Extraction of anti-analog giant dipole resonance and neutron skin thickness for $^{208}\text{Pb}$

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The anti-analog giant dipole resonance (AGDR) was separated from other excitations such as the spin-dipole resonance by multipole decomposition analysis of the $^{208}\text{Pb}(\vec{p}, \vec{n})$ reaction at a bombarding energy of $T_p = 296 \text{MeV}$. The polarization transfer observables were found to be useful for carrying out this separation. The energy difference between the AGDR and the isobaric analog state (IAS) was determined to be $\Delta E = 8.69 \pm 0.36 \text{MeV}$, where the uncertainty includes both statistical and systematic contributions. Theoretical calculations using the proton-neutron relativistic quasi-particle random phase approximation predicted a strong correlation between $\Delta E$ and the neutron skin thickness $\Delta R_{pn}$. Under the assumption that the correlation predicted in this model is correct, the present $\Delta E$ value corresponds to a neutron skin thickness of $\Delta R_{pn} = 0.216 \pm 0.046 \pm 0.015 \text{fm}$, where the first and second uncertainties are the experimental and theoretical uncertainties, respectively.

1. Introduction

For an atomic nucleus with $N > Z$, the neutron density distribution extends somewhat further than the proton density distribution. The neutron skin thickness $\Delta R_{pn}$ is generally defined as the difference between the neutron and proton root-mean-square radii. The $\Delta R_{pn}$ value depends not only on the neutron excess $(N - Z)$ but also on the symmetry energy in the nuclear equation of state (EOS). In fact, an almost linear relation between $\Delta R_{pn}$ and the symmetry energy has been suggested in various mean-field models [1–3]. Therefore, precise information on $\Delta R_{pn}$ has become important for obtaining the symmetry energy, which is essential for constraining neutron star models [4,5]. In particular, $\Delta R_{pn}$ for $^{208}\text{Pb}$ has been studied by various methods such as dipole polarizability [6], elastic proton scattering [7], pygmy response [8], and parity-violating elastic electron scattering [9].

Another interesting method for studying $\Delta R_{pn}$ is excitation of the isovector giant dipole resonance (IVGDR). In a macroscopic picture, the IVGDR is a collective vibration mode in which protons and neutrons oscillate out of phase, with the symmetry energy acting as the restoring force. Therefore,
the excitation energy of the IVGDR is sensitive to both the symmetry energy and $ΔR_{pn}$ [10,11]. The anti-analog giant dipole resonance (AGDR) [12] in the charge exchange $(p, n)$ reaction is also sensitive to $ΔR_{pn}$ [13–16]. The energy difference $ΔE$ between the AGDR and the isobaric analog state (IAS) can be obtained by measuring direct $γ$-decay between these states. It should be noted that the spin-dipole resonance (SDR) is also excited in the $(p, n)$ reaction. However, the SDR contribution may be suppressed because the $γ$-decay branching ratio of the SDR is estimated theoretically to be small [17]. If the suppression of the SDR in the $γ$-coincident spectra is indeed strong, a reliable $ΔE$ value can be derived. Thus the corresponding $ΔR_{pn}$ value can be obtained by measuring direct $γ$-decay from the AGDR under the assumption that the theoretical models for the correlation between $ΔE$ and $ΔR_{pn}$ are correct. Note that the use of the $(p, n)$ reaction to study the AGDR can be extended to unstable nuclei due to the progress made in the development of new experimental techniques involving radioactive beams in inverse kinematics [18,19].

Recently, Krasznahorkay et al. [15,16] investigated the reliability of determining $ΔR_{pn}$ using $ΔE$ for the AGDR. They obtained $ΔE = 9.12 ± 0.20$ MeV and $ΔR_{pn} = 0.161 ± 0.042$ fm for $^{208}$Pb. This $ΔR_{pn}$ value is in good agreement with previous results [6–9]. Note that they used the cross section for the $^{208}$Pb$(p, n)$ reaction in order to obtain the $ΔE$ value, and thus the SDR contribution was corrected only in a phenomenological manner [15,16]. Therefore, obtaining a precise experimental $ΔE$ value for the AGDR is very important for confirming the reliability of this approach.

