ABSTRACT  Our primary objective was to improve on an existing model for the individual weekly egg production curve by modeling the curve as a sum of logistic functions: one for the increasing phase of production and a sum for the decreasing phases. To illustrate the model, we used four data sets from two pairs of individuals.

For these data, the model consisted of an increasing phase and a single decreasing phase of production:

\[ y_t = m k_1 \left( \frac{1 - e^{-t}}{1 + e^{-t}} \right) - m(k_1 - k_2) \left( \frac{1 - e^{-t}}{1 + e^{-(t - c_2)}} \right) \]

where \( y_t \) is egg production at week \( t \), \( m \) is maximum production during a specific time interval, \( k_1 \) is proportion of maximum production for the increasing phase, \( k_2 \) is proportion of maximum production for the decreasing phase, and \( c_2 \) is point of inflection from the upper level of the increasing phase to the lower level of the decreasing phase; thus, \( c_2 \) is a measure of persistency of egg production.

For one pair of individuals, production was about 88% of maximum (\( k_1 \)) for the increasing phase and about 76% of maximum (\( k_2 \)) for the decreasing phase. For the other pair, production was about 91% of maximum (\( k_1 \)) for the increasing phase and about 75% of maximum (\( k_2 \)) for the decreasing phase. Persistency (\( c_2 \)) was about 25 wk for one pair and about 28 wk for the other. Predicted total 52-wk production was within one or two eggs of actual production.

The secondary objective was to improve estimation of model parameters by summarizing egg production data by 1-wk, 2-wk, or 4-wk intervals and by using cumulative egg production instead of weekly production. For weekly production, estimates of parameters changed only slightly, as intervals increased from 1 wk to 2 wk or to 4 wk. Predicted total 52-wk production, however, decreased up to five eggs as interval increased from 1 wk to 4 wk.

For cumulative egg production by time \( t \), \( Y_t \), the model was

\[ Y_t = 7 k_1 \left[ 2 \log \left( \frac{1 + e^t}{2} \right) - t \right] - 7(k_1 - k_2) \left( 1 + e^{-3} \right) \log \left( \frac{e^2 + e^t}{1 + e^{2c_2}} \right) - t e^{-c_2}. \]

For cumulative production, estimates of parameters changed only slightly, if at all, as intervals increased from 1 wk to 4 wk. Predicted 52-wk production, however, approached the actual number as interval increased from 1 wk to 4 wk.

Prediction of annual (52-wk) egg production based on part-record production for only the first 22 wk might lead to over-prediction because persistency of production lasted longer than the part record. Genetic gain from selection to improve annual production, therefore, might be increased if selection accounted for persistency of production and for the multiphasic shape of the individual egg production curve, and if data were summarized by 4-wk intervals and cumulated.

(\textit{Key words}: mathematical model, egg production, persistency, individual egg production curve, chicken)

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INTRODUCTION

One major weakness of the usual analysis of egg production data is “failure to recognize the entire [production] curve either by the analysis of production at each period or by accumulated production” (Tonkinson et al., 1969). Mathematical models have been used to better understand the biology of egg production (Etches and Schoch, 1984; Koops and Grossman, 1992), to describe egg production curves for a flock (for a review, see Fairfull...
A typical egg production curve for a flock increases during the first 8 or 9 wk of production and then decreases to the end of the production period (North and Bell, 1990). For flock production, there is variability in age at first egg for individual hens, so that increase in weekly egg production is relatively slow and relatively smooth (North and Bell, 1990). The increase in the flock production curve has been modeled by a logistic function (Adams and Bell, 1980; Cason and Britton, 1988; Yang et al., 1989; Cason and Ware, 1990) and by the first compartment of a two-compartmental model (Gavora et al., 1971; McMillan, 1981; Gavora et al., 1982; McMillan et al., 1986). The decrease in the flock production curve has been modeled by a linear function (Adams and Bell, 1980), an exponential function (Gavora et al., 1971; McNally, 1971; Foster et al., 1987; Cason and Britton, 1988; Yang et al., 1989; Cason, 1990; Cason and Ware, 1990), and a polynomial function (Lokhorst, 1996). Other approaches to modeling the egg production curve for flocks include use of growth functions for the increasing term combined with linear or curvilinear functions for the decreasing term (Cason and Ware, 1990), a cyclic function (Bell and Adams, 1992), segmented polynomials (Fialho and Ledur, 1997), and smoothed intersecting straight lines (Grossman et al., 2000).

