Orientation dependence of the Andreev transport in d-wave superconductor–ferromagnet–d-wave superconductor trilayers

Zorica Popović\textsuperscript{1,}\textsuperscript{*}, Radomir Zikic\textsuperscript{2}, and Ljiljana Dobrosavljević-Grujić\textsuperscript{2}

\textsuperscript{1}University of Belgrade, Faculty of Physics, Studentski trg 12, 11001 Belgrade, Serbia
\textsuperscript{2}University of Belgrade, Institute of Physics, Pregrevica 118, 11080 Belgrade, Serbia
\textsuperscript{*}E-mail: pzorica@ff.bg.ac.rs

Received March 27, 2015; Revised August 7, 2015; Accepted August 17, 2015; Published October 1, 2015

The quasiparticle transport through ballistic voltage-biased d-wave superconductor–ferromagnet–d-wave superconductor (DFD) junctions with misoriented superconducting electrodes is studied theoretically. We found that at low bias the coherence in the quasiparticle transport could be enhanced by the magnetic field in F. This new feature originates from the interplay between the phase-coherent propagation of Andreev pairs in the barrier and decoherence mechanisms due to the electrode misorientation $\theta$, the presence of the exchange field $h$ in F, the bias voltage, and the temperature. For $\theta = \pi/4$, where the proximity effect is strongest, there is a distinct enhancement of the current $I(h)$, starting from $h = 0$ up to a maximum at $h \approx \Delta(T)$. For other misorientations, a similar increase of the current appears only for thin F barriers, or for large enough $\theta$, where the proximity effect is also strong.

Subject Index I60, I63, I64

1. Introduction

One of the remarkable differences between conventional and unconventional superconductors is the anisotropy of the pair potential. In d-wave superconductors, the quasiparticles experience different pair potentials depending on the direction of motion, and, at the interface, interference effects of quasiparticles, which are absent in the s-wave superconductors, are expected. Due to Andreev reflection (AR)\textsuperscript{[1]} the quasiparticle will retrace its path and form a closed loop, leading to a bound state when the order parameter before and after scattering differs in sign\textsuperscript{[2]}. Hu’s predictions of the formation of midgap states at the surface of d-wave superconductors called for reinvestigation of a number of transport problems involving d-wave superconductors. This reinvestigation has to some extent been carried out, starting with the normal metal–superconductor junction\textsuperscript{[3–5]}, followed by the dc Josephson effect\textsuperscript{[6–9]}, the ac Josephson effect\textsuperscript{[10–14]}, and tunneling effects in unconventional superconductors\textsuperscript{[15]}. In parallel, there has also been much interest in the theoretical studies of the effects of spin-polarized transport on the current–voltage characteristic and conductance in ferromagnet–unconventional superconductor junctions\textsuperscript{[16–18]}. Recent progress in the nanofabrication of hybrid ferromagnet (F)–d-wave superconductor (D) structures at relatively high temperatures, as well as the possibility of making high-transmissivity F–D interfaces, has renewed interest in studying the interplay between superconductivity and ferromagnetism\textsuperscript{[19–24]}. In voltage-biased DFD junctions, the non-equilibrium transport properties are...
governed by Andreev reflection. The introduction of superconductivity in the ferromagnet may be attributed to the coherent coupling of electrons and holes in F, whereas the suppression of the superconductivity near the F–D interface in D is due to the pair-breaking effect coming from the injection of spin-polarized electrons from F. Both processes are determined by AR.

As a consequence of the pair-potential anisotropy for d-wave symmetry, the transport properties of a DFD junction depend on the angle of orientation of the d-wave electrodes with respect to the F–D interface. Until now, most experimental and theoretical investigations have been done for F–D junctions with (100) or (110) d-wave superconductor orientation [16,22,25–29]. In the present work, we consider DFD trilayers with transparent interfaces, and study the orientational effects assuming (100) orientation of the left electrode, whereas the right electrode may have an arbitrary orientation $\theta$.

As expected, the resulting orientation dependence gives rise to a number of interesting phenomena, and this is the main topic of our study.

We calculate in the clean limit the current–voltage characteristic (CVC, i.e. $I(V)$), the corresponding conductance $G(V) = dI/dV$, and the dependence of the current $I$ on the exchange energy $h$ in the barrier. The calculation is based on the non-equilibrium microscopic theory of transport in isotropic s-wave superconductor–normal metal (SNS) junctions by Kümmel, Gunsenheimer, and Nikolsky (KGN) [30] in the relaxation-time approximation using the time-dependent Bogoliubov–de Gennes equations (BdGEs) [30–32].