In this paper, we present the strength distribution and the mean energy of the AGDR obtained by multipole decomposition (MD) analysis for $^{208}$Pb$(\bar{p}, \bar{n})$ data at a bombarding energy of $T_p = 296$ MeV [20]. The polarization transfer observables are very useful for separating the non-spin-flip AGDR from the spin-flip SDR in MD analysis [21–23]. The energy difference $ΔE$ between the AGDR and the IAS is used to extract $ΔR_{pn}$ for $^{208}$Pb by comparison with theoretical predictions using the proton-neutron relativistic quasi-particle random phase approximation (pn-RQRPA) [15,16].

2. Experimental data and results

We used the experimental data and MD analysis results from Ref. [20]. In the following, only a brief description of the experiment is given and only those MD analysis results that are relevant to the present analysis are discussed.

The measurements were performed using neutron time-of-flight (NTOF) [24] and neutron detector and polarimeter (NPOL3) [25] systems at the Research Center for Nuclear Physics (RCNP) of Osaka University. Cross sections $I$ and analyzing powers $A_y$ at reaction angles of $θ_{lab} = 0.0^\circ–10.0^\circ$ and complete sets of polarization transfer observables at $θ_{lab} = 0.0^\circ–7.0^\circ$ were measured for the $^{208}$Pb$(p, n)$ reaction at $T_p = 296$ MeV with a neutron flight path length of 70 m. Typical beam currents used for the cross section and polarization transfer measurements were 30 and 500 nA, respectively, with an average beam polarization of 0.59.

The obtained cross section $I$ was separated into non-spin $ID_0$, spin-longitudinal $ID_L$, and spin-transverse $ID_T$ polarized cross sections by using the polarization transfer observables [26,27]. Figure 1 shows a typical result at $θ_{lab} = 2.0^\circ$, where low angular momentum transfer transitions with $ΔL = 0$ and 1 are predominant. The energy transfer $ω$ is the total energy difference defined as $ω = (T_p + m_p) − (T_n + m_n)$, where $T_p$ and $m_p$ ($T_n$ and $m_n$) is the proton (neutron) kinetic energy and mass, respectively. For the well-established $ΔL = 0$ IAS, the peak is clearly observed at an energy transfer $ω ≃ 17$ MeV in the $ID_0$ spectrum. The bump at $ω ≃ 26$ MeV in $ID_0$ would be the contribution of the $ΔL = 1$ AGDR. Significant spin-flip contributions, such as the SDR and the
Gamow–Teller resonance (GTR) in $I D_0$, are expected due to distortion effects \[28\]. We therefore also conducted an MD analysis in order to separate $I D_0$ into the individual spin-parity $\Delta J^\pi$ components. In the MD analysis, the experimental $I D_{i}^{\text{expt}}(\theta, \omega) (i = 0, L, T)$ are fit using the least-squares method, based on the linear combination of the calculated $I D_{i}^{\text{calc}}(\theta, \omega) (i = 0, L, T)$ for various $\Delta J^\pi$ as

$$I D_{i}^{\text{expt}}(\theta, \omega) = \sum_{\Delta J^\pi} a_{\Delta J^\pi}(\omega) I D_{i;\Delta J^\pi}(\theta, \omega),$$  \hspace{1cm} (1)$$

where $a_{\Delta J^\pi}(\omega)$ are the fitting coefficients. The $\chi^2$ value in the fitting procedure is defined as

$$\chi^2 = \sum_{i=0, L, T} \sum_{\theta} \left( \frac{I D_{i}^{\text{expt}}(\theta) - I D_{i}^{\text{calc}}(\theta)}{\delta I D_{i}(\theta)} \right)^2,$$  \hspace{1cm} (2)$$