A typical egg production curve for an individual, in contrast to a flock, increases within the first 2 wk of production, maintains a level of production, and then decreases to the end of the production period (North and Bell, 1990). For individual hen production, there is no variability in age at first egg, so that increase in weekly egg production is relatively rapid and relatively abrupt (North and Bell, 1990). The individual weekly egg production curve has been described by a two-compartment model (Gavora et al., 1971), which modeled the increase and exponential decrease in production, and by smoothed intersecting straight lines (Grossman et al., 2000), which modeled the increase, the constant level, and the linear decrease in production.

A model for an individual egg production curve is necessary in order to select individuals based on specific parameters of the egg production curve, rather than on total egg production, which is a function of all parameters (McMillan, 1981). The discrete nature of individual egg production, however, requires the level of weekly production to be an integer value. The existence of clutches, moreover, results in short periods of lowered weekly egg production over the production period. The decrease in weekly egg production, therefore, occurs as a series of nonlinear phases. Until now, there was no model that described the multiple phases of decrease in an individual egg production curve and that also accounted for integer values or clutches.

The primary objective of this study was to improve on an existing model for the individual weekly egg production curve (Grossman et al., 2000) by modeling the egg production curve as a sum of logistic functions: one function describing an increasing phase of production and a sum of functions describing a series of decreasing phases. The secondary objective was to improve estimation of model parameters by summarizing weekly egg production data by 1-wk, 2-wk, or 4-wk intervals and by using cumulative egg production instead of weekly production. To illustrate the model, and for the purpose of comparison, we used the four data sets from two pairs of individuals that were used by Grossman et al. (2000).

**MATERIALS AND METHODS**

**General Model**

**Weekly Egg Production.** We described the ideal weekly egg production curve for an individual by a multiphasic curve with an increasing phase of production and a series of decreasing phases of production (Figure 1). The increasing phase was described by an increasing logistic function of time $t$, and the decreasing phases were described by a sum of decreasing logistic functions of time. The logistic function is continuous for $-\infty < t < +\infty$, is symmetric around the inflection point, and has a lower asymptote of zero when $t = -\infty$ and an upper asymptote of unity when $t = +\infty$.

The rapidly increasing first phase in weekly egg production during the first 2 wk was expressed as an increasing logistic function of time $t$:

$$y_t = A_1 \frac{1}{1 + e^{-(t - c_1)/b_1}} \quad [1]$$

where $y_t =$ egg production at time $t$ (wk), $A_1 =$ upper level of production (eggs), $c_1 =$ time of maximum increase or time of point of inflection (wk), $b_1 =$ a measure of duration of the increasing phase (wk), and $e =$ base of
the natural logarithm. Note that when \( t = c_1 \), then \( y_{c_1} = A_1/2 \) so that half of the upper level of production is achieved by \( t = c_1 \), and the remaining half is achieved by \( t = +\infty \).

For our purposes, we assumed that starting at \( t = 0 \) a hen’s egg production increased rapidly at a decreasing rate toward an upper level of production, and so we set the time of point of inflection \( c_1 = 0 \). We assumed also that the first production datum for a hen was not available until \( t = 1 \) so that at \( t = 0, y_0 = 0 \). Applying these assumptions to Equation [1], and substituting \( a_i \) for \( A_1/2 \), yielded

\[
y_t = a_i \left( \frac{1 - e^{-t/b_i}}{1 + e^{-t/b_i}} \right)
\]

which has a lower level of production of 0 when \( t = 0 \) and an upper level now of \( a_i \) when \( t = +\infty \).

