merely as an aid to calculation, but as a picture of the physical process which gives rise to that matrix element. For example, an electron line joining x_1 to x_2 represents the possible creation of an electron at x_1 and its annihilation at x_2, together with the possible creation of a positron at x_2 and its annihilation at x_1. (Dyson 1949, p. 496)

In the 1960s, however, Feynman seemed to take a different view. In an interview with Charles Weiner, Feynman seems to deny that FDs are in any way a picture or model of reality but are instead an aid to calculation and nothing more.

> Feynman: I can't tell you when I first wrote them. […] I probably made diagrams to help me think about [perturbation expressions]. […] It was probably not any specific invention but just a sort of a shorthand with which I was helping myself think, which gradually developed into specific rules for some diagrams. […]

> Weiner: For helping you think physically? In other words, you were seeing in physical—
> Feynman: No, mathematical expressions. Mathematical expressions. A diagram to help write down the mathematical expressions. (Quoted in Wüthrich 2010, p. 6)

These are very different accounts of FDs coming from Feynman himself. Did he change his mind? Was he aware of the tension between these two views? It is not crucial that we know what he thought about the issue. But it does mean we can't turn to the creator of FDs as an unimpeachable authority who might pronounce infallibly on how these things were then intended to work or how they are intended to work today.

4. The Issues

FDs are useful for calculating, but do they do anything beyond this? The principle philosophical issues involving Feynman diagrams are these:

1. Are FDs pictures of physical reality?
2. Do FDs represent (in any sense) physical processes?
3. If FDs do not represent physical objects or processes, what do they do and why are they successful at it?

Of the more sophisticated views that I consider wrong are Adrian Wüthrich's and Letitia Meynell. Wüthrich contends that FDs "can function simultaneously as idealized representation of the phenomena under study and as a tool for deriving statements about these phenomena" (Wüthrich 2010, p. 13). Letitia Meynell (2008) makes a similar claim that FDs can be both computationally useful and in some important sense representational. My reply is the same to both: Tool? — yes. Ideal representation? — no.

5. Pictures of Physical Reality

I won't try to define "picture," but take it as understood. Photos and drawings that closely resemble their targets are obvious examples. They can be more abstract and still be pictures (think of a Picasso painting), but there is not likely to be a sharp boundary between pictures and other forms of representation. With this rough characterization of picture in mind, we can ask: Are FDs pictures of physical reality, specifically of quantum systems?

At first glance it might seem so. Freeman Dyson (as I quoted above), in his famous paper that unified the theories of Tomonaga, Schwinger, and Feynman, wrote, "In Feynman's theory the graph corresponding to a particular matrix element is regarded, not merely as an aid to calculation, but as a picture of the physical process…" (Dyson 1949, p. 496). FDs are depicted as processes in space and time; indeed, they first appeared in Feynman's article titled "A Space-time Approach…" and they look like cloud chamber tracks. There is even an interesting similarity with knot diagrams and the notations used to represent them. Eventually the analogy breaks down, but it is instructive to compare them, which I will do momentarily.

A number of people have objected to the idea that FDs are pictures of physical objects or processes. One of these objections is based on a familiar feature of quantum mechanics, namely, the non-existence of trajectories. In an earlier article on this theme I wrote,

> Feynman diagrams look like cloud chamber pictures, and they are often called space-time diagrams. This leads to the confusion. In fact, the diagrams do not picture physical processes at all. Instead, they represent probabilities (actually, probability amplitudes). The argument for this is very simple. In quantum mechanics (as normally understood) the Heisenberg uncertainty relations imply that no particle could have a position and a momentum simultaneously, which means there are no such things as trajectories, paths, through space-time. So the lines in a Feynman diagram cannot be representations of particles and their actual paths through space-time. (Brown 1996, pp. 265–67)

This point is certainly not new; Bohr made it at Feynman's first presentation in 1948 of his account of QED. Feynman, it seems, was perfectly aware; he was not claiming otherwise. This rejection of FDs as pictures of motion in space and time seems right as far as it goes, but the conclusion that FDs are merely calculation tools is not yet warranted and has been rightly criticized (Meynell 2008; Wüthrich 2010). There are other representational possibilities to consider (most recently by Stöltzner 2017).

