Experimental Identification and Analysis of the Dynamics of a PHANToM Premium 1.5A Haptic Device

Abstract

The dynamics of a PHANToM Premium 1.5A haptic device from SensAble Technologies, Inc. is experimentally identified and analyzed for different installations of the device and its accessories, such as the typical upright, upside down, with gimbal and counterbalance weight, and with force sensor. An earlier formulation of the robot dynamic model is augmented with a friction model, linearly parameterized, and experimentally identified using least squares. The identified dynamics are experimentally evaluated with an inverse dynamics controller and verified by comparing user hand force estimates with the measured values. The contribution of different dynamic terms such as inertial, Coriolis and centrifugal, gravitational, and Coulomb and viscous friction are demonstrated and discussed. The identified model can be used for a variety of haptic applications, such as hand force estimation, accurate active gravity compensation and counterbalance weight determination for various installation conditions, and model-based control for haptic simulation and teleoperation.

1 Introduction

The PHANToM Premium 1.5A is a desktop haptic device by SensAble Technologies, Inc. that provides force feedback with three degrees of freedom (3 DOF; Massie & Salisbury, 1994). Although the PHANToM has been widely used in haptic and telerobotic applications (Cavusoglu, Sherman, & Tendick, 2002; Mobasser & Hashtrudi-Zaad, 2008; Tavakoli, Patel, & Moalem, 2004), its functionality is not satisfactory for some high performance applications, partly because its electrical and software subsystems are unknown. Accurate dynamic models of the device are desired for control, simulation, and contact force observation.

There are two main approaches for identifying the device dynamics: (1) piecewise and (2) experimental identification methods. In the piecewise method, the mechanical properties of various components of a manipulator such as the mass and inertia of linkages, and stiffness of transmission systems, are individually measured (Cavusoglu & Feygin, 2001; Cavusoglu, Feygin, &
Tendick, 2002). A major challenge for applying this approach is the need for disassembling the robot, which may not be easy or applicable in many cases. Alternatively, Computer Aided Design (CAD) models have been used to provide an estimate of the mechanical properties based on the geometry and material composition. A challenge in applying such a technique is the introduction of inaccuracies stemming from geometric simplifications, ignoring loose or small components, assuming uniform densities, and inaccuracy in robot hardware assembly. While piecewise methods can identify mechanical properties of each component, they do not consider complex dynamic effects such as joint friction.

In experimental approaches, an implicit or explicit model of the dynamics of a manipulator in its entirety is derived based on a series of measurements. An example of implicit experimental system identification is training a neural network to encapsulate the dynamics of the robot and to avoid explicit estimation of individual parameters (Smith, Mobasser, & Hashtrudi-Zaad, 2006; Xing & Pham, 1995). In explicit methods, which have been used for industrial robots, the manipulator dynamics is formulated in terms of several dynamic parameters to be identified, based on a series of measurements throughout the robot workspace (Atkeson, An, & Hollerbach, 1986; Bona & Curatella, 2005; Khosla & Kanade, 1985; Tafazoli, Lawrence, & Salcudean, 1999; Yoshida, Ikeda, & Mayeda, 1991). It has been shown that the dynamics of manipulators can be linearly parameterized and identified using least squares estimation (Astrom & Wittenmark, 1994; Sciavicco & Siciliano, 2001). In Mayeda, Osuka, and Kangawa (1984), a sequential identification method is presented. Similarly, in Nakamura and Ghodoussi (1989) and Mayeda, Yoshida, and Osuka (1988), a subset of all robot parameters are identified such that they include no redundant parameter, and thus, prevent the combinatorial explosion of the total number of parameters to be identified as the number of DOF grows. The accuracy of experimental methods depends on the comprehensiveness of the dynamic model, the accuracy of the recorded measurements, and the richness of the robot trajectory.

It has been noted in Ma and Hollerbach (1996) that accurate identification of gravitational parameters for some manipulators is sufficient for many tasks. A rotation command is applied to each joint separately, a sinusoidal curve is fit to the resulting data, and the link mass parameters are estimated statically and separately. The algorithm avoids the need for numerical derivative calculations as required in previous studies (Atkeson et al., 1986; Bona & Curatella, 2005; Khosla & Kanade, 1985; Tafazoli et al., 1999; Yoshida et al., 1991). While the approach is accurate and robust, it is not suitable for the PHANToM haptic device, since the gravity effect is less important than other dynamic components such as inertia, as will be shown in Section 4.1. An alternative approach for isolating link dynamics is presented in Tam, Kubica, and Wang (2005) in order to simplify the identification process for haptic devices. In the developed technique, joint isolation is achieved by iteratively locking all the joints but one during the identification process, thus identifying parameters of each joint separately. Determining the sequence of joints to lock is established by observing the parameters that affect the configuration of each locked joint.

Unlike piecewise methods, the explicit experimental methods can only identify a subset of parameters that are a combination of the individual properties of various components. However, even though the mass and inertial parameters of each component are not individually identified, the estimated set of parameters is sufficient for many applications, including torque and force estimation and model-based control. The important advantages of experimental over piecewise methods are in their ability to include friction dynamic effects, and to account for any change in manipulator dynamics due to various configurations or added accessories.

In this work, the dynamic parameter identification of a PHANToM Premium 1.5A is experimentally investigated for various configurations. The proposed method in this paper uses the dynamic structure derived previously (Cavusoglu & Feygin, 2001; Cavusoglu, Feygin et al., 2002). The method avoids some of the assumptions made in these earlier works, such as uniformity of link densities, and it can be applied to some other models of PHANToM series devices such as the PHANToM Premium 1.0 and 3.0. The method can easily be applied every time the dynamics of the device is modified. This happens frequently, as researchers often use
the PHANToM in different configurations (e.g., upside down) or with different sensors and tools, which changes mass, inertia, and the lengths of different segments of the robot. In such cases, piecewise methods are time-consuming and are often not sufficiently accurate, particularly if the added tools do not have simple geometric shapes.