Following a period of constant production at the level of \( a_i \), the decreasing phases in weekly egg production throughout the production period were expressed as a sum of \( n \) decreasing logistic functions of time \( t \), each phase \( i \) starting from the upper level of the previous phase \( a_{i-1} \) and decreasing to the lower level of the current phase \( a_i \):

\[
-\sum_{i=2}^{n} (a_{i-1} - a_i) \left( \frac{1 - e^{-t/b_i}}{1 + e^{-t/c_i/b_i}} \right)
\]

which is 0 when \( t = 0 \) and is \( - (a_1 - a_n) \) when \( t = +\infty \), and where for each phase \( i, c_i = \) time of maximum decrease or point of inflection (wk), and \( b_i = \) a measure of duration of the transition to the decreasing phase (wk).

To describe the entire increasing and decreasing phases of individual weekly egg production, therefore, Equations [2] and [3] were summed:

\[
y_1 = a_i \left( \frac{1 - e^{-t/b_i}}{1 + e^{-t/b_i}} \right) - \sum_{i=2}^{n} (a_{i-1} - a_i) \left( \frac{1 - e^{-t/b_i}}{1 + e^{-t/c_i/b_i}} \right)
\]

which is 0 when \( t = 0 \) and is \( a_i \) when \( t = +\infty \), and where the number of decreasing phases \( n \) depends on the individual hen.

Total egg production over a period \( 0 \leq t \leq T \), \( y_T \), was computed by integrating Equation [4] from 0 to \( T \), with respect to week \( t \):

\[
y_T = a_i \int_0^T \frac{1 - e^{-t/b_i}}{1 + e^{-t/b_i}} dt - \sum_{i=2}^{n} (a_{i-1} - a_i) \int_0^T \frac{1 - e^{-t/b_i}}{1 + e^{-t/c_i/b_i}} dt
\]

which yielded

\[
y_T = a_1 \left[ 2b_1 \ln\left( \frac{1 + e^{T/b_1}}{2} \right) - T \right] - \sum_{i=2}^{n} (a_{i-1} - a_i) \left( \frac{1 + e^{-T/b_i}}{1 + e^{-c_i/b_i}} \right) \ln\left( \frac{e^{c_i/b_i} + e^{T/b_i}}{1 + e^{c_i/b_i}} - Te^{-c_i/b_i} \right)
\]

where \( \ln \) denotes the natural logarithm. To compute total egg production to a fixed time \( T \), e.g., annual egg production to 52 wk, set \( T = 52 \).

**Cumulative Egg Production.** To study the use of cumulative egg production instead of weekly production, cumulative egg production by variable time \( t, Y_b \), was modeled based on Equation [6]:

\[
Y_t = a_1 \left[ 2b_1 \ln\left( \frac{1 + e^{t/b_1}}{2} \right) - t \right] - \sum_{i=2}^{n} (a_{i-1} - a_i) \left( \frac{1 + e^{-t/c_i/b_i}}{1 + e^{-c_i/b_i}} \right) \ln\left( \frac{e^{c_i/b_i} + e^{t/b_i}}{1 + e^{c_i/b_i}} - te^{-c_i/b_i} \right)
\]

with \( f_i = \) the number of phases, \( k_i = \) the sum of each phase, \( f_i <= k_i \), \( f_i = \) the time of maximum increase of \( f_i \), and \( k_i = \) the time of point of inflection of \( f_i \).

**Reduced Model**

**Weekly Egg Production.** Preliminary analyses (not shown) of data on individual hens led to results that helped to characterize the curve for weekly egg production in these data. First, the maximum number of phases that could be fit, with statistically significant estimates for parameters, was two \( (n = 2) \), one increasing phase and one decreasing phase. Second, the estimate of the measure of duration of the increasing phase \( b_1 \) was not significantly different from unity, meaning that transition from one level of production to the next was 95% complete within about 7 wk (Gupta and Gnanadesikan, 1966). Assuming that the measure of duration of the increasing phase \( b_1 \) was equal to the measure of duration of the decreasing phase \( b_2 \), we set \( b_1 = b_2 = 1 \). These results led to a simplification of the weekly egg production model, Equation [4], to a reduced model:

\[
y_t = a_1 \left[ 1 - e^{-t/b_1} \right] - (a_1 - a_2) \left[ 1 - e^{-t/c_2} \right]
\]

where \( c_2 \) is the point of inflection from the upper level of the increasing phase of production to the lower level of the decreasing phase and, thus, is a measure of persistency (in weeks) of egg production. Note that the first expression on the right-hand side of Equation [8] has the scaling factor \( a_1 \) as its only parameter, indicating that differences in the increasing phase are only in the upper level of production. These results led also to a simplification of the computation of total egg production to a fixed time \( T \), \( y_T \) of Equation [6]:

\[
y_T = a_1 \left[ 2 \ln\left( \frac{1 + e^{T/2}}{2} \right) - T \right] - (a_1 - a_2) \left( 1 + e^{-c_2} \right) \ln\left( \frac{e^{c_2} + e^{T/2}}{1 + e^{c_2}} - Te^{-c_2} \right)
\]
To generalize the model to intervals of production other than weekly, e.g., 2-wk intervals or 4-wk intervals, it might be useful to express each asymptotic level of production as a proportion of maximum production during a specific time interval, e.g., as a proportion of 14 eggs in a 2-wk interval, assuming one egg is produced per day. If maximum production in an interval of m days is assumed to be m eggs, then \( a_1 = m k_1 \) and \( a_2 = m k_2 \), where \( k_1 \) is the proportion of maximum production m for the increasing phase, and \( k_2 \) is the proportion of maximum production m for the decreasing phase. Substituting these proportions into Equation [8] yielded

\[
y_t = m \left( k_1 \left( \frac{1 - e^{-t}}{1 + e^{-t}} \right) - m(k_1 - k_2) \left( \frac{1 - e^{-t}}{1 + e^{-t(k_1-k_2)}} \right) \right)
\]

for egg production during the specific interval. For production summarized by 1-wk intervals, \( m = 7 \); by 2-wk intervals, \( m = 14 \); or by 4-wk intervals, \( m = 28 \).

For total egg production to a fixed time T, from Equation [9]:

\[
y_T = 7 k_1 \left( 2 \ln \left( \frac{1 + e^T}{2} \right) - T \right) - 7(k_1 - k_2) \left( (1 + e^{-c_2}) \ln \left( \frac{e^{c_2} + e^T}{1 + e^{c_2}} \right) - T e^{-c_2} \right)
\]

where, e.g., \( T = 52 \) for annual egg production to 52 wk. Note in Equation [11] that 7 denotes the maximum number of eggs per week and not the number of days per week.

**Cumulative Egg Production.** It is possible to characterize egg production on the basis of cumulative egg production (Koops and Grossman, 1992). Applying assumptions for the reduced model to Equation [7] for cumulative egg production by variable time t, \( Y_t \), and simplifying, we obtained

\[
Y_t = 7 \left( k_1 \left( 2 \ln \left( \frac{1 + e^t}{2} \right) - t \right) - 7(k_1 - k_2) \left( (1 + e^{-c_2}) \ln \left( \frac{e^{c_2} + e^t}{1 + e^{c_2}} \right) - t e^{-c_2} \right) \right)
\]

### Data

To illustrate the model, four data sets from two pairs of individuals were used. Data were provided by W. M. Muir (1999, Purdue University, West Lafayette, IN 47907, personal communication) and were used in a previous study to model persistency of egg production (Grossman et al., 2000). Individuals selected for this study started production between 15 and 19 wk of age. Eggs were collected daily, and number of eggs was summarized weekly for each hen for 52 wk from the start of production.

Pairs of hens were selected that produced the same number of eggs over the 52-wk production period; Hens 3073 and 3127 each produced 291 eggs, and Hens 3129 and 3272 each produced 299 eggs. Although each hen per pair produced the same number of eggs over 52 wk, they did so with different patterns of egg production curves (Grossman et al., 2000). The slightly lower level of constant production for Hen 3073 compared with Hen 3127 was compensated for by a longer persistency. The lack of persistency of Hen 3129, however, was compensated for by a slower rate of decline for Hen 3129 compared with Hen 3272.

