Searching for the origin of CP violation in Cabibbo-suppressed $D$-meson decays

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The recent evidence of relatively large direct CP violation in $D^0$ decay at LHCb suggests that CP studies in the $D$ system may become an important new avenue for understanding CP just as studies in the $B$ system have proven to be. The current level of CP violation could be consistent with the Standard Model or, perhaps, contain evidence of new physics. A clean Standard Model prediction of the CP violation in these decays would, of course, be important in understanding these results but hadronic uncertainties make such a prediction difficult. In this paper, we make several suggestions to try to seek the role of new physics. We propose that the hadronic enhancement needed to attribute the observed CP violation in $D$ to two pseudoscalar modes may not operate for inclusive final states, where it is likely that we will see asymmetries at the quark level expectation provided the source is the Standard Model. A simple way to implement this is to search for CP asymmetries in final states containing $K$ and $ar{K}$ but where the sum of their energies is less than the energy of the parent $D$. This is meant to ensure that the event belongs to an inclusive and not an exclusive sample. We also propose that CP asymmetries may be enhanced in modes where the tree is color suppressed. In particular, the final state $\rho^0\rho^0$ is of special interest because it consists of charged pions only and, in addition, it can have C-even P-odd triple product correlations; similarly, $D_s \to \rho^0K^+$ and $\rho^0K^{*+}$ also appear interesting. We also emphasize the use of CPT constraints leading to interesting correlations. We then consider how isospin symmetry can provide observables that are sensitive to certain classes of new physics and are small in the Standard Model. In particular, we discuss using isospin analysis in the decays $D \to \pi\pi$, $\rho\pi$, and $\rho\rho$ as well as in $D_s \to K^*\pi$. We also consider how such analysis may eventually be supplemented by information about the weak phases in $D^0$ decay. In order to obtain this information experimentally, we consider various methods for preparing an initial state that is a quantum mechanical mixture of $D^0$ and $\bar{D}^0$. This may be done through the use of natural $D^0/\bar{D}^0$ oscillations; observing $D^0$ mesons that arise from $B_d$ or $B_s$ mesons, which themselves are oscillating, or from quantum correlations in $D^0$ pairs that arise from either $\psi^\prime$ decay or $B$-meson decay. Observing CP violation in the magnitudes of decay amplitudes should be within the capability of experiments in the near future; however, obtaining the weak phases through the methods we discuss will likely require future generations of machines due to the large statistics that are likely to be needed.

1. Introduction

Recent results [1] from the LHCb provide evidence for CP violation in $D$-meson decays, in particular, $A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) = -0.82 \pm 0.21$ (stat) $\pm 0.11$ (syst)$\%$, giving a 3.5 sigma signal of CP violation. CDF has reported [2] the modes separately obtaining
\[ A_{CP}(K^+K^-) = -0.24 \pm 0.22 \pm 0.09\% \text{ and } A_{CP}(\pi^+\pi^-) = +0.22 \pm 0.24 \pm 0.11\%. \] CDF has also directly measured the difference previously measured by LHCb and obtained [3] \[ A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) = -0.62 \pm 0.21 \text{(stat)} \pm 0.10 \text{(syst)}\%. \] A similar result of \[ A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) = -0.87 \pm 0.41 \text{(stat)} \pm 0.06 \text{(syst)}\% \] was also recently reported by BELLE at ICHEP2012 [4]. BELLE [4] also gave \[ A_{CP}(K^+K^-) = -0.32 \pm 0.21 \pm 0.09\% \text{ and } A_{CP}(\pi^+\pi^-) = +0.55 \pm 0.36 \pm 0.09\%. \] In the LHCb result, the cancellation of experimental uncertainties between the two modes plays an important role in the extraction of a significant signal for the difference in the two asymmetries. These measurements dominate the world average for the difference given by the HFAG group [5]: \[ \Delta A_{CP} = A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) = -0.678 \pm 0.147\%. \] The Belle results are particularly significant for individual modes, because the leptonic environment allows better detection of these two final states, so super KEK [6,7] and the Super B Factory [8] should be able to produce more precise results in the future, especially for individual modes.

The LHCb result for the difference in the asymmetries appears to be large compared to the Standard Model (SM) [9] based on early expectations, as we will discuss below. The weak phase arises in the SM from the Cabibbo–Kobayashi–Maskawa (CKM) matrix. The relevant combination of CKM elements that gives the weak phase between the tree and penguin graphs is

\[ |\theta_W| \approx 5.6 \times 10^{-4}, \quad (1) \]

where we have expanded this in terms of the the Wolfenstein parametrization [10]. As discussed in Sect. 2, we expect the CP asymmetry on the quark level to be roughly of this size. For specific final states, hadronic effects will alter this expectation appreciably, especially since the charm quark is so light. Thus the central value for \[ A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) \] may not be inconsistent with the SM [11–16] but this is far from proof that the SM is fully adequate and therefore the role of new physics cannot yet be ruled out [11,12,17,18]. There is, in fact, the intriguing possibility that various models for new physics (NP) are also able to contribute significantly to direct CP violation in \( D \)-decays [11,18–30]. For this reason and others, it is important to devise observables that can distinguish between SM and NP origins for this CP violation.

Broadly speaking, for singly Cabibbo-suppressed (SCS) decays it is useful to divide potential models of NP into two categories. The first is “penguin-like”, where the effective Hamiltonian is strictly \( \Delta I = \frac{1}{2} \). Generally this kind of contribution will result only when the NP contributes through a gluonic penguin. The second is “tree-like”, where the effective Hamiltonian contains a \( \Delta I = \frac{3}{2} \) component. This includes models where there are extra massive scalar or vector bosons that enter at tree level as well as photon, Z, or W penguin topologies. In principle, electroweak penguins could contribute in this way but in the SM such contributions are negligible.

Depending on whether the new physics is tree-like or penguin-like, different kinds of studies may help identify the underlying mechanism. In this paper we consider strategies that would be helpful in both cases.

In Sect. 2 the requirements of CPT symmetry motivate us to devise a general test for SM versus NP. If the large observed CP asymmetry is due to the SM alone, then we would expect CP violation in a more inclusive final state to converge to the SM quark level expectation given in Eq. (1). Using the number of kaons as a surrogate for the number of \( s \)-quarks, we suggest that CP violation in inclusive \( K \bar{K} + X \) final states would provide a good test of this idea. We also suggest that hadronic matrix element enhancements mostly occur in only exclusive two-body modes (especially pseudoscalars). With that in mind, a simple method is suggested to experimentally identify inclusive events.
In Sect. 3 we focus on finding additional two-body decay modes that are likely to also show large CP asymmetries in the case of penguin-like NP. The key point here is to go after color-suppressed tree modes. In selecting potentially useful modes, we also consider which final states are more easily detected because they appear in the final state as charged particles only. Using these criteria, modes of particular interest include \( D^0 \rightarrow \rho^0 \rho^0 \), \( D_s \rightarrow \rho^0 K^{(*)+} \), and \( D^0 \rightarrow K^{(*)0} \bar{K}^{(*)0} \). In addition, the final states \( \eta \eta \), \( \eta \phi \), and \( \eta \phi \) may be interesting because of the “\( s \)-quark rich” nature of the final states, even though they contain neutrals in the final state; these could be of special interest to upcoming super \( B \) factories. We briefly discuss radiative final states, which are considered in Ref. [31]. These modes may show CP violation in a large class of penguin NP models; however, through CPT arguments we show that \( A_{\text{CP}} \) is likely to be suppressed.

In all of the above, CP violation is observed through \( A_{\text{CP}} \), a difference in decay rate between \( D \) and \( \bar{D} \) to a given final state. For this form of CP violation a strong phase is also required. It would be very useful to also be able to measure the weak phase directly, independently of the strong phase. In Sect. 4 we propose various methods to measure this phase. Three methods are considered: (1) using \( D^0 \) oscillation just as oscillation in \( B \) mesons is used to measure the weak phase in \( B^0 \rightarrow \psi K_s \); (2) using oscillation in \( B \)-mesons where \( B \rightarrow \bar{D}^0 \rho^0 \) (or similar final states); and (3) using correlations in \( D^0 \bar{D}^0 \) pairs. If we assume that the phase between \( D^0 \) and \( \bar{D}^0 \) decay to a given final state is of the same magnitude as the observed value of \( A_{\text{CP}} \) in \( D \rightarrow \pi \pi \) and \( D \rightarrow K K \) then the statistics required is \( \sim 10^{11} \) mesons for the methods using oscillation in \( D \)-mesons, oscillation in \( D \)-mesons, and correlations in \( D \)-pairs originating from \( B \)-meson decay. In the case where the \( D \)-pairs arise in a \( \psi'' \) factory, then \( \sim 10^9 \) \( D \)-mesons are required. More realistically, weak phases an order of magnitude larger than the currently observed \( A_{\text{CP}} \) may be observable in the foreseeable future. This would indicate a situation where the strong phase is small and yet there is a large NP weak phase.

In Sect. 5 we consider tests for NP based on isospin that would apply to tree-like NP models. In some cases isospin can be used to isolate CP violation in the \( \Delta I = \frac{3}{2} \) channel. Since the SM predicts no CP violation in this channel, such a signal would indicate the presence of NP.

Tests of this form, where the magnitude of a \( D \) amplitude is compared to that of a \( \bar{D} \) decay amplitude, are proposed for \( D \rightarrow \pi \pi \), \( D \rightarrow \rho \pi \), \( D_s \rightarrow \pi K^* \), and \( D \rightarrow \rho \rho \). In the case of \( D \rightarrow \rho \pi \) there are two separate amplitudes that can be used, one derived only from the \( D^0 \rightarrow \pi^+ \pi^- \pi^0 \) overall reaction and one that also includes input from \( D^+ \rightarrow \rho^+ \pi^0 \), \( \rho^0 \pi^+ \). In the case of \( D \rightarrow \rho \rho \) each polarization can provide a separate test.

For the final states \( \pi \pi \), \( \rho \rho \), and \( \rho \pi \), we can also combine this analysis with weak phase determination in Sect. 4, then additional tests are possible where the phase of a \( \Delta I = \frac{3}{2} \) amplitude is compared to its conjugate. Again such a phase would be indicative of tree-like NP.

In Sect. 6 we discuss the statistical requirements for testing the SM, particularly for the determination of weak phases. In Sect. 7 we give our summary and conclusion.

2. CPT and flavor symmetry considerations

2.1. Rough estimate for quark level expectations

The effective Hamiltonian for SCS charm decay can be written:

\[
H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left\{ \frac{1}{2} (\lambda_s - \lambda_d) \sum_{i=1,2} C_i (Q_i^s - Q_i^d) - \lambda_b \left( \sum_{i=1,2} \frac{1}{2} C_i (Q_i^s + Q_i^d) + \sum_{i=3,6} C_i Q_i \right) \right\} + h.c.
\]

(2)
where \( \lambda_q = V_{cq} V_{q}^* \) (note that \( \lambda_d + \lambda_s + \lambda_b = 0 \) by CKM unitarity). The operators are

\[
Q_1^d = (\bar{q}u)_{V-A} (\bar{c}q)_{V-A} \quad Q_2^d = (\bar{q}_d u_\beta)_{V-A} (\bar{c}_\beta q_a)_{V-A} \\
Q_3 = \sum_{q=u,d,s} (\bar{c}u)_{V-A} (\bar{q}q)_{V-A} \quad Q_4 = \sum_{q=u,d,s} (\bar{c}_d u_\beta)_{V-A} (\bar{q}_\beta q_a)_{V-A} \\
Q_5 = \sum_{q=u,d,s} (\bar{c}u)_{V-A} (\bar{q}q)_{V+A} \quad Q_6 = \sum_{q=u,d,s} (\bar{c}_d u_\beta)_{V+A} (\bar{q}_\beta q_a)_{V-A}.
\]

(3)

The first term proportional to \( \lambda_s - \lambda_d \) is the tree contribution and the term proportional to \( \lambda_b \) is the penguin contribution. If we assume that there is a strong phase difference between the tree and penguin of \( \phi_{\text{strong}} \), then we can write the CP asymmetry in the quark level process \( c \to d\bar{u} \) as [33]:

\[
|A_{\text{CP}}(c \to d\bar{u})| = \left| \text{Im} \left( \frac{2\lambda_b}{\lambda_s - \lambda_d} \right) \right| R \sin \phi_{\text{strong}} \approx \left| \text{Im} \left( \frac{V_{ub} V_{cb}^*}{V_{us} V_{cs}^*} \right) \right| R \sin \phi_{\text{strong}} \\
\approx A^2 \lambda^4 \eta R \sin \phi_{\text{strong}} \approx 6.4 \times 10^{-4} \sin \phi_{\text{strong}}.
\]

(4)

Here \( R \) is a number of order 1 that depends on the Wilson coefficients. If we neglect the mass of the \( s \)-quark and hadronization effects and use the one loop evolution of the Wilson coefficients given in Refs. [34,35] then numerically \( R \approx 1.2 \). The resultant asymmetry at the quark level is thus expected to be about the same as given in Eq. (1) if the strong phase is near maximal. As discussed below, CPT implies \( \Delta \Gamma(c \to d\bar{u}) = -\Delta \Gamma(c \to s\bar{s}u) \) where for a given decay \( A \to B, \Delta \Gamma(A \to B) = \Gamma(A \to B) - \Gamma(\bar{A} \to \bar{B}) \).