For evaluating the identified model, an inverse dynamics position controller using the identified model is experimentally shown to be more stable and more responsive than the controller that uses the CAD-based identified dynamic parameters. In addition, hand force estimation including device dynamics proves to be more accurate than the one obtained using only the device geometrical Jacobian derived in Cavusoglu and Feygin (2001). A more in-depth analysis of the robot dynamics is also provided for each joint and configuration by looking at the contribution of various torque components in the dynamic model, such as inertial, Coriolis and centrifugal, gravitational, and Coulomb and viscous frictions. The above dynamic dissection can also be utilized for accurate determination of the counterbalancing weights by observing the isolated gravitational term for each joint.

This paper is organized as follows. The dynamics equations of the PHANToM robot, including friction effects, are presented and linearly parameterized in Section 2. The parameters of the dynamic model for various configurations of PHANToM are experimentally identified and verified in Section 3. The identified dynamic model is used in Section 4 to obtain insight into the system dynamic components, to estimate the counterbalance weights required to compensate for the gravitational effect of the device attachments, to implement an inverse dynamics controller, and to provide an estimate of the user hand force. Finally, Section 5 draws conclusions.

2 Dynamic Model

The PHANToM Premium 1.5A has three degrees of mobility (i.e., three joints) and provides three translational DOF at its endpoint. Figure 1 shows the schematics of the device with three motors and the corresponding joint angles, \( \theta_1 \), \( \theta_2 \), and \( \theta_3 \), and a Cartesian frame attached to the endpoint of the manipulator.

The dynamics of a robotic manipulator can be formulated as shown in Equation 1:

\[
\tau = M(\Theta)\ddot{\Theta} + C(\Theta, \dot{\Theta})\dot{\Theta} + N(\Theta)
\]  

(1)

where \( M \), \( C \), and \( N \) represent the inertial matrix, the Coriolis and centrifugal matrix, and the gravitational vector, respectively, defined in terms of the inertial and kinematic properties of the robot individual components (Sciavicco & Siciliano, 2001). Here, \( \tau = [\tau_1 \ \tau_2 \ \tau_3]^T \) and \( \Theta = [\theta_1 \ \theta_2 \ \theta_3]^T \) are torque vectors delivered by the motors and the vector of joint angles derived from the encoders, respectively (Cavusoglu & Feygin, 2001; Cavusoglu, Feygin, et al., 2002).

Using the Euler-Lagrange method, Cavusoglu et al. derived the dynamics structure and equations of motion for the PHANToM 1.5A as given in Equation 2:
\[
\begin{bmatrix}
\tau_1 \\
\tau_2 \\
\tau_3
\end{bmatrix} =
\begin{bmatrix}
M_{11} & 0 & 0 \\
0 & M_{22} & M_{23} \\
0 & M_{32} & M_{33}
\end{bmatrix}
\begin{bmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2 \\
\dot{\theta}_3
\end{bmatrix} \\
+ \begin{bmatrix}
C_{11} & C_{12} & C_{13} \\
C_{21} & 0 & C_{23} \\
C_{31} & C_{32} & 0
\end{bmatrix}
\begin{bmatrix}
\ddot{\theta}_1 \\
\ddot{\theta}_2 \\
\ddot{\theta}_3
\end{bmatrix} \\
+ \begin{bmatrix}
0 \\
N_2 \\
N_3
\end{bmatrix}
\] (2)

The reader should note that since the electric dynamics of the motors and the voltage amplifier box are assumed to be much faster than the mechanical dynamics of the motors and the linkages, the torque command voltages sent out to the amplifier box are proportional to the motor induced torques. Therefore, the actual motor induced torques reported in this paper, that is, \(\tau\), are found by scaling down the torque command voltages by about a factor of \(\alpha = 2.7\). From now on, “torque” refers to the actual motor induced torque.

To identify the device dynamics, Equation 2 is linearly parameterized as:

\[
\tau = Y(\Theta, \dot{\Theta}, \ddot{\Theta}) \pi
\] (3)

where \(Y\) is the regressor matrix and \(\pi\) is the vector of eight unknown parameters, defined as in Equations 4 and 5:

\[
Y^T = \begin{bmatrix}
Y_{d}^T \\
Y_{g}^T
\end{bmatrix} = \begin{bmatrix}
\ddot{\theta}_1 & 0 & 0 & \ddot{\theta}_2 & 0 & 0 & \ddot{\theta}_3 & 0 \\
0 & \ddot{\theta}_2 & 0 & \ddot{\theta}_3 & 0 & \ddot{\theta}_3 & \ddot{\theta}_3 & \ddot{\theta}_3 \\
0 & 0 & \frac{1}{2} \ddot{\theta}_2 s_3 & 0 & \frac{1}{2} \ddot{\theta}_2 s_3 & 0 & \frac{1}{2} \ddot{\theta}_2 s_3 & \frac{1}{2} \ddot{\theta}_2 e_3 \\
0 & 0 & 0 & \text{etc.} & 0 & 0 & \text{etc.} & 0 \\
0 & 0 & 0 & 0 & \text{etc.} & 0 & 0 & \text{etc.}
\end{bmatrix}
\]