For this study, it was of practical interest to examine the effect of the interval of summary on estimates of model parameters. Number of eggs, therefore, was summarized by 2-wk and by 4-wk intervals. It was also of practical interest to examine the use of cumulative egg production, instead of weekly egg production, to estimate model parameters. Number of eggs, therefore, was cumulated and was also summarized by 2-wk and by 4-wk intervals.

### Statistical Analysis

Nonlinear regression (Sherrod, 1998) was used to fit Equation [10] to observed weekly egg production for each individual, for 1-wk, 2-wk, and 4-wk intervals. The SWEEP statement was used for model parameter \( c_2 \) to ensure that the algorithm found a global minimum and not a local minimum. Total 52-wk egg production for each individual was computed using Equation [11]. Nonlinear regression was used also to fit Equation [12] to cumulative egg production for each individual, for 1-wk, 2-wk, and 4-wk intervals.

The convergence criterion for the iterative nonlinear estimation procedure was set at \( 1 \times 10^{-10} \). Goodness-of-fit criteria were adjusted \( R^2 \), residual SE, and Durbin-Watson statistic (D-W), for which a value around 2 indicates lack of first-order autocorrelation (Neter et al., 1985).

### RESULTS

**Weekly Egg Production**

Estimates of model parameters, SE of parameters, total 52-wk egg production \( y_{52} \), adjusted \( R^2 \), residual SE, and D-W for 1-wk, 2-wk, and 4-wk intervals are in Table 1, for individual egg production for the two pairs of hens. Each parameter estimate was significantly different from zero \( (P \leq 0.001) \). Goodness-of-fit criteria indicated that the model provided a better fit to egg production data summarized by 1-wk intervals, generally, than to data summarized by 2-wk or 4-wk intervals; \( R^2 \) was higher and RSE was lower, but D-W was not always closest to 2.

Actual and predicted weekly egg production are in Figure 2 for Hens 3073 and 3127. Recall that each hen produced 291 eggs in 52 wk. Each hen (Table 1) reached an upper level of about 88% of maximum \( (k_i) \) during the rapidly increasing phase, maintained that level until
TABLE 1. Estimates of model parameters, {\textsuperscript{1}} SE of parameters, total 52-wk egg production \( y_{52} \), adjusted \( R^2 \), residual SE (RSE), and Durbin-Watson statistic (D-W) for 1-wk, 2-wk and 4-wk intervals for individual egg production for two pairs of hens

<table>
<thead>
<tr>
<th>Interval (wk)</th>
<th>Parameter</th>
<th>( y_{52} ) (eggs)</th>
<th>Goodness-of-fit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( k_1 )</td>
<td>( c_2 ) (wk)</td>
<td>( k_2 )</td>
</tr>
<tr>
<td>Hen 3073</td>
<td>1</td>
<td>0.878 (0.020)</td>
<td>25.0 (2.0)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.855 (0.012)</td>
<td>28.6 (2.7)</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.846 (0.031)</td>
<td>28.6 (3.8)</td>
</tr>
<tr>
<td>Hen 3127</td>
<td>1</td>
<td>0.878 (0.027)</td>
<td>25.7 (2.8)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.864 (0.037)</td>
<td>26.4 (4.5)</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.840 (0.040)</td>
<td>27.3 (6.3)</td>
</tr>
<tr>
<td>Hen 3129</td>
<td>1</td>
<td>0.911 (0.020)</td>
<td>26.1 (1.8)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.895 (0.024)</td>
<td>27.6 (2.4)</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.877 (0.032)</td>
<td>28.9 (3.6)</td>
</tr>
<tr>
<td>Hen 3272</td>
<td>1</td>
<td>0.914 (0.024)</td>
<td>30.1 (1.9)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.900 (0.025)</td>
<td>31.8 (2.1)</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.883 (0.038)</td>
<td>33.7 (4.6)</td>
</tr>
</tbody>
</table>

\[
y_t = m_k \left( \frac{1 - e^{-t}}{1 + e^{-t}} \right) - m(k_1 - k_2) \left( \frac{1 - e^{-t}}{1 + e^{-c_2(t - c_2)}} \right)
\]

about 25 wk of production (\( c_2 \)), i.e., persistency was about 25 wk, and declined to about 76% of maximum (\( k_2 \)). Predicted total 52-wk production (\( y_{52} \)) was about 289 eggs for Hen 3073 and about 290 eggs for Hen 3127, a difference from actual production of only one or two eggs.