This quark level result need not be the same as the CP asymmetry in any given exclusive hadronic final state. Due to the significant hadronic uncertainties in the formation of specific final states, in order to characterize the CP violation in \( D \)-meson decay it is useful to consider symmetries that may be partially respected by strong interactions. The most obvious such symmetry is \( SU(3)_{\text{flavor}} \), which unfortunately is badly broken [18]. We will discuss the use of the isospin subgroup of \( SU(3) \) in Sect. 5. For the current experimental results, a handier subgroup of \( SU(3) \) to consider is \( U \)-spin, since it directly relates the \( K^+ K^- \) and \( \pi^+ \pi^- \) final states. If this symmetry were strictly observed then \( \text{Br}(D^0 \to \pi^+ \pi^-) = \text{Br}(D^0 \to K^+ K^-) \) and \( A_{\text{CP}}(D^0 \to \pi^+ \pi^-) = -A_{\text{CP}}(D^0 \to K^+ K^-) \). Since \( \text{Br}(D^0 \to \pi^+ \pi^-) = (1.397 \pm 0.026) \times 10^{-3} \) while \( \text{Br}(D^0 \to K^+ K^-) = (3.94 \pm 0.07) \times 10^{-3} \), more than a factor of 2 discrepancy, it is clear that \( U \)-spin is badly broken. Due to the large experimental error in the individual CP asymmetries, no firm conclusion with respect to \( U \)-spin applied to CP violation can be drawn, though the central values tend to show opposite sign.

2.2. CPT relations

Another symmetry that must, of course, be respected, is CPT, which implies that the width of the \( D \) and \( \bar{D} \) mesons must be the same. This means that, when summed over all final states of \( D \)-meson decay, the partial rate differences must vanish:

\[
\sum_X \Delta \Gamma(X) = 0.
\]

(5)

In detail, this means [36,37] that \( \Delta \Gamma \) must be exchanged between the various final states. This exchange is caused by rescattering between at least two final states (say \( X_1 \) and \( X_2 \)) with different strong and weak phases. If \( X_2 \) rescattering into \( X_1 \) provides a strong phase for \( X_1 \) it will give rise to a contribution to \( \Delta \Gamma(X_1) \). This partial rate asymmetry will be exactly canceled by the contribution to \( \Delta \Gamma(X_2) \) proportional to the strong phase produced when \( X_1 \) rescatters into \( X_2 \).
At the quark level, the SM maintains CPT in SCS decays by an exchange of $\Delta \Gamma$ between $c \rightarrow u\bar{d}u$ and $c \rightarrow u\bar{s}s$, hence

$$\Delta \Gamma(c \rightarrow u\bar{d}u) = -\Delta \Gamma(c \rightarrow u\bar{s}s). \tag{6}$$

The “double penguin” unitarity graph in Fig. 1 shows how this compensation arises where the two cuts indicate the two final states. Thus, cut #1 gives a final state with $u\bar{s}s$ where one of the amplitudes has an internal loop with an $u\bar{s}s$ final state. The magnitude of this graph is the same as that given by cut #2, giving an $u\bar{s}s$ final state with an intermediate $d\bar{d}u$, but the sign is opposite due to the internal loop being on the left side in this case.

To draw conclusions concerning specific groups of decay modes, it is useful to break down Eq. (6) according to quantum numbers conserved by strong interactions, since the exchange of $\Delta \Gamma$ between final states can only occur between states that can rescatter into each other. Such rescattering is via strong interactions so the general statement in Eq. (5) can be refined to:

$$\Delta \Gamma(P, C, I, G, S) = 0, \tag{7}$$

where $P, C, I, G, S$ are the quantum numbers, parity, charge conjugation, isospin, G-parity and strangeness respectively.

In applying this to SCS modes, where $S = 0$ in the final state, we can further classify final states according to the number, $N_K$, of kaons and anti-kaons they contain. Notice that in general $N_K \in \{0, 1, 2, 3\}$ because $M_D < 4M_K$ and, for $S = 0$ in particular, $N_K \in \{0, 2\}$, therefore

$$\Delta \Gamma(PCIG, S = 0, N_K = 0) = -\Delta \Gamma(PCIG, S = 0, N_K = 2). \tag{8}$$

The left and right sides of these equations should represent $d\bar{d}u$ and $s\bar{s}s$ quark content respectively, since it is expected that $c \rightarrow u\bar{d}u$ couples dominantly to $N_K = 0$ while $c \rightarrow u\bar{s}s$ couples dominantly to $N_K = 2$, so $\Delta \Gamma(c \rightarrow u\bar{d}u) \sim \Delta \Gamma(N_K = 0)$ and $\Delta \Gamma(c \rightarrow u\bar{s}s) \sim \Delta \Gamma(N_K = 2)$.

Implementing Eq. (8) directly for each combination of quantum numbers is difficult since many of the $D$ decay modes contain multiple charged pions and so the quantum numbers may be difficult to determine on an event-by-event basis.

What may be a more practical experimental test of CP violation is to look for CP violation in the inclusive case summed over $P, C, I, G$. In this case CPT implies:

$$\hat{\Delta} \Gamma(S = 0, N_K = 0) + \Delta \Gamma(\pi + \pi) = -\hat{\Delta} \Gamma(S = 0, N_K = 2) - \Delta \Gamma(KK), \tag{9}$$

where $\hat{\Delta}$ means that two-body pseudoscalars are not included, as we explain further below. CP asymmetry in both $\hat{\Delta} \Gamma(S = 0, N_K = 0)$ and $\hat{\Delta} \Gamma(S = 0, N_K = 2)$ should approximate the quark level CP asymmetry in $d\bar{d}u$ and $s\bar{s}s$ respectively.
If CP violation in the \( P \bar{P} \) final states is due to the SM then there must be some hadronic enhancement for those exclusive final states such as \( \pi^+\pi^- \), \( K^+K^- \), which we would not expect to be present in the inclusive Eq. (9). Recall also that for exclusive two pseudoscalar modes, in particular, there are well known reasons to expect large QCD corrections, e.g. chiral enhancements. It is quite unlikely that inclusive modes will receive such large enhancements. Thus, the inclusive CP asymmetry should be smaller, \( \sim 6 \times 10^{-4} \). On the other hand, if the largeness of the CP asymmetry in the PP final states is due to NP, then one would expect the inclusive asymmetry to be roughly of the same order as the exclusive. The larger statistics of the inclusive state may provide more accurate results for this channel, leading to an important indication of the nature of the observed CP violation.

In practice, observing a quantity like \( \Delta\Gamma(S = 0, N_K = 2) \) is subject to the problem that it is not possible to catch every final state. Thus it is useful to rephrase the relation \( \Delta\Gamma(S = 0, N_K = 2) \sim \Delta\Gamma(c \to s\bar{s}u) \) as

\[
\lim_{\chi \to I} \Delta\Gamma(S = 0, N_K = 2; \chi) \sim \Delta\Gamma(c \to s\bar{s}u),
\]  

(10)

where \( \chi \) is some CP invariant acceptance cut on the final states and \( I \) represents the cut where all events are accepted. In any case \( \Delta\Gamma(S = 0, N_K = 2; \chi) \) is a CP-violating quantity.

Actually, given that in the sample of inclusive \((N_K = 2)\) final states we do not want to include the exclusive \( K\bar{K} \) mode, a simple working definition of inclusive is all those final states in which the sum of \( K \) and \( \bar{K} \) energies is less than the energy of the parent \( D \).

Equation (8) can be broken down further with approximate symmetries, allowing us to gain some understanding of the pattern of CP violation in exclusive decay modes. For instance, if \( U \)-spin were a good symmetry then Eq. (8) could be broken down into groups of final states related by this symmetry. In particular, it would follow that \( \Delta\Gamma(\pi^+\pi^-) = -\Delta\Gamma(K^+K^-) \). Of course, \( U \)-spin is broken but in Fig. 2 of Ref. [11] they fit the experimental data including \( U \)-spin breaking allowed within the SM. This fit favors a solution where \( \left| A_{CP}(\pi^+\pi^-)/A_{CP}(K^+K^-) \right| > 1 \), although the statistics are not yet good enough to draw any firm conclusion. Analogously, in Ref. [15] (see Eq. 34), the above ratio of asymmetries is predicted to be \( \approx 1.8 \). This suggests a pattern where the partial rate asymmetry exchange is mostly between these two final states, in which case we would expect \( A_{CP}(\pi^+\pi^-)/A_{CP}(K^+K^-) \approx -B(\pi^+\pi^-)/B(\pi^+\pi^-) \approx -2.82 \pm 0.14 \).

In Fig. 2 the current weighted average results for \( A_{CP}(\pi^+\pi^-) \) and \( A_{CP}(K^+K^-) \) are shown as 1σ bands on an \( A_{CP}(\pi^+\pi^-) \) versus \( A_{CP}(K^+K^-) \) plot (indicated by the diagonally hatched and square hatched regions respectively) as well as the world average for the \( A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) \) result (vertically hatched band). The dashed line indicates the \( U \)-spin result \( A_{CP}(K^+K^-) = -A_{CP}(\pi^+\pi^-) \) while the black wedge indicates the result where we assume that CPT is maintained within PP final states by exchange between \( K^+K^- \) and \( \pi^+\pi^- \), i.e. that \( \Delta\Gamma(K^+K^-) + \Delta\Gamma(\pi^+\pi^-) = 0 \). For comparison, the circle in the lower left corner indicates the naive expectation where we assume that the meson asymmetry is the same as the quark level asymmetry. In particular, we suppose here that the quark level asymmetry is given by Eq. (4) with maximal strong phase and that \( A_{CP}(K^+K^-) = A_{CP}(s\bar{s}u) \) and \( A_{CP}(\pi^+\pi^-) = A_{CP}(d\bar{d}u) \).

In order to facilitate distinguishing SM from NP contributions, in Sect. (5) we will discuss relations that rely on isospin only, which, unlike the more general \( SU(3) \), should be good to the level of a few %.
Suppose that CP violation is the result of a large amplitude \( A \) interfering with a smaller amplitude \( a \). If we normalize the amplitudes in units of square root of branching ratio, then \( \text{Br}(D \to f) \approx |A|^2 \) while \( A_{CP}(f) \propto a/A \). If we want to observe the CP violation with a significance of \( N_{\sigma} \), the number of mesons required is \( N = N_{\sigma}^2/(\text{Br} A_{CP}^2) \). In terms of the amplitudes then,

\[
N = N_{\sigma}^2/(\text{Br} A_{CP}^2) \propto \frac{N_{\sigma}^2}{|A|^2 |a/A|^2} \propto \frac{N_{\sigma}^2}{|a|^2}. \tag{11}
\]

So that, generally, \( N \) depends on \( a \) but is independent of \( A \), but a smaller value of \( A \) does enhance \( A_{CP} \); \( N \) is not affected because this is at the expense of the branching ratio. Going to a mode that has a smaller branching ratio with higher asymmetry has the advantage of reducing the effects of systematic errors and other errors that are not statistical in nature, all other things being equal.

