Revisiting the model predicting maximal 2–3 mixing and CP violation for neutrinos

Eiichi Takasugi$^{1,2,*}$

$^1$Maskawa Institute, Kyoto Sangyo University, Kamigamo, Kyoto, Japan
$^2$Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan
$^*$E-mail: takasugi@cc.kyoto-su.ac.jp, takasugi@het.phys.sci.osaka-u.ac.jp

Received August 21, 2015; Revised September 15, 2015; Accepted September 24, 2015; Published November 16, 2015

The model of the neutrino mass matrix that we proposed in 2000 is revisited in the light of the recent T2K experiments. This model has the special property that it predicts maximal 2–3 mixing and CP violation under some simple condition. In this model, if the condition is relaxed, the 2–3 angle and the CP violation deviate from their maximal values and are related. We present such relations for typical cases.

Subject Index B52, B54

1. Introduction

In view of the recent T2K experiments [1], the mode predicting maximal 2–3 mixing and CP violation is attracting more attention and seems to require further investigation. In 2000 and 2001, we produced a series of papers [2–4] where the neutrino mass matrix was proposed in the mass eigenstate basis of charged leptons and made the following two findings.

One is that this neutrino mass matrix predicts maximal 2–3 mixing and maximal CP violation under some simple condition. This is due to the fact that elements of the derived neutrino mixing matrix satisfy a special relation, which is the same as that discussed by Grimus and Lavoura [5] in 2003$^1$. Also, this neutrino mass matrix has the same property as that recently discussed by Ma [7].

The second is that we found a new mixing that is essentially the same as the one that is now called tribimaximal mixing, proposed by Harrison et al. [8] in 2002. These points are explained in Sect. 2.

In Sect. 3, we relax the condition and discuss how the 2–3 mixing and CP violation deviate from the maximal values and derive their relations for some typical cases. In Sect. 4, we discuss which neutrino mass matrices realize the situation discussed in Sect. 3. Concluding remarks are given in Sect. 5.

2. A brief survey of our papers

In Refs. [2–4], we proposed a neutrino mass matrix in the diagonal basis of the charged lepton mass matrix.

---

$^1$The information on the recent discussions about the neutrino mass matrix was obtained from the review article by King et al. [6].
Our neutrino mass matrix

The neutrino mass matrix was given by the combination of \( S_i \) and \( T_i \) as

\[
m_\nu = m_1^0 S_1 + m_2^0 S_2 + m_3^0 S_3 + \tilde{m}_1 (T_1 - S_1) + \tilde{m}_2 (T_2 - S_2) + \tilde{m}_3 (T_3 - S_3),
\]

where

\[
S_1 = \frac{1}{3} \begin{pmatrix} 1 & \omega^2 & \omega \\ \omega^2 & \omega & 1 \\ \omega & 1 & \omega^2 \end{pmatrix}, \quad S_2 = \frac{1}{3} \begin{pmatrix} 1 & \omega & \omega^2 \\ \omega & 1 & \omega \\ \omega^2 & \omega & 1 \end{pmatrix}, \quad S_3 = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix},
\]

\[
T_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \quad T_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \quad T_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.
\]

In general, the mass parameters \( m_i^0 \) and \( \tilde{m}_i \) are complex and thus \( m_\nu \) becomes a general complex symmetric matrix.

Then we found that, if the mass matrix is transformed by the trimaximal mixing matrix \( V_T \),

\[
V_T = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ \omega & \omega & \omega \\ \omega^2 & \omega^2 & \omega^2 \end{pmatrix},
\]

with \( \omega = e^{2\pi/3} \), the transformed neutrino mass matrix has the following form:

\[
m'_\nu = V_T^T m_\nu V_T = \begin{pmatrix} m_1^0 & \tilde{m}_3 & \tilde{m}_2 \\ \tilde{m}_3 & m_2^0 & m_1^0 \\ \tilde{m}_2 & m_1^0 & m_3^0 \end{pmatrix}.
\]

From this, we found that if all mass parameters \( m_i^0 \) and \( \tilde{m}_i \) are real, \( m'_\nu \) becomes a real symmetric matrix and is diagonalized by a real orthogonal matrix. As a result, the neutrino mass matrix \( m_\nu \) is diagonalized by the matrix that is the trimaximal matrix multiplied by a real orthogonal matrix \( O \), i.e., \( V = V_T O \). Next, we showed that this matrix has a property

\[
V_{2i} = V_{3i}^* (i = 1, 2, 3)
\]

and discussed the fact that, by the phase redefinition of the mixing matrix, \( V \) can be converted to the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix [9] and the condition in Eq. (5) leads to the constraint \(|(U_{PMNS})_{2i}| = |(U_{PMNS})_{3i}|\), from which we found [2]

\[
s_{23}^2 = c_{23}^2, \quad \cos \delta_{CP} = 0,
\]

where \( c_{ij} = \cos \theta_{ij} \) and \( s_{ij} = \sin \theta_{ij} \). That is, the maximal 2–3 mixing \( \theta_{23} = \pi/4 \) and the maximal CP violation phase \( \delta_{CP} = \pm \pi/2 \). It is noted that the condition in Eq. (5) is exactly the same as the one discussed by Grimus and Lavoura [5].