with

$$\delta I D_{i}(\theta) = \max[\delta I D_{i}^{\text{expt}}(\theta), \alpha \times I D_{i}^{\text{expt}}(\theta)],$$  \hspace{1cm} (3)$$

where $\delta I D_{i}^{\text{expt}}(\theta)$ are the statistical uncertainties of $I D_{i}^{\text{expt}}(\theta)$. Here, we have introduced the parameter $\alpha$ to avoid trapping in an unphysical local $\chi^2$ minimum. The $\alpha$ dependence for the final result was investigated in the range of $\alpha \leq 0.06$ and taken into account as the systematic uncertainty. Note that the experimental $I^{\text{expt}}(\theta, \omega)$ and $A^{\text{expt}}(\theta, \omega)$ are also fit as described in Ref. \[20\]. Figure 2 shows a typical result of the MD analysis at $\theta_{\text{lab}} = 2.0^\circ$ with $\alpha = 0.03$. For the peak at $\omega \simeq 17$ MeV, the IAS contribution is dominant and the lower and higher energy tails are attributed to the SDR and GTR contributions. For the bump at $\omega \simeq 26$ MeV, the AGDR contribution is concentrated around the peak, whereas the significant SDR and other $\Delta L \geq 2$ contributions are found at the lower and higher energy sides, respectively.

3. Analysis and discussion

Figure 3 shows the IAS and AGDR cross sections at $\theta_{\text{lab}} = 2.0^\circ$, represented by the filled circles and squares, respectively. In this figure, the cross sections have been obtained by the MD analysis with the different selection of $\alpha$ discussed in Ref. \[20\]. As seen in Fig. 3, the peak energy is insensitive to the choice of $\alpha$. The IAS distribution was fitted using a Gaussian function. The full width at
Fig. 2. Results of MD analysis for $I D_0$ at $\theta_{lab} = 2.0^\circ$.

Fig. 3. IAS (circles) and AGDR (squares) cross sections for the $^{208}$Pb$(\vec{p}, \vec{n})$ reaction at $\theta_{lab} = 2.0^\circ$ obtained by MD analysis with the different selection of $\alpha$ discussed in Ref. [20]. The vertical bars at the data points indicate the statistical uncertainties. The solid curves represent the results of peak fitting discussed in the text. The vertical dashed lines represent the mean values of the IAS and AGDR peak energies.

Half maximum (FWHM) was estimated to be about 1.7 MeV (standard deviation $\sigma \sim 0.7$ MeV), which represents the intrinsic energy resolution of the experiment. For the giant dipole resonance, the distribution can be well described by a Lorentzian function given by [29]

$$L(\omega) = \frac{A}{1 + [(\omega^2 - \omega_0^2)^2 / (\omega \Gamma)^2]}$$

where $A$, $\omega_0$, and $\Gamma$ are the peak height, energy, and width, respectively. To account for the intrinsic energy resolution, we fitted the AGDR distribution using a Voigt function, which is a convolution of a Lorentzian and Gaussian function, and is given by

$$V(\omega) = \int L(\omega - \omega') \times \exp\left[-\frac{\omega'^2}{2\sigma^2}\right] d\omega'.$$

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The fitting results are presented in Fig. 3 by the solid curves. The systematic uncertainties coming from the selection of $\alpha$ in the heights and widths are significantly large for both the IAS and the AGDR. On the other hand, the uncertainties in the peak energies have relatively small values of $\delta\omega_0 \lesssim 0.1$ MeV.

The intrinsic width of the AGDR was estimated to be about $\Gamma \sim 1$ MeV, which was narrower than the previously reported value of $\Gamma = 2.9$ MeV [12,30]. In the MD analysis, the spin-flip SDR contribution for the non-spin ID$_0$ spectrum is sensitive to the optical model potential (OMP), thereby the width of the AGDR would be sensitive to the choice of the OMPs. To investigate the OMP dependence, we also performed the MD analysis by using other global OMPs ($E$-dependent – $A$-independent (EDAI) and $E$-dependent – $A$-dependent (EDAD) Fit 1) parametrized by Cooper et al. [31,32]. It should be noted that the OMP dependence is significantly small for the spin-flip ID$_L$ and ID$_T$ cross sections presented in the previous paper [20].