If we assume that the observed CP violation in \( D^0 \to \pi^+\pi^- \), \( K^+K^- \) is due to penguin-like NP, it may be that larger signals of CP asymmetries will be present in similar decays where the SM tree contribution is suppressed. Following this rationale, in this section we focus on the cases where the SM tree is color suppressed.

Color suppression in two-body final states is a pattern that is often borne out in \( B \)-meson decays. For example, in decays to charm mesons, the color-allowed \( B^0 \to \pi^+D^- \) has a branching ratio of \((2.6 \pm 0.13) \times 10^{-3}\) while the analogous color-suppressed mode \( B^0 \to \pi^0\bar{D}^0 \) has a branching ratio an order of magnitude smaller of \((2.61 \pm 0.24) \times 10^{-4}\). A similar pattern obtains for related decays. In contrast, in the case of \( B \)-meson decay to two light pseudoscalar mesons, color suppression fails. Thus \( \text{Br}(B^0 \to \pi^+\pi^-) = (5.13 \pm 0.24) \times 10^{-6} \) while \( \text{Br}(B^0 \to \pi^0\pi^0) = (1.62 \pm 0.31) \times 10^{-6} \), but with PV and PP final states, for instance \( \text{Br}(B^0 \to \pi^+\rho^- + \pi^-\rho^+) = (2.3 \pm 0.23) \times 10^{-5} \) versus \( \text{Br}(B^0 \to \pi^0\rho^0) = (2.0 \pm 0.5) \times 10^{-6} \), likewise \( \text{Br}(B^0 \to \rho^+\rho^-) = (2.42 \pm 0.31) \times 10^{-5} \) versus \( \text{Br}(B^0 \to \rho^0\rho^0) = (7.3 \pm 2.6) \times 10^{-7} \); color suppression of these modes seems to hold.
Table 1. The singly Cabibbo-suppressed decays of $D$ mesons to two ground state are listed. Note that the notation $\pi^{(*)0}$ stands for $\pi^0$ or $\rho^0$; $\pi^{(*)}$ stands for $\pi^0$, $\rho^0$, or $\omega^0$; $\phi^{(*)}$ stands for $\phi$ or $\eta^{'(0)}$ to the extent that $\eta^{'(0)}$ is an $s\bar{s}$ state. For each group of decays, we have indicated whether the tree contribution is color suppressed with “X” and if it is both color and Zweig suppressed with “XX”. The instances that lead to an all charged final state are listed. Note that the branching ratio is known from Ref. [38], we have included it in the last column; this is the branching ratio including the subsequent decays to the final all charged state indicated.

<table>
<thead>
<tr>
<th>Decay</th>
<th>Suppressed</th>
<th>Charged final state</th>
<th>Favored</th>
<th>Total ( \text{Br} ) (10^{-3})</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D_s \to \pi^{(<em>)0} K^{(</em>)+} )</td>
<td>X</td>
<td>( \rho^0 \to \pi^+\pi^- K^+ )</td>
<td>X</td>
<td>2.7 ± 0.05</td>
</tr>
<tr>
<td>( D_s \to \phi^{(<em>)} K^{(</em>)+} )</td>
<td></td>
<td>( \phi \to K^+K^- \eta )</td>
<td>&lt;0.3</td>
<td></td>
</tr>
<tr>
<td>( D^+ \to \pi^{(<em>)+} \phi^{(</em>)} )</td>
<td>X</td>
<td>( \pi^+\phi \to K^+K^- )</td>
<td>2.65 ± 0.08</td>
<td></td>
</tr>
<tr>
<td>( D^+ \to K^{(<em>)+} K^{(</em>)0} )</td>
<td>( K^+[K_s \to \pi^+\pi^-] )</td>
<td>1.98 ± 0.13</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>( K^+[K^{(*)0} \to K^+\pi^-] )</td>
<td>2.45 ± 0.14</td>
<td></td>
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<tr>
<td></td>
<td>( [K^{(*)+} \to \pi^+[K_s \to \pi^+\pi^-]][K_s \to \pi^+\pi^-] )</td>
<td>5.7 ± 2.3</td>
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<tr>
<td></td>
<td>( [K^{(<em>)+} \to \pi^+[K_s \to \pi^+\pi^-]][K^{(</em>)0} \to K^+\pi^-] )</td>
<td></td>
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<td></td>
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<tr>
<td>( D^+ \to \pi^{(<em>)+} \pi^{(</em>)0} )</td>
<td>( \pi^+[\rho^0 \to \pi^+\pi^-] )</td>
<td>0.81 ± 0.15</td>
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<td></td>
</tr>
<tr>
<td>( D^0 \to K^{(<em>)0} \bar{K}^{(</em>)0} )</td>
<td>XX</td>
<td>( [K_s \to \pi^+\pi^-][K_s \to \pi^+\pi^-] )</td>
<td>0.085 ± 0.014</td>
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<tr>
<td></td>
<td>( [K^{(*)0} \to K^+\pi^-][K_s \to \pi^+\pi^-] )</td>
<td>X</td>
<td>&lt;0.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( [K^{(*)0} \to K^-\pi^+][K_s \to \pi^+\pi^-] )</td>
<td>X</td>
<td>&lt;0.35</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( [K^{(<em>)0} \to K^+\pi^-][K^{(</em>)0} \to K^-\pi^+] )</td>
<td>X</td>
<td>0.07 ± 0.05</td>
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</tr>
<tr>
<td>( D^0 \to \pi^{(<em>)0} \pi^{(</em>)0} )</td>
<td>( \rho^0 \to \pi^+\pi^- )</td>
<td>1.82 ± 0.10</td>
<td></td>
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</tr>
<tr>
<td>( D^0 \to \pi^{(<em>)+} \pi^{(</em>)-} )</td>
<td>( \pi^+\pi^- )</td>
<td>1.400 ± 0.026</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( D^0 \to \phi^{(<em>)} \pi^{(</em>)0} )</td>
<td>X</td>
<td>( D^0 \to \phi \rho^0 )</td>
<td>1.40 ± 0.12</td>
<td></td>
</tr>
<tr>
<td>( D^0 \to K^{(<em>)+} K^{(</em>)-} )</td>
<td>( K^+K^- )</td>
<td>3.96 ± 0.08</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>( [K^{(*)+} \to \pi^+[K_s \to \pi^+\pi^-]][K^- \to \pi^-\pi^+] )</td>
<td>2.19 ± 0.1</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>( K^+[K^{(*)-} \to \pi^-[K_s \to \pi^+\pi^-]] )</td>
<td>0.78 ± 0.06</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( [K^{(<em>)+} \to \pi^+[K_s \to \pi^+\pi^-]][K^{(</em>)-} \to \pi^-\pi^+] )</td>
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</table>

It is to be expected that color suppression is less effective in $D$ decays because of greater nonperturbative effects and increased meson rescattering at the charm mass scale. Indeed this appears to be the case, e.g., $\text{Br}(D^0 \to K^-\pi^+) = 3.89\%$ while $\text{Br}(D^0 \to \bar{K}^{(*)0}\pi^0) = 2.44\%$, showing no color suppression, likewise $\text{Br}(D^0 \to K^{(*)-}\pi^+) = 1.73\%$ while $\text{Br}(D^0 \to \bar{K}^{(*)0}\pi^0) = 2.28\%$, again showing no color suppression. Conversely, the $K\rho$ channel does seem to show the effect since $\text{Br}(D^0 \to K^-\rho^+) = 10.8\%$ while $\text{Br}(D^0 \to \bar{K}^{(*)0}\rho^0) = 1.32\%$.

In spite of the unreliable evidence that color suppression is universally operative in $D$-meson decays, used with care and caution, it may provide us with a guide in searching for other decay modes for future searches for enhanced CP violation. In particular, modes where the SM amplitude tends to be suppressed and a possible NP penguin amplitude may be enhanced may be of particular interest in NP searches. In Table 1 we list all the two-body Cabibbo-suppressed decay modes of $D$-mesons to ground state mesons. Here we use the notation $\pi^{(*)\pm}$ to indicate either $\pi^\pm$ or $\rho^\pm$ and $\pi^{(*)0}$ to indicate either $\pi^0$, $\rho^0$, or $\omega^0$. Likewise we use $\phi^{(*)}$ to indicate either $\phi$ or $\eta^{'(0)}$ (or at least the $s\bar{s}$ component of the latter).

Decays of the form $D_s \to \pi^{(*)0} K^{(*)+}$, $D^+ \to \pi^{(*)+} \phi^{(*)}$, $D^0 \to K^{(*)0} \bar{K}^{(*)0}$, and $D^0 \to \pi^{(*)0} \phi^{(*)}$ have the tree color-suppressed in this way. Also, modes of the form $D^0 \to K^{(*)0} \bar{K}^{(*)0}$
have an additional suppression of the tree contribution due to the fact that the $d\bar{s}s\bar{s}$ final state quark content is not the same as produced by the tree graph. This is borne out by the smallness of the branching ratios of $D^0 \rightarrow K_sK_s$ and $D^0 \rightarrow K^{*0}\bar{K}^{*0}$. Therefore, most promising from this list is $D_s \rightarrow \pi^{(s)0}K^{(s)+}$.

As an example of how color suppression can work in the realm of Cabibbo-allowed $D$-meson decays, consider such decays to two vector final states. The decay $D^0 \rightarrow K^{*+}\rho^-$ has no suppression and its branching ratio is $10.8 \pm 0.7\%$. The related color-suppressed decays $D^0 \rightarrow \bar{K}^{*0}\rho^0$ and $D^0 \rightarrow \bar{K}^{*0}\omega$ have branching ratios $1.58 \pm 0.35\%$ and $1.1 \pm 0.5\%$ respectively.

In the table, we also enumerate the cases where the mode cascades down to a final state that contains all charged particles (i.e. $\pi^{\pm}$ and $K^{\pm}$), and give their branching ratios from Ref. [38], where known. Final states with all charged states will generally be easier to detect, particularly at the LHCb. Based on these criteria, $D^0 \rightarrow \rho^0\rho^0$, $D_s \rightarrow \rho^0 K^+$, and $D_s \rightarrow \rho^0 K^{*+}$ are perhaps the most favorable channels to find CP violation due to penguin-like new physics.

From this point of view, the case of $D^0 \rightarrow \rho^0\rho^0$ is perhaps of particular interest to search for enhanced CP violation due to NP. An additional feature of this VV final state is the spin degree of freedom; there are three polarization states: transverse parallel ($A_\parallel$), transverse perpendicular ($A_\perp$), and longitudinal ($A_\ell$). Each amplitude could have different CP violation. Already existing measurements [38] of the polarization fractions show that the longitudinal mode dominates with a fraction of $67\%$ longitudinal. Further measurements of the angular distribution using the methods of Ref. [39] will allow the extraction of the phases between the polarization amplitudes.

As discussed in Refs. [40,41] and in Ref. [42] for the analogous $B$ decays, a qualitatively different feature of VV final states is that there can be $P$-odd triple product observables. Such observables can lead to either CP-odd or CP-even correlations depending on the combination of $D$ and $\bar{D}$ decays. If the C-even combination is formed (adding the triple product of $D$ and $\bar{D}$) then the combination is CP-odd; conversely, the C-odd combination is CP-even. CPT then requires C-odd, CP-violating amplitudes to be real whereas C-odd, CP-violating amplitudes need a rescattering strong phase.

CP-odd observables of this form that are C-even can be formed from untagged samples of $D^0$ mesons. This is an advantage for $e^+e^-$ $B$ factories where the initial state is self-conjugate so the $D^0$ samples obtained at such machines could be used directly (except for the small asymmetry in the $D$ meson production mechanism between inclusive $B \rightarrow D + X$ versus $\bar{B} \rightarrow D + X$, which will have to be determined from separate studies). In the case of LHCb this would not be true, since the $pp$ initial state is not self-conjugate so tagging will, in any case, be necessary.