\[
\pi = \begin{bmatrix}
\pi_d \\
\pi_g
\end{bmatrix} = \begin{bmatrix}
\frac{\sqrt{3}}{2}(4I_{mxy} + 4I_{mzx} + 8I_{bewy} + 4I_{bewx} + 4I_{byx} + 4I_{byz} + 4I_{byx} + 4I_{byy} + 4I_{byz} + 4I_{byx} + 4I_{byy} + 4I_{byz} + 4I_{byx} + 4I_{byy} + 4I_{byz}) \\
\frac{\sqrt{3}}{2}(4I_{cxy} + 4I_{cyy} + 4I_{czz} + 4I_{bwy} + 4I_{bwy} + 4I_{bwy} + 4I_{bwy} + 4I_{bwy} + 4I_{bwy} + 4I_{bwy} + 4I_{bwy} + 4I_{bwy} + 4I_{bwy} + 4I_{bwy} + 4I_{bwy} + 4I_{bwy}) \\
\frac{\sqrt{3}}{2}(4I_{mxy} + 4I_{mzx} + 8I_{bewy} + 4I_{bewx} + 4I_{byx} + 4I_{byz} + 4I_{byx} + 4I_{byy} + 4I_{byz} + 4I_{byx} + 4I_{byy} + 4I_{byz} + 4I_{byx} + 4I_{byy} + 4I_{byz} + 4I_{byx} + 4I_{byy} + 4I_{byz}) \\
\text{etc.} & 0 & 0 & \text{etc.} & 0 & 0 & \text{etc.}
\end{bmatrix}
\]

where \(\pi_d\) and \(\pi_g\) represent dynamic and gravitational parameter vectors, respectively. The inertial and kinematic parameters \(I_{11}, I_{22}, I_{33}, I_{55}, I_{66}, I_{12}, I_{15}, I_{16}, I_{25}, I_{26}, I_{35}, I_{36}, I_{56}, I_{125}, I_{126}, I_{156}, I_{256}, I_{356}, I_{1256}\) and \(m_{d}\) and \(m_{f}\) are the same as the ones defined previously (Cavusoglu, Feygin, et al., 2002) and \(g\) is the gravity acceleration. The \(Y\) matrix, given in Equation 4, contains all the terms in Equation 1 that are functions of the robot configuration vector \(\Theta\). Here, \(s_i, c_i, s_{ij}, c_{ij}, s_{2i},\) and \(c_{2i}\), \(i, j = 1, 2, 3\), represent the shorthand notation for \(\sin(\theta_i), \cos(\theta_i), \sin(\theta_i - \theta_j), \cos(\theta_i - \theta_j), \sin(2\theta_i),\) and \(\cos(2\theta_i)\), respectively.

The following Coulomb and viscous friction models

\[
\tau_i = \pi_{f_i} \text{sgn}(\dot{\theta}_i) + \pi_{f_i} \dot{\theta}_i, i = 1, 2, 3
\] (6)

or

\[
\tau_j = \begin{bmatrix}
\tau_{j1} \\
\tau_{j2} \\
\tau_{j3}
\end{bmatrix} = \begin{bmatrix}
\pi_{f_{j1}} \text{sgn}(\dot{\theta}_1) + \pi_{f_{j1}} \dot{\theta}_1 \\
\pi_{f_{j2}} \text{sgn}(\dot{\theta}_2) + \pi_{f_{j2}} \dot{\theta}_2 \\
\pi_{f_{j3}} \text{sgn}(\dot{\theta}_3) + \pi_{f_{j3}} \dot{\theta}_3
\end{bmatrix} = Y_j \pi_f
\] (7)
are also employed to include the effect of friction, where \( \text{sgn}(\cdot) \) denotes the signum function, and \( \pi_{fi} \) and \( \pi_{vi} \) represent the Coulomb and viscous friction coefficients for joint \( i \), respectively, and \( Y_f \) and \( \pi_f \) are defined as:

\[
Y_f = \begin{bmatrix}
\dot{\theta}_1 & 0 & 0 & \text{sgn}(\dot{\theta}_1) & 0 & 0 \\
0 & \dot{\theta}_2 & 0 & 0 & \text{sgn}(\dot{\theta}_2) & 0 \\
0 & 0 & \dot{\theta}_3 & 0 & 0 & \text{sgn}(\dot{\theta}_3)
\end{bmatrix}
\]

\[\pi_f = [\pi_{fi} \quad \pi_{vi} \quad \pi_{fi} \quad \pi_{vi} \quad \pi_{fi} \quad \pi_{vi}]^T\]

(8)

(9)

Considering friction effects in the dynamics model, the total \( Y \) and \( \pi \) expand to:

\[
\pi = \begin{bmatrix}
\pi_d \\
- \pi_f \\
\pi_d \\
- \pi_f
\end{bmatrix}
\]

(10)

and

\[
Y_{[3 \times 14]} = [Y_{[2 \times 4]} \quad Y_{[2 \times 2]} \quad Y_{[3 \times 6]}]
\]

(11)

The linear system of equations in Equation 3 can be solved using the least squares estimation method if several independent data points are available. To collect this data, the robot is moved along a trajectory and its joint angles and motor torques are recorded for a period of time to create \( \tau_N \) and \( Y_N \), which are an ensemble of the torque vectors and regression matrices \( \tau \) and \( Y \) stacked over \( N \) samples. The least squares solution for \( \pi \) is

\[
\hat{\pi} = [(Y_N^T Y_N)^{-1} Y_N^T] \tau_N
\]

(12)

where \( \hat{\pi} \) is the estimate of the \( \pi \) vector, which minimizes the torque error in the sense of mean square, and \( (Y_N^T Y_N)^{-1} Y_N^T \) is called the left pseudoinverse of \( Y_N \). The number of measurements should be large enough to avoid ill-conditioning of matrix \( Y_N \) and to ensure the existence of the left pseudoinverse matrix.