Before looking in very different places, it is worth considering the similarity or lack of it in a specific physics example. The Compton effect, concerning the interaction of light with electrons, was one of the most important discoveries in the development of quantum mechanics (Compton 1923). Figure 6 is the diagram that appeared in Compton's original article.

When this diagram was published in 1923, Heisenberg's Principle had not yet arrived on the scene. At that time it would have been taken as a schematic picture of the actual physical process as it happened in space. Other diagrams from that period are pretty much the same. It is interesting to compare them with contemporary FDs of the Compton effect (Figure 7).

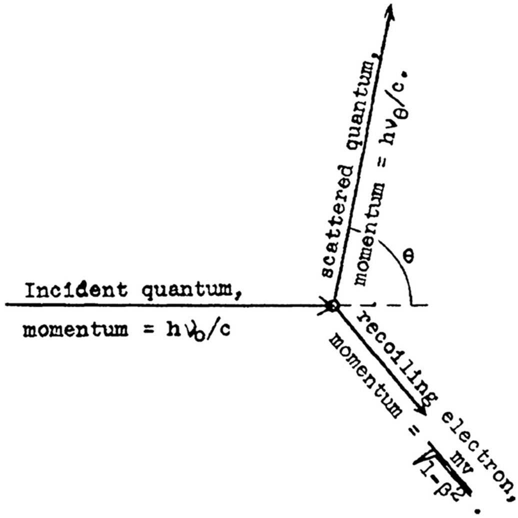


Figure 6. Compton's original diagram (Compton 1923).

As representations, they look similar, and so they should both be thought of as pictures of physical processes or neither should be. As a matter of fact, neither should for the reason already given — quantum mechanics is incompatible with trajectories through spacetime. In the case of the Compton effect diagram (Figures 6 and 7 [left]) we do take it to be a picture of the physical process, but the mistake of assuming trajectories is harmless, since the picture does no serious work. We should not make the same mistake in the FD version (Figure 7 [right]), since the diagram is instrumental only in making the relevant calculations.

A second objection to FDs being pictures of the physical realm is based on their use of virtual particles. In may FDs, a process is depicted that could not be observed in any sense, because, for instance, it violates the

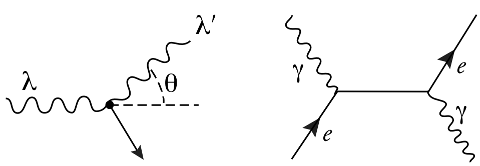


Figure 7. Two contemporary diagrams of the Compton effect: A standard diagram (left), and a Feynman diagram (right). (Time runs from left to right.)

conservation of energy. However, if this process lasts less than a given small time (i.e., does not violate the time-energy uncertainty relation), then its existence is not logically ruled out. A number of critics argue that such virtual entities do not exist. Two attitudes are possible in light of this objection. One is that physics should reject the use of virtual particles. The other is that they may be used but we should recognize their merely instrumental or fictional nature. In either case, diagrams with virtual entities do not picture reality. Of course, this objection to FDs being pictures is only as good as the objection to virtual particles, but the case against them is strong. We will not repeat these arguments, but instead refer readers to a sample from the literature. See, for instance, Bunge (1970) or Teller (1995).

There is a third objection, weaker than the others but worth mentioning. FDs assume a particle interpretation of QED. Such an interpretation is far from obviously correct and many would dispute it. The point, however, is that FDs would work perfectly well, even if such a particle interpretation is wrong. So, if FDs were pictures, they would be pictures of point particles, but if point particles don't exist, then FDs would still be successful. Still, their success would be a mystery.

6. Free Body Diagrams

There is an interesting analogy with so called free body diagrams (also known as force diagrams).[4] We start with the actual physical situation, which might be visible, or a fairly realistic diagram or photo. Then we draw a free body diagram. Usually this is a separate diagram, but it might be superimposed on the realistic picture. Finally, guided by the diagram, we attach numbers and use them in the appropriate equations to solve some problem of interest.

