Analysis of forward–backward and lepton polarization asymmetries in $B \to K_1 \ell^+\ell^-$ decays in the two-Higgs-doublet model

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The exclusive semileptonic $B \to K_1(1270)\ell^+\ell^-$ ($\ell = \mu, \tau$) decays are analyzed in variants of two-Higgs-doublet models (THDMs). The mass eigenstates $K_1(1270)$ and $K_1(1400)$ are a mixture of two axial-vector SU(3) $^1P_1$ and $^3P_1$ states with the mixing angle $\theta_{K_1}$. Making use of the form factors calculated in the light-cone QCD approach and by taking a mixing angle of $\theta_{K_1} = -34^\circ$, the impact of the parameters of the THDMs on different asymmetries in the above-mentioned semileptonic $B$ meson decays is studied. In this context, the forward–backward asymmetry and different lepton polarization asymmetries have been analyzed. We have found comprehensive effects of the parameters of the THDMs on the above-mentioned asymmetries. Therefore, precise measurements of these asymmetries at the LHC and different $B$ factories, for the above-mentioned processes, can serve as a good tool to put some indirect constraints on the parametric space of the different versions of the THDM.

1. Introduction

The Standard Model (SM) of particle physics successfully explains the observed data so far and the recent observation of a Higgs(-like) boson with a mass of 126 GeV supports it further. However, it is still difficult to accept it as an ultimate theory of nature. The Large Hadron Collider (LHC) data are now ready to undergo further analysis that will possibly check the SM in more detail and also probe for new physics (NP). This is, therefore, an exciting time to test the predictions of the SM in different sectors and try to identify the nature of physics beyond it. The study of the rare decays of $B$ mesons induced by flavor-changing neutral-current transitions (FCNC), being loop suppressed in the SM, provides us with a natural ground to look for the possible existence of NP on the TeV scale associated with the hierarchy problem.

The measurements of inclusive $b \to s \ell^+\ell^-$ transitions are preferred because of their lower theoretical uncertainties. However, they are most challenging to measure experimentally. The branching fractions and various asymmetries of the inclusive $B \to X_s \ell^+\ell^-$ decays, where $\ell$ can be any of the three leptons and $X_s$ is any hadronic state with an $s$ quark, have been measured at Belle [1] and...
BABAR [2]. The theoretical studies of the rare $B$ meson decays give an opportunity to investigate the physics beyond SM, where these decays are purposefully used to test these models and to constrain their parameter space.

It is remarkable that most of the experimental results are in agreement with the SM predictions, but problems such as neutrino oscillations, the matter–antimatter asymmetry, and the problems of dark matter cannot be explained in this model. It is, therefore, widely believed that it is an effective theory on an electroweak scale. In order to understand these unfinished mysteries of nature, there exist some physics that lies beyond the scope of the SM or needs its extensions. In this context, some possible extensions of the SM including the little Higgs model [3,4], the extra-dimension model [5–7], and multi-Higgs models like the supersymmetric standard model [8] have been extensively studied in the literature.

The two-Higgs-doublet model (THDM) is among the most popular extensions of the SM. In contrast to the SM, in which we have only one Higgs doublet, in the THDM we consider two complex Higgs doublets. Generally, the THDM possesses tree-level FCNC transitions, which can be avoided by imposing an ad hoc discrete symmetry [9]. This results in two different possibilities:

1. The first possibility to keep the flavor conservation at the tree level is to couple all the fermions to only one of the Higgs doublets. This is called model I.
2. The second possibility is when the up-type quarks are coupled to one Higgs doublet and the down-type quarks are coupled to the second one; this is called model II. This is a popular choice because, in this case, the Higgs sector coincides with the supersymmetric model.

The physical contents of the Higgs sector are two neutral scalar Higgs bosons $H^0, h^0$, a pseudo-scalar Higgs $A^0$, and a pair of charged Higgs bosons $H^\pm$. The vacuum expectation values of the two Higgs doublets are denoted by $v_1$ and $v_2$, respectively, and the interaction of the fermions with the Higgs fields depends on $\tan \beta = v_2/v_1$, which is a free parameter of the model.

There is another possibility where the discrete symmetry is not imposed, which in turn leads to the most general form of the THDM, i.e., to, say, model III. In this version, the FCNC transitions are allowed at the tree level. The indirect constraints on the masses of the charged Higgs bosons $m_{H^\pm}$, the neutral scalars $m_{H^0}, m_{h^0}$, and the neutral pseudo-scalar $m_{A^0}$, as well as on the fermion Higgs interaction vertex, $\tan \beta$, are obtained from experimental observation of the branching ratios of $b \rightarrow s \gamma$, $B \rightarrow D \tau \nu$ decays and $K^-\bar{K}$ and $B^-\bar{B}$ mixing in the literature [10]. Consistent with the low-energy constraints, the FCNCs involving the third generation are not as severely suppressed as that involving the first two generations. In contrast to the SM and THDMs I and II, there exists a single CP phase of vacuum, which is a rich source for phenomenological studies of CP-violating observables [11].

In connection with the FCNC transitions mediated by $b \rightarrow s \ell^+\ell^-$, like the rare semileptonic decays involving $B \rightarrow (X_s, K^*, K)\ell^+\ell^-$, the $B \rightarrow K_1(1270, 1400)\ell^+\ell^-$ decays are also rich in phenomenology giving some hints of the NP [12–16]. In some senses, they might be even more interesting and sophisticated than NP because of the mixture of $K_{1A}$ and $K_{1B}$, where $K_{1A}$ and $K_{1B}$ are the members of two axial-vector SU(3) octet $^3P_1$ and $^1P_1$ states, respectively. The physical states $K_1(1270)$ and $K_1(1400)$ can be obtained by the mixing of $K_{1A}$ and $K_{1B}$ as