We find that $I(V)$ changes with increasing misorientation, diminishing in amplitude with increasing $\theta$. Also, due to the importance of AR processes, the magnitude of the proximity effect in the d-wave case depends to a great extent on the electrode misorientation (see below) [33]. This is reflected in the quasiparticle current dependence on $h$. At low bias, the interplay between the phase-coherent propagation of Andreev pairs in F and various decoherence mechanisms may lead to the increase of the current with $h$ (up to $h \approx \Delta$). Outside of the low-voltage region where multiple Andreev reflection (MAR) takes place, at high voltages we find a few isolated dips in the conductance dependence on the bias voltage, the locations of which are independent of $\theta$, and for a given temperature depend only on $h$. Observation of such dips would provide the possibility of extracting the value of $h$ from experiments.

In Sect. 2, we discuss the model describing the quasiparticle dynamics. The results are presented in Sect. 3, and Sect. 4 is devoted to the conclusion.

2. Model and formalism

The geometry where two long d-wave superconductor banks are separated by a ferromagnetic layer of thickness $d = 2a$ is considered. In F, we use the Stoner model with an exchange energy shift $2h$ between two sub-bands and, in D, an anisotropic BCS model of superconductivity. For simplicity, we assume that the Fermi velocities are equal in F and D. The barrier is assumed perpendicular to the $\hat{a}$ axis in the $\hat{a}$–$\hat{b}$ plane of the left-hand superconducting electrode $D_L$, which is misoriented by the angle $\theta$ with respect to the right-hand one $D_R$. The pair potential in $D_L$ is $\Delta_L(\varphi) = \Delta_0(T) \cos 2\varphi$, where $\varphi$ is the angle that the quasiparticle momentum makes with the $\hat{a}$ axis, and $\Delta(T) = \Delta \tanh (1.74\sqrt{T_c/T - 1})$ [34]. In the d-wave case, the pair potential changes its sign on the Fermi surface, the transmitted electron-like quasiparticle (EQL) and the hole-like quasiparticle (HQL) feel different signs of the pair potential in the $\hat{a}$–$\hat{b}$ plane of the junction, and the effective pair potentials in $D_R$ for ELQ and HLQ are $\Delta_R^+(\varphi, \theta) = \Delta(T) \cos 2(\varphi - \theta)$ and $\Delta_R^-(\varphi, \theta) = \Delta(T) \cos 2(\varphi + \theta)$, respectively. We take a step-function variation of the pair potential
along the z axis perpendicular to the barrier $\Delta(z) = \Delta_L \Theta(-a-z) + \Delta_R \Theta(z-a)$, where $\Theta(z)$ stands for the Heaviside step function.

Approximate solutions of the time-dependent BdGEs are given by four-component spinor wave functions, $\Psi(r, t) = [u_\uparrow(r, t), u_\downarrow(r, t), v_\uparrow(r, t), v_\downarrow(r, t)]^T$, in both the ferromagnet layer and superconducting banks. Since the parallel component of the wave vector, $k_\parallel = k_x \mathbf{e}_x + k_y \mathbf{e}_y$, is conserved due to translational invariance of the junction in directions perpendicular to the z axis, we can write $\Psi(r, t) = \psi(z, t) e^{i k_\parallel r}$, where $\psi(z, t) = [u_\uparrow(z, t), u_\downarrow(z, t), v_\uparrow(z, t), v_\downarrow(z, t)]^T$. Electrons and holes with momenta in the same direction are coupled together by Andreev reflection, and are decoupled from electrons and holes with opposite momenta. As a consequence, the wave function in the F layer (as well as in the D layers) splits into two independent solutions, which refer to positive (+) and negative (−) z momenta. In these solutions the quasiparticle wave packets $u_{k\sigma}^\pm$ and $v_{k\sigma}^\pm$, which move in the electric field due to the applied voltage $V$, are $u_{k\sigma}^\pm(z, t) = \Sigma_{n=-\infty}^{+\infty} u_{n\sigma}^\pm(z, t, k)$ and $v_{k\sigma}^\pm(z, t) = \Sigma_{n=-\infty}^{+\infty} v_{n\sigma}^\pm(z, t, k), \sigma = \uparrow (\downarrow)$ being the spin projection. Here, $u_{n\sigma}^\pm$ represents the wave packet of the quasiparticle that is an electron during the time interval between the 2nth and 2n + 1st Andreev reflection, then becomes a hole described by the wave packets $v_{n\sigma}^\pm$ until the next Andreev reflection, which produces an electron again, and the energy of the quasiparticle is steadily increasing (+) or decreasing (−) [30–32].