Examination of residuals for Hens 3073 and 3127 (Figure 3) revealed a more or less random distribution, more so for Hen 3073 (D-W = 2.41) than for Hen 3127 (D-W = 1.83). A value for D-W greater than 2 (Hen 3073) indicates a negative autocorrelation, so that weekly egg production tended to be over- or underestimated one week if it was under- or overestimated the previous week. A value for D-W less than 2 (Hen 3127), however, indicates a positive autocorrelation, so that weekly egg production tended to be over- or underestimated one week if it was over- or underestimated the previous week. A negative autocorrelation, therefore, is more desirable than a positive autocorrelation, hence a D-W value greater than 2 is more desirable than a D-W value less than 2.

Actual and predicted weekly egg production are in Figure 4 for Hens 3129 and 3272. Recall that each hen produced 299 eggs in 52 wk. Each hen (Table 1) reached about 91% of maximum (\( k_1 \)) during the rapidly increasing phase. Hen 3129, however, maintained that level until about 26 wk of production (\( c_2 \)), i.e., persistency was about 26 wk, whereas Hen 3272 maintained that level until about 30 wk of production, i.e., persistency was about 30 wk. Hen 3129 declined to about 77% of maximum (\( k_2 \)), whereas Hen 3272 declined to about 74% of maximum. Predicted total 52-wk production (\( y_{52} \)) was about 298 eggs.
for Hen 3129 and about 297 eggs for Hen 3272, a difference from actual production of only one or two eggs.

Examination of residuals for Hens 3129 and 3272 (Figure 5) again revealed a more or less random distribution, more so for Hen 3272 ($D-W = 2.50$) than for Hen 3129 ($D-W = 1.86$); again, a $D-W$ value greater than 2 is more desirable than a $D-W$ value less than 2.

When intervals of summary increased from 1 wk to 2 wk or to 4 wk, estimates of parameters changed only slightly. Upper level of production ($k_1$) decreased, persistency of production ($c_2$) generally increased, and lower level of production ($k_2$) remained the same. Predicted total 52-wk production ($y_{52}$), however, decreased up to five eggs as interval increased from 1 to 4 wk.

**Cumulative Egg Production**

Estimates of model parameters, SE of parameters, 52-wk egg production $Y_{52}$, adjusted $R^2$, residual SE, and $D-W$ for 1-wk, 2-wk, and 4-wk intervals are in Table 2, for cumulative egg production for the two pairs of hens. Each parameter estimate was significantly different from zero ($P \leq 0.001$). Goodness-of-fit criteria indicated that the model provided an excellent fit to cumulative egg production data summarized by 1-wk, 2-wk, or 4-wk intervals, perhaps better for 4-wk intervals when judged only on $D-W$.

Recall that Hens 3073 and 3127 each produced 291 eggs in 52 wk. Each hen reached a level of about 90% of maximum ($k_1$) during the increasing phase, maintained that level until about 23 wk of production ($c_2$), i.e., persistency was about 23 wk, and declined to about 76% of maximum ($k_2$). Predicted 52-wk production ($Y_{52}$) was about 291 eggs for each hen.

Examination of residuals for Hens 3073 and 3127 (Figure 6) revealed a positive first-order autocorrelation, hence a nonrandom distribution, more so for Hen 3073 ($D-W = 0.48$) than for Hen 3127 ($D-W = 0.68$). Residuals became more random as interval of summary increased from 1 to 4 wk.