\begin{align}
|K_1(1270)\rangle &= |K_{1A}\rangle \sin \theta_K + |K_{1B}\rangle \cos \theta_K, \\
|K_1(1400)\rangle &= |K_{1A}\rangle \cos \theta_K - |K_{1B}\rangle \sin \theta_K.
\end{align}
where the magnitude of the mixing angle $\theta_K$ has been estimated to be $34^\circ \leq |\theta_K| \leq 58^\circ$ [20–22]. Recently, from the studies of $B \to K_1(1270)\gamma$ and $\tau \to K_1(1270)\nu$, the value of $\theta_K$ has been estimated to be $\theta_K = -(34 \pm 13)^\circ$, where the minus sign of $\theta_K$ is related to the chosen phase of $|K_{1A}\rangle$ and $|K_{1B}\rangle$ [17,18]. Getting an independent confirmation of this value of the mixing angle $\theta_K$ is by itself interesting. It has already been pointed out that this particular choice suppresses the branching ratio (BR) for $K_1(1400)$ in the final state compared to $K_1(1270)$, which can be tested in some ongoing and future experiments [19].

There exist extensive studies showing that observables such as the branching ratio, the forward–backward asymmetry ($A_{FB}$), lepton polarization asymmetries ($P_{li}$), and the helicity fractions of the final-state meson $f_{L,T}$ for semileptonic $B$ decays are greatly influenced in different scenarios beyond the SM [12–16]. Therefore, the precise measurement of these observables will play an important role in the indirect searches for NP and possibly the signatures of the THDM. The purpose of the present study is to address this question, i.e., to investigate the possibility of searching for NP due to variants of THDMs in $B \to K_1(1270, 1400)\ell^+\ell^-$ decays with $\ell = \mu, \tau$ through forward–backward asymmetry and lepton polarization asymmetry.

The manifestation of the NP due to the THDM is twofold in the sense that it modifies the Wilson coefficients as well as introducing new operators in the effective Hamiltonian in addition to the SM operators. In the present study, the NP effects are analyzed by studying the forward–backward asymmetry $A_{FB}$ and the lepton polarization asymmetries for $B \to K_1(1270)\ell^+\ell^-$ decays in all three THDM variants, namely, models I, II, and III.

This paper is organized as follows: In Sect. 2, we fill our toolbox with the theoretical framework needed to study the said process in the THDM. In Sect. 2.1, we present the mixing of $K_1(1270)$ and $K_1(1400)$ and the form factors used in this study. In Sect. 3, we discuss the observables of $B \to K_1\ell^+\ell^-$ in detail, and, in Sect. 4, present a numerical analysis of our observables and discuss the sensitivity of these observables to the THDM parameters. We conclude the findings of the present study in Sect. 5.

2. Theoretical framework

At quark level, the semileptonic decays $B \to K_1(1270, 1400)\ell^+\ell^-$ are governed by the transition $b \to s\ell^+\ell^-$ for which the general effective Hamiltonian in the SM and THDM can be written, after integrating out the heavy degrees of freedom in the full theory, as [23]:

$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu) + \sum_{i=1}^{10} C_{Q_i}(\mu) Q_i(\mu) ,$$

(2)

where $O_i(\mu)$ ($i = 1, 2, \ldots, 10$) are the four quark operators and $C_i(\mu)$ are the corresponding Wilson coefficients at the energy scale $\mu$, which is usually taken to be the $b$-quark mass ($m_b$). The theoretical uncertainties related to the renormalization scale can be reduced when the next-to-leading logarithm corrections are included. Also, the contribution from the charged Higgs boson in the case of the THDM is absorbed in these Wilson coefficients. The new operators $Q_i$ ($i = 1, 2, \ldots, 10$) come from the neutral Higgs boson (NHB) exchange diagrams, whose manifest forms and corresponding Wilson coefficients can be found in Ref. [23]; at the scale $\mu = m_W$, these can be
summarized as [11]:

\[ C_{Q_i}(m_W) = \frac{m_{h_i}}{m_{A_0}^2} \frac{1}{|\lambda_{tH1}|^2} \frac{1}{\sin^2 \theta_W} \left[ x \left( \sin^2 \alpha + h \cos^2 \alpha \right) f_1(x, y) + \left( \frac{m_{h_0}^2}{m_W^2} + \sin^2 \alpha + h \cos^2 \alpha \right) (1 - z) \right] f_2(x, y) + \frac{\sin^2 2\alpha}{2m_{H^\pm}^2} \left[ m_{h_0}^2 - \frac{m_{h_0}^2 - m_{H^0}^2}{2m_W^2} \right] f_3(y) \]

(3)

\[ C_{Q_2}(m_W) = \frac{m_{h_0}}{m_{A_0}^2} \frac{1}{|\lambda_{tH1}|^2} f_1(x, y) + \left[ 1 + \frac{m_{H^\pm}^2 - m_{A_0}^2}{2m_W^2} \right] f_2(x, y) \]

(4)

\[ C_{Q_i}(m_W) = \frac{m_{h_0}^2}{m_{A_0}^2} \left[ C_{Q_i}(m_W) + C_{Q_2}(m_W) \right] \]

(5)

\[ C_{Q_i}(m_W) = \frac{m_{h_0}^2}{m_{A_0}^2} \left[ C_{Q_i}(m_W) - C_{Q_2}(m_W) \right] \]

(6)

\[ C_{Q_i}(m_W) = 0 \quad i = 5, \ldots, 10 \]

(7)

where

\[ x = \frac{m_{h_0}^2}{m_W^2}, \quad y = \frac{m_{h_0}^2}{m_{H^\pm}^2}, \quad z = \frac{x}{y}, \quad h = \frac{m_{h_0}^2}{m_{H^0}^2}, \]

\[ f_1(x, y) = \frac{x \ln x - y \ln y}{x - 1} - \frac{y \ln y}{y - 1}, \]

\[ f_2(x, y) = \frac{x \ln y - \ln z}{(z - x)(x - 1)} - \frac{\ln z}{(z - 1)(x - 1)}, \]

\[ f_3(y) = \frac{1 - y + y \ln y}{(y - 1)^2}. \]

(8)

The evolution of the coefficients \( C_{Q_1} \) and \( C_{Q_2} \) is performed by the anomalous dimensions of \( Q_1 \) and \( Q_2 \), respectively:

\[ C_{Q_i}(m_b) = \gamma_{Q_i}^{\gamma^5/m_0} C_{Q_i}(m_W), \quad i = 1, 2 \]

(9)

where \( \gamma_{Q_i} = -4 \) is an anomalous dimension of the operator \( s_{LBR} \).