As stated earlier, experimental parameter identification cannot identify the mass and inertia properties of each component of the manipulator. Specifically, Equation 5 illustrates that the first eight elements of the \( \pi \) vector are each a combination of several mass, inertial, and length properties for various links of the robot. It should be noted that while the properties of robot joints and components are not individually identified, the set of identified parameters is sufficient for model-based control and hand force estimation.

## 3. Experiments

In this section, the PHANToM’s dynamic parameters are experimentally estimated and the identified dynamics are discussed and analyzed. The experimental setup, various configurations of the robot, and a filtering technique for avoiding double differentiation for estimating joint acceleration are explained in Section 3.1. The identification of the dynamic parameters for various configurations of the robot, with and without friction considerations, is conducted. An initial verification of the derived dynamics based on torque estimation and torque error analysis are presented in Section 3.2.

### 3.1 Experimental Setup

The experimental setup consists of a PHANToM Premium 1.5A, an ATI Industrial Automation Nano-17 force/torque sensor, and a real-time open architecture control system developed in-house for full functionality of the robot. The developed open architecture platform that utilizes the PHANToM amplifier box uses a Quanser Q8 data acquisition board and a WinCon/RTX real-time control system, which links with MATLAB Real-Time Workshop Toolbox. The sensory information provided by the setup are the three motor angles read by encoders, and the six endpoint generalized forces provided by the force sensor.
in three Cartesian directions. The data collection and control processes run at the rate of 1 kHz.

Joint velocities are calculated from joint angles by using a high-pass filter numerical differentiator. Since these calculations are performed off-line, the filter need not be causal, allowing more accurate estimation of joint velocities (Sciavicco & Siciliano, 2001). Numerical calculation of joint accelerations significantly amplifies noise and is not recommended. Instead, a filtering technique was proposed (Hsu, Bodson, Sastry, & Paden, 1987) to eliminate the need for acceleration measurement. To this end, the system total dynamics model in Equations 3, 10, and 11 is passed through a strictly stable low-pass filter with a transfer function $L(s) = \frac{\omega}{\omega + s}$, $\omega > 0$ to obtain

$$\tau_L = Y_L(\Theta, \dot{\Theta})\pi$$  \hspace{1cm} (14)

where $Y_L(\Theta, \dot{\Theta})$ and $\tau_L$ are filtered $Y$ and $\tau$, respectively. Therefore, $Y_L$ and $\tau_L$ can be used to find $\tilde{\pi}$. Figure 2 illustrates the block diagram of the filtering operation.

The regression matrix $Y_L$ is only a function of the joint angle and velocity vectors $\Theta$ and $\dot{\Theta}$. This is because the filtered acceleration can be written as a function of joint velocity

![Low-Pass Filter](image)

\[ \tau = Y(\Theta, \dot{\Theta}, \ddot{\Theta})\pi \]

\[ \tau_L = Y_L(\Theta, \dot{\Theta})\pi \]

Figure 2. Passing the system dynamic equation through a low-pass filter.

and filtered velocity, that is, $\ddot{\theta}_i = \omega(\dot{\theta}_i - \dot{\theta}_i)$, $i = 1, 2, 3$. The elements of $Y_L$ are given in Equation 13, where the subscript $L$ signifies that the argument has passed through the low-pass filter $L(s)$. In general, by designing the filter such that its cutoff frequency lies between the system bandwidth and the noise frequency,
it is possible to attenuate the degrading effects of measurement noise on the identification performance if the spectrum of the excitation input spans over the entire system bandwidth.

For our identification experiments, an angular position tracking PD controller was implemented. The desired joint trajectories were chosen for the high levels of system excitation required for the convergence of the identified parameters to the true values (Astrom & Wittenmark, 1994; Otani, Kakizaki, & Kogure, 1992). Toward this end, the desired joint trajectories were constructed from several sinusoids with various frequencies and amplitudes for each joint, such that the robot traversed throughout its workspace without any attempt to violate its boundaries. Since for a linear system with \( n \) unknown parameters, the input can be a linear combination of at least \( n/2 \) sinusoids with different phases and frequencies, a sum of 10 sinusoidal signals with frequencies ranging from 0.11 to 5 rad/s was chosen as the position command for each link.

As deriving the exact workspace boundaries and joint dependencies was beyond the scope of our work, simple joint angle constraints were imposed on the range of the desired joint angles to make sure that the robot did not violate its workspace and its physical limitations. The amplitudes and phase shifts of the sinusoidal components were chosen to comply with the constraints. The cutoff frequency of the low-pass filter was set to \( \omega = 10 \) rad/s, which is higher than the input signal bandwidth and much lower than the noise frequency range. Since the sampling frequency was 1 kHz, the chosen sine frequencies were well below the Nyquist rate of 500 Hz, such that the elements of the regression matrix \( Y_L \) contained most of their energy below the Nyquist rate.

The robot dynamics are nonlinear and applying a simple PD position controller guarantees neither accurate tracking nor asymptotic stability for free motion. In the experiments, the gains of the PD controller were empirically set to values that would preserve system stability throughout the commanded trajectory. Accurate position tracking was not an issue as long as the robot path did not reach the boundaries of the workspace.

The reader should note that the proposed identification method is an off-line process, which runs with a dataset that has already been collected experimentally. Therefore, either the above-mentioned open architecture solution or OpenHaptics Toolkit (Itkowitz et al., 2005) can be used to implement a simple position PD controller to enable commanding the robot joints through persistently exciting trajectories and to record the angular position, angular velocity, and joint torques of the manipulator required for off-line parameter identification.\(^3\)

The dynamic parameters of the PHANToM device were identified for the five common configurations of the robot as described below.