To calculate the current density from these solutions in the barrier, we use the relaxation-time model [30], so that the expression for the average current density is

$$
\langle j \rangle = -\frac{e}{4m} \sum_k \left\{ f_0(E_k + h) \left[ \langle u_{k\sigma}^+ \mathbf{P} u_{k\sigma}^- \rangle + \langle u_{k\sigma}^- \mathbf{P} u_{k\sigma}^+ \rangle \right] + f_0(E_k - h) \left[ \langle u_{k\sigma}^+ \mathbf{P} u_{k\sigma}^- \rangle + \langle u_{k\sigma}^- \mathbf{P} u_{k\sigma}^+ \rangle \right] \right. 
+ (1 - f_0(E_k + h)) \left[ \langle v_{k\sigma}^+ \mathbf{P} v_{k\sigma}^- \rangle + \langle v_{k\sigma}^- \mathbf{P} v_{k\sigma}^+ \rangle \right] + (1 - f_0(E_k - h)) \left[ \langle v_{k\sigma}^+ \mathbf{P} v_{k\sigma}^- \rangle + \langle v_{k\sigma}^- \mathbf{P} v_{k\sigma}^+ \rangle \right] \right\},
$$

where $f_0$ is the Fermi distribution function and $\mathbf{P} = [-i \mathbf{\nabla} + e \mathbf{A}/c]$ is the gauge-invariant momentum operator, while $\mathbf{A}$ is the time-dependent vector potential $\mathbf{A} = e_\perp V t / (2a) \Theta(a - |z|)$ [30], which is defined via an externally applied bias voltage $V$ that exists across the ferromagnetic layer only. After spatial and time integration, the averaged momentum densities are angular dependent and proportional to the corresponding angular-dependent probability amplitudes

$$
\langle u_{k\sigma}^+ \mathbf{P} u_{k\sigma}^- \rangle \propto \prod_{r=1}^{n} \left| \gamma_R^+(E_k + \rho_\sigma h + (2r - 1)eV) \right|^2 \prod_{r=1}^{n} \left| \gamma_L(E_k + \rho_\sigma h + 2reV) \right|^2,
$$

$$
\langle u_{k\sigma}^- \mathbf{P} u_{k\sigma}^+ \rangle \propto \prod_{r=1}^{n} \left| \gamma_R^-(E_k + \rho_\sigma h - 2reV) \right|^2 \prod_{r=1}^{n} \left| \gamma_L(E_k + \rho_\sigma h - (2r - 1)eV) \right|^2,
$$

$$
\langle v_{k\sigma}^+ \mathbf{P} v_{k\sigma}^- \rangle \propto \prod_{r=1}^{n} \left| \gamma_R^+(E_k - \rho_\sigma h + 2reV) \right|^2 \prod_{r=1}^{n} \left| \gamma_L(E_k - \rho_\sigma h + (2r - 1)eV) \right|^2,
$$

$$
\langle v_{k\sigma}^- \mathbf{P} v_{k\sigma}^+ \rangle \propto \prod_{r=1}^{n+1} \left| \gamma_R^-(E_k - \rho_\sigma h - (2r - 1)eV) \right|^2 \prod_{r=1}^{n} \left| \gamma_L(E_k - \rho_\sigma h - 2reV) \right|^2,
$$

where $\rho_\sigma = +1(-1)$ is related to the spin projection $\sigma = \uparrow (\downarrow)$. In contrast to the d-wave case with $\theta = 0$ [32], the important difference is that in the present case these amplitudes depend on the
misorientation angle $\theta$ as well, since

$$\gamma_L(\varphi) = \frac{E - i\left(\Delta_L(\varphi)^2 - E^2\right)^{1/2}}{\Delta_L(\varphi)} \quad \text{for} \quad E < \Delta_L(\varphi),$$

$$\gamma_L(\varphi) = \frac{E - \left(\Delta_L(\varphi)^2 - E^2\right)^{1/2}}{\Delta_L(\varphi)} \quad \text{for} \quad E > \Delta_L(\varphi),$$

and

$$\gamma_R^+(\varphi, \theta) = \frac{E - i\left(\Delta^+_R(\varphi, \theta)^2 - E^2\right)^{1/2}}{\Delta^+_R(\varphi, \theta)} \quad \text{for} \quad E < \Delta^+_R(\varphi, \theta),$$

$$\gamma_R^+(\varphi, \theta) = \frac{E - \left(\Delta^+_R(\varphi, \theta)^2 - E^2\right)^{1/2}}{\Delta^+_R(\varphi, \theta)} \quad \text{for} \quad E > \Delta^+_R(\varphi, \theta).$$