Recall that Hens 3129 and 3272 each produced 299 eggs in 52 wk. Hen 3129 reached a level of about 98% of maximum ($k_1$), whereas Hen 3272 reached a level of about 93% of maximum. Hen 3129 maintained the level of production until about 16 wk ($c_2$), i.e., persistency was about 16 wk, whereas Hen 3272 maintained the level of production until about 29 wk, i.e., persistency was about 29 wk. Hen 3129 declined to about 79% of maximum ($k_2$), whereas Hen 3272 declined to about 74% of maximum. Predicted 52-wk production ($Y_{52}$) was about 301 eggs for Hen 3129 and about 300 eggs for Hen 3272, a difference from actual production of only one or two eggs.

Examination of residuals for Hens 3129 and 3272 (Figure 7) again revealed a positive first-order autocorrelation, and hence a nonrandom distribution, more so for Hen 3129 ($D-W = 0.27$) than for Hen 3272 ($D-W = 1.03$). Residuals became more random as interval of summary increased from 1 to 4 wk.

When intervals of summary increased from 1 to 2 wk and to 4 wk, estimates of parameters changed only slightly. Upper level ($k_1$) decreased, persistency of production ($c_2$) generally increased, and lower level ($k_2$) remained about the same. Predicted 52-wk egg production ($Y_{52}$), however, approached the actual number as interval increased from 1 to 4 wk.

Compared with weekly egg production (Table 1), results for cumulative egg production (Table 2) indicated lower SE for parameters, higher values for $R^2$, and lower values for $D-W$, as expected from cumulative data with correlated errors. Predicted 52-wk egg production for cumulative data, however, was similar to that predicted for weekly data.

**DISCUSSION**

The primary objective of this study was to improve on an existing model for the individual weekly egg production curve (Grossman et al., 2000). The existing model
Table 2. Estimates of model parameters, SE of parameters, 52-wk egg production Y, adjusted R², residual SE (RSE), and Durbin-Watson statistic (D-W) for 1-wk, 2-wk and 4-wk intervals for cumulative egg production for two pairs of hens

<table>
<thead>
<tr>
<th>Interval (wk)</th>
<th>Parameter</th>
<th>Yₜₑ (eggs)</th>
<th>Goodness-of-fit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>k₁</td>
<td>c₂ (wk)</td>
<td>k₂</td>
</tr>
<tr>
<td>Hen 3073</td>
<td>1</td>
<td>0.901 (0.002)</td>
<td>23.26 (0.59)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.902 (0.004)</td>
<td>23.23 (0.95)</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.900 (0.006)</td>
<td>23.51 (1.44)</td>
</tr>
<tr>
<td>Hen 3127</td>
<td>1</td>
<td>0.887 (0.002)</td>
<td>23.89 (0.69)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.888 (0.004)</td>
<td>23.65 (1.07)</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.891 (0.005)</td>
<td>22.62 (1.34)</td>
</tr>
<tr>
<td>Hen 3129</td>
<td>1</td>
<td>0.981 (0.006)</td>
<td>16.42 (0.61)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.972 (0.008)</td>
<td>17.61 (0.95)</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.968 (0.012)</td>
<td>18.40 (1.49)</td>
</tr>
<tr>
<td>Hen 3272</td>
<td>1</td>
<td>0.931 (0.002)</td>
<td>29.00 (0.38)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.931 (0.002)</td>
<td>29.28 (0.57)</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.931 (0.003)</td>
<td>29.97 (0.84)</td>
</tr>
</tbody>
</table>

\[ Y_t = 7k_1 \left[ 2 \ln \left( \frac{1 + e^{t/k_1}}{2} \right) \right] - 7 \left( k_1 - k_2 \right) \left[ \ln \left( \frac{e^{t/k_1} + e^{t/k_2}}{1 + e^{t/k_1}} \right) - te^{t/k_2} \right]. \]

A MODEL FOR INDIVIDUAL EGG PRODUCTION

We achieved our objective in this study by treating the egg production curve as a sum of logistic functions, one function for the increasing phase of production and a sum of functions for the decreasing phases. The model for individual weekly egg production for these data consisted of only two phases, an increasing phase and a decreasing phase. The proportion of variance explained by the model (R²) averaged 0.48 for egg production summarized by 1-wk intervals and was similar to that of Gavora et al. (1971) (0.48) and to that of Grossman et al. (2000) (0.51). The improvement in this model, over that presented by Grossman et al. (2000), was not in the R² but in the recognition that decrease in weekly egg production is discrete in nature, and hence in the introduction of a sum of nonlinear logistic functions to describe that discrete decline.