In some tree-like NP models, even if the SM tree is color suppressed the NP contribution is not. This will tend to enhance the NP contribution to CP violation. In particular, if a $q\bar{q}$ pair is produced by a color neutral object (e.g. a $Z'$ or higgs-like boson) then the effective Hamiltonian will have a different color structure from the SM and so color suppression may not apply. In addition to the two families of modes mentioned above, $D_s \rightarrow \pi^{(s)0}K^{(s)+}$ and $D^0 \rightarrow \pi^{(*)0}\pi^{(*)0}$, modes of the form $D^+ \rightarrow \pi^{(*)+}\phi^{(*)}$ should have enhanced CP asymmetries in this scenario.

Tree-like CP violation of this form should contribute to the $\Delta I = 1/2$ channel and so the isospin analysis in Sect. 5 should reveal this, though with some unknown hadronization effects, and thus may reveal a contradiction with the SM. In this scenario, the $\pi\pi$, $\rho\pi$, and $\rho\rho$ systems are particularly suited since there are enough charge distributions to allow isospin analysis and the final state with two neutral mesons has a color-suppressed tree graph, which may enhance NP CP violation. Of these
cases, the \( \rho^0 \rho^0 \) state also has the advantage that it leads to an observed final state with all charged mesons.

Another class of final states that may be of interest in some models of new physics are those that are rich in \( s \bar{s} \), in particular if they contain \( \phi, \eta, \) and \( \eta' \). The only three such two-body modes that are kinematically allowed are \( D^0 \to \phi \eta, D^0 \to \eta' \eta \) and \( D^0 \to \eta \eta \). Of course none of these modes leads to an all charged final state but the tree graph is color suppressed. There is some additional suppression since the quark content only couples to the \( u \bar{u} \) part of the \( \eta^{(')} \) wave function, which makes up only 20–30\% of these mesons.

Another manifestation of penguin-like NP that could lead to CP-violating signals is radiative decays. At the quark level, such decays would proceed through c \( \to u \gamma \), which leads to modes like \( D^0 \to \rho^0 \gamma, D^0 \to \omega \gamma, D^+ \to \rho^+ \gamma, \) and \( D_s \to K^{*+} \gamma \). Other radiative \( D \)-meson decays that would not be expected to receive large contributions from short distance radiative penguins are \( D^0 \to K^{0*} \gamma \left( \text{Br} = (3.28 \pm 0.35) \times 10^{-4} \right), D^0 \to \phi \gamma \left( \text{Br} = (2.70 \pm 0.34) \times 10^{-5} \right) \). In Ref. [31] these kind of radiative decays are discussed in the context of new physics. According to their analysis, if the QCD dipole \( c \to u \) transition operators:

\[
Q_8 = \frac{g_s m_c}{4 \pi^2} \bar{u}_L \sigma_{\mu \nu} T^a G^a_{\mu \nu} c_R \quad Q_8' = \frac{g_s m_c}{4 \pi^2} \bar{u}_R \sigma_{\mu \nu} T^a G^a_{\mu \nu} c_L, \tag{12}
\]

then operator evolution from the NP scale to the charm scale would lead to comparable coefficients for the electromagnetic dipole \( c \to u \) transition operators:

\[
Q_7 = e Q_u \frac{m_c}{4 \pi^2} \bar{u}_L \sigma_{\mu \nu} F_{\mu \nu} c_R \quad Q_7' = e Q_u \frac{m_c}{4 \pi^2} \bar{u}_R \sigma_{\mu \nu} F_{\mu \nu} c_L. \tag{13}
\]

Even if the coefficient of \( Q_7^{(')} \) were much smaller than \( Q_8 \) at the high NP scale, the coefficients may be comparable at the charm scale. Assuming that the observed LHCb result is due to the effects of \( Q_8^{(')} \) they [31] suggest that the induced coefficient of \( Q_7^{(')} \) would generate \( A_{\text{CP}} \) in \( D \to \rho \gamma \) of \( O(10\%) \) provided the strong phases involved were maximal (see, however, Ref. [32]). Using vector dominance one can estimate that \( \text{Br}(D \to \rho \gamma) \approx 10^{-5} \), perhaps making this mode a good test for new physics.

It is, however, unlikely that the strong phase will be \( O(1) \) because the kinematics of the rescattering forces there to be an \( \alpha_s \) correction to satisfy CPT. To see this, consider the unitarity diagrams in Fig. 3 for the quark level process \( c \to u \gamma \). Diagram 1 in this figure shows the interference of an NP penguin with the SM penguin (having an internal \( s \)-quark) at lowest order. Note that while cut #1 corresponds to the \( u \gamma \) final state, cut #2 does not correspond to an on-shell state since the \( s \bar{s} \) pair must rescatter into a single photon. There is, therefore, no strong phase and so there cannot be a CP asymmetry from this diagram. Diagram 2 shows an order \( \alpha_s \) correction to the diagram where there can be a strong phase, since now cut #2 corresponds to an on-shell state. This diagram, therefore, can give rise to a CP asymmetry but that asymmetry will be suppressed by \( \alpha_s \) since there is an extra loop. Model calculations carried out in Ref. [32] appear to bear out this situation.

4. Weak phase determination by initial state mixing

In this section we will consider CP violation, which arises in the phase between \( D^0 \) and \( \bar{D}^0 \) decaying to a common final state \( f \). Using the interference between the two amplitudes, the weak phase can be directly measured without relying on the the existence of a strong rescattering phase. Thus it is useful to find new examples of CP violation as well as elucidating the source of CP violation in \( D \)-meson decays.
Fig. 3. This unitarity graph illustrates CPT conservation for the quark level process $c \to u \gamma$ due to NP. Diagram 1 shows the lowest order interference between NP and SM where cut #1 is for the $c \gamma$ final state and cut #2 is for an $s \bar{s} u$ final state. Cut #2 cannot be on shell. Diagram 2 shows an example of an order $\alpha$, correction to diagram 1 where, in contrast, cut #2 can be on shell.

In all cases, these kinds of measurements are difficult and most likely cannot be carried out in the near future if the weak phase is of the same order of magnitude as the currently observed CP asymmetry in $P P$ final states. It is possible that the small direct CP asymmetry seen in $D^0 \to P P$ results from a large weak phase in combination with a small (i.e. $< O(10\%)$) strong phase. If this proves to be the case, then a weak phase ($O(10\%)$) may be measured by the various methods considered here. A phase of this magnitude in any mode would be hard to explain in the SM and so would be an indication of NP.

If the CP violation in $D$-mesons is presumed to be from the SM source, the weak phase measurement can also tell us the magnitude of the penguin contribution. To see this, recall that the SM decay amplitude receives a dominant contribution from the tree that has no weak phase and a penguin contribution with weak phase $\gamma$. We can therefore write the amplitude for any SCS decay and its conjugate as:

$$A = T + P e^{i\gamma},$$

$$\bar{A} = T + P e^{-i\gamma},$$  \hspace{1cm} (14)

where $T$ and $P$ are generally complex numbers because they contain strong phases.

Thus, if we know $|A|$ and $|\bar{A}|$ from measurement of $D^0$ and $\bar{D}^0$ decays to the final state as well as the phase $\theta = \arg(\bar{A}A^*)$, then

$$|A| - |\bar{A}| e^{i\theta} = 2|P| \sin \gamma.$$  \hspace{1cm} (15)

In the context of the SM, this allows us to extract the magnitude of the penguin $|P|$ using the value of $\gamma$ known independently from the global fit to the unitarity triangle. To the extent that the SM contribution to the exclusive final state can be calculated from QCD, admittedly a very challenging task, this can be compared to such a measured value of $|P|$.

In any case, a precise determination of the weak phase in some modes may also allow us to test the SM by feeding it into the isospin analysis discussed in Sect. 5. In particular, the weak phase in
the neutral $D$ decay can allow us to determine the weak phase of the $\Delta I = \frac{3}{2}$ channel. Such a phase would be an indication of tree-like NP.

In this section, we consider three methods to accomplish a weak phase measurement in $D^0$ decays by producing mixed initial states of $D^0$ and $\bar{D}^0$ mesons. First we consider the "conventional approach" of taking advantage of the natural mixing of the two flavor eigenstates, $D^0$ versus $\bar{D}^0$. Since the rate of this mixing is small compared to the $D^0$ lifetime, we consider using the much larger relative mixing in the $B^0$ and $\bar{B}^0$ mesons if they subsequently decay to a $D^0$ meson. Finally we consider methods using entangled $D^0\bar{D}^0$ states produced at tau-charm factories, where the meson pair arises from a $\psi''$ resonance, and also using (super) $B$ factories, where the pair arises in a two- or three-body decay of a $B$ meson (e.g. $B \rightarrow D\bar{D}$ or $D\bar{D}K$). First, however, we quickly review the oscillation formalism for neutral mesons.

### 4.1. Oscillation formalism

Let us consider a generic neutral flavored meson $X$ (i.e. $X = K^0, D^0, B_d, or B_s$). Defining the light eigenstate ($X_L$) with mass $m_{X_L}$ and width $\Gamma_{X_L}$ and heavy eigenstate ($X_H$) with mass $m_{X_H}$ and width $\Gamma_{X_H}$, we have:

\[
\begin{align*}
|X_L\rangle &= p^X|X\rangle + q^X|\bar{X}\rangle \\
|X_H\rangle &= p^X|X\rangle - q^X|\bar{X}\rangle.
\end{align*}
\] (16)

Thus the flavor eigenstates evolve with time $t_X$ according to

\[
\begin{align*}
|X(t)\rangle_{\text{phys}} &= g_+^X|X\rangle - \frac{q^X}{p^X}g_-^X|\bar{X}\rangle \\
|\bar{X}(t)\rangle_{\text{phys}} &= g_+^X|\bar{X}\rangle - \frac{p^X}{q^X}g_-^X|X\rangle,
\end{align*}
\] (17)

where the time-dependent mixing coefficients $g_+^X$ are given by:

\[
ge_\pm^X = e^{-(im_{X_H}+\frac{1}{2}\Gamma_{X_H})t} \pm e^{-(im_{X_L}+\frac{1}{2}\Gamma_{X_L})t}
\] (18)

Let $f$ be a final state that both $X$ and $\bar{X}$ can decay to. If $A_f^X$ is the amplitude for $X \rightarrow f$ and $\bar{A}_f^X$ is the amplitude for $\bar{X} \rightarrow f$ then the time-dependent rates of $X$ and $\bar{X}$ to $f$ are:

\[
\begin{align*}
\frac{d}{dt_X}\Gamma(X(t) \rightarrow f) &= \frac{1}{2}e^{-\tau_X} \left[ (C_y^X + C_z^X)|A_f^X|^2 + (C_y^X - C_z^X)\left|\frac{q^X}{p^X}\bar{A}_f^X\right|^2 \right. \\
&\left. + 2S_x^X \text{Re}\left(\frac{q^X}{p^X}A_f^X\bar{A}_f^X\right) + 2S_{x'}^X \text{Im}\left(\frac{q^X}{p^X}A_f^X\bar{A}_f^X\right)\right] \\
\frac{d}{dt_X}\Gamma(\bar{X}(t) \rightarrow f) &= \frac{1}{2}e^{-\tau_X} \left[ (C_y^X + C_z^X)|\bar{A}_f^X|^2 + (C_y^X - C_z^X)\left|\frac{p^X}{q^X}A_f^X\right|^2 \right. \\
&\left. + 2S_x^X \text{Re}\left(\frac{p^X}{q^X}A_f^X\bar{A}_f^X\right) + 2S_{x'}^X \text{Im}\left(\frac{p^X}{q^X}A_f^X\bar{A}_f^X\right)\right].
\end{align*}
\] (19)

Here $\Delta m_X = m_{X_H} - m_{X_L}$, $\Delta \Gamma_X = \Gamma_{X_H} - \Gamma_{X_L}$, $\tau_X = \Gamma_X^{-1}$, $x_X = \Delta m_X/\Gamma_X$, $y_X = \Delta \Gamma_X/(2\Gamma_X)$, $C_x^X = \cos(x_X\tau_X)$, $S_x^X = \sin(x_X\tau_X)$, $C_{x'}^X = \cosh(y_X\tau_X)$, $S_{x'}^X = \sinh(y_X\tau_X)$, and $z_X = x_X - iy_X$.