We calculate the total current density $\langle \mathbf{j} \rangle$ as the sum of the ohmic current density

$$\langle \mathbf{j}_N \rangle = -e_z \frac{V}{R_N L_x L_y},$$

where $R_N$ is the normal resistance, $L_x L_y$ is the cross-sectional area, and the current density due to Andreev reflection is

$$\langle \mathbf{j}_{AR} \rangle = -e_z \frac{\hbar}{2m} \int_{-\pi/2}^{\pi/2} d\varphi \sum_{n=1}^{\infty} e^{-2\pi n} \left\{ \int_{B_1}^{C_1} dE g(E) f_0(E + h) k_{e\uparrow} \right. \left. - \int_{B_2}^{C_2} dE g(E) f_0(E + h) k_{e\downarrow} + \int_{B_3}^{C_3} dE g(E) f_0(E - h) k_{e\downarrow} \right. \left. - \int_{B_4}^{C_4} dE g(E) f_0(E - h) k_{e\uparrow} - \int_{B_5}^{C_5} dE g(E)(1 - f_0(E + h)) k_{h\uparrow} + \int_{B_6}^{C_6} dE g(E)(1 - f_0(E - h)) k_{h\downarrow} \right. \left. + \int_{B_7}^{C_7} dE g(E)(1 - f_0(E - h)) k_{h\uparrow} - \int_{B_8}^{C_8} dE g(E)(1 - f_0(E + h)) k_{h\downarrow} + \int_{B_9}^{C_9} dE g(E)(1 - f_0(E + h)) k_{h\uparrow} \right. \left. - \int_{B_{10}}^{C_{10}} dE g(E)(1 - f_0(E - h)) k_{h\downarrow} - \int_{B_{11}}^{C_{11}} dE g(E)(1 - f_0(E + h)) k_{h\uparrow} + \int_{B_{12}}^{C_{12}} dE g(E)(1 - f_0(E + h)) k_{h\downarrow} - \int_{B_{13}}^{C_{13}} dE g(E)f_0(E - h) k_{e\uparrow} + \int_{B_{14}}^{C_{14}} dE g(E)f_0(E - h) k_{e\downarrow} \right. \left. + \int_{B_{15}}^{C_{15}} dE g(E)f_0(E + h) k_{e\uparrow} - \int_{B_{16}}^{C_{16}} dE g(E)f_0(E + h) k_{e\downarrow} \right\},$$

where the integration limits $B_i$ and $C_i$ ($B_i \geq 0, C_i > B_i, i = 1, \ldots, 16$, are given in the Appendix A. Here,

$$k_{e\sigma} = k_{eF} + \frac{E + \rho_e h}{h v_{eF}}, \quad k_{h\sigma} = k_{eF} - \frac{E - \rho_e h}{h v_{eF}},$$
where \( k_z F = k_F \cos \varphi \) and \( v_z F = v_F \cos \varphi \) are the \( z \) component of the Fermi wave vector and Fermi velocity, respectively. The density of states \( g(E) \) is calculated numerically using the method of Ref. [35]. The mean free path \( l \) for inelastic scattering enters into the calculation via the relaxation-time model [30]. The integration limits are found by noticing that the scattering probabilities can be approximated by a step-function product,

\[
\prod_{r=1}^{\alpha} \big| \gamma_R^\beta (E \pm h + (2r - 1)eV) \big|^2 \prod_{r=1}^{n} \big| \gamma_L (E \pm h + 2reV) \big|^2 \\
\approx \Theta \left( \Delta_R^\beta (\varphi, \theta) + E \mp h + eV \right) \Theta \left( \Delta_R^\beta (\varphi, \theta) - E \mp h - (2n - 1)eV \right) \\
\times \Theta \left( \Delta_L (\varphi) + E \mp h + 2eV \right) \Theta \left( \Delta_L (\varphi) - E \mp h - 2neV \right),
\]

\[
\prod_{r=1}^{\alpha} \big| \gamma_R^\beta (E \mp h - (2r - 1)eV) \big|^2 \prod_{r=1}^{n} \big| \gamma_L (E \mp h - 2reV) \big|^2 \\
\approx \Theta \left( \Delta_R^\beta (\varphi, \theta) - E \mp h + 2eV \right) \Theta \left( \Delta_R^\beta (\varphi, \theta) + E \mp h + 2neV \right) \\
\times \Theta \left( \Delta_L (\varphi) - E \mp h + eV \right) \Theta \left( \Delta_L (\varphi) + E \mp h - (2n - 1)eV \right),
\]

\[
\prod_{r=1}^{\alpha} \big| \gamma_R^\beta (E \mp h + (2r - 1)eV) \big|^2 \prod_{r=1}^{n} \big| \gamma_L (E \mp h + 2reV) \big|^2 \\
\approx \Theta \left( \Delta_R^\beta (\varphi, \theta) - E \mp h + eV \right) \Theta \left( \Delta_R^\beta (\varphi, \theta) + E \mp h - (2n - 1)eV \right) \\
\times \Theta \left( \Delta_L (\varphi) + E \mp h + 2eV \right) \Theta \left( \Delta_L (\varphi) - E \mp h - 2neV \right).
\]

where \( \alpha = n(n + 1) \) is related to \( \beta = +[-] \) and \( \bar{\beta} = -[+] \). Therefore, the integrands are finite only in the range of non-vanishing scattering probabilities. We underline that in the present case there is a nontrivial modification of the current calculation [31,32], due to the angular-dependent integration limits, as well as different pair potentials in the two superconducting electrodes. To understand this modification, which leads to interesting new results, we present here only the corresponding generalization of the previously developed theory in Refs. [31] and [32].