**Upright (UR).** The most common configuration of the robot is illustrated in Figure 3(a).

**Gimbal and Counterbalance Weight (GCW).** The gimbal and the counterbalance weight supplied by the manufacturer are mounted on the PHANToM.

\(^3\)The proposed identification algorithm, programmed in MATLAB, is available through the corresponding author’s website.
for three added passive rotational DOFs, as shown in Figure 3(b).

**Force Sensor (FS).** Measured hand force can be used for the design of transparent haptic or telerobotic bilateral controllers, as seen in Figure 3(c).

**Upside Down (USD).** This configuration of PHANToM has especially been used in medical simulators for larger user workspace and enhanced ergonomics.

**Upside Down Plus Gimbal and Counterbalance Weight (USD + GCW).** Figure 3(d) shows the upside down configuration of the robot with gimbal and counterbalance weight mounted for added passive DOFs and higher force dynamic range. This configuration has been reported in an ultrasound-based training simulator (Tahmasebi, Abolmaesumi, Thompson, & Hashtrudi-Zaad, 2005).

Data acquisition was performed for a duration of over 40 s in all experiments. Since the low-pass filter was implemented using a memory unit in the discrete time domain, the first few seconds of the filtered data were discarded to allow for the output of the filter to converge. The first half of the remaining collected data was utilized to identify the dynamic parameters. For cross validation, the estimated parameters were then employed to predict joint torques with the second half of the collected data. Since filtered values of torque and joint velocities were used in the estimation process, the filtered torques were compared with their corresponding predicted values.

### 3.2 Dynamic Parameter Identification and Torque Estimation

In the first experiment that was conducted in the UR configuration, friction parameters were not included and only eight dynamic parameters, $\pi_{ji}$ and $\pi_{ji}'$, in Equation 5 were identified. Figure 4 compares the filtered measured torque ($\tau_i$) with the two estimated filtered torques ($\hat{\tau}_i = Y_{ij}'\hat{\pi}_j$, one using the identified parameters and the other one using calculated parameters (Cavusoglu & Feygin, 2001; Cavusoglu, Feygin, et al., 2002). The large errors between the actual and the estimated filtered torques for joints one and two were expected and they represent the unaccounted friction effect in the joints and the power transmission system.

To better visualize the effect of friction, the filtered torque prediction errors, $\tau_i - \hat{\tau}_i$, are plotted versus joint velocities in Figure 5. As can be observed from Figure 5, substantial friction exists in all joints, particularly in joint one. Therefore, six additional friction parameters were included in the dynamics model. In total, 14 $\pi$ parameters in Equation 11 were identified. Figure 6 illustrates the estimated torques for the UR configuration with the identified parameters after including friction in the model. It can be observed that the estimation errors are substantially less than those without friction consideration and the predicted torques follow the actual values closely. The filtered torque prediction errors are plotted versus joint velocities in Figure 7. The residual error can correspond to other effects such as those unaccounted in the friction model, flexibilities in joints and linkages, measurement and rounding errors, as well as ignored dynamics in sensors and the amplifier.
The predicted torques closely follow the actual values in other configurations of the device as well. Figure 8, for instance, compares the actual filtered torque and the estimated torque in the USD + GCW configuration.

The 14 element parameter vector \( \boldsymbol{\pi} \), including the friction coefficients, for all five configurations and the eight parameters calculated from the piecewise method...
(Cavusoglu, Feygin, et al., 2002) are listed in Table 1. The standard deviations of the estimates are small, in the range of $10^{-3}$ to $10^{-5}$. For the entire workspace of the robot and for all configurations, the estimate of the inertia matrix was confirmed to be positive definite with the minimum eigenvalue of $1.8 \times 10^{-3}$. The percent relative root mean square (RMS%) of the filtered torque estimation error (Tafazoli et al., 1999), that is

$$\text{RMS\%} = \sqrt{\frac{\sum_{i=1}^{N} (\hat{\tau}_i - \tau_i)^2}{\sum_{i=1}^{N} \tau_i^2}} \times 100$$

(15)

has been calculated for all configurations and listed in Table 2, where $N$ is the total number of samples. The following observations can be made from studying Tables 1 and 2, and Figures 8 and 9.

1. The corresponding $\hat{\pi}_d$ parameters (Cavusoglu, Feygin, et al., 2002) and the UR column have the same signs and are relatively close. This validates the assumptions made in the piecewise method (Cavusoglu, Feygin, et al., 2002). The largest difference is observed in $\hat{\pi}_{d1}$, perhaps because $\pi_{d1}$ includes nearly all the system’s mass and inertia parameters. Therefore, the errors in the piecewise estimation of these dynamics parameters accumulate, resulting in such a difference.

2. Comparing $\pi_{d1}$ to $\pi_{d6}$ in UR and GCW columns shows that adding gimbal and counterbalance weight adds substantially to the device’s inertia.