The explicit forms of the operators responsible for the decay \( B \to K_1(1270, 1400)\ell^+\ell^- \), in the SM and THDMs, are

\[ O_7 = \frac{e^2}{16\pi^2} (m_b - m_\pi) (s_\sigma_{\mu\nu} Rb) F_{\mu\nu} \]

(10a)

\[ O_9 = \frac{e^2}{16\pi^2} (s_\gamma_{\mu\nu} Lb) \tilde{\ell} \gamma_{\mu\nu} \ell \]

(10b)

\[ O_{10} = \frac{e^2}{16\pi^2} (s_\gamma_{\mu\nu} Lb) \tilde{\ell} \gamma_{\mu\nu} \gamma^5 \ell \]

(10c)

\[ Q_1 = \frac{e^2}{16\pi^2} (s Rb) \tilde{\ell} \ell \]

(10d)

\[ Q_2 = \frac{e^2}{16\pi^2} (s Rb) \tilde{\ell} \gamma^5 \ell \]

(10e)

with \( L, R = \frac{1}{2} (1 \mp \gamma^5) \).
Using the effective Hamiltonian given in Eq. (2), the free quark amplitude for $b \rightarrow s \ell^+ \ell^-$ can be written as

$$
\mathcal{M}(b \rightarrow s \ell^+ \ell^-) = -\frac{G_F \alpha}{\sqrt{2} \pi} V_{tb} V_{ts}^* \left[ \tilde{C}_7^{\text{eff}}(\mu) (\bar{s} \gamma_\mu L b)(\bar{\ell} \gamma^\mu \ell) + \tilde{C}_{10}(\bar{s} \gamma_\mu L b)(\bar{\ell} \gamma^\mu \gamma^5 \ell) \\
- 2 \tilde{C}_7^{\text{eff}}(\mu) \frac{m_b}{s} (\bar{s} i \sigma_{\mu \nu} q^\nu R b) \bar{\ell} \gamma^\mu \ell + C_{Q_1}(\bar{s} R b) (\bar{\ell} c) C_{Q_2}(\bar{s} R b) (\bar{\ell} \gamma^5 \ell) \right],
$$

(11)

where $q$ is the momentum transfer. By using the knowledge of the Wilson coefficients $C_7$, $\tilde{C}_9$, and $\tilde{C}_{10}$ calculated at scale $m_W$, the Wilson coefficients $\tilde{C}_7^{\text{eff}}$, $\tilde{C}_9^{\text{eff}}$, $\tilde{C}_{10}$, $C_{Q_1}$, and $C_{Q_2}$ are calculated at the scale $m_b$. After adding the contribution from the charged Higgs diagrams to the SM results, the Wilson coefficients $\tilde{C}_7^{\text{eff}}$, $\tilde{C}_9^{\text{eff}}$, and $\tilde{C}_{10}$ can take the form [11,23]:

$$
\tilde{C}_7(m_W) = C_7^{\text{SM}}(m_W) + |\lambda_{tt}|^2 \left( \frac{y(7 - 5y - 8y^2)}{72(y - 1)^3} + \frac{y^2(3y - 2)}{12(y - 1)^4} \ln y \right) \\
+ \lambda_{tt} \lambda_{bb} \left( \frac{y(3 - 5y)}{12(y - 1)^2} + \frac{y(3y - 2)}{6(y - 1)^3} \ln y \right),
$$

(12)

$$
\tilde{C}_9(m_W) = C_9^{\text{SM}}(m_W) + |\lambda_{tt}|^2 \left[ \frac{1 - 4 \sin^2 \theta_W x y}{\sin^2 \theta_W} \frac{1}{8} \left( \frac{1}{y - 1} - \frac{1}{(y - 1)^2} \ln y \right) \\
- y \left( \frac{47y^2 - 79y + 38}{108(y - 1)^3} - \frac{3y^3 - 6y^2 + 4}{18(y - 1)^4} \right) \right],
$$

(13)

$$
\tilde{C}_{10}(m_W) = C_{10}^{\text{SM}}(m_W) + |\lambda_{tt}|^2 \frac{1}{\sin^2 \theta_W} \frac{xy}{8} \left( \frac{1}{y - 1} + \frac{1}{(y - 1)^2} \ln y \right).
$$

(14)

It can be easily seen that, in the limit $y \rightarrow 0$, along with $C_{Q_{1,2}} \rightarrow 0$, the SM results of the Wilson coefficients can be recovered.

Note that the operator $O_{10}$ given in Eq. (10c) cannot be induced by the insertion of four quark operators because of the absence of $Z$-bosons in the effective theory. Therefore, the Wilson coefficient $C_{10}$ does not renormalize under QCD corrections and is independent of the energy scale $\mu$. Additionally, the above quark-level decay amplitude can get contributions from the matrix element of the four quark operators, $\sum_{i=1}^{6} \langle \bar{\ell}^+ \ell^- s | O_i | b \rangle$, which are usually absorbed into the effective Wilson coefficient $C_9^{\text{eff}}(\mu)$ and can be written as [24–27]

$$
\tilde{C}_9^{\text{eff}}(\mu) = \tilde{C}_9(\mu) + Y_{\text{SD}}(z, s') + Y_{\text{LD}}(z, s'),
$$

where $z = m_c/m_b$ and $s' = s/m_b^2$. $Y_{\text{SD}}(z, s')$ describes the short-distance contributions and the long-distance contribution is $Y_{\text{LD}}(z, s')$. The manifest expressions of these contributions are given as:

$$
Y_{\text{SD}}(z, s') = h(z, s') \left[ 3C_1(\mu) + C_2(\mu) + 3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu) \right] \\
- \frac{1}{2} h(1, s') \left[ 4C_3(\mu) + 4C_4(\mu) + 3C_5(\mu) + C_6(\mu) \right] - \frac{1}{2} h(0, s') \left[ 3C_1(\mu) + 3C_4(\mu) \right] \\
+ \frac{2}{5} \left[ 3C_5(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu) \right],
$$