3. Results and discussion

In the anisotropic d-wave case, the proximity effect depends on the angle \( \theta \) between the crystal axis and the interface normal [33]. In D, with increasing misorientation from \( \theta = 0 \) toward \( \theta = \pi / 4 \), the superconducting order parameter (OP)\(^1\) decreases near the interface, which arises from

\(^1\) Note that the pair potential \( \Delta = \lambda F \cos(2\varphi - 2\theta) \) is proportional to the angle-dependent order parameter \( F(x, \theta) \).
a pair-breaking effect due to the injection of spin-polarized electrons from F to D. For the \( a \) axis of D perpendicular to the interface (\( \theta = 0 \)), there is only a small decrease in \( \Delta_0 \), which is somewhat similar to the case of an s-wave superconductor. For \( \theta = \pi/4 \), the pair-breaking effect is strongest, since the \( x \) axis is a nodal direction of \( \Delta_0 \), so that OP in D has the biggest decrease, i.e. the proximity effect is also strongest. On the ferromagnetic side, there appears an oscillating superconducting OP induced by the proximity of D. As Cooper pairs are injected from the superconductor to the ferromagnet, electron-like and hole-like quasiparticles interfere with each other, producing a damped oscillation of OP. The period of this oscillation makes another difference between the \( \theta = 0 \) and \( \theta = \pi/4 \) cases. For \( \theta = 0 \), this period is close to \( 2\pi\xi_F \), where \( \xi_F = \hbar v_F/2\hbar \) is the coherence length in the ferromagnet; for \( \theta = \pi/4 \), the period is significantly shorter [33]. The orientation dependence of the proximity effect strongly influences the quasiparticle current in DFD junctions.

To illustrate this effect, we calculate numerically the total current, including the contributions of the ohmic current and the AR current, \( I = I_N + I_{AR} = L_x L_y (\langle j_N \rangle + \langle j_{AR} \rangle) \). In the following, we introduce reduced units: \( \tilde{h} = h/\Delta(T) \), \( \tilde{V} = eV/\Delta(T) \), whereas \( \tilde{d} = d/\xi_0 \) and \( \tilde{l} = l/\xi_0 \), with \( \xi_0 \) being the superconducting zero-temperature coherence length. The current is normalized with respect to the temperature-dependent current \( I_0 = 2\Delta(T)/eR_N \). The results correspond to the clean case \( \tilde{l} > \tilde{d} \), \( \tilde{l} > (\pi/\tilde{h})(\Delta(0)/\Delta(T)) \). The model can be used for more general cases, but for arbitrary inelastic scattering with a mean free path \( l \), a modification of the expression for the current would be necessary [30].

We start with the current–voltage characteristics (CVC) presented in Fig. 1. The CVC change with increasing misorientation \( \theta \), diminishing in amplitude when \( \theta \) varies from 0 to \( \pi/2 \). In contrast to the isotropic s-wave case, in unconventional junctions with gap nodes in the superconductor, the CVC are smooth, even for small \( h \), because AR are not allowed for quasiparticle momenta directions corresponding to the positions of gap nodes. This results in broadening of the sharp structures found in the isotropic case. With increasing \( \theta \), CVC gradually approach linear normal characteristics, without a “foot” (or “shoulder”) occurring for small misorientations at low voltages, due to MAR.

Analyzing the current \( I \) as a function of the exchange field \( h \), we find an interesting interplay between the phase-coherent propagation of Andreev pairs in F and decoherence mechanisms originating from misorientation, bias voltage, temperature, and exchange-field-induced pair breaking. This interplay leads to non-monotonic behavior of the transport properties as a function of \( h \) (Figs. 2–4).
Fig. 2. The exchange-field dependence of the current $I(h)$ for $\tilde{d} = d/\xi_0 = 3$, $l/\xi_0 = 27$, $T/T_C = 0.05$, and four values of the bias voltage $\tilde{V} = eV/\Delta(T) = 0.2$ (black solid line), 0.8 (pink dotted line), 1.5 (blue dashed line), and 2.5 (orange dash-dotted line). (a) $\theta = 0$. (b) $\theta = \pi/4$.