In the case of $D$ mesons, both $x_D$ and $y_D \leq O(10^{-2})$, so we will expand observables to first order in $x_D$ and $y_D$. 

12/25
In this limit, the above time-dependent rate becomes:
\[
\frac{d}{d\tau_X} \Gamma(X(t) \rightarrow f) = e^{-\tau_X} \left[ |A_f^X|^2 + \tau \text{Re} \left( -iz^*A_f^X\bar{A}_f^X\frac{q^*}{p^*} \right) \right] + O(x^2, y^2)
\]
\[
\frac{d}{d\tau_X} \Gamma(\bar{X}(t) \rightarrow f) = e^{-\tau_X} \left[ |\bar{A}_f^X|^2 + \tau \text{Re} \left( -iz^*A_f^X\bar{A}_f^X\frac{p^*}{q^*}\right) \right] + O(x^2, y^2).
\]

In some of the examples below we will consider the time-integrated effect of oscillation. To first order in \( x_X, y_X \) this can be accomplished by replacing the decay amplitudes with “effective” decay amplitudes:

\[
B_f^X = A_f^X + \frac{i}{2} \bar{A}_f^X \left( \frac{q^X}{p^X} \right) z
\]
\[
\bar{B}_f^X = \bar{A}_f^X + \frac{i}{2} A_f^X \left( \frac{q^X}{p^X} \right) z.
\]

The time-integrated rate for a \( D^0 \) meson to decay to \( f \) is given, up to first order in \( x_X \) and \( y_X \), by using this effective amplitude “without oscillation”.

Thus, for instance,

\[
\int_0^\infty d\Gamma(X \rightarrow f) \ d\tau = |B_f^X|^2 + O(x^2, y^2)
\]
\[
\int_0^\infty d\Gamma(\bar{X} \rightarrow f) \ d\tau = |\bar{B}_f^X|^2 + O(x^2, y^2).
\]

### 4.2. Weak phases from \( D^0/\bar{D}^0 \) oscillation

As discussed in Refs. [43,44], in \( D^0 \) decay to a given final state one must consider both direct CP violation and indirect CP violation due to \( D^0 \) oscillation. Conversely, assuming that the oscillation parameters are known from separate studies, we can use oscillation to extract the phase between \( A_f^X \) and \( \bar{A}_f^X \). To do this, it is necessary to observe the time dependence of the decays.

From Eq. (20) if we know \( x_D, y_D, p^D, \) and \( q^D \), we see that the constant term gives the magnitudes of the amplitudes \( |A_f^D| \) and \( |\bar{A}_f^D| \). The slope of the decay rate gives the phase between these two amplitudes. If \( f \) is self-conjugate like \( \pi^+\pi^- \), such a phase difference will be CP-odd. If \( f \) is not self-conjugate, such as \( \rho^+\pi^- \), the phase will be a combination of CP-odd and CP-even phase differences. If both \( x_D \) and \( y_D \) are non-zero, the phase can be determined in this way without ambiguity. If one of these is zero, there is a twofold ambiguity in the phase determination.

### 4.3. Weak phases from \( B^0_\ell/\bar{B}^0_\ell \) oscillation

Another way to accomplish the measurement of the relative phase is to look at a two-body decay of a neutral \( B \)-meson, \( B_q \) for \( q = d, s \), to a neutral \( D \)-meson where the \( D \)-meson subsequently decays to the final state \( f \). If we observe this overall reaction \( B \rightarrow M^0[D^0 \rightarrow f] \) (where \( M^0 \) is a self-conjugate neutral meson and \( B \) is either \( B_d \) or \( B_s \)) as a function of the time of the \( B_q \) decay then the \( D \) state involved in the second decay will generally be a mixture of the flavor eigenstates.

Of course, once the \( D \)-meson is spawned, it will oscillate as described above. In the following we will assume that only the \( B \)-meson decay time is observed and therefore the \( D \)-meson decay time is integrated over.

Let us denote by \( T_{DB} \) the amplitude for \( B \rightarrow M^0D \), \( T_{\bar{D}B} \) the amplitude for \( B \rightarrow M^0\bar{D} \), \( T_{D\bar{B}} \) the amplitude for \( \bar{B} \rightarrow M^0D \), and \( T_{D\bar{B}} \) the amplitude for \( \bar{B} \rightarrow M^0\bar{D} \); thus, we can define the effective
amplitudes for \( B \) and \( \bar{B} \) cascading down to the final state \( f \). Using the formalism in Eq. (21) we can define the effective amplitudes:

\[
D^B_f = B^D_f T_{DB} + \bar{B}^D_f T_{DB} \\
\bar{D}^B_f = B^D_f T_{DB} + \bar{B}^D_f T_{DB}.
\] (23)

Thus the time-dependent decay rate integrated over the \( D \)-meson decay time as a function of the \( B \)-meson decay time is given by Eq. (19):

\[
\frac{d}{dt_B} \Gamma(B(t_B) \to M[D \to f]) = \frac{1}{2} e^{-t_B} \left[ (C^B_y + C^B_x)|D^B_f|^2 + (C^B_y - C^B_x) \left| \frac{q^B}{p^B} \bar{D}^B_f \right|^2 \\
+ 2S^B_y \Re \left( \frac{q^{B\ast}}{p^{B\ast}} D^B_f \bar{D}^{B\ast}_f \right) + 2S^B_x \Im \left( \frac{q^{B\ast}}{p^{B\ast}} D^B_f \bar{D}^{B\ast}_f \right) \right]
\]

\[
\frac{d}{dt_B} \Gamma(\bar{B}(t_B) \to M[D \to f]) = \frac{1}{2} e^{-t_B} \left[ (C^B_y + C^B_x)|\bar{D}^B_f|^2 + (C^B_y - C^B_x) \left| \frac{p^B}{q^B} D^B_f \right|^2 \\
+ 2S^B_y \Re \left( \frac{p^{B\ast}}{q^{B\ast}} D^B_f \bar{D}^{B\ast}_f \right) + 2S^B_x \Im \left( \frac{p^{B\ast}}{q^{B\ast}} D^B_f \bar{D}^{B\ast}_f \right) \right].
\] (24)

From the above equation, assuming that \( x_B, y_B, \) and \( p^B/q^B \) are known, then the magnitudes and relative phase of \( D^B_f \) and \( \bar{D}^B_f \) can be determined. Assuming that \( T_{ij} \) is also known, then by inverting Eq. (23) we determine \( B^D_f \) and \( \bar{B}^D_f \). As in the last section, we can then invert the relations contained in Eq. (21) to determine the magnitudes and relative phases of \( A^D_f \) and \( \bar{A}^D_f \). In most cases, the amplitudes \( T_{DB} \approx \pm T_{DB} \) will dominate over \( T_{DB} \) and \( T_{DB} \) and since \( \{B^D_f, \bar{B}^D_f\} \) differs from \( \{A^D_f, \bar{A}^D_f\} \) by \( O(10^{-2}) \), the phase between \( D^B_f \) and \( \bar{D}^B_f \) will, to a good approximation, be the negative of the phase between \( A^D_f \) and \( \bar{A}^D_f \).

Let us consider some particular cases of the parent \( B \to MD \) decay. In the case of \( B_d \) some candidates are \( B_d \to \pi^0 \tilde{D}^0 \) (Br = 2.61 ± 0.24 × 10\(^{-4}\)) and \( B_d \to \rho^0 \tilde{D}^0 \) (Br = 3.2 ± 0.5 × 10\(^{-4}\)). The latter is probably easier to observe since \( \rho^0 \) decays to \( \pi^+ \pi^- \). Indeed, if the final state of the \( D^0 \) decay is either \( \pi^+ \pi^- \) or \( \rho^0 \rho^0 \) then the entire event has an all charged final state. Other decays of this type are \( B_d \to \eta \tilde{D}^0 \) (Br = 2.02 ± 0.35 × 10\(^{-4}\)), \( B_d \to \eta' \tilde{D}^0 \) (Br = 1.25 ± 0.23 × 10\(^{-4}\)), and \( B_d \to \omega \tilde{D}^0 \) (Br = 2.59 ± 0.3 × 10\(^{-4}\)). In principle, the results from these modes can be combined (taking into account the CP of the final state). In this case we could have an aggregate branching ratio of \( \sim 10^{-3} \).

We can also consider \( \tilde{D}^{0\ast} \) instead of \( \tilde{D}^0 \) and that can augment the effective branching ratio.

Another choice is to consider decays such as \( B_d \to K_s \tilde{D}(\text{Br} = 5.2 \times 10^{-5}) \) and related modes, but these are an order of magnitude smaller in branching ratio due to Cabibbo suppression.

It is also possible to start with a \( B_s \) state. The analogous decays are \( B_s \to K_s \tilde{D} \) and related processes, which likely have roughly the same branching ratios. These would include \( B_s \to K_s \tilde{D}^0 \) and \( B_s \to K^* \tilde{D}^0 \) with branching ratios at the \( 10^{-4} \) level (note the \( K^* \) would need to decay to \( K_s \pi^0 \) to collapse the \( B_s \) flavor wave function) and decays such as \( B_s \to \phi \tilde{D}^0 \) at the \( 10^{-5} \) level.

4.4. Correlations at charm and \( B \) factories

Let us now consider the case of a \( D^0 \tilde{D}^0 \) pair that is initially in a single, correlated, quantum state. Let us arbitrarily label the mesons \( D_1 \) and \( D_2 \) and consider reactions where \( D_1 \to f; D_2 \to g \) where \( f \) is the state of interest and \( g \) is an “index” decay (which needs to be a decay state of both \( D^0 \) and \( \tilde{D}^0 \)),
for instance $f = \pi^+\pi^-$ and $g = K_s\pi^0$, where the weak phase of $\pi^+\pi^-$ is to be measured and it is assumed that the weak phase in $K_s\pi^0$ is small and known (i.e. just from $K_s$).

In such a scenario, the initial wave function together with the observation of $D_2 \to g$ determines the wave function of the $D_1$ state as a mixture of the flavor eigenstates. In this way we are able to observe the interference of $D^0$ and $\bar{D}^0$ decay amplitudes to $f$.

Starting with the wave function of the meson pair:

$$\Psi = a|D_1\rangle|\bar{D}_2\rangle + \bar{a}|\bar{D}_1\rangle D_2\rangle,$$

where $|a|^2 + |\bar{a}|^2 = 1$, the amplitude for the combined decay $(D_1 \to f)(D_1 \to g)$ is therefore

$$|A_{fg}|^2 = \frac{1}{2}(|A_f|^2|\bar{A}_g|^2 + |\bar{A}_f|^2|A_g|^2) + \frac{1}{2}(|a|^2 - |\bar{a}|^2)(|A_f|^2|\bar{A}_g|^2 - |\bar{A}_f|^2|A_g|^2)
\quad + 2\text{Re}(a\bar{a}^* A_f \bar{A}_g^* A_g \bar{A}_f).$$

Thus if $|A_f|$, $|\bar{A}_f|$, $A_g$, $\bar{A}_g$, $\theta$, and $\delta$ are known, then the phase between $A_f$ and $\bar{A}_f$ can be determined.

This equation takes into account only the entanglement of the initial state; we can also take into account the time-integrated neutral $D$ oscillations by integrating $|A_{fg}|^2$ to first order in $x$, $y$. As above, this is equivalent to replacing $A_{fg}$ in Eq. (26) with the effective amplitudes $B_{fg}$ given by Eq. (21).