(15)

$$
Y_{\text{LD}}(z, s') = \frac{3}{\alpha_s(z)} \left( 3C_1(\mu) + C_2(\mu) + 3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu) \right) \\
\times \sum_{j=\psi, \psi'} \omega_j(s) k_j \frac{\pi \Gamma(j \rightarrow l^+ l^-) M_j}{s - M_j^2 + i M_j \Gamma_{\text{tot}}^j},
$$

(16)
with

\[ h(z, s') = -\frac{8}{9} \ln z + \frac{8}{27} + \frac{4}{9} x - \frac{2}{9} (2 + x) |1 - x|^{1/2} \times \begin{cases} \
\ln \left| \frac{\sqrt{1 - x} + 1}{\sqrt{1 - x} - 1} \right| - i\pi & \text{for } x \equiv 4z^2/s' < 1 \\
2 \arctan \frac{1}{\sqrt{x - 1}} & \text{for } x \equiv 4z^2/s' > 1 
\end{cases} \]

\[ h(0, s') = -\frac{8}{27} - \frac{8}{9} \ln \frac{m_b}{\mu} - \frac{4}{9} \ln s' + \frac{4}{9} i\pi. \]  

(17)

Here \( M_j(\Gamma^\text{tot}_j) \) are the masses (widths) of the intermediate resonant states and \( \Gamma(j \rightarrow l^+l^-) \) denotes the partial decay width for the transition of the vector charmonium state to a massless lepton pair, which can be expressed in terms of the decay constant of charmonium through the relation [28]

\[ \Gamma(j \rightarrow \ell^+\ell^-) = \frac{\pi}{16} f_j^2 \frac{M_j}{\mathcal{A}_\text{em}}. \]

The phenomenological parameter \( k_j \) in Eq. (16) is to account for the inadequacies of the factorization approximation, and it can be determined from

\[ \mathcal{B}R(B \rightarrow K J/\psi \rightarrow K_1 \ell^+\ell^-) = \mathcal{B}R(B \rightarrow K_1 J/\psi) \cdot \mathcal{B}R(J/\psi \rightarrow \ell^+\ell^-). \]

The function \( \omega_j(s) \) introduced in Eq. (16) is to compensate for the naive treatment of long-distance contributions due to the charm-quark loop in the spirit of quark–hadron duality, which can significantly overestimate the genuine effect of the charm quark at small \( s \).\(^1\) The quantity \( \omega_j(s) \) can be normalized to \( \omega_j(M^2_{\psi_i}) = 1 \), but its exact form is unknown at present. Since the dominant contribution of the resonances is in the vicinity of the intermediate \( \psi_i \) masses, we will simply use \( \omega_j(s) = 1 \) in our numerical calculations. In addition, for the resonances, \( J/\psi \) and \( \psi' \) are taken to be \( \kappa = 1.65 \) and \( \kappa = 2.36 \), respectively [29].

Moreover, the nonfactorizable effects from the charm-quark loop brings further corrections to the radiative transition \( b \rightarrow s \gamma \), and these can be absorbed into the effective Wilson coefficient \( C_7^{\text{eff}} \), which then takes the form [28,30–34]

\[ C_7^{\text{eff}}(\mu) = C_7(\mu) + C_{b \rightarrow s \gamma}(\mu) \]

with

\[ C_{b \rightarrow s \gamma}(\mu) = i\alpha_s \left[ \frac{2}{9} \eta^{14/23} (G_1(x_t) - 0.1687) - 0.03 C_2(\mu) \right] \]  

(18)

\[ G_1(x_t) = \frac{x_t (x_t^2 - 5x_t - 2)}{8 (x_t - 1)^3} + \frac{3x_t^2 \ln^2 x_t}{4 (x_t - 1)^4}, \]  

(19)

where \( \eta = \alpha_s(m_W)/\alpha_s(\mu), x_t = m_t^2/m_W^2 \), and \( C_{b \rightarrow s \gamma} \) is the absorptive part for the \( b \rightarrow s c \bar{c} \rightarrow s \gamma \) rescattering.

\(^1\) For extensive discussions on long-distance and short-distance contributions from the charm loop, one can refer to Refs. [28,30,35–39].
2.1. Form factors and mixing of $K_1(1270)$–$K_1(1400)$

The exclusive $B \to K_1(1270, 1400)\bar{\ell}^+\bar{\ell}^-$ decays involve the hadronic matrix elements of quark operators given in Eq. (11). The different matrix elements can be parametrized in terms of the form factors as:

$$\begin{align*}
\langle K_1(k, \varepsilon) | V_\mu | B(p) \rangle &= \varepsilon^*_\mu (M_B + M_{K_1}) V_1(s) - (p + k)_\mu (\varepsilon^* \cdot q) \frac{V_2(s)}{M_B + M_{K_1}} \\
&\quad - q_\mu (\varepsilon \cdot q) \frac{2M_{K_1}}{s} [V_3(s) - V_0(s)] \\
\langle K_1(k, \varepsilon) | A_\mu | B(p) \rangle &= \frac{2i\epsilon_{\mu\nu\alpha\beta}}{M_B + M_{K_1}} \varepsilon^{\nu\beta} p^\alpha k^\beta A(s) \\
\langle K_1(k, \varepsilon) | S | B(p) \rangle &= \pm \frac{2M_{K_1}}{m_b + m_s} (\varepsilon^* \cdot p) V_0(s),
\end{align*}$$

(20)

(21)