Fig. 3. The exchange-field dependence of the current $I(h)$ for $\tilde{V} = eV/\Delta(T) = 0.2$, $l/\xi_0 = 27$, $T/T_C = 0.05$, and three values of misorientation angle $\theta = 0$ (black solid line), $\pi/8$ (blue dotted line), and $\pi/4$ (pink dashed line). (a) $\tilde{d} = d/\xi_0 = 1$. (b) $\tilde{d} = d/\xi_0 = 3$. Inset: The same as (a) for $\theta = 0$ and two values of the bias voltage $\tilde{V} = 0.2$ and 0.3.
Fig. 4. The exchange-field dependence of the current $I(h)$ for $d = d/\xi_0 = 3$, $l/\xi_0 = 27$, $\theta = \pi/8$, $
abla = eV/\Delta(T) = 0.2$, and three values of temperature $t = T/T_C = 0.05$ (black solid line), 0.4 (blue dotted line), and 0.82 (pink dashed line).

At the interface of the superconductor and ferromagnet, the mechanism of charge transport is modified since the incoming electron and reflected hole belong to different spin bands. Therefore, Cooper pairs penetrating into the F layer from the superconducting layer (both conventional and unconventional) have nonzero momentum due to the exchange field $h$. One expects that by increasing $h$ the electron–hole coherence would be suppressed and hence the quasiparticle current $I$ is reduced. However, this picture does not always stand for a subgap current. For a sufficiently strong ferromagnet, the exchange field always breaks the induced Cooper pairs, while for a weak exchange field the quasiparticle current can be enhanced at low voltage. This is illustrated in Figs. 2(a) and (b), where $I(h)$ curves for different applied bias voltages at $\theta = 0$ and $\theta = \pi/4$ are presented, respectively. For large enough voltages, the decoherence first causes the current to diminish with increasing $h$; it then becomes nearly constant for both orientations. For small voltages (e.g. $eV/\Delta(T) = 0.2$, which is the region of MAR [31]), the form of the $I(h)$ curves is determined by the proximity effect in both D and F.

For weak exchange fields and sufficiently thin barriers, so that $d < \xi_F$ ($d/\xi_0 = 3$ in our examples, Figs. 2(a) and (b)), the proximity effect in D depends on the misorientation, increasing with $\theta$, together with the gapless region penetrated by spin-polarized electrons. In F, the Cooper pairs injected from D are not broken immediately, but can travel up to a distance of the order of $\xi_F$. With increasing misorientation $\theta$, the magnetic coherence length decreases ($\xi_F(\theta \neq 0) < \xi_F(\theta = 0)$), while, at the same time, the induced OP near the barrier (in the first half-period of induced oscillations, $\pi\xi_F$) becomes larger, so that the proximity effect in F increases as well [33]. This is in accordance with our numerical results, which show that for $\theta = \pi/4$, where the proximity effect in D is strongest, as discussed above, the current first increases to a maximum at $h \approx \Delta(T)$, and then drops by further increase of $h$, the exchange field becoming large enough to break the Cooper pairs; see Fig. 2(b). This enhancement at low voltages is absent for $\theta = 0$; see Fig. 2(a).

For weak exchange fields and sufficiently thin barriers, so that $d$ is smaller than the magnetic coherence length, $d < \xi_F(\theta)$, in the whole interval $0 \leq \theta \leq \pi/4$, there is always a strong proximity effect in F, independent of the misorientation. This is because in this case there is a significant induced OP in the whole ferromagnet. After an eventual small decrease of the current for $h < eV$, there is always an enhancement up to $h \approx \Delta$, even for $\theta = 0$ (Fig. 3(a)).
We expect that above current enhancement mechanisms are similar to those in the s-wave case [36–38]. The main contribution to the two electrons tunneling from F does come from nearly time-reversed electrons with momenta $k_1 \approx -k_2$ located in an energy window proportional to $eV$, close to the Fermi energy. For $h = 0$, in the presence of a finite small-bias voltage $eV$, all the “candidates” for pairing in the $k$-space are not time-reversed [36]. With increasing $h$, the energy bands for spin-up and spin-down electrons are shifted, so that $|k_1| \rightarrow |k_2|$, and the relevant excitations become more and more coherent, leading to an increase in the current; see Figs. 2(b), 3(a), and 3(b). Such compensation is not possible for large $eV$, since it would need correspondingly large pair-breaking values of $h$.

This is particularly corroborated by the behavior of $I(h)$, presented in the inset of Fig. 3(a), where the curves for two values of the bias voltage $eV/\Delta(T) = 0.2$ and 0.3, with minima at $h/\Delta(T) = 0.2$ and 0.3, respectively, are shown. A minimum at the $I(h)$ curve occurs because the exchange field cannot compensate the depairing effect of the applied bias voltage up to $h = eV$.