We note that, under our assumption that all mass parameters are real, the mass matrix \( m_\nu \) becomes a matrix with the property that \( (m_\nu)_{11} \) and \( (m_\nu)_{23} \) are real, and \( (m_\nu)_{22} = (m_\nu)_{33}^* \),

\[\text{ref}[2]\]

\[\text{ref}[2]\]

2 In Ref. [2], the mass matrix is given by \( m_\nu = \Sigma_i (m_i^0 S_i + \tilde{m}_i T_i) \), which is equivalent to the present expression.

3 These matrices are obtained from \( S_3 \) and \( T_3 \) by the transformation \( P S_i P = S_i + 1 \) and \( P T_i P = T_i + 1 \) (mod 3) with \( P = T_2 \).
\((m_\nu)_{12} = (m_\nu)_{13}^*\). This property of the neutrino mass matrix is exactly the same as that recently discussed by Ma [7].

(2) Tribimaximal mixing

In Refs. [3,4], we found a new mixing by taking a special orthogonal rotation for \(O\) (see Eqs. (28), (33), and (41) in Ref. [3], and also the equation in Sect. 3 in Ref. [4]):

\[
V_T \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{3}} & -\frac{\sqrt{2}}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & i \end{pmatrix}, \tag{7}
\]

which gave the predictions \(\sin^2 2\theta_{\text{sol}} = 8/9\) and \(\sin^2 2\theta_{\text{atm}} = 1\). Since neutrino masses are free parameters in this model, the mixing matrix is essentially that called tribimaximal mixing; i.e., if we interchange the 1st and 2nd columns, we reach it.

We emphasize that, if the maximal 2–3 mixing and CP violation are confirmed by the experimental observations, as the T2K experiment implies, the model is perfect because other mixing angles and neutrino masses are obtained in any precision by choosing the mass parameters. If the maximality for the 2–3 mixing and the CP violation is violated, some modification is necessary, which we discuss in the next section.

3. The relation between the 2–3 mixing angle and the CP violation phase in the modified condition

If some of the mass parameters are complex, then the orthogonal matrix \(O\) above Eq. (5) becomes a unitary matrix and then the 2–3 mixing and CP violation shift from their maximal values. In this section, we discuss the effect of changing the orthogonal matrix into a unitary one.

We start from the basis where the trimaximal mixing matrix is converted to a tribimaximal mixing matrix. Then we consider two typical cases: 1) 2–3 rotation followed by 1–2 rotation, and 2) 1–3 rotation followed by 1–2 rotation. We assume that the 2–3 (or 1–3) rotation is made by a unitary matrix and the 1–2 rotation by a real orthogonal matrix.

To reach tribimaximal mixing, we can use the 2–3 rotation given in Eq. (7) followed by the exchange of the 1st and 2nd columns. Here we use a direct method such that

\[
\tilde{V} = V_T \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}
\]

\[
= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -i \end{pmatrix}. \tag{8}
\]

(1) 2–3 rotation followed by 1–2 rotation

We consider

\[
V' \equiv \tilde{V} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c & s e^{-i\rho} \\ 0 & -s e^{i\rho} & c \end{pmatrix} \begin{pmatrix} c' & s' & 0 \\ -s' & c' & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{9}
\]
 where \( c = \cos \theta, \ c' = \cos \theta' \) and \( s = \sin \theta, \ s' = \sin \theta' \); we take \( c > 0 \) and \( c' > 0 \) and assume that \( s \) and \( s' \) are small from the observation that tribimaximal mixing reproduces the data fairly well. We find \( V' = \text{diag}(1, \omega^2, \omega)V \text{diag}(1, 1, -i) \), where