(22)

where $V_\mu = \bar{s}\gamma_\mu b$, $A_\mu = \bar{s}\gamma_\mu\gamma_5 b$, and $S = \bar{s}(1 \pm \gamma_5) b$ are the vector, axial-vector, and (pseudo-) scalar currents involved in the transition matrix, respectively. Also, $p(k)$ are the momenta of the $B(K_1)$ mesons, $q = p - k$ is the momentum transfer, and $\varepsilon_\mu$ corresponds to the polarization of the final-state axial-vector $K_1$ meson. In Eq. (20), we have

$$V_3(s) = \frac{M_B + M_{K_1}}{2M_{K_1}} V_1(s) - \frac{M_B - M_{K_1}}{2M_{K_1}} V_2(s),$$

(23)

with

$$V_3(0) = V_0(0).$$

In addition, there is also a contribution from the penguin form factors, which can be expressed as

$$\begin{align*}
\langle K_1(k, \varepsilon) | \bar{s}\sigma_{\mu\nu} q^\nu b | B(p) \rangle &= \left[ \left( M_B^2 - M_{K_1}^2 \right) \varepsilon_\mu - (\varepsilon \cdot q)(p + k)_\mu \right] F_2(s) \\
&\quad + (\varepsilon^* \cdot q) \left[ q_\mu - \frac{s}{M_B^2 - M_{K_1}^2} (p + k)_\mu \right] F_3(s) \\
\langle K_1(k, \varepsilon) | \bar{s}\sigma_{\mu\nu} q^\nu \gamma_5 b | B(p) \rangle &= -i\epsilon_{\mu\nu\alpha\beta} \varepsilon^{\nu\beta} p^\alpha k^\beta F_1(s),
\end{align*}$$

(24)

(25)

with $F_1(0) = 2F_2(0)$.

As the physical states $K_1(1270)$ and $K_1(1400)$ are a mixture of the $K_{1A}$ and $K_{1B}$ states with mixing angle $\theta_K$, as defined in Eqs. (1a) and (1b), we can write

$$\begin{align*}
\begin{pmatrix} \langle K_1(1270) | \bar{s}\gamma_\mu (1 - \gamma_5) b | B \rangle \\ \langle K_1(1400) | \bar{s}\gamma_\mu (1 - \gamma_5) b | B \rangle \end{pmatrix} &= M \begin{pmatrix} \langle K_{1A} | \bar{s}\gamma_\mu (1 - \gamma_5) b | B \rangle \\ \langle K_{1B} | \bar{s}\gamma_\mu (1 - \gamma_5) b | B \rangle \end{pmatrix},
\end{align*}$$

(26)

$$\begin{align*}
\begin{pmatrix} \langle K_1(1270) | \bar{s}\sigma_{\mu\nu} q^\nu (1 + \gamma_5) b | B \rangle \\ \langle K_1(1400) | \bar{s}\sigma_{\mu\nu} q^\nu (1 + \gamma_5) b | B \rangle \end{pmatrix} &= M \begin{pmatrix} \langle K_{1A} | \bar{s}\sigma_{\mu\nu} q^\nu (1 + \gamma_5) b | B \rangle \\ \langle K_{1B} | \bar{s}\sigma_{\mu\nu} q^\nu (1 + \gamma_5) b | B \rangle \end{pmatrix},
\end{align*}$$

(27)

where the mixing matrix $M$ is

$$M = \begin{pmatrix} \sin \theta_K & \cos \theta_K \\ \cos \theta_K & -\sin \theta_K \end{pmatrix}.$$
With these definitions, the corresponding form factors \( A_{K_1} \), \( V_{0,1,2} \), and \( F_{0,1,2} \) in \( B \to K_1 \) can be parametrized in terms of the following relations:

\[
\begin{align*}
\frac{A_{K_1}(1270)}{m_B + m_{K_1}(1270)} & = M \left( \frac{A_{K_1A}}{m_B + m_{K_1A}} \right), \\
\frac{V_1^{K_1(1270)}}{m_B + m_{K_1(1270)}} & = M \left( \frac{V_1^{K_1A}}{m_B + m_{K_1A}} \right), \\
\frac{V_2^{K_1(1270)}}{m_B + m_{K_1(1270)}} & = M \left( \frac{V_2^{K_1A}}{m_B + m_{K_1A}} \right), \\
\frac{F_1^{K_1(1270)}}{m_B + m_{K_1(1270)}} & = M \left( \frac{F_1^{K_1A}}{m_B + m_{K_1A}} \right), \\
\frac{F_2^{K_1(1270)}}{m_B + m_{K_1(1270)}} & = M \left( \frac{F_2^{K_1A}}{m_B + m_{K_1A}} \right), \\
\frac{F_3^{K_1(1270)}}{m_B + m_{K_1(1270)}} & = M \left( \frac{F_3^{K_1A}}{m_B + m_{K_1A}} \right),
\end{align*}
\]

where we have supposed that \( k_{K_1}^{K_1(1270),K_1(1400)} \approx k_{K_1A}^{K_1A,K_1B} \).

For the numerical analysis, we have used the light-cone QCD sum-rule form factors [40], summarized in Table 1, where the momentum dependence dipole parametrization is:

\[
T_i^X(s) = \frac{T_i^X(0)}{1 - a_i^X (s/m_B^2) + b_i^X (s/m_B^2)^2},
\]

where \( T \) is the \( A, V, \) or \( F \) form factors and the subscript \( i \) can take a value 0, 1, 2, or 3; the superscript \( X \) belongs to the \( K_1A \) or \( K_1B \) state.