In general, our results indicate (Figs. 3(a) and (b)) that compensation of the voltage $eV$-induced decoherence by the exchange field $h$ is always possible for a strong proximity effect, e.g. for large misorientations with thick barriers, and for any misorientation with thin barriers. In the last case, this compensation, with corresponding increase of the current $I(h)$, for $\theta = 0$ may occur for $h$ larger than $eV$, while, for $\theta = \pi/4$, the current increases with $h$ starting from $h = 0$.

For $h > \Delta(T)$, the Andreev current decreases by increasing $h$ in all cases; see Figs. 3(a) and (b). The two-electron tunneling is absent, and the total current is carried by the ohmic component only, independent of $h$.

Comparing Figs. 3(a) and (b), we can see another effect of the barrier thickness. For a thicker barrier, Fig. 3(b), there are oscillations of the current in low field for small $\theta$. This would correspond to the subsequent breaking and formation of the Andreev pairs. For a large orientation proximity effect ($\theta = \pi/4$), the coherence is restored with increasing $h$ (Fig. 3(b)).

Another decoherence factor is the temperature. This is seen in Fig. 4 for $\theta = \pi/8$, $d/\xi_0 = 3$ and at low voltage $eV/\Delta(T) = 0.2$. At very low temperatures, the coherence is established by increasing $h$, with some oscillations and a maximum at $h \approx \Delta(T)$. For larger $h$, the Andreev current is gradually suppressed. At higher temperatures, $I(h)$ is a smooth curve, with a similar shape, but with a significantly diminished amplitude.

The influence of misorientation angle $\theta$ on the conductance curves, $G(V) = \frac{dI}{dV}(V)$, is presented in Fig. 5. In the MAR region at low voltage, the $G(V)$ curves show pronounced nonlinear structures, while, at higher voltage, only a few harmonics contribute to $G(V)$, as shown by numerical calculation. In this higher-voltage region, the shapes of $G(V)$ are similar for various misorientation angles (Figs. 5(a)–(c)) with a few distinct dips. The locations of these nonlinearities, independent of $\theta$ and of the pair-potential symmetry [32], vary with exchange field (see Fig. 5(c) and its inset) and temperature according to the simple law $neV = 2\Delta(T) \mp h$, with $n = 0, 1, \ldots$. Using this result, by measuring the location of a dip on $G(V)$ at a given temperature, one could determine the value of the exchange field (up to $h = 2\Delta(T)$) in the barrier.

Several other detection methods have recently been suggested. In particular, in SF bilayers at zero temperature, $h < \Delta$ can be determined from the position of the peak on the subgap conductance at $eV = h$ [36,39], whereas the critical current measurements allow the assessment of the magnitude of $h$ in FISIS Josephson junctions, where I is an insulator [40]. In ferromagnet and d-wave superconductor bilayers with (110) orientation, the determination of the degree of spin polarization in the ferromagnet using the value of zero-bias conductance has been proposed [41].
Fig. 5. Conductance $G(V)$ for $T / T_c = 0.05$, $\tilde{d} = d / \xi_0 = 3, l / \xi_0 = 27, \tilde{h} = h / \Delta(T) = 0.5$, and three values of misorientation angle (a) $\theta = 0$, (b) $\theta = \pi / 8$, and (c) $\theta = \pi / 4$. Inset: The same as (c) for $\tilde{h} = 1$.

The detection method of experimental exchange-field determination proposed here for clean tri-layers has the advantage that it can be performed in transport measurements up to $h = 2\Delta(T)$, independently of the superconductor anisotropy and misorientation.

4. Conclusion

In conclusion, we have investigated theoretically the quasiparticle transport in ballistic voltage-biased DFD junctions, with misoriented superconducting electrodes. Using the generalized KGN theory for non-equilibrium transport, we found a strong influence of electrode misorientation $\theta$ on the quasiparticle current through the junction. Due to the interplay between the phase-coherent propagation of Andreev pairs in F and decoherence mechanisms originating from electrode misorientation, the presence of the exchange field, the bias voltage, and the temperature, new features in the transport have been found. In particular, at low bias, a non-monotonic behavior of the quasiparticle current dependence on the exchange field $I(h)$ can be seen. For thick F barriers, this is the case for $\theta = \pi / 4$, when the proximity effect is strongest, and, for thin barriers, for every $\theta$, with an increase of $I(h)$ (up to $h \approx \Delta(T)$), beginning from $h = 0$ for $\theta = \pi / 4$ only. In addition, at higher bias, a few distinct dips arise in the differential conductance. The positions of these dips are not influenced by the type of OP anisotropy in the superconducting electrodes, as mentioned in our previous papers. Here we find that they do not depend on the electrode misorientation either, but vary only with the exchange field in F and the temperature-dependent pair potential in D, according to the simple law $neV = 2\Delta(T) \mp h$. This provides the possibility of experimental determination of exchange fields $h \leq 2\Delta(T)$ at finite temperatures for both s-wave and d-wave types of clean superconductor–ferromagnet–superconductor junctions.