The conceptually simplest example is applying this at a tau-charm factory using the method of Refs. [45,46]. In this case, the $D^0$ pair arises from the decay of the $\psi(3770)$ and so in the initial state,

$$a = +\frac{1}{\sqrt{2}}, \quad \bar{a} = -\frac{1}{\sqrt{2}}.$$

An evenly mixed $D_1$ state will thus arise when $|A_g| \approx |\bar{A}_g|$. As an example, if we take $g = K_s\pi^0$ then $A_g = -\bar{A}_g$ if, as the SM predicts, there is no CP violation in this pure tree decay mode except for the well understood $O(10^{-3})$ CP violation in the mixing of the $K_s$. (CP violation in $D$ decay to states that contain $K_s$ has been observed in the related decay $D^+ \to K_s\pi^+$ by BELLE [47] and has been shown to be consistent with CP violation only due to the well understood mixing in the neutral kaon.)

This method may be generalized somewhat to the case where $g$ is a three-body decay such as $K_s\pi^+\pi^-$. Here, the decay amplitude is a function of the kinematic variables. In this case we can specify the kinematics by the variables $E_\pm$ being the energies of the $\pi^\pm$ in the rest frame of the $D^0$. The amplitudes $A_g$ and $\bar{A}_g$ are functions of these variables: $A_g(E^+, E^-)$ and $\bar{A}_g(E^+, E^-)$.

If the decay to $g$ is CP-invariant then the relation $\bar{A}_g(E^+, E^-) = A_g(E^-, E^+)$. If we assume that $A_g(E^+, E^-)$ and $\bar{A}_g(E^+, E^-)$ are known from other studies then Eq. (26) can be use to find the phase between $A_f$ and $\bar{A}_f$.

Another potential way to generate correlated neutral $D$-meson pairs is at a $B$ factory through decays such as $B^+ \to D^0\bar{D}^0K^+$ (Br = $2.10 \pm 0.26 \times 10^{-3}$). More generally, it should be possible to adapt this analysis to the decays $B^+ \to D^{*0}\bar{D}^{*0}K^+$ (Br = $4.7 \pm 1.0 \times 10^{-3}$), $B^+ \to D^{*0}\bar{D}^{*0}K^+$ (Br = $5.3 \pm 1.6 \times 10^{-3}$), and $B^+ \to D^0\bar{D}^{*0}K^+$ (Br not yet known), which may increase the statistics by a factor of $\sim 5$.

Observing the Dalitz plot of $B^+ \to D^0\bar{D}^0K^+$ decay and fitting it to a resonance and background will give a model for the phase of the decay amplitude as a function of the Dalitz plot variables.
Let us take the Dalitz plot variables to be $E_D$ and $\bar{E}_D$, the energies of the $D^0$ and $\bar{D}^0$ in the $B^+$ frame respectively, so that the decay amplitude will have the dependence $A(E_D, \bar{E}_D)$. Let $E_f$ be the energy in the $B^+$ frame of state $f$ and $E_g$ be the energy of state $g$. The wave function of the $D$-meson pair is therefore given in terms of $E_f$ and $E_g$ by (note $f \leftrightarrow g$ between the two equations):

$$a(E_f, E_g) = A(E_f, E_g)/\sqrt{|A(E_f, E_g)|^2 + |A(E_g, E_f)|^2}$$

$$\bar{a}(E_f, E_g) = A(E_g, E_f)/\sqrt{|A(E_f, E_g)|^2 + |A(E_g, E_f)|^2}.\quad (27)$$

This is because there is interference between the cases where $D^0 \to f$ with $\bar{D}^0 \to g$ and $\bar{D}^0 \to f$ with $D^0 \to g$. Using Dalitz plot phases in this way is similar to a method used by BaBar to find the phase $\gamma$ in the $B$-meson decay to $D^0 K$ where the $D$ meson subsequently decays to $3\pi$ [48].

5. Isospin decompositions

Since isospin is a very good symmetry of strong interactions, conclusions reached based on isospin alone should hold quite accurately in spite of some theoretical uncertainties due to hadronic interactions. In a recent application of isospin to $D$ decays [15] it is argued that, although generally isospin breaking is $O(1\%)$, the isospin breaking contribution to CP violation should be second order in the isospin breaking parameter.

The SM predicts that there is no CP violation in the $\Delta I = \frac{3}{2}$ channel because the contribution to this channel is only through the tree graph $c \to d\bar{d}u$ while the QCD penguin that has the CP-violating phase is pure $\Delta I = \frac{1}{2}$. In principle, the electroweak penguin could introduce CP violation into the $\Delta I = \frac{3}{2}$ channel but, as discussed below, this amplitude is negligibly small in $D$ decays.

In principle, at higher order in the SM, the electroweak penguin (EWP) graphs could also contribute to CP violation in the $\Delta I = \frac{3}{2}$ channel. We can see, however, that such contributions will be very small, as follows: First, one expects that these will be suppressed compared to the QCD penguin by a factor of $\alpha_W/\alpha_s \sim O(1\%)$. In the analogous case of $B$ physics, the EW penguins are thought to be large in part due to enhancement $\propto m_t$, which is not the case in charm decays. For example, in the decay $B \to K^+\pi^-$, $|\mathcal{A}_{CP}| = 9\%$, so if we assume that all of this CP violation is due to EWP interfering with the tree graph, then we can crudely estimate the corresponding EWP contribution to the asymmetry in $D$ decay as follows:

$$A_{CP}^{EWP}(D \to \pi^+\pi^-) \approx \frac{\text{(EWP}(D)\text{ (Penguin}(B))}{\text{(Tree}(D)\text{ (EWP}(B))} A_{CP}(B \to \pi^+K^-)$$

$$\approx \frac{\text{Penguin}(B)}{\text{Tree}(B)} \frac{|V_{cb}| |V_{ub}| m_b}{|V_{ub}| |V_{us}|} A_{CP}(B \to \pi^+K^-)$$

$$\approx \frac{\text{Penguin}(B)}{\text{Tree}(B)} |V_{ub}| \frac{m_b}{m_t} \sim O(10^{-5}),\quad (28)$$

which suggests an even smaller contribution. There are a number of channels where we can directly test the premise that isospin is a good symmetry in $D$-meson decays to two-body final states. As we discuss below, the relative phases in decays to $\rho\rho$ provide a test of isospin conservation.

Turning now to the isospin decomposition of SCS $D$-meson decays, we proceed in analogy to previous work in the case of $B \to \pi\pi$ and related processes [49,50]. In our expansion we will adopt a notation similar to Ref. [50]. For each particular final state we will denote the isospin amplitude
by $A_{\Delta I}^f I = A_{T: \Delta I}^f I + A_{P: \Delta I}^f I$, which indicates the amplitude for a transition through an effective Hamiltonian with isospin change $\Delta I$ leading to a final state of type $f$ with total isospin $I$. The right hand side indicates the further decomposition of the given amplitude into tree and penguin contributions respectively. Likewise the notation $A_{ij}^f = A_{T:ij}^f + A_{P:ij}^f$ where $i, j \in \{+0\}$ indicates the amplitude for a decay with the indicated charge distribution. The corresponding amplitudes for $\bar{D}$ decay are indicated by $\bar{A}$.

Using this notation, we find for the $\pi \pi$ final state:

$$A^{\pi \pi}_{+0} = \frac{\sqrt{3}}{2} A^{\pi \pi}_{2,2}$$
$$A^{\pi \pi}_{+-} = \frac{A^{\pi \pi}_{7,1}}{\sqrt{6}} + \frac{A^{\pi \pi}_{1,0}}{\sqrt{3}}$$
$$A^{\pi \pi}_{00} = \frac{A^{\pi \pi}_{1,2}}{\sqrt{3}} - \frac{A^{\pi \pi}_{1,0}}{\sqrt{6}}. \tag{29}$$

where the analogous relations also apply for the charge conjugate amplitudes.

This leads to the following “isospin triangle” relationships:

$$\frac{1}{\sqrt{2}} A^{\pi \pi}_{+-} + A^{\pi \pi}_{00} - A^{\pi \pi}_{+0} = 0 = \frac{1}{\sqrt{2}} \bar{A}^{\pi \pi}_{+-} + \bar{A}^{\pi \pi}_{00} - \bar{A}^{\pi \pi}_{-0}. \tag{30}$$

Figure 4 shows a sketch of such a triangle where we use the central values for the branching ratios involved. In the sketch we also show $A^{\pi \pi}_{7,0}$ and $A^{\pi \pi}_{3,0}$.

The case of the $K K$ final state can also be expanded in a similar way. The resulting relations are:

$$A^{KK}_{+0} = -\frac{1}{2} A^{KK}_{3,1} + A^{KK}_{1,1}$$
$$A^{KK}_{+-} = \frac{1}{2} A^{KK}_{7,1} + \frac{1}{2} A^{KK}_{3,1} + \frac{1}{2} A^{KK}_{1,0}$$
$$A^{KK}_{00} = \frac{1}{2} A^{KK}_{1,2} + \frac{1}{2} A^{KK}_{1,1} - \frac{1}{2} A^{KK}_{1,0}. \tag{31}$$

In this case, there are three isospin amplitudes determining three decay amplitudes, so we cannot construct a triangle relation such as Eq. (30).

In the case of the $\pi \pi$ final state, we can see from Fig. 4 that the two isospin amplitudes have a strong phase between them due to rescattering. In particular, this illustrates the failure of color suppression in this system; for color suppression to be realized, the two amplitudes must cancel, thus be collinear in the complex plane, and have magnitudes related by $\sqrt{2} A^{\pi \pi}_{7,2} = A^{\pi \pi}_{1,0}$.

In contrast, for the $K_1 K_s$ case, evidently there is considerable suppression of the branching ratio. This makes sense on the quark level since not only is the decay color suppressed but it is also Zweig suppressed. For this to happen, the isospin 1 tree amplitude and the isospin 0 tree amplitude must cancel fairly well: $\frac{1}{2} A^{KK}_{T:3,1} + \frac{1}{2} A^{KK}_{T:3,1} \approx \frac{1}{2} A^{KK}_{T:1,0}$. Since the dominant tree amplitude is suppressed, this suggests the possibility that $A_{CP}$ in $D^0 \rightarrow K_1 K_s$ could be enhanced compared to the $K^+ K^-$ case at the expense of the total rate.
The $\rho\rho$ final states have the same form as $\pi\pi$ except that there are three polarization states that can arise from the decay of a scalar, $A_\parallel$, $A_\perp$, and $A_\ell$. There will thus be a separate set of isospin amplitudes, so the analog to the above decomposition is:

\begin{align}
A_{\rho\rho}^{\rho\rho(i)+0} &= \frac{\sqrt{3}}{2}A_{3,2}^{\rho\rho(i)} \\
A_{\rho\rho}^{\rho\rho(i)+-} &= \frac{1}{\sqrt{6}}A_{3,2}^{\rho\rho(i)} + \frac{1}{\sqrt{3}}A_{1,0}^{\rho\rho(i)} \\
A_{\rho\rho}^{\rho\rho(i)00} &= \frac{1}{\sqrt{3}}A_{3,2}^{\rho\rho(i)} - \frac{1}{\sqrt{6}}A_{1,0}^{\rho\rho(i)},
\end{align}

(32)

where $i \in \{\perp, \parallel, \ell\}$ indexes the polarization state.

For each polarization state we therefore also have an isospin triangle relation:

\begin{equation}
\frac{1}{\sqrt{2}}A_{\rho\rho}^{\rho\rho(i)+-} + A_{\rho\rho}^{\rho\rho(i)00} - A_{\rho\rho}^{\rho\rho(i)+0} = 0 = \frac{1}{\sqrt{2}}\bar{A}_{\rho\rho}^{\rho\rho(i)+-} + \bar{A}_{\rho\rho}^{\rho\rho(i)00} - \bar{A}_{\rho\rho}^{\rho\rho(i)+0}.
\end{equation}

(33)

For each $\rho\rho$ final state, an angular analysis [39] provides the magnitude of the three polarization amplitudes and the cosine of the phase angle between them. Thus (up to a twofold ambiguity) the relative phases between these three amplitudes can be determined. The relation (Eq. (33)) gives the phase between the three amplitudes with the same charge distribution in terms of their magnitudes (again up to a twofold ambiguity). Combining the two kinds of information, we have 18 phase differences for 9 amplitudes in $D$-decays (taking into account an overall phase, the system is over-determined by 8 degrees of freedom), which checks the validity of Eq. (33). Since this relation was derived using isospin conservation, the validity of this symmetry is thus quantified.