Figure 4(a) shows typical results of the MD analysis with $\alpha = 0.01$ by using the EDAI OMP and the global OMP parametrized by Shen et al. [33] for incident and exit channels, respectively, which are the same as those in Fig. 3. Figures 4(b) and 4(c) represent the results obtained by using the EDAI and EDAD OMPs for both channels, respectively. As seen in Fig. 4, the width of the AGDR was sensitive to the choice of the OMPs. By using the EDAI OMP for both channels, the width was deduced to be $\Gamma = 2.7 \pm 0.6$ MeV, which is consistent with the previous result [12,30]. The use of the EDAD OMP also provides a similar result of $\Gamma = 2.3 \pm 0.6$ MeV. Note that the deduced widths also have significant systematic uncertainties coming from the selection of $\alpha$ as shown in Fig. 3. Consequently, it is hard to discuss here the width of the AGDR quantitatively. On the contrary, the peak energy is insensitive to the choice of the OMPs. Therefore, the present results are useful for determining the energy difference $\Delta E$ with reasonable accuracy. In the following, we used the results
Fig. 5. pn-RQRPA predictions (triangles) for the energy difference $\Delta E$ between the AGDR and the IAS as a function of the neutron skin thickness $\Delta R_{pn}$ for $^{208}$Pb [15,16]. The thin solid line is the result of linear regression, and the band represents the theoretical uncertainty [15,16]. The horizontal bands show the uncertainties in the experimental $\Delta E$ value in the present analysis.

The mean energy of the AGDR was derived using the Lorentzian function $L(\omega)$ as

$$\omega_{\text{AGDR}} = \frac{\int_{\omega_l}^{\omega_u} \omega L(\omega) \, d\omega}{\int_{\omega_l}^{\omega_u} L(\omega) \, d\omega}. \quad (6)$$

Here, the lower and upper limits of the integration, $\omega_l$ and $\omega_u$, are taken to be 22.5 and 32.5 MeV, respectively, which are the same values used in the theoretical calculations [15,16]. The $\omega_{\text{AGDR}}$ value was determined to be $\omega_{\text{AGDR}} = 26.42 \pm 0.21(\text{stat.}) \pm 0.18(\text{syst.})$ MeV, where the first and second uncertainties are the statistical and systematic uncertainties, respectively. For the IAS, the mean energy $\omega_{\text{IAS}}$ was deduced from the peak energy of the Gaussian function and obtained as $\omega_{\text{IAS}} = 17.73 \pm 0.13(\text{stat.}) \pm 0.20(\text{syst.})$ MeV. This result is consistent with the established value of $\omega_{\text{IAS}} = 17.53 \pm 0.02$ MeV [34]. In the following, we used this fitting value instead of the established value since the uncertainty of the energy calibration could be largely reduced by taking the energy difference. The energy difference $\Delta E = \omega_{\text{AGDR}} - \omega_{\text{IAS}}$ between the AGDR and the IAS is then given by $\Delta E = 8.69 \pm 0.24(\text{stat.}) \pm 0.26(\text{syst.})$ MeV.

We also derived the centroids of the experimental data for both the IAS and the AGDR in order to evaluate $\Delta E$ without making any assumptions about the strength distributions. The obtained result is $\Delta E = 8.76 \pm 0.70(\text{stat.}) \pm 0.16(\text{syst.})$ MeV, where the systematic uncertainty is associated with the MD analysis [20]. This value is consistent with that obtained by fitting, and the difference is taken into account as the systematic uncertainty.

The final result is $\Delta E = 8.69 \pm 0.24(\text{stat.}) \pm 0.27(\text{syst.})$ MeV. This is slightly smaller than the previously reported value of $\Delta E = 8.97 \pm 0.20$ MeV [12], most likely due to the SDR contribution in the previous analysis.