From Eq. (11), one can get the decay amplitudes for \( B \to K_1(1270)\ell^+\ell^- \) as

\[
\mathcal{M}(B \to K_1(1270)\ell^+\ell^-) = -\frac{G_F \alpha}{2\sqrt{2} \pi} V_{tb} V_{ts}^* \left[ T_A^\mu \bar{\tau}_\mu \ell + T_A^\mu \bar{\tau}_\mu \gamma_5 \ell + T_S \bar{\ell} \ell \right],
\]

where the functions \( T_A^\mu \) and \( T_V^\mu \) can be written in terms of matrix elements as:

\[
T_A^\mu = \tilde{C}_{10} \left( K_1(k, e) \right) \bar{\ell} \gamma^\mu \left( 1 - \gamma^5 \right) b \mid B(p) \rangle,
\]

\[
T_V^\mu = \tilde{C}_{9} \left( K_1(k, e) \right) \bar{\ell} \gamma^\mu \left( 1 - \gamma^5 \right) b \mid B(p) \rangle,
\]

\[
\tilde{C}_{9}^{\text{eff}} \Rightarrow 2im_b \left( K_1(k, e) \right) \bar{\ell} \sigma^{\mu\nu} \left( 1 + \gamma^5 \right) q_\nu b \mid B(p) \rangle,
\]

\[
- \tilde{C}_{9}^{\text{eff}} \Rightarrow 2im_b \left( K_1(k, e) \right) \bar{\ell} \sigma^{\mu\nu} \left( 1 + \gamma^5 \right) q_\nu b \mid B(p) \rangle,
\]

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which will take the form

\[
T_{\mu}^V = f_1 \epsilon^{\mu
u
\rho
\sigma} \epsilon_{\nu}^* p_{\rho} k_{\sigma} - i f_2 \epsilon^{\nu \mu} f_3 (q \cdot \epsilon) (p^\mu + k^\mu)
\] (40)

\[
T_A^\mu = f_3 \epsilon^{\mu
\nu
\rho
\sigma} \epsilon_{\nu}^* p_{\rho} k_{\sigma} + i f_5 (q \cdot \epsilon) (p^\mu + k^\mu) + i f_7 (q \cdot \epsilon) q^\mu
\] (41)

\[
T_S = 2i f_8 (q \cdot \epsilon^* \cdot q).
\] (42)

The auxiliary functions appearing in Eqs. (40) and (41) are defined as:

\[
f_1 = 4(m_b + m_s) \frac{\tilde{C}_7^{\text{eff}}}{s} \left[ F_1^{K_{1A}} \sin \theta_K + F_1^{K_{1B}} \cos \theta_K \right] + 2 \tilde{C}_9^{\text{eff}} \left[ \frac{A_1^{K_{1A}} \sin \theta_K}{m_B + m_{K_{1A}}} + \frac{A_1^{K_{1B}} \cos \theta_K}{m_B + m_{K_{1B}}} \right]
\] (43)

\[
f_2 = 2(m_b + m_s) \frac{\tilde{C}_7^{\text{eff}}}{s} \left[ \left( m_B^2 - m_{K_{1A}}^2 \right) F_2^{K_{1A}} \sin \theta_K + \left( m_B^2 - m_{K_{1B}}^2 \right) F_2^{K_{1B}} \cos \theta_K \right] + \tilde{C}_9^{\text{eff}} \left[ (m_B + m_{K_{1A}}) V_1^{K_{1A}} \sin \theta_K + (m_B + m_{K_{1B}}) V_1^{K_{1B}} \cos \theta_K \right]
\] (44)

\[
f_3 = 2(m_b + m_s) \frac{\tilde{C}_7^{\text{eff}}}{s} \left[ \left( F_2^{K_{1A}} + \frac{s F_3^{K_{1A}}}{m_B^2 - m_{K_{1A}}^2} \right) \sin \theta_K + \left( F_2^{K_{1B}} + \frac{s F_3^{K_{1B}}}{m_B^2 - m_{K_{1B}}^2} \right) \cos \theta_K \right] + \tilde{C}_9^{\text{eff}} \left[ \frac{A_1^{K_{1A}} \sin \theta_K}{m_B + m_{K_{1A}}} + \frac{A_1^{K_{1B}} \cos \theta_K}{m_B + m_{K_{1B}}} \right]
\] (45)

\[
f_4 = 2 \tilde{C}_9^{\text{eff}} \left[ \frac{A_1^{K_{1A}} \sin \theta_K}{m_B + m_{K_{1A}}} + \frac{A_1^{K_{1B}} \cos \theta_K}{m_B + m_{K_{1B}}} \right]
\] (46)

\[
f_5 = \tilde{C}_{10} \left[ (m_B + m_{K_{1A}}) V_1^{K_{1A}} \sin \theta_K + (m_B + m_{K_{1B}}) V_1^{K_{1B}} \cos \theta_K \right]
\] (47)

\[
f_6 = \tilde{C}_{10} \left[ \frac{V_2^{K_{1A}} \sin \theta_K}{m_B + m_{K_{1A}}} + \frac{V_2^{K_{1B}} \cos \theta_K}{m_B + m_{K_{1B}}} \right]
\] (48)

\[
f_7 = \frac{2}{s} \left[ m_{K_{1A}} \left( V_3^{K_{1A}} - V_0^{K_{1A}} \right) \sin \theta_K + m_{K_{1B}} \left( V_3^{K_{1B}} - V_0^{K_{1B}} \right) \cos \theta_K \right]
\] (49)

\[
f_8 = -C_{Q_2} \left[ \frac{-m_{K_{1A}} V_0^{K_{1A}} \sin \theta_K + m_{K_{1B}} V_0^{K_{1B}} \cos \theta_K}{m_e (m_b + m_s)} \right]
\] (50)
3. Physical observables for $B \to K_1 \ell^+ \ell^-$

In this section, we will present the calculations of the physical observables such as the branching ratios $BR$, the forward–backward asymmetries $\mathcal{A}_{FB}$, and the lepton polarization asymmetries for the decays $B \to K_1 \ell^+ \ell^-$. 