For the $\rho\pi$ final state, the decomposition is:

\begin{align}
A_{\rho\pi}^{\rho\pi+0} &= \frac{\sqrt{3}}{\sqrt{8}}A_{3,2}^{\rho\pi} - \frac{1}{\sqrt{8}}A_{3,1}^{\rho\pi} + \frac{1}{\sqrt{2}}A_{1,1}^{\rho\pi} \\
A_{\rho\pi}^{\rho\pi0+} &= \frac{\sqrt{3}}{\sqrt{8}}A_{3,2}^{\rho\pi} + \frac{1}{\sqrt{8}}A_{3,1}^{\rho\pi} - \frac{1}{\sqrt{2}}A_{1,1}^{\rho\pi}
\end{align}
\[
    A_{-+}^{\rho\pi} = \frac{1}{\sqrt{12}} A_{3/2,2}^{\rho\pi} + \frac{1}{2} A_{3/1,1}^{\rho\pi} + \frac{1}{\sqrt{6}} A_{2,0}^{\rho\pi},
\]
\[
    A_{++}^{\rho\pi} = \frac{1}{\sqrt{12}} A_{3/2,2}^{\rho\pi} - \frac{1}{2} A_{3/1,1}^{\rho\pi} - \frac{1}{\sqrt{6}} A_{2,0}^{\rho\pi},
\]
\[
    A_{00}^{\rho\pi} = \frac{1}{\sqrt{3}} A_{3/2,2}^{\rho\pi} - \frac{1}{\sqrt{6}} A_{1/2,0}^{\rho\pi},
\]

which in turn leads to the following pentagonal isospin relationships:
\[
    \sqrt{3} A_{3/2}^{\rho\pi} = \sqrt{2} (A_{+0}^{\rho\pi} + A_{0+}^{\rho\pi}) = A_{++}^{\rho\pi} + A_{-+}^{\rho\pi} + 2A_{00}^{\rho\pi},
\]
\[
    \sqrt{3} A_{1/2}^{\rho\pi} = \sqrt{2} (A_{-0}^{\rho\pi} + A_{0-}^{\rho\pi}) = \tilde{A}_{++}^{\rho\pi} + \tilde{A}_{-+}^{\rho\pi} + 2\tilde{A}_{00}^{\rho\pi}.
\]

Also, the \(\Delta I = 3/2\) contribution to the \(I = 1\) final state follows from the relation:
\[
    3 A_{3/1}^{\rho\pi} = \sqrt{2} (A_{0+}^{\rho\pi} - A_{+0}^{\rho\pi}) + 2 (A_{+-}^{\rho\pi} - A_{-+}^{\rho\pi}),
\]
\[
    3 \tilde{A}_{3/1}^{\rho\pi} = \sqrt{2} (\tilde{A}_{0+}^{\rho\pi} - \tilde{A}_{+0}^{\rho\pi}) + 2 (\tilde{A}_{+-}^{\rho\pi} - \tilde{A}_{-+}^{\rho\pi}).
\]

In the case of the decay \(D_s \to \pi K^*\), the isospin decomposition of the amplitudes is:
\[
    A_{++}^{\pi K^*} = \frac{1}{\sqrt{3}} A_{3/2}^{\pi K^*} + \sqrt{\frac{2}{3}} A_{1/2}^{\pi K^*},
\]
\[
    A_{00}^{\pi K^*} = \frac{\sqrt{2}}{\sqrt{3}} A_{3/2}^{\pi K^*} - \frac{1}{\sqrt{3}} A_{1/2}^{\pi K^*}.
\]

Thus,
\[
    \sqrt{3} A_{3/2}^{\pi K^*} = A_{++}^{\pi K^*} + \sqrt{2} A_{00}^{\pi K^*}.
\]

In this case, the two decay amplitudes depend on two isospin amplitudes so there is no isospin triangle relation as in the case of \(\pi\pi\) and \(\rho\pi\).

5.1. Phases in Dalitz plots

In the two-body decays \(D \to \rho\pi\) and \(D_s \to K^*\pi\), the vectors decay in turn to two pseudoscalars, \(\rho \to \pi\pi\) and \(K^* \to K\pi\). The final states are therefore three-body Dalitz decays [51,52]. The same three scalar final state will, in general, receive contributions from a number of different pseudo two-body channels. For example, in the case of \(D_s \to K^*\pi\), the two charge distributions will contribute to the same three-body final state, in particular \(D_s \to K^{*+}\pi^0 \to K^0\pi^+\pi^0\) and \(D_s \to K^{*0}\pi^+ \to K^0\pi^0\pi^+\). Thus the \(K^0\pi^0\pi^+\) final state receives contributions from both the \(K^{*0}\pi^+\) and \(K^{*+}\pi^0\) channels. A fit to the the distribution in the Dalitz plot variables will therefore determine both the magnitudes of the two-body amplitudes and also the relative phase between them, as well as other channels that contribute to this final state, such as \(K^0\rho^+\). Note that the other decay of the \(K^*\) in the above will not involve interference between these two channels, in particular \(D_s \to K^{*+}\pi^0 \to K^+\pi^0\pi^0\) and \(D_s \to K^{*0}\pi^+ \to K^+\pi^0\pi^+\).

The same situation also applies to \(D^0 \to \rho\pi\), which leads to the final state \(\pi^+\pi^-\pi^0\). In this case the pseudo two-body channels \(\rho^0\pi^0\), \(\rho^+\pi^-\), and \(\rho^-\pi^+\) all contribute, so in fitting the Dalitz plot one obtains the magnitude and relative phases of each of these channels.
5.2. Standard Model tests using isospin

The main test of the SM origin for CP violation in hadronic $D$ decays that can be accomplished using isospin analysis is to test the SM prediction that the tree graph, which is the only contribution to the $\Delta I = 3/2$ Hamiltonian, has no phase in the Wolfenstein phase convention. Thus, assuming EWP are negligible, any CP violation in phase or magnitude is contained in the $\Delta I = 1/2$ component, which receives contributions both from the tree and the penguin.

In each system of decays there are therefore two kinds of tests that, in principle, can be performed.

1. The magnitude of the $\Delta I = 3/2$ transition amplitude is the same for the decay and its charge conjugate.
2. The phase of the $\Delta I = 3/2$ transition amplitude is the same for the decay and its charge conjugate.

In the $\pi\pi$ final state, the system is sufficiently simple to allow us to cleanly extract three isospin-related CP asymmetries. To fully characterize CP violation in this system, a fourth quantity must be determined by a phase measurement of the type described in Sect. 4.

Let us denote by $\delta_{ij}$ the partial rate difference for final state $f$ with charge distribution $ij$, i.e. $\delta_{ij} = \left| A^X_f \right|^2 - \left| \bar{A}^X_f \right|^2$. Likewise, for the isospin amplitudes denote:

$$
\delta_{\Delta I I} = \left| A^X_f \right|^2 - \left| \bar{A}^X_f \right|^2,
$$

$$
\delta_{[\Delta I I \Delta I J]} = \text{Re} \left( A^f_{\Delta I I} A^{f*}_{\Delta I J} - \bar{A}^f_{\Delta I I} \bar{A}^{f*}_{\Delta I J} \right). \quad (38)
$$

Using this notation, Eq. (28) implies that the CP violation in the $D \to \pi\pi$ decays can be rewritten as:

$$
\delta_{\pi\pi} = \frac{3}{4} \delta_{\pi\pi},
$$

$$
\delta_{\pi\pi} = \frac{1}{6} \delta_{\pi\pi} + \frac{1}{2} \delta_{\pi\pi} - \frac{1}{3} \sqrt{2} \delta_{\pi\pi},
$$

$$
\delta_{\pi\pi} = \frac{1}{3} \delta_{\pi\pi} + \frac{1}{6} \delta_{\pi\pi} - \frac{1}{3} \sqrt{2} \delta_{\pi\pi}.
$$

Since this gives each of the observed partial rate differences in terms of three different CP-violating underlying isospin quantities, we can invert these and obtain:

$$
\frac{4}{3} \delta_{\pi\pi} = \frac{4}{3} \delta_{\pi\pi},
$$

$$
\frac{1}{2} \delta_{\pi\pi} = \frac{1}{2} \delta_{\pi\pi} - \frac{1}{2} \delta_{\pi\pi}.
$$

(40)

As discussed in Ref. [15], the first expression for $\delta_{\pi\pi}$ implies that $\delta_{\pi\pi} = 0$ is a test of type (1) due to the evident fact that the decay to $\pi^+\pi^0$ is governed only by the $\Delta I = 3/2$ Hamiltonian.

The other two combinations indicate different features of CP violation in the $\Delta I = 1/2$ channel, which could be entirely due to SM physics. As discussed in Ref. [15], for $\delta_{\pi\pi}$ to be non-zero requires that there are two contributions to this isospin channel that have different strong phases and also different weak phases. This would generally be expected to be the case in the SM since both tree
and penguin contribute to $\Delta I = 1/2$. It could, however, happen that $\delta^{\pi\pi}_{22}^{3}$ is small due to the strong phase difference between the two contributions to $\Delta I = 1/2$ being small, but in this case the quantity $\delta^{\pi\pi}_{22;\,10}$ could be non-zero due to strong and weak phase differences between the two different isospin channels.

To make this clear, consider, e.g., what happens if $\delta^{\pi\pi}_{22} = \delta^{\pi\pi}_{10} = 0$ but $\delta^{\pi\pi}_{22;\,10} \neq 0$. This would imply first that $|A^{\pi\pi}_{22}| = |A^{\pi\pi}_{12}|$ and $|A^{\pi\pi}_{10}| = |A^{\pi\pi}_{20}|$ but that the phase between $A^{\pi\pi}_{22}$ and $A^{\pi\pi}_{10}$ is different from the phase between $A^{\pi\pi}_{12}$ and $A^{\pi\pi}_{20}$, resulting from a different weak phase between the two isospin channels.

The measurement of the phase difference between either of the neutral amplitudes and their charge conjugates, i.e. $A^{\pi\pi}_{+\,+}$ versus $\bar{A}^{\pi\pi}_{+\,+}$ or $A^{\pi\pi}_{0\,0}$ versus $\bar{A}^{\pi\pi}_{0\,0}$, using the methods in Sect. 4 allows the complete determination of all the amplitudes and therefore all the CP violation in this system. This follows from relation Eq. (30), which implies that the three amplitudes form a triangle in the complex plane. The phase between $A^{\pi\pi}_{+\,0}$ and either of the neutral modes is therefore determined (up to a twofold ambiguity) and so the phase of $A^{\pi\pi}_{+\,0}$ is known. The same is also true for the charge conjugate amplitudes, so ultimately the weak phase between $A^{\pi\pi}_{+\,0}$ and $\bar{A}^{\pi\pi}_{+\,0}$ is determined (up to a fourfold ambiguity). This then is a test of the SM of type (2).

Note that if the phase difference with the conjugates is measured for both $\pi^+\pi^-$ and $\pi^0\pi^0$ final states, then there is a consistency check for the isospin relations because the isospin triangle for $D^0 \rightarrow \rho\rho$ decay fixes the phase between $A^{\pi\pi}_{+\,+}$ and $A^{\pi\pi}_{0\,0}$ while the the isospin triangle for $\bar{D}^0 \rightarrow \bar{\rho}\rho$ decay fixes the phase between $\bar{A}^{\pi\pi}_{+\,+}$ and $\bar{A}^{\pi\pi}_{0\,0}$. In addition, having both phase measurements will resolve the fourfold ambiguity with respect to the orientation of the isospin triangles. Of course, measuring the weak phase directly with the $\pi^0\pi^0$ final state using the methods described above will likely be experimentally difficult.

The same discussion also applies to each polarization of the final state. the $\rho\rho$ final state. Because all of the relative phases of the 9 $D \rightarrow \rho\rho$ amplitudes can be measured as discussed above (and likewise for the $\bar{D} \rightarrow \rho\rho$ amplitudes), if one weak phase measurement is made then all of the weak phase differences are known. In principle, there are six possible weak phase differences (2 modes $\times$ 3 polarizations) that can be measured in $D^0$ decay to $\rho^0\rho^0$ or $\rho^+\rho^-$ so there are multiple checks on this kind of measurement.