Start with a child on a slide. Let's suppose that the slide is frictionless, the angle of the slide with the ground is 45°, and the mass of the child is 25 Kg. We want to know the acceleration a of the child. (Figure 8)

There is a straightforward way to do this. The free body diagram shows all the forces. The force due to gravity $F_{gravity}$ is resolved into a force that is normal F_{normal} to the slide and a force along the slide F_{slide}. The last of these is $F_{slide} = F_{gravity} \times sin\ 45$. Since $F = ma$, the force $F_{gravity} = mg = 25 \times 9.8\ Newtons$. Thus, the force on the child is $F_{slide} = 25 \times 9.8 \times sin\ 45 = 173\ Newtons$. Consequently, the acceleration of the child down the slide is $a = F/m = 173/25 = 6.9\ m/s^2$.

4. Wüthrich (2010, p. 14) uses a similar example, but draws the opposite conclusion, claiming it is similar to an FD.

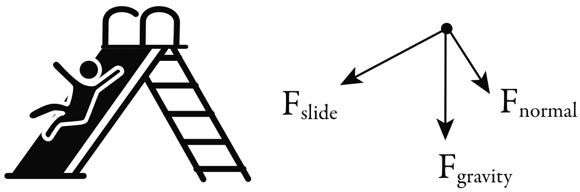


Figure 8. Child on slide and free body diagram representing the forces.

The reasoning pattern is simple:

physical system (or realistic picture)
↓
free body diagram
↓
Mathematics

If FDs were analogous to free body diagrams, then the pattern would be similar:

physical system (or realistic picture)
↓
Feynman diagram
↓
Mathematics

Of course, there is one obvious dis-analogy. In the typical example drawn from classical physics we have a clear understanding of the physical system in the sense that we know how to draw a free body diagram after looking at it (or at a realistic picture) and we know from seeing the actual system how to attach numbers to the elements of the free body diagram. This we cannot do in the quantum case. Instead, we are presented with something of a mystery. Instead of an independently accessible physical system, we really have little or no idea of how these processes work. We have little or no idea what the system could be or how the FDs might be linked to it. Instead of the diagram above, the situation seems more like this:

Inaccessible physical system
↓
Feynman diagram ↔ mathematics

The Feynman rules tell us how to link FDs to mathematics, but there is no guidance from the physical system. In fact, a significant problem becomes apparent. What link, if any, is there between the world and FDs? Why should we think any FD has any relation to reality at all that we could think of as a realistic representation or picture? It seems that FDs are just a way of helping with calculations. The Feynman rules keep track of the mathematics by utilizing the FDs. This neatly answered the question, how do FDs work? Still it leaves us with a major puzzle that we can express as a dilemma.

A free body diagram mediates between reality (or realistic picture of reality) and the mathematical realm with which we calculate measurable quantities. There are three independently accessible entities involved: the physical system, the diagram, and the mathematics. With a FD there are only two, the diagram and the mathematics. We could get a third element by having the FD do double duty: we assume it is a realistic picture of reality and that it is akin to a free body diagram. Or we could be content with there just being two components, the diagram and the mathematics. If we take the first route, we run into the acknowledged problem of violating the uncertainty principle, since electrons, and other elementary particles do not have trajectories. If we take the second route, then we have no connection between the FD and reality. It would be similar to having a free body diagram with no idea of what it represents, that is, no connection to the world at all. This is our dilemma.

7. Representations

Pictures of physical reality are not the only way to represent. Could FDs be representations of the physical realm in some more abstract way? Pictures, of course, form a range; they can be very similar to their targets or they can be rather schematic. But there are forms of representation that are not pictures in any sense. For instance, some periodic process might be represented by a sine curve (Figure 9), which in turn might be represented by a Taylor series:

$$\sin x = x - x^3/3! + x^5/5! - x^7/7! \ \ldots$$

Perhaps FDs represent physical processes in this more abstract way, like a Taylor series. This, I think, is Wüthrich's view.