\[
V = \begin{pmatrix}
\sqrt{\frac{2}{3}}c' - \frac{c' e^{i \rho}}{\sqrt{3}} & \sqrt{\frac{2}{3}}s' + \frac{c' e^{i \rho}}{\sqrt{3}} & i\frac{s}{\sqrt{3}} e^{-i \rho} \\
\frac{c'}{\sqrt{3}} - \frac{i s' e^{i \rho}}{\sqrt{2}} & \frac{s'}{\sqrt{3}} - \frac{i c' e^{i \rho}}{\sqrt{2}} & \frac{c'}{\sqrt{2}} + i\frac{s}{\sqrt{3}} e^{-i \rho} \\
\frac{c'}{\sqrt{3}} - \frac{i s' e^{i \rho}}{\sqrt{2}} & \frac{s'}{\sqrt{3}} + \frac{c' e^{i \rho}}{\sqrt{2}} & \frac{c'}{\sqrt{2}} + i\frac{s}{\sqrt{3}} e^{-i \rho}
\end{pmatrix}.
\]

From this, we find

\[
s_{13}^2 = \frac{s^2}{3},
\]

\[
s_{23}^2 s_{13}^2 = \frac{1}{2} - \frac{s^2}{6} - \frac{\sqrt{2}}{3} s c \sin \rho,
\]

which lead to

\[
\sin \rho = \frac{\text{sgn}(s) \cos 2 \theta_{23} c_{13}^2}{2 \sqrt{2} s_{13} \sqrt{1 - 3 s_{13}^2}}.
\]

Next, we compute the Jarlskog invariant:

\[
J_{CP} = \text{Im} [V_{11} V_{22}^* V_{12} V_{21}^*] = V_{11} V_{12} \frac{s \cos \rho}{2 \sqrt{3}} = s_{23} c_{23} s_{12} c_{12} s_{13} c_{13}^2 \sin \delta_{CP},
\]

where the 2nd equality is computed by using the mixing matrix \( V \) and the 3rd one by using the PMNS matrix from the Particle Data Group [9]. Since we take \( c > 0 \) and \( c' > 0 \) and assume that \( s \) and \( s' \) are small, \( V_{11} > 0 \) and \( V_{12} > 0 \), so that we find

\[
\cos \rho = \text{sgn}(s) \sin 2 \theta_{23} \sin \delta_{CP},
\]

where we have used \(|s| = \sqrt{3} s_{13}|. From Eqs. (12) and (14), we find

\[
|\cos \delta_{CP}| \tan 2 \theta_{23} = \sqrt{\frac{c_{13}^4}{8 s_{13}^2 (1 - 3 s_{13}^2)} - 1}.
\]

That is, \(|\cos \delta_{CP}| \) is proportional to \(|1/ \tan 2 \theta_{23}|\) and this is the relation between the deviations of \( \theta_{23} \) and \( \delta_{CP} \) from their maximal values. It is thus noted that, in the limit of \( \theta_{23} = \pi/4 \), \( \sin \delta_{CP} = 0 \) is reproduced. By convention, we take \( \cos \rho > 0 \), then, for \( s > 0 \), \( \sin \delta_{CP} > 0 \) and, for \( s < 0 \), \( \sin \delta_{CP} < 0 \).

As for \( \theta_{12} \),

\[
\tan \theta_{12} = \frac{\sqrt{2} \tan \theta' + \sqrt{1 - 3 s_{13}^2}}{\sqrt{2} - \sqrt{1 - 3 s_{13}^2} \tan \theta'}.
\]

In the limit of \( s' = \sin \theta' = 0 \), this relation reduces to a well known one:

\[
s_{12}^2 = 1 - \frac{2}{3} \frac{1}{c_{13}^2},
\]

which seems to reproduce the data well, so that the assumption that \(|s'| \) is small is valid.
(2) 1–3 rotation followed by 1–2 rotation

We consider

\[ V' \equiv \tilde{V} \begin{pmatrix} c & 0 & s e^{-i\rho} \\ 0 & 1 & 0 \\ -s e^{i\rho} & 0 & c \end{pmatrix} \begin{pmatrix} c' & s' & 0 \\ -s' & c' & 0 \\ 0 & 0 & 1 \end{pmatrix}, \]

where we take \( c > 0 \) and \( c' > 0 \) and assume that \( s \) and \( s' \) are small. We find

\[ V' = \text{diag}(1, \omega^2, \omega) V \text{diag}(1, 1, -i), \]

where

\[ V = \begin{pmatrix} \sqrt{\frac{2}{3}} c c' - \frac{s'}{\sqrt{3}} & \sqrt{\frac{2}{3}} c s' + \frac{c'}{\sqrt{3}} & i \sqrt{\frac{2}{3}} s e^{-i\rho} \\ \frac{c c'}{\sqrt{6}} - s' \frac{c}{\sqrt{3}} - i \frac{s c'}{\sqrt{2}} \theta & \frac{c c'}{\sqrt{6}} + s' \frac{c}{\sqrt{3}} - i \frac{s c'}{\sqrt{2}} \theta & \frac{c c'}{\sqrt{6}} + s' \frac{c}{\sqrt{3}} + i \frac{s c'}{\sqrt{2}} \theta \end{pmatrix}. \]