3. Comparing $\pi_{d1}$ to $\pi_{g8}$ for UR and USD configurations shows that changing the PHANToM configuration to USD does not change the dynamics parameters except for the sign of the gravitational parameters $\pi_{g7}$ and $\pi_{g8}$. This confirms the fact that in UR and USD configurations the arm

<table>
<thead>
<tr>
<th>$\pi \times 1000$</th>
<th>Piecewise</th>
<th>UR</th>
<th>GCW</th>
<th>FS</th>
<th>USD</th>
<th>USD + GCW</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_{d1}$</td>
<td>2.78</td>
<td>1.42</td>
<td>3.40</td>
<td>1.02</td>
<td>1.20</td>
<td>3.04</td>
</tr>
<tr>
<td>$\pi_{d2}$</td>
<td>1.09</td>
<td>1.35</td>
<td>4.43</td>
<td>2.16</td>
<td>1.61</td>
<td>3.29</td>
</tr>
<tr>
<td>$\pi_{d3}$</td>
<td>-0.40</td>
<td>-0.40</td>
<td>-1.17</td>
<td>-0.76</td>
<td>-0.51</td>
<td>-0.73</td>
</tr>
<tr>
<td>$\pi_{d4}$</td>
<td>0.91</td>
<td>0.69</td>
<td>9.23</td>
<td>3.00</td>
<td>0.65</td>
<td>9.19</td>
</tr>
<tr>
<td>$\pi_{d5}$</td>
<td>2.41</td>
<td>2.08</td>
<td>7.54</td>
<td>3.47</td>
<td>2.85</td>
<td>6.37</td>
</tr>
<tr>
<td>$\pi_{d6}$</td>
<td>0.91</td>
<td>0.95</td>
<td>3.55</td>
<td>1.13</td>
<td>1.28</td>
<td>3.15</td>
</tr>
<tr>
<td>$\pi_{g7}$</td>
<td>-16.30</td>
<td>-19.23</td>
<td>26.92</td>
<td>46.84</td>
<td>21.24</td>
<td>-16.59</td>
</tr>
<tr>
<td>$\pi_{g8}$</td>
<td>-73.80</td>
<td>-109.96</td>
<td>46.52</td>
<td>-69.08</td>
<td>108.75</td>
<td>-112.45</td>
</tr>
<tr>
<td>$\pi_{g9}$</td>
<td>—</td>
<td>-2.08</td>
<td>-2.71</td>
<td>-0.91</td>
<td>-1.35</td>
<td>-0.10</td>
</tr>
<tr>
<td>$\pi_{g10}$</td>
<td>—</td>
<td>-1.28</td>
<td>0.10</td>
<td>-0.32</td>
<td>-0.14</td>
<td>0.54</td>
</tr>
<tr>
<td>$\pi_{g11}$</td>
<td>—</td>
<td>-0.18</td>
<td>1.13</td>
<td>0.43</td>
<td>0.91</td>
<td>0.73</td>
</tr>
<tr>
<td>$\pi_{g12}$</td>
<td>—</td>
<td>25.89</td>
<td>26.22</td>
<td>27.06</td>
<td>26.08</td>
<td>24.35</td>
</tr>
<tr>
<td>$\pi_{g13}$</td>
<td>—</td>
<td>9.19</td>
<td>8.35</td>
<td>9.34</td>
<td>8.24</td>
<td>7.28</td>
</tr>
<tr>
<td>$\pi_{g14}$</td>
<td>—</td>
<td>9.08</td>
<td>9.04</td>
<td>10.00</td>
<td>8.09</td>
<td>8.71</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>RMS%</th>
<th>UR</th>
<th>GCW</th>
<th>FS</th>
<th>USD</th>
<th>USD + GCW</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_1$</td>
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<td>19.0</td>
<td>17.0</td>
<td>13.7</td>
<td>19.2</td>
</tr>
<tr>
<td>$\tau_2$</td>
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<td>28.2</td>
<td>10.4</td>
<td>9.0</td>
<td>11.3</td>
</tr>
<tr>
<td>$\tau_3$</td>
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<td>14.6</td>
<td>13.0</td>
<td>4.5</td>
<td>17.0</td>
</tr>
</tbody>
</table>
moves in opposite directions when the motors are not activated.

4. By adding gimbal and counterbalance weight to the USD configuration, \( g_7 \) and \( g_8 \) change their signs, and thus, the added weight overcompensates for gravity.

5. The gravitational parameters \( g_7 \) and \( g_8 \), change from \(-19.23\) and \(-109.96\) to \(46.84\) and \(-69.08\) for the FS configuration. The change of sign in \( g_7 \) and the reduction in \( g_8 \) imply that force sensor at the tip acts as a counterbalance weight for the UR configuration. For bulkier force sensors, the gravitational effect needs to be compensated for by a counterbalance weight on the motors.

6. Comparing \( \pi_9 \) to \( \pi_{14} \) for all configurations shows that joint one has the most significant Coulomb and viscous friction among all three, followed by joint three and then joint two. Also, the effect of Coulomb friction is consistently higher than the effect of viscous friction for such ranges of speed. The absolute values of \( \pi_{f_i} / \pi_{v_i}, i = 1, 2, 3 \), according to Equation 6, provide an upper bound on joint velocities, up to which the Coulomb friction remains larger than the viscous friction. For instance, for the UR configuration, the angular velocity limits for joints one, two, and three are 710 deg/s, 412 deg/s, and 2,841 deg/s, respectively. In order to grasp a better view on the effect of each friction component, Coulomb and viscous friction terms are illustrated separately in Figure 9 for the UR configuration. As illustrated, Coulomb friction is the dominant friction term in all three joints.

7. Considering the RMS% values in Table 2, it is clear that the GCW and USD + GCW configurations have the most significant errors. This can be due to the movement of the gimbal, since it is difficult to rigidly fix the gimbal to the robot arm while the arm is moving. Any small movement of the gimbal can noticeably change the dynamics of the robot.

### 4 Applications of the Identified Dynamics

In this section, the identified dynamics of the PHANToM are used to obtain (i) a general understanding of the major contributors to each joint dynamics, (ii) an estimate of the counterbalance weight needed for each robot configuration and added tool attachment, (iii) a more precise and robust inverse dynamics control, and (iv) an online estimate of contact forces at the tip position of the robot.