> …it needs to be made clear that Feynman diagrams (certainly by the time that Dyson systematized the diagrams) are not classical representations of scattering events. They represent the latter's relevant aspects in a more abstract way: they show the connections between the creation and annihilation events, and these connections are understood to be the vacuum expectation values of field operators. (Wüthrich 2010, p. 31)

Letitia Meynell has argued at length for such an account (Meynell 2008). She acknowledges the argument that says FDs cannot in any ordinary sense

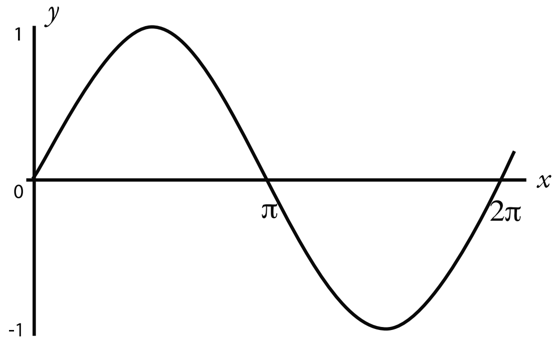


Figure 9. Sine curve.

be pictures of physical processes but claims that they might be representations nevertheless at some more abstract level. To this end she develops and applies the aesthetic views of Kendall Walton on fictions and make-believe. The crucial distinction she employs is that between "denotation" and "representation." The former refers to something but the latter need not. She concludes that FDs are indeed calculation devices, but that they are also representations of physical processes. "…we play a kind of make-believe with the diagrams that allows one to imagine the subatomic realm" (Meynell 2008, p. 55). Could it be that FDs are calculation tools and represent physical processes in some highly abstract sense? I will consider some proposals, but first, a brief discussion of the nature of applied mathematics.

8. How Does Mathematics Hook onto the World?[5]

Let us assume two distinct realms: one is a mathematical realm, which is rich enough to represent the second, a physical realm. We pick out an aspect of the physical world and find a similar mathematical structure to represent it. For example, weight is represented on a numerical scale. The main physical relations among objects that have weight determined, say, by a balance beam, are that some have more weight than others and that when objects are combined, their joint weight is greater than either of their individual weights. Weight can then be represented by any mathematical structure, such as the positive real numbers in which there is a greater-than relation matching the physical greater-than relation, and an addition relation matching the physical combination relation.

More generally, a mathematical representation of a non-mathematical realm occurs when there is a homomorphism between a relational system

5. This section is adapted from Brown (2008).

P and a mathematical system **M**. **P** will consist of a domain D and relations R_1, R_2, ... defined on that domain; **M** similarly consists of a domain D* and relations $R*_1$, $R*_2$, ... on that domain. A homomorphism is a mapping φ from D to D* that preserves the structure in an appropriate way. Consider a simplified example. Let D be a set of bodies with weight, let D* $=$ **R**, the set of real numbers; furthermore, let $\preccurlyeq$ and $\oplus$ be the relations of physically weighs the same or less than and physical composition. The relations $\leq$ and $+$ are the usual relations on real numbers of equal or less than and addition, respectively. The two systems, then, are $\mathbf{P} = \langle \mathrm{D}, \preccurlyeq, \oplus \rangle$ and $\mathbf{M} = \langle \mathbf{R}, \leq, + \rangle$. Numbers are then associated with the bodies (a, b, ... $\in$ D) by the homomorphism φ: D $\rightarrow$ **R** which satisfies the two conditions:

(1) $$a \preccurlyeq b \rightarrow \varphi(a) \leq \varphi(b)$$

(2) $$\varphi(a \oplus b) = \varphi(a) + \varphi(b).$$

In plain English, (1) says that if a weighs the same or less than b, then the real number associated with a is equal to or less than the number associated with b, and (2) says that the number associated with the weight of the combined object $a \oplus$ b is equal to the sum of the numbers associated with the objects separately. In other words, the relations that hold among physical bodies get encoded into the mathematical realm and are there represented by relations among real numbers. One of the objects can be singled out arbitrarily (but usually with an eye to convenience) to serve as the unit weight, u, so that $\varphi(u) = 1$.

The crucial point is that mathematics applies to the physical world by providing models or analogies, not by directly describing things. Strictly speaking, mass is not a number and force is not a vector, but they are represented by real numbers and vectors, respectively. The common phrase "a mathematical description of reality" is misleading if it means anything more than mathematics being used by a theory in this analogical way.

The physical combination of two bodies with weight, as noted above, is represented by the addition of two real numbers. But the embedding homomorphism is not always as simple as in this case. The relativistic addition of two velocities, for example, is constrained by an upper limit on their joint velocity. It is instructive to consider the difference.