The improved model contains a measure of persistency that is similar, but not identical, to the measure of persistency proposed by Grossman et al. (2000). This new measure of persistency is the time of transition (point of inflection) from the upper level of the increasing phase of production to the lower level of the decreasing phase.

FIGURE 6. Residual cumulative egg production for Hen 3073 (○) and for Hen 3127 (+).

FIGURE 7. Residual cumulative egg production for Hen 3129 (○) and for Hen 3272 (+).
Persistency was about 25 wk for one pair of hens and about 26 and 30 wk for the other pair. This result has an important consequence for predicting annual egg production from a part record. If the part record were only the first 22 wk, then annual production would be overpredicted because time of transition would be after the part record. Different parts of the egg production curve, furthermore, have different heritabilities (Flock, 1977; Muir, 1990), which supports the notion of multiple phases of egg production. Failure to recognize the multiphasic shape of the individual egg production curve might account for mixed results to improve annual egg production by selecting for part-record production (Bohren et al., 1970).

The secondary objective was to improve estimation of model parameters by summarizing weekly egg production data by 1-wk, 2-wk, or 4-wk intervals and by using cumulative egg production instead of weekly egg production. For weekly production, total 52-wk production was predicted within one or two eggs of actual production. Prediction decreased between about one to five eggs, however, as interval of summary increased from 1 to 4 wk, which was similar to that of Grossman et al. (2000). The $R^2$ decreased as interval increased to 4-wk, which is in contrast to that of Gavora et al. (1971) and of Grossman et al. (2000). A decrease in $R^2$ might be expected, in fact, because the multiphasic pattern of egg production, characteristic of weekly data, was obscured when data were cumulated. Residual SE for weekly egg production averaged 0.76, which was similar (0.74) to that of Grossman et al. (2000) for the same data, and increased as interval increased, as expected, because number of data points decreased.

For cumulative egg production, predicted 52-wk production was within one or two eggs of actual production, but prediction approached the actual number as interval of summary increased from 1 wk to 4 wk. The $R^2$ was high (about 0.9998) and remained high as interval increased. Residual SE for weekly egg production averaged 0.91 and generally increased slightly as interval increased, as expected. These results suggest that accurate prediction of 52-wk egg production can be achieved by summarizing data at 4-wk intervals and cumulating them.

One limitation of this study was the fact that, although the distribution of residuals for weekly egg production was more or less random (Figures 3 and 5), the distribution of residuals for cumulative egg production revealed a positive autocorrelation (Figures 6 and 7), which indicated that long runs with negative residuals were followed by long runs with positive residuals or vice versa.

The presence of long runs in the residuals might be associated with the presence of clutches during the ovulatory cycle of the hen. Because the end of a clutch might come at an arbitrary (perhaps random) day during a 1-wk interval, egg production for that week will be always less than maximum, assuming that a hen lays no more than one egg per day. Thus variation around a level of production will not be random and probably will not be normally distributed but, rather, will be skewed to the right. Recall, furthermore, that the $R^2$ for weekly egg production was only about 0.48. The challenge to increase $R^2$ can be met with further research to account for clutches and the resulting skewness in modeling the individual egg production curve.

Heritability of total egg production is low. Our findings might have important consequences if selection to improve annual egg production is based on a part record. Part-record production for only the first 22 wk might lead to over-prediction of annual production using this model, because persistency of production lasted longer than the part record. Genetic gain from selection to improve annual production, therefore, might be increased if selection were based on a part record that accounted for the multiphasic shape of the individual egg production curve, and if data were summarized by 4-wk intervals and cumulated. Further research in this area is necessary.

**ACKNOWLEDGMENTS**

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**REFERENCES**