Krasznahorkay et al. [15,16] found an almost linear relationship between $\Delta E$ and the neutron skin thickness $\Delta R_{pn}$. In Fig. 5, the triangles show their pn-RQRPA results using relativistic effective interactions with five different symmetry energies at a saturation density of $a_4 = 30$–38 MeV. The thin solid line shows the result of linear regression, and the band represents a 10% theoretical uncertainty.
Table 1. Neutron skin thickness $\Delta R_{pn}$ of $^{208}$Pb determined in the present work, and previously measured values.

<table>
<thead>
<tr>
<th>Method</th>
<th>Ref.</th>
<th>Date</th>
<th>$\Delta R_{pn}$ (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pygmy response</td>
<td>[8]</td>
<td>2010</td>
<td>0.194 ± 0.024</td>
</tr>
<tr>
<td>$(p, p)$</td>
<td>[7]</td>
<td>2010</td>
<td>0.211 ± 0.063</td>
</tr>
<tr>
<td>Dipole polarizability</td>
<td>[6]</td>
<td>2011</td>
<td>0.153 ± 0.025</td>
</tr>
<tr>
<td>Parity-violating $(e, e)$</td>
<td>[9]</td>
<td>2012</td>
<td>0.33 ± 0.17</td>
</tr>
<tr>
<td>AGDR</td>
<td>This work</td>
<td>2013</td>
<td>0.216 ± 0.046 ± 0.015</td>
</tr>
</tbody>
</table>

in $\Delta R_{pn}$ [15,16]. In the pn-RQRPA calculation, the energy transfer is defined as $\omega_{int} = m_{rs} - m_{tg}$, where $m_{rs}$ and $m_{tg}$ are the residual and target mass, respectively. Note that the difference between $\omega$ and $\omega_{int}$ is negligibly small in the present work.

The extracted $\Delta E$ value gives a neutron skin thickness of $\Delta R_{pn} = 0.216 ± 0.046 ± 0.015$ fm, where the first and second uncertainties are the experimental and theoretical uncertainties, respectively. The experimental uncertainty includes both statistical and systematic contributions. The present $\Delta R_{pn}$ value is consistent with previous results as shown in Table 1. It should be noted that the obtained $\Delta R_{pn}$ value is based on the model-dependent method and the true uncertainty is hard to estimate. Therefore, systematic studies with various theoretical models and experimental investigations in other nuclei would be important. Note that the parity-violating elastic electron scattering [9] is useful to obtain $\Delta R_{pn}$ for $^{208}$Pb almost model-independently. Therefore, a follow-up result with high accuracy should be very important not only to provide stringent constraints on the theoretical models but also to assess the theoretically predicted correlation between $\Delta R_{pn}$ and experimental observables such as $\Delta E$.

4. Summary and conclusion

The strength distribution and mean energy of the AGDR were investigated by MD analysis for the $^{208}$Pb$(\vec{p}, \vec{n})$ reaction at $T_p = 296$ MeV. The polarization transfer observables were found to be very useful in separating the non-spin-flip AGDR from the spin-flip SDR. The energy difference $\Delta E$ between the AGDR and the IAS was determined to be $\Delta E = 8.69 ± 0.36$ MeV, where the uncertainty includes both statistical and systematic contributions. Theoretical calculations using the pn-RQRPA predict a strong correlation between $\Delta E$ and the neutron skin thickness $\Delta R_{pn}$. The obtained $\Delta E$ value corresponds to a neutron skin thickness of $\Delta R_{pn} = 0.216 ± 0.046 ± 0.015$ fm, where the first and second uncertainties are the experimental and theoretical uncertainties, respectively. This $\Delta R_{pn}$ value is in good agreement with previous results obtained using a variety of other methods. However, this result relies on the model-dependent theoretical calculation. Therefore, systematic experimental investigations in various nuclei are required.

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