Imagine a ball thrown forward with speed W inside an airplane that is flying at speed V with respect to the ground. Take W and V to be speeds, not the numbers representing them, just as we distinguished weight from their representing numbers. As outlined above, we associate real numbers with these speeds: $\varphi(W) = w$ and $\varphi(V) = v$. In Newtonian physics the composition of speeds takes a simple and familiar form: $\varphi(W \oplus V) = \varphi(W) + \varphi(V) = w + v$. However, in relativistic physics the composition of speeds is more complicated:[6]

$$\varphi(W \oplus V) = \frac{\varphi(W) + \varphi(V)}{1 + \frac{\varphi(W) \times \varphi(V)}{c^2}} = \frac{w + v}{1 + \frac{wv}{c^2}}$$

The shift from Galilean to relativistic addition of speed should be easy to follow, even though the latter is a departure from common sense. This is not always the case. Often, after an initial association between the physical realm and the mathematical, there is still much that is left open. We have yet to discover the full range of consequences of the initial association. Major mathematical innovations may be required. But we have said enough about the application of mathematics to the physical realm to pose our problem clearly.

With this account of mathematics behind us, we can now more clearly state the problem of FDs as follows: Do FDs represent the physical system **P** or the mathematical system **M**? That is, if FDs represent something, then what? Do they represent the physical, or the mathematical, both, or neither? I am going to argue that the correct answer is neither. Before getting to that, a brief digression to help set the stage.

9. Divergent Series

The perturbation series used by QED does not converge. This was proven long ago by Dyson (1952). It is, however, an asymptotic series, so it will work in the sense of giving useful answers for low order approximations, but it will eventually diverge as more terms are included. Thus, the mathematical representation of QED is in an important sense not consistent. This means that FDs cannot be a consistent representation of anything in the physical world. They could still be a representation that, perhaps, is pragmatically useful but not "intelligible."

A word of explanation about divergent series might be useful. A simple example of a convergent series is the series $\sum_1^\infty 1/2^n = 1/2 + 1/4 + 1/8 + \ldots$, since it equals 1, a finite number. The series $1 + 2 + 3 + 4 + \ldots$ is divergent; it obviously sums to infinity, as does the series $\sum_1^\infty 1/n = 1 + 1/2 + 1/3 + \ldots$.

6. Note that the range of φ is different in the two cases. Letting N be the set of all possible speeds in the Newtonian case, we have $\varphi : \mathrm{N} \rightarrow \mathbf{R}$, while in the Einsteinian case, with E the set of possible speeds, we have $\varphi : \mathrm{E} \rightarrow (-c, c)$, since c is a bound for any frame of reference.

The perturbation series of QED is divergent, but asymptotic, which means that we can truncate it after a few terms and get a useful, finite answer. Divergent series are, to say the least, dodgy. The great mathematician Abel called them the Devil's work and said they should shunned. Since his day, however, mathematicians have grown comfortable with their use, a use that has become quite fruitful.

In spite of their utility, the divergence of the perturbation series makes even the following modest attempt at a representational account impossible.

> Please understand: these Feynman diagrams are purely symbolic; they do *not represent particle* trajectories (as you might see them in, say, a bubble chamber photograph). The vertical dimension is *time, and horizontal spacings do not correspond to physical separations*. … All the diagram says is: "Once there was an electron and a positron; they exchanged a photon; then there was an electron and a positron again." (Griffiths 1987, p. 59. Cited in Wüthrich 2010, p. 3)

The author wants (quite rightly) to warn readers not to be naïve about FDs; they do not represent processes in space and time. However, he does say, "Once there was an electron and a positron; they exchanged a photon; then there was an electron and a positron again." Even this might be saying too much, as the following considerations suggests.

I now want to put two things together that were mentioned earlier: first, Dyson's argument for the one-one correspondence between diagrams and series terms, and second, Dyson's proof that the series is divergent. To be precise,

1. There is a one-one correspondence between diagrams and series terms.
2. The series diverges.
3. Thus, the series cannot coherently and correctly represent any physical process.
4. Therefore, diagrams cannot represent any physical process (ie, cannot represent in any reasonable sense of the term).