From this, we find

\[ s_{13}^2 = \frac{2s^2}{3}. \]

Next, from \( s_{23} c_{13} = |V_{23}| \), we find

\[ \sin \rho = -\frac{\text{sgn}(s)}{\sqrt{2}} \frac{\cos 2\theta_{23} c_{13}^2}{s_{13} \sqrt{1 - \frac{3}{2} s_{13}^2}}. \]

Now, we compute the Jarlskog invariant and find, similarly to the previous case,

\[ \cos \rho = \text{sgn}(s) \sin 2\theta_{23} \sin \delta_{\text{CP}}, \]

where we have used \( |s| = \sqrt{3/2} s_{13} \). From Eqs. (21) and (22), we find

\[ |\cos \delta_{\text{CP}}| \tan 2\theta_{23} = \frac{c_{13}^4}{2 s_{13}^2 (1 - \frac{3}{2} s_{13}^2)} - 1. \]

It is thus noted that, in the limit of \( \theta_{23} = \pi/4 \), \( \sin \delta_{\text{CP}} = 0 \) is derived. By convention, we take \( \cos \rho > 0 \), then, for \( s > 0 \), \( \sin \delta_{\text{CP}} > 0 \) and, for \( s < 0 \), \( \sin \delta_{\text{CP}} < 0 \).

As for \( \theta_{12} \),

\[ \tan \theta_{12} = \sqrt{2 - 3 s_{13}^2 \tan \theta' + 1} \]

\[ \sqrt{2 - 3 s_{13}^2 - \tan \theta'}. \]

In the limit of \( s' = \sin \theta' = 0 \), this relation reduces to the well known relation

\[ s_{12}^2 = \frac{1}{3 c_{13}}, \]

which seems not to reproduce the data. If \( \tan \theta_{12} < 1/\sqrt{2} \), as the recent data [9] imply, then \( \tan \theta' < -\left(\sqrt{2} - 1\right) / \left(\sqrt{2} + 1\right) \approx -0.17 \), but this is still small enough for our argument.
4. The mass parameters that realize the rotation given in the previous section

In this section, we discuss what kind of mass parameters produce the transformation given in the previous section. At first, we note that the mass matrix after the transformation by $\tilde{V}$ is

$$\tilde{m}_v \equiv \tilde{V}^T m_v \tilde{V} = \begin{pmatrix} \frac{m_0^0 + m_1^0 + 2m_2^0}{2} & \frac{m_1^0 + m_3^0}{\sqrt{2}} & \frac{m_3^0 - m_1^0}{2} \\ \frac{m_1^0 + m_3^0}{\sqrt{2}} & m_2^0 & \frac{m_3^0 - m_1^0}{\sqrt{2}} \\ \frac{m_1^0 - m_3^0}{2} & \frac{m_1^0 - m_3^0}{\sqrt{2}} & \frac{m_0^0 + m_3^0 - 2m_2^0}{2} \end{pmatrix}. \quad (26)$$

(1) We want to realize the 2–3 rotation case given in Eq. (9). If we take $m_3^0 = m_1^0$ and choose $m_1^0$ and $m_2^0$ to be real, and also

$$\tilde{m}_1 = \frac{1}{\sqrt{2}} \left( d + be^{i \alpha} \right), \quad \tilde{m}_3 = \frac{1}{\sqrt{2}} \left( d - be^{i \alpha} \right), \quad (27)$$

where $b$ and $d$ are real, we obtain the mass matrix

$$\tilde{m}_v = \begin{pmatrix} a_1 & d & 0 \\ d & a_2 & be^{i \alpha} \\ 0 & be^{i \alpha} & a_3 \end{pmatrix}. \quad (28)$$

where $a_1 = m_1^0 + m_2^0$, $a_2 = m_2^0$, and $a_3 = m_1^0 - m_2^0$, and they are real.