3.1. Branching ratio

The double differential decay rate for $B \to K_1 \ell^+ \ell^-$ can be written as [17,18]

$$\frac{d\Gamma(B \to K_1 \ell^+ \ell^-)}{ds} = \frac{1}{(2\pi)^3} \frac{1}{32M_B^3} \int_{-u(s)}^{+u(s)} du |\mathcal{M}|^2,$$  

(51)

where

$$s = (p_{l^+} + p_{l^-})^2$$  

(52)

$$u = (p - p_{l^+})^2 - (p - p_{l^-})^2.$$  

(53)

Now the limits on $s$ and $u$ are

$$4m^2 \leq s \leq (M_B - M_{K_1})^2$$  

(54)

$$-u(s) \leq u \leq u(s)$$  

(55)

with

$$u(s) = \sqrt{\lambda \left(1 - \frac{4m^2}{s}\right)}$$  

(56)

and

$$\lambda \equiv \lambda \left(M_B^2, M_{K_1}^2, s\right) = M_B^4 + M_{K_1}^4 + q^4 - 2M_B^2M_{K_1}^2 - 2M_{K_1}^2s - 2sM_B^2.$$  

(57)

Here, $m$ corresponds to the mass of the leptons, which, for our case, are $\mu$ and $\tau$. The total decay rate for the decay $B \to K_1 \ell^+ \ell^-$ can be expressed as

$$\frac{d\Gamma}{ds} = \frac{G_F^2 |V_{tb}V_{ts}^*|^2 \alpha^2}{2^{11}\pi^5 3M_B^3M_{K_1}^3} u(s) \times \mathcal{A}(s).$$  

(58)

The function $u(s)$ is defined in Eq. (56) and $\mathcal{M}(s)$ can be parametrized as

$$\mathcal{A}(s) = 8M_{K_1}^2 \lambda \left\{ \left(2m^2 + s\right) |f_1(s)|^2 - \left(4m^2 - s\right) |f_4(s)|^2 \right\}$$

$$+ 4M_{K_1}^2 s \left\{ \left(2m^2 + s\right) \left(3 |f_2(s)|^2 - \lambda |f_3(s)|^2\right) - \left(4m^2 - s\right) \left(3 |f_5(s)|^2 - \lambda |f_6(s)|^2\right) \right\}$$

$$+ \lambda \left(2m^2 + s\right) \left|f_2(s) + \left(M_B^2 - M_{K_1}^2 - s\right) f_3(s)\right|^2 + 24m^2M_{K_1}^2 \lambda |f_7(s)|^2$$

$$- \left(4m^2 - s\right) \left|f_5(s) + \left(M_B^2 - M_{K_1}^2 - s\right) f_6(s)\right|^2 + \left(s - 4m^2\right) \lambda |f_8(s)|^2$$

$$- 12m^2s \left[ \Re \left(f_5 f_7^*\right) - \Re \left(f_6 f_8^*\right) \right].$$  

(59)

It is also very useful to define the ratio of the branching fractions ($R_\ell$) as:

$$R_\ell = \frac{BR(B \to K_1(1400)\ell^+\ell^-)}{BR(B \to K_1(1270)\ell^+\ell^-)},$$  

(59)

where $\ell = \mu, \tau$. 

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3.2. Forward–backward asymmetries

In this section, we investigate the forward–backward asymmetry ($A_{FB}$) of leptons. In the context of THD models, the $A_{FB}$ can also play a crucial role in $B \rightarrow K_1 \ell^+ \ell^-$ transitions. The differential $A_{FB}$ of final-state leptons for the said decays can be written as

$$\frac{dA_{FB}(s)}{ds} = \int_0^1 \frac{d^2\Gamma}{dsd\cos\theta} d\cos\theta - \int_{-1}^0 \frac{d^2\Gamma}{dsd\cos\theta} d\cos\theta.$$  

(60)

From the experimental point of view, the normalized forward–backward asymmetry is more useful, i.e.,

$$A_{FB} = \frac{\int_0^1 \frac{d^2\Gamma}{dsd\cos\theta} d\cos\theta - \int_{-1}^0 \frac{d^2\Gamma}{dsd\cos\theta} d\cos\theta}{\int_{-1}^1 \frac{d^2\Gamma}{dsd\cos\theta} d\cos\theta}.$$

The normalized $A_{FB}$ for $B \rightarrow K_1 \ell^+ \ell^-$ can be obtained from Eq. (51) as

$$A_{FB} = \frac{1}{d\Gamma/ds} \frac{G_F^2 \alpha^2}{2\pi^5 m_B} |V_{tb}V_{ts}^*|^2 s u(s)$$

$$\times \left\{ 4\Re[f_4 f_7 + f_6 f_8] + 2\Re[f_3 f_7] + 4\Re[f_2 f_8] \left( -m_{B^*}^2 + m_{D_s}^2 + s \right) \right\},$$

(61)

where $d\Gamma/ds$ is given in Eq. (57).

3.3. Single lepton polarization asymmetries

In the rest frame of the lepton and antilepton, the unit vectors along the longitudinal, normal, and transversal components of the lepton polarization to the c.m. frame of the lepton pair as

$$s_L^- = (0, \vec{e}_L) = \left(0, \frac{\vec{p}_-}{|\vec{p}_-|}\right),$$

(62a)

$$s_N^- = (0, \vec{e}_N) = \left(0, \frac{\vec{k} \times \vec{p}_-}{|\vec{k} \times \vec{p}_-|}\right),$$

(62b)

$$s_T^- = (0, \vec{e}_T) = \left(0, \vec{e}_N \times \vec{e}_L\right),$$

(62c)

where $\vec{p}_-$ and $\vec{k}$ are the three-momenta of the lepton $\ell^-$ and $K_1$ meson, respectively, in the center-of-mass (c.m.) frame of the $\ell^+ \ell^-$ system. Lorentz transformation is used to boost the longitudinal component of the lepton polarization to the c.m. frame of the lepton pair as

$$\left(s_L^-\right)_{\text{c.m.}} = \left(\frac{|\vec{p}_-|}{m}, \frac{E\vec{p}_-}{m|\vec{p}_-|}\right),$$