10. Flowcharts

We need a different way of thinking about FDs. I suggest that they are flow charts. To spell this out, I first need to distinguish two kinds of flow charts: descriptive and prescriptive. A prescriptive flow chart is not a model of a physical system or process. Instead it is a set of rules, instructions, or commands. By contrast, a descriptive flow chart is (at least typically) a model or representation of some process. Here are two simple illustrations.

Prescriptive flowchart: A recipe

Preheat oven and grease two pans

↓

Stir sugar, flour, eggs, milk,
and coco in large bowl

↓

Pour into the greased pans

↓

Bake for 35 mins at 190 C

↓

Let cool

↓

Stack the two layers and cover
with chocolate frosting

Descriptive flowchart: Digestion

Mouth

↓

Pharynx

↓

Esophagus

↓

Stomach

↓

Small intestine

↓

Large intestine

↓

Rectum (excretion)

It only takes a moment's reflection to see a huge difference. The descriptive flowchart is a description of reality (a purported description, of course; it is fallible). As representations go, it is perfectly straightforward. By contrast, the recipe does not even try to represent or model reality; it tells us how to achieve something.

With the descriptive-prescriptive distinction in mind, a FD is, I would suggest, a prescriptive flowchart. It is a set of commands. It is (implicitly) a set of instructions for assigning equations to physical processes. Wüthrich suggests something similar, calling them "algorithms."

> That the mathematical expressions M can be reduced to a diagram, described by Dyson as the "graphical representation of matrix elements," might do away with the need to write down a long integral expression in some cases, but this is not at all the main function of the diagrams. This visualization of the mathematical expressions is secondary. Rather, the most important function of the diagrams is that the graphical rules of connecting vertices and the subsequent translation into a mathematical expression form an algorithm to find all the non-zero matrix elements, and these alone. (Wüthrich 2010, p. 155)

There may be a sense in which even a recipe could be said to be representational: Reality is such that the recipe (above) for a chocolate cake will indeed result in a chocolate cake. Not all recipes do. Particularly unpromising is this recipe from *Macbeth*: "Eye of newt, and toe of frog / Wool of bat, and tongue of dog, / Adder's fork, and blind-worm's sting, / Lizard's leg, and howlet's wing, / For a charm of powerful trouble, / Like a hell-broth boil and bubble." As charms go, it has no power. As for making soup for lunch—don't even ask.

11. Concluding Remarks

We have three things to think about: the physical world, the mathematical realm that represents or models the physical world, and FDs, which seem to exist independently from the other two. The key question concerns the status of those diagrams. Are they models of the physical realm? If so, how do they model? If they are not representational, then what is their relation to both the physical realm and the mathematical realm? My answer has been that FDs are not descriptive or representational in any significant sense. Instead, they are prescriptions, instructions for how to write down the perturbation series that is a mathematical model of reality. The distinction between descriptive and prescriptive flow charts seems to capture this well.

So, what then is being visualized? I think the answer is simply this: FDs are geometric representations of probability functions. They are not pictures of phenomena. We should not confuse the visualization of the technique for constructing the perturbation series with a visualization of the physical process modelled by the perturbation series.

Understanding is a nebulous and subjective concept. A spacetime diagram of a physical process often provides considerable understanding of what is going on. FDs seem to provide that sort of understanding. But it is an illusion. They are wonderful at what they do, but they explain nothing and they provide nothing in the way of understanding the workings of nature.

FDs are excellent tools to help with calculations. They are no more than that. They are not pictures, they do not represent anything, and they do not provide some sort of insight into the quantum realm. Josef Jauch was right when he complained long ago:

> The pragmatic tendency of modern research has often obscured the difference between *knowing the usage of a language* and *understanding the meaning of its concepts*. There are many students everywhere who passed their examinations in quantum mechanics with top grades without really understanding what it all means. Often it is even worse than that. Instead of learning quantum mechanics in parrot-like fashion, they may learn in this fashion only particular approximation techniques (such as perturbation theory, Feynman diagrams or dispersion relations), which then lead them to believe that these useful techniques are identical with the conceptual basis of the theory. (Jauch 1968, p. v)

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