For the normal hierarchy (NH) case, we perform a seesaw calculation by assuming that $a_3$ is large in comparison with the others. This gives the 2–3 rotation and we obtain $|b/a_3| \simeq \sin \theta_{13}$ and $\alpha \simeq -\rho$. The effect on the submatrix of the 1st and 2nd generations is the change of the 2–2 element $a_2$ to $a_2 - b^2 e^{2i \alpha}/a_3$. Therefore, the effect on the complex phase is of the order of $| (b^2/a_2a_3) \sin 2\alpha | \lesssim 0.1 | \sin 2\alpha |$, because $(b^2/a_2a_3) \lesssim s_{13}^2 \sqrt{\Delta m^2_{\text{sol}}/\Delta m^2_{\text{atm}}}$. If the shift of $\theta_{23}$ from $\pi/4$ is small, then $\alpha \simeq -\rho \sim 0$, so that this effect is small. Therefore, the 1–2 rotation can be considered to be a real orthogonal one to a good approximation.

For the inverted hierarchy (IH) case, since $a_1 \simeq \pm a_2$ are large, the effect after the seesaw calculation is only on the 3–3 element $a_3$. Therefore, the real 1–2 rotation is achieved by the above mass matrix.

Thus, the matrix given in Eq. (28) gives the rotation in Eq. (9) to a good approximation, so that the relation between $| \cos \delta_{\text{CP}} |$ and $| \tan 2\theta_{23} |$ is realized to a good approximation.

(2) We want to realize the 1–3 rotation case given in Eq. (18). If we take $\tilde{m}_1 = \tilde{m}_3 = d/\sqrt{2}$, $\tilde{m}_2 = (a_1 - a_2)/2$, and

$$m_3^0 = \frac{a_1 + a_2}{2} + be^{i \alpha}, \quad m_1^0 = \frac{a_1 + a_2}{2} - be^{i \alpha}, \quad (29)$$

where $a_1$ and $b$ are real, then we obtain

$$\tilde{m}_v = \begin{pmatrix} a_1 & d & be^{i \alpha} \\ d & a_2 & 0 \\ be^{i \alpha} & 0 & a_3 \end{pmatrix}. \quad (30)$$

A similar argument holds for this case. For the NH case, the effect after the seesaw calculation, $a_1$, is shifted to $a_1 - (b^2/a_3)e^{2i \alpha}$. However, for the 1–2 rotation, the complex phase that
contributes to the Dirac CP violation phase enters as $a_1 + a_2 - b^2 e^{2i\alpha}/a_3$, so that the same argument as discussed in the previous case holds. Also, for the IH case, the same argument as in the previous case holds. Therefore, the above mass matrix will give the preferable rotation given in Eq. (18) to a good approximation.

5. Concluding remarks

In this note, we have revisited our old papers from 2000 and 2001 and presented the findings given there, i.e., our mass matrix predicts the maximal 2–3 mixing angle and the maximal Dirac CP violation under the condition that all mass parameters are real. If this condition is relaxed, both the 2–3 mixing and the CP violation deviate from their maximal ones. We have considered two typical cases and obtained the relation between these deviations, which are expressed by

$$|\cos \delta_{\text{CP}}| \tan 2\theta_{23} = k(s_{13}),$$  \hspace{1cm} (31)

where $k$ is the function of $\theta_{13}$ given in Eqs. (15) and (23) for the two cases. If we take $s_{13} = 0.15$ [9], $k = 2.1$ for the 2–3 rotation case and $k = 4.5$ for the 1–3 rotation case. For the case of the small deviation case, $\Delta_{\text{CP}} \simeq 2k\Delta_{23}$, where $\Delta_{\text{CP}} = |\delta_{\text{CP}} - \pi/2|$ and $\Delta_{23} = |\theta_{23} - \pi/4|$.

Our predictions in Eq. (31) give a good test of the model if precision measurements of $\delta_{\text{CP}}$ and $\theta_{23}$ are made.

From a model-building point of view, we explored the situation in which these cases are realized. In Sect. 4, we gave some simple choices of mass parameters that reproduce the rotations used in Sect. 3 to a good approximation.

Finally, we comment that the relations in Eqs. (15) and (23) are valid for some other classes of models. Suppose that we construct a neutrino mass matrix model that realizes tribimaximal mixing without the phase matrices appearing in Eq. (8) and then rotate the mixing matrix by the unitary matrix discussed in the text by adding some small mass terms. This case is realized by the change $\rho \rightarrow \sigma - \pi/2$. Since the relations are independent of this phase, the relations hold for these cases. The no-1–2 rotation cases are discussed by Shimizu and Tanimoto [10]. It may be interesting to examine the relation between the CP violation phase and mixing angles for the general unitary matrix case numerically.

Acknowledgements

I would like to thank Dr Atsushi Watanabe for giving me the recent information on neutrino physics; I enjoyed our discussions.

Funding

Open Access funding: SCOAP³.

References