For the $\rho\rho$ final state, then, we have 3 type (1) tests of the SM by comparing the magnitude of each of the $\rho^+\rho^0$ amplitudes with their conjugates. There are two other independent tests that can be made by comparing the phase differences between the amplitudes with the phase differences of their conjugates. Finally, with an absolute weak phase determination and the isospin relationships Eq. (32), we can have 3 type (2) tests for the weak phase difference between each of the $\rho^+\rho^0$ polarizations and their conjugates.

In the case of $D \rightarrow \rho\pi$, the phase between the three $D^0 \rightarrow \rho\pi$ amplitudes can be determined from analysis of the Dalitz plot distributions of $D^0 \rightarrow \pi^+\pi^-\pi^0$. These amplitudes are therefore part of a more general isospin analysis of $D \rightarrow 3\pi$, as considered in Ref. [53]. Using the relationship Eq. (35), we can see that $A^{\rho\pi}_{\mp\,0}$ is determined as a linear combination of these three amplitudes and so is determined up to an overall weak phase. Likewise we can extract the charge conjugate so the SM can be tested by comparison of $|A^{\rho\pi}_{\mp\,0}|$ with $|\bar{A}^{\rho\pi}_{\mp\,0}|$.

Furthermore, the relation Eq. (35) gives $A^{\rho\pi}_{\mp\,0}$ as a linear combination of the related charged $D$-meson decays $A^{\rho\pi}_{+\,0}$ and $A^{\rho\pi}_{0\,+}$ so that the phase of these two decays relative to the neutral decays can

21/25
be determined up to a twofold ambiguity. We can thus use Eq. (36) to find the magnitude of $|A_{3/2}^0|$. Likewise we can extract the charge conjugate of the same amplitude and so the SM can be tested by comparison of $|A_{3/2}^0|$ with $|\bar{A}_{3/2}^0|$. Thus we have two tests of type (1) in this system.

We can generate the corresponding type (2) tests for both of the $\Delta I = 3/2$ amplitudes in $D \to \rho\pi$, if we know the weak phase difference between at least one of the neutral modes and its conjugate using the methods of Sect. 4. Since the relative phases between all the $D$ decays are determined by the construction above, the weak phase difference will then be determined. The weak phases of the other two neutral cases (all of which are found in the same Dalitz plot) would then provide consistency checks.

In the case of $D_s \to K^*\pi$, there is just an SM check of type (1). In this case, the phase between the two amplitudes $A_{3/0}^{\pi K^0}$ and $A_{0+}^{\pi K^+}$ can be determined from the $K^+\pi^-\pi^0$ Dalitz plot. Thus, using Eq. (37), we obtain the magnitude of $|A_{3/2}^{\pi K^+}|$. Again, we can test the SM through verifying $|A_{3/2}^{\pi K^+}| = |\bar{A}_{3/2}^{\pi K^+}|$. Unlike the above cases, there is no way to determine the weak phase of this amplitude because there is no neutral decay related by isospin.

6. General requirements for testing CP violation in SCS decays of $D$-mesons

In order to form a rough estimate of the requirements to find CP violation and test the SM through the modes above, let us assume that the CP violation in these SCS modes is generally at the same level as seen in the SCS modes (e.g. $\pi\pi$ and $KK$), on the order of 0.1–1%. In terms of raw statistics, a sample of $10^5$–$10^7$ would be required. Since the branching ratios of these modes is typically $10^{-3}$, this would mean that $10^8$–$10^{10}$ $D$-mesons would be required; and probably an order of magnitude more depending on the acceptance for various decay modes. Indeed this is roughly true in the LHCb results [1]; based on an integrated luminosity of $0.62 \text{ fb}^{-1}$, the yield of $K^+K^-$ was $1.44 \times 10^6$ and the yield of $\pi^+\pi^-$ was $0.38 \times 10^6$. These results point out important challenges that must be overcome to carry out such studies at LHCb and more generally at other facilities.

At the LHCb it is, of course, crucial to overcome the fact that the initial state is not charge conjugate. This, of course, is less of a problem at $e^+e^-$ colliders such as $B$ factories or tau-charm factories. In any case, aside from the requirement of raw statistics, it is necessary to identify and tag the initial $D$-meson and find the various final states. To this end, as discussed in Sect. 3, final states with all charged final state particles (i.e. $\pi^\pm$ and $K^\pm$) will be easier to detect.

Determining the phase through any of the methods discussed in Sect. 4 may require statistics somewhat beyond currently planned facilities. First consider using straight $D$ oscillation with Eq. (20). Obviously the first requirement is the ability to track the time dependence of the decay to a precision $\ll 1/\Gamma_D$. The relevant term in the time dependence that we need to extract is the term $\propto \tau$. This term, of course, is multiplied by the relative rate of mixing $|z| \sim 10^{-2}$. Furthermore, if the weak phase is similar to the observed level of CP violation in magnitude for $\pi^+\pi^-/K^+K^-$ then we would expect $\text{arg}(A\bar{A}^*) \sim 1%$. If this is indeed the case, you would need $10^9$ final states in order to see the decay and, since the branching ratio is $10^{-3}$, you would therefore need $\sim 10^{12}$ $D$-mesons to start with.

Using the double oscillation method, e.g., Eq. (24) in the $B_d$ case where $y$ is small and $p/q = e^{2i\beta}$, the relevant term would be the one proportional to $S_y^B$. If there was no weak phase then this would be proportional to the same $\sin 2\beta$ as $B \to \psi K_s$. In effect, then, we would be looking for a deviation from the SM value of this coefficient by $O(1\%)$, so we would expect to need $\sim 10^5$ decays to perform the measurement. In the case of the $\pi\pi$ final state, the combined branching ratio would
be $4.2 \times 10^{-7}$ for the channel through either $D^0\pi^0$ or $D^0\rho^0$. This gives an initial requirement for the number of $B$-mesons to be $\sim 2 \times 10^{11}$. By combining a number of modes (e.g. $B^0 \to \bar{D}^0\pi$, $B^0 \to \bar{D}^0\rho$, $B^0 \to \bar{D}^0\rho\pi$ etc.) it may be possible to reduce this to the $10^{10}$–$10^{11}$ range.

Using the correlation method, if we take the decay $B^+ \to K^+D^0\bar{D}^0$ and use the index decay $g = K_\pi\pi^0$ with the final state $f = \pi^+\pi^-$, and assuming that we need to observe $10^5$ events, then, not including acceptance, the number of $B$ mesons needed is $3 \times 10^{12}$. If we broaden the method to include $B^+ \to K^+D^{*0}\bar{D}^{*0}$ (assuming a total Br=1%) and use as an index state $g = K_\pi\pi^+$ and a target state $f = \rho^0\rho^0$, the number is reduced to $1.5 \times 10^{11}$.

Using correlations at a $\psi$ factor, the number of $DD$ pairs required using the above assumptions with index state $g = K_\pi\pi^+$ and a target state $f = \rho^0\rho^0$ is $5 \times 10^9$.

It seems then that each of these methods requires an input of $\sim 10^{11}$ mesons if the phase is of the same order of magnitude as $A_{CP}$ for $\pi\pi$ and $KK$. This is probably beyond the capability of machines in the foreseeable future. If, however, the CP-violating phase is an order of magnitude larger than $A_{CP}$ (i.e. because the strong phase was $O(10\%)$), then these requirements would be reduced by 2 orders of magnitude and perhaps such experiments could become possible at super $B$ factories or the LHCb upgrades.

Perhaps the cleanest environment to measure such phases would be at high luminosity charm factories where $\sim 10^{10}$ meson pairs would be needed if the phase is $O(1\%)$. Again, if the phase were $10\%$, this would be reduced by two orders of magnitude.

7. Summary and conclusion

$D^0$ mixing is unique as it is the only charge 2/3 bound system, providing us with a great opportunity to search for new physics. In many interesting BSM scenarios, enhanced mixing and also enhanced CP asymmetries are expected; warped extra dimension models are a well known example. The recent discovery of direct CP violation in $D^0$-decays by the LHCb collaboration gives a huge impetus to these searches. The observed CP asymmetry of $O(0.5\%)$ is somewhat bigger than some estimates, though it seems that an SM explanation is quite plausible. Hadronic uncertainties make precise predictions exceedingly difficult, therefore, for now, the possible role of new physics cannot be ruled out. More experimental information may well be pivotal in this instance. This is the basic rationale behind this work, leading us to make several suggestions.

We suggest that the observed enhanced effects due to non-perturbative physics may be most pronounced for the exclusive two pseudoscalar modes only, e.g., $\pi\pi$ and $K\bar{K}$. For multiparticle (inclusive) final states, the quark level CP asymmetry of about $6 \times 10^{-4}$ may be relevant. A simple way to implement this experimentally may be to look for, say, decays of $D$ to final states with a $K$ and a $\bar{K}$ where the sum of their energies is less than the energy of the parent $D$. If these inclusive final states also show enhanced CP asymmetries (say, at the level seen in exclusive $K^+K^-$, $\pi^+\pi^-$), then that would mean that it has a new physics origin; otherwise, it will give support to an SM explanation.

Since the tree contribution is likely suppressed in color-suppressed final states, it is likely that CP asymmetries will be enhanced therein. To facilitate experimental detection, final states leading to charged $\pi$ s may be best to focus on. These twin considerations lead us to suggest $D^0 \to \rho^0\rho^0$, $D_s \to \rho K^+$, and $D_s \to \rho^0 K^{*+}$ as especially interesting. The vector–vector final states have the additional bonus that angular correlations can also be used for additional CP-violating observables.

The importance of CPT constraints on CP-violating observables is emphasized and illustrated with regard to exclusive, inclusive, and radiative modes.
While $SU(3)$ and $U$-spin symmetries seem quite badly broken in $D$ decays, isospin likely holds quite well, motivating us to investigate its use, especially in decays such as $D \rightarrow \pi \pi, \rho \pi$, and $\rho \rho$, as well as for $D_s \rightarrow K^* \pi$.

We have also studied how such analysis may be augmented by information about the weak phase in $D^0$ decays. To do this, it is necessary to study a sample of $D$-mesons that are in a mixed state of $D^0$ and $\bar{D}^0$. Such a state may result from $D^0 \bar{D}^0$ oscillation or from the decay of a $B$ or $B_s$ meson that itself is in a mixed state due to its oscillation. Alternatively, if a $D^0 \bar{D}^0$ pair is in an entangled state, the observation of the decay of one neutral $D$-meson implies that the other $D$-meson is in a mixed state. Such entangled pairs may be produced in charm factories through the decay of $\psi''$ or as the result of $B$-meson decays.

**Note added I**

Since the role of hadronic matrix elements of penguin operators in charm CP has been the subject of considerable discussions and speculation, we take this opportunity to briefly draw attention to recent work on the lattice (Ref. [54]) by the RBC and UKQCD collaborations, on the origin of the large enhancement in $K \rightarrow \pi \pi$ decays in the $I = 0$ final state relative to $I = 2$, which often goes under the name of the “$\Delta I = 1/2$ puzzle”. That work finds that, at a renormalization scale of around 1.7 GeV or more, the entire enhancement originates from non-perturbative matrix elements of simple tree operators (i.e. $Q_1$ and $Q_2$) and the contribution of the penguin operators is quite negligible.

**Note added II**

We want to take the opportunity to briefly mention two new experimental results from LHCb. In LHCb-CONF-2013-003, they give a new preliminary result using the soft pion technique as in Ref. [1], but now with 1.0 fb$^{-1}$ of data; they find $\Delta A_{CP} = (-0.34 \pm 0.15 \pm 0.10)\%$. On the other hand, using $B$-semileptonic tags on the same amount of data, they report [55] $\Delta A_{CP} = (+0.49 \pm 0.30 \pm 0.14)\%$, yielding a new world average, $\Delta A_{CP} = (-0.33 \pm 0.12)\%$ [56].

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**References**

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