Acknowledgements

The work was supported by the Serbian Ministry of Education, Science and Technological Development, Project Nos. 171033 and 41028.
Appendix A

In this appendix, we give integration limits in the Andreev current density, Eq. (11):

\[ B_1 = \max \left[ -\Delta_R^+ - h - eV, -\Delta_L - h - 2eV \right], \]
\[ C_1 = \min \left[ \Delta_R^+ - h - neV + eV, \Delta_L - h - neV \right], \]
\[ B_2 = \max \left[ -\Delta_L - h + neV - eV, -\Delta_R^- - h - neV \right], \]
\[ C_2 = \min \left[ \Delta_L - h + eV, \Delta_R^- - h + 2eV \right], \]
\[ B_3 = \max \left[ -\Delta_R^+ + h - eV, -\Delta_L + h - 2eV \right], \]
\[ C_3 = \min \left[ \Delta_R^+ + h - neV + eV, \Delta_L + h - neV \right], \]
\[ B_4 = \max \left[ -\Delta_L + h + neV - eV, -\Delta_R^- + h + neV \right], \]
\[ C_4 = \min \left[ \Delta_L + h + eV, \Delta_R^- + h + 2eV \right], \]
\[ B_5 = \max \left[ -\Delta_L + h - eV, -\Delta_R^+ + h - 2eV \right], \]
\[ C_5 = \min \left[ \Delta_L + h - neV, \Delta_R^+ + h - neV + eV \right], \]
\[ B_6 = \max \left[ \Delta_R^+ - h - neV + eV, \Delta_L - h - neV \right], \]
\[ C_6 = \min \left[ \Delta_R^- + h + eV, \Delta_L + h + 2eV \right], \]
\[ B_7 = \max \left[ -\Delta_L - h - eV, -\Delta_R^+ - h - 2eV \right], \]
\[ C_7 = \min \left[ \Delta_L - h - neV, \Delta_R^+ - h - neV + eV \right], \]
\[ B_8 = \max \left[ -\Delta_R^- - h + neV, -\Delta_L - h + neV - eV \right], \]
\[ C_8 = \min \left[ \Delta_R^- - h + eV, \Delta_L - h + 2eV \right], \]
\[ B_9 = \max \left[ -\Delta_R^+ + h + neV - eV, -\Delta_L + h + neV \right], \]
\[ C_9 = \min \left[ \Delta_R^+ + h + eV, \Delta_L + h + 2eV \right], \]
\[ B_{10} = \max \left[ -\Delta_L + h - eV, -\Delta_R^- + h - 2eV \right], \]
\[ C_{10} = \min \left[ \Delta_L + h - neV + eV, \Delta_R^- + h - neV \right], \]
\[ B_{11} = \max \left[ -\Delta_R^+ - h + neV - eV, -\Delta_L - h + neV \right], \]
\[ C_{11} = \min \left[ \Delta_R^+ - h + eV, \Delta_L - h + 2eV \right], \]
\[ B_{12} = \max \left[ -\Delta_L - h - eV, -\Delta_R^- - h - 2eV \right], \]
\[ C_{12} = \min \left[ \Delta_L - h - neV + eV, \Delta_R^- - h - neV \right], \]
\[ B_{13} = \max \left[ -\Delta_L - h + neV, -\Delta_R^+ - h + neV - eV \right], \]
\[ C_{13} = \min \left[ \Delta_L - h + eV, \Delta_R^+ - h + 2eV \right], \]
\[ B_{14} = \max \left[ -\Delta_R^- + h - eV, -\Delta_L - h - 2eV \right], \]
\[ C_{14} = \min \left[ \Delta_R^- - h - neV, \Delta_L - h - neV + eV \right], \]
\[ B_{15} = \max \left[ -\Delta_L + h + neV, -\Delta_R^+ + h + neV - eV \right], \]
\[ C_{15} = \min \left[ \Delta_L + h + eV, \Delta_R^+ + h + 2eV \right], \]
\[ B_{16} = \max \left[ -\Delta_R^- + h - eV, -\Delta_L + h - 2eV \right], \]
\[ C_{16} = \min \left[ \Delta_R^- + h - neV, \Delta_L + h - neV + eV \right]. \tag{A1} \]

References