#### 4.1 Dynamic Dissection

In order to further analyze system dynamics, the estimated \( \pi \) parameters are employed to calculate the joint torques and the contributions of inertia, Coriolis, and centrifugal, gravity, and friction torque terms for each configuration for random free motion operation with a maximum velocity \( 3.5 \text{ rad/s} = 200.6 \text{ deg/s} \). Figure 10 shows the contribution of each dynamics term in each joint torque for the UR configuration, whereas Figure 11 demonstrates the contribution of each term in the joint three torque for all configurations. The following observations can be made:

1. Joint one: For all configurations, the most significant contribution comes from friction and then
2. Joint two: The main contribution comes from the inertial term and then the Coriolis and centrifugal terms, especially when the gimbal and counterbalance weight are added (not shown in the figures).

3. Joint three: In general, the contribution of gravity to torque is the most significant in joint three compared to the other two joints. Adding extra mass, such as a force sensor, to the UR configuration cuts down the effect of gravity by half, whereas adding a gimbal and counterbalance weight practically nullifies the effect of gravity at the expense of a significant increase in inertial torque.

4.2 Counterbalance Weight Estimation

The identified dynamics can also be utilized to obtain a relatively accurate initial estimate of the counterbalance weight required for any configuration and added accessory.

The gravitational parameters $\pi_{g7}$ and $\pi_{g8}$ in Equation 5 are determined by the mass and length properties of various links of the robot as described in detail elsewhere (Cavusoglu & Feygin, 2001; Cavusoglu, Feygin, et al., 2002). Adding a lumped mass $m_{ep}$ at the end-point (EP), such as a gimbal or a force sensor, and mounting predesigned counterbalance weights $m_{cb2}$ and $m_{cb3}$ on motors two and three will result in the following changes to the gravitational parameters:

$$
\pi_{g7}' = \pi_{g7} - g m_{cb2} I_{cb} + g m_{ep} L_1 \\
\pi_{g8}' = \pi_{g8} - g m_{cb3} I_{cb} + g m_{ep} L_{ep}
$$

where $I_{cb} \geq 0$ is the distance between the center of gravity of the counterbalance weight and the horizontal axis of the vertical capstan, and $L_{ep}$ is the distance between the center of gravity of the EP attachment and the distal end of the last link, as shown in Figure 12. It is assumed that the center of gravity of the attachment is along the last link. To investigate Equations 16–17, the gravitational parameters for the USD + GCW configuration are calculated from their corresponding parameters from the UR configuration column of Table 1, that is, $\pi_{g7} = -0.01923$ and $\pi_{g8} = -0.10996$. For the gimbal with approximate weight $m_{ep} = 0.09$ kg, the manufacturer has provided a counterbalance weight $m_{cb2} = 0.216$ kg. Considering $I_{cb} \approx 0.075$ m, $I_{ep} \approx 0.195$ m, $L_1 = 0.216$ m and $g = 9.81$ m/s$^2$, and using Equations 16–17, the gravitational parameters for the GCW configuration...
are calculated as $\pi_{g7}' = 0.0223$ and $\pi_{g8}' = 0.0622$, which are a close match to the estimated parameters $0.0269$ and $0.0465$ from the fourth column of Table 1.

From Equation 17, it is clear that the provided counterbalance weight for motor two cannot have any effect on $\pi_{g8}$. To compensate for this shortcoming, a second counterbalance weight ($m_{cb3}$) needs to be mounted on motor three. The mass of the counterbalance weights that cancels the effect of gimbal on both gravitational parameters such that $\pi_{g7} = \pi_{g8} = 0$ can be calculated from:

\begin{equation}
(m_{cb2})_{net} = \frac{\pi_{g7} + \alpha m_{ep} L_1}{g I_{cb}} \tag{18}
\end{equation}

\begin{equation}
(m_{cb3})_{net} = \frac{\pi_{g8} + \alpha m_{ep} L_{ep}}{g I_{cb}} \tag{19}
\end{equation}

If the mass of the EP tool or accessory is substantial, the mass of the counterbalance weights become proportional to the mass of the tool according to $m_{cb2} \approx (L_1/I_{cb}) m_{ep}$ and $m_{cb3} \approx (L_{ep}/I_{cb}) m_{ep}$. From Equations 18–19, the recommended counterbalance weights for motors two and three are $(m_{cb2})_{net} = 0.248$ kg and $(m_{cb3})_{net} = 0.091$ kg. It has been noted that the calculated weights are sensitive to the center of gravity distances $L_{ep}$ and $L_{cb}$, however, the calculated values provide an initial estimate, which can be fine-tuned for any setup and application.

### 4.3 Inverse Dynamics Control

The identified dynamics of the PHANToM can be employed to implement inverse dynamics control for haptic or telerobotic bilateral controllers that are designed based on such a control strategy or require linear dynamics for master stability or performance analysis. As a slave, a PHANToM robot controlled with inverse dynamics can provide more accurate position tracking. Finally, the inverse dynamics experiments in this section also provide a qualitative verification of the proposed parameter estimation technique (Bona & Curatella, 2005).

Figure 13 illustrates a block diagram of a position controlled PHANToM robot, in which the control system consists of an inverse dynamics inner control loop and a PD outer control loop (Sciavicco & Siciliano, 2001). The inverse dynamics, which is computed online using the estimated dynamic parameters, tries to ideally cancel the nonlinear dynamics effects and to decouple the resulting linear dynamics. Here $F$ represents the friction terms.

If the nonlinearities are fully compensated for, then adjusting the gains of the PD controller can place the poles of the system for each of the decoupled joints at any desired location as long as the motors are not saturated. In practice, since nonlinear compensation is not perfect, there could be issues with the responsiveness and stability of the overall system. In order to empirically analyze the accuracy of the estimated parameters from the piecewise and the proposed experimental...
methods, an inverse dynamics + PD controller was implemented in various control parameter settings and for various configurations of the robot. The tracking error of the overall system with low gains and the system’s stability with high gains signify the responsiveness and robustness of the controller and show how well the nonlinearities of the robot dynamics are canceled, which in turn indicate how accurate the dynamics parameters are estimated.

In the experiments, the proportional gain was varied from 100 to 1,200, and the derivative gain was adjusted to maintain critically damped or slightly underdamped or overdamped \((0.7 \leq \xi \leq 1.2)\) closed-loop decoupled joint dynamics. Since full cancelation of friction could lead to instability, the linearizing feedback loop was tried with 50% friction compensation. The experiments showed that the identified parameters were able to effectively cancel nonlinearities and the performance of the controller was often satisfactory and close to the ideal system, that is, a double integrator stabilized with a PD controller. The following observations were made based on the experimental results.

1. In the UR configuration with light to medium proportional gains \((100 \leq K_p \leq 800)\), the identified parameters and those calculated based on the piecewise method (Cavusoglu, Feygin, et al., 2002) lead to close set point tracking with position control, especially for joints one and two. The largest tracking error was observed in joint three. This indicates that the nonlinearities in this joint were not compensated for as well as those in the other two joints. Figure 14 illustrates the position tracking results for joint three when \(K_p = 625\) and \(K_d = 35\).

2. The tracking error was smaller for larger proportional gains since tighter PD control can diminish the nonlinear effects as long as the system does not become unstable. However, increasing the gains to tighter values eventually made the controller unstable. The controller with parameters from the piecewise method (Cavusoglu, Feygin, et al., 2002) became unstable with smaller values of \(K_p\) than the experimentally estimated parameters. In other words, the estimated parameters provided stability for a larger range of PD gains. Figure 15 illustrates the joint three angle when \(K_p = 1,200\) and \(K_d = 80\).
gin, et al., 2002) lead to unsatisfactory position tracking with small to medium PD gains, whereas the experimentally identified parameters provided more acceptable tracking even with low proportional gains. Figure 16 illustrates the joint three angle for the USD + GCW configuration, when $K_P = 100$ and $K_D = 14$.

Overall, the results from the experiments with an inverse dynamics controller illustrated that the identified parameters provided more responsive and more stable position control in comparison to the parameters from the piecewise method.

### 4.4 Contact Force Estimation

Haptic interfaces aim to produce a sense of contact between a human and virtual objects by displaying bidirectional forces. To design transparent haptic or tele robotic controllers, high bandwidth contact forces need to be measured or estimated. Using an accurate estimate of the PHANToM’s dynamics and kinematic parameters, the external force at the EP, that is, $F_{ext}$, can be estimated according to:

$$
\hat{F}_{ext} = (J_p)^T (r - \hat{Y}_\mathbf{\hat{r}})
$$

(20)

where $J_p$, the translational Jacobian in the EP frame (see Figure 1) relating joint angles to the EP position, is found (Cavusoglu & Feygin, 2001; Cavusoglu, Feygin, et al., 2002) as:

$$
J_p = \begin{bmatrix}
  l_1 c_2 + l_2 s_3 & 0 & 0 \\
  0 & l_1 c_3 & 0 \\
  0 & -l_1 s_3 & l_2 
\end{bmatrix}
$$

(21)

Note that in Equation 20 the effect of external torque is neglected since the PHANToM is a point-based device with 3-DOF active force feedback; thus, it cannot apply any resistive torque about any axis. The estimated forces are validated by comparing the estimated hand forces with the output of an ATI Nano-17 force sensor, mounted at the EP of the device, as shown in Figure 3(c). For consistency with the filtered torque values, the measured forces provided by the force sensor are passed through the low-pass filter of Equation 2, as well.

Figure 17 compares the actual force in the $y$-direction to the estimated forces obtained from Equation 20 without using the dynamic model (i.e., $\hat{F}_{ext}(J_p)^T \mathbf{\hat{r}}$). As can be seen in the dynamic condition, especially when the direction of motion changes, force estimation with the dynamic model produces slightly better results. Some of the factors that contribute to the observed error in the estimated force are the same as the ones discussed in Section 3.2.

Another possible source of error is the fact that the force sensor frame needs to be registered with the EP frame. In the experiments, the sensor was mounted such that the two frames were visually aligned and the elementary transformation for converting the measurements from the force sensor frame to the EP frame was experimentally derived using least squares.

### 5 Conclusions

The dynamics of a SensAble Technologies PHANToM Premium 1.5A are experimentally identified. The advantages of the applied experimental method over the previously developed piecewise techniques are accuracy (partly due to the inclusion of friction in the dynamics model), ease of use for handling alterations to the device dynamics, and applicability to other models of the PHANToM. The haptic
device dynamics for different configurations were also identified, validated, and analyzed. The parameter estimates produced an inertia matrix that was confirmed to be positive-definite within the device workspace. The identified model also demonstrated noticeable Coulomb friction, especially in joint one.

The identified dynamics were used to compute the contribution of various dynamic terms in each joint for different configurations. It was noticed that two counterbalance weights are required to cancel the gravitational effect of an endpoint tool or accessories such as the gimbal. In addition, the required weights on motors two and three to cancel the effect of the gimbal were calculated. Experiments with model-based position control showed that the identified dynamics provided more robust and more responsive position control than earlier piecewise (CAD-based) estimations of the robot inertial parameters. Finally, the identified dynamics were utilized to obtain a more accurate estimate of external hand (contact) force.

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References