(63)

where $E$ and $m$ are the energy and mass of the lepton. The normal and transverse components remain unchanged under the Lorentz boost. The longitudinal ($P_L$), normal ($P_N$), and transverse ($P_T$) polarizations of the lepton can be defined as:

$$P_i^{(\pm)}(s) = \frac{d\Gamma}{ds} \left( \xi^{(\pm)} = \pm \vec{e}_i \right) - \frac{d\Gamma}{ds} \left( \xi^{\mp} = -\vec{e}_i \right).$$

(64)

where $i = L, N, T$ and $\xi^{\pm}$ is the spin direction along the leptons $\ell^{\pm}$. The differential decay rate for a polarized lepton $\ell^{\pm}$ in $B \rightarrow K_1 \ell^{\pm} \ell^-$ decay along any spin direction $\xi^{\pm}$ is related to the unpolarized
decay rate \((57)\) with the following relation:

\[
\frac{d\Gamma(\vec{\xi}^\pm)}{ds} = \frac{1}{2} \left( \frac{d\Gamma}{ds} \right) \left[ 1 + \left( P_L^\pm \vec{e}_L^\pm + P_N^\pm \vec{e}_N^\pm + P_T^\pm \vec{e}_T^\pm \right) \cdot \vec{\xi}^\pm \right].
\]

(65)

The expressions of the longitudinal, normal, and transverse lepton polarizations can be written as

\[
P_L(s) \propto \frac{4\lambda}{3M_{K_1}^2} \sqrt{\frac{s-4m^2}{s}} \times \left\{ 2\Im(f_2f_5^*) + \lambda \Im(f_3f_6^*) + 4\sqrt{s} \Re(f_1f_4^*) \left( 1 + \frac{12sM_{K_1}^2}{\lambda} \right) \right.
\]
\[
+ \left( -M_B^2 + M_{K_1}^2 + s \right) \left[ \Re(f_3f_6^*) + \Re(f_2f_5^*) \right]
\]
\[
+ \frac{3}{2}m_{\ell}\left[ \Re(f_5f_8^*) + \Re(f_6f_8^*) \left( -M_B^2 + M_{K_1}^2 \right) - \Re(f_7f_8^*) \right] \right\}
\]

(66)

\[
P_N(s) \propto \frac{m_{\ell}\pi}{M_{K_1}^2} \sqrt{\frac{s}{s-4m^2}} \times \left\{ -\lambda s \Re(f_3f_6^*) + \lambda \left( M_B^2 - M_{K_1}^2 \right) \Re(f_3f_6^*) - \lambda \Re(f_3f_5^*) \right.
\]
\[
+ \left( -M_B^2 + M_{K_1}^2 + s \right) \left[ s \Re(f_2f_5^*) + \left( M_B^2 - M_{K_1}^2 \right) \Re(f_2f_5^*) + \left( s - 4m^2 \right) \Re(f_5f_8^*) \right]
\]
\[
- 8sM_{K_1}^2 \Re(f_1f_2^*) + \sqrt{\lambda} \left( s - 4m^2 \right) \Re(f_6f_8^*) \right\}
\]

(67)

\[
P_T(s) \propto i \frac{m_{\ell}\pi}{M_{K_1}^2} \sqrt{\frac{s-4m^2}{s}} \lambda \left\{ M_{K_1} \left[ 4\Im(f_2f_4^*) + 4\Im(f_1f_3^*) + 3\Im(f_5f_6^*) \right] - \lambda \Im(f_6f_2^*) \right.
\]
\[
+ \left( -M_B^2 + M_{K_1}^2 + s \right) \left[ \Im(f_5f_8^*) + \Im(f_2f_5^*) \right] - s \Im(f_5f_8^*) \right\}
\]

(68)

where the auxiliary functions \(f_1, f_2, \ldots, f_8\) are defined in Eqs. \((43)\)–\((50)\). Here we have dropped the constant factors that are, however, understood.

4. Numerical results and discussion

In order to perform a numerical analysis of the forward–backward asymmetry \((A_{FB})\) and the lepton polarization asymmetries \(P_{L,N,T}\) for the \(B \rightarrow K_1(1270)\ell^+\ell^-\) decays, with \(\ell = \mu, \tau\), we first give the numerical values of the input parameters and the SM Wilson coefficients that are used in our numerical calculations in Tables 2 and 3, respectively. In principle, the asymmetries listed above can also be studied when we have a \(K_1(1400)\) meson instead of a \(K_1(1270)\) meson in the final state. It has already been pointed in the literature \([19]\) that the branching ratio of \(B \rightarrow K_1(1400)\ell^+\ell^-\) is an order of magnitude smaller than its partner \(B \rightarrow K_1(1270)\ell^+\ell^-\) decay; therefore, we will limit our study to the case when the \(K_1(1270)\) meson occurs in the final state.

Of course, to perform the numerical analysis, another important ingredient is the form factors. The values of the form factors used in the upcoming analysis are the ones calculated using the QCD sum rules; they are summarized in Table 1.

Coming to the THDM, the free parameters in these models are the masses of the charged Higgs boson \(m_{H^\pm}\), the coefficients \(\lambda_{tt}, \lambda_{bb}\), and the ratio of the vacuum expectation values of the two Higgs
experimental results for the branching ratio of $b \to S\gamma$ and $B \to D\ell\nu_{\ell}$ decays as well as $B \to \bar{B}$ and $K \to \bar{K}$ mixing in the literature [49]. In addition, the parameters $|\lambda_{tt}|, |\lambda_{bb}|$ and the phase $\delta$ are restricted by experimental results for the electric dipole moment of the neutron, $B \to \bar{B}$ mixing, $\rho_0, R_b$, and $Br (b \to S\gamma)$ [50–53]. The value of $\lambda_{tt} \lambda_{bb}$ is constrained to be 1 and $\delta$ is restricted to the range 60–90° by using the experimental limits on the electric dipole moment of the neutron and $Br (b \to S\gamma)$, plus the constraint on $M_{H^+}$ from the LEP II. Using the constraints from $B \to \bar{B}$ mixing as well as from $R_b$, an analysis of various lepton polarization asymmetries in $B \to K^0_s \ell^+ \ell^-$ has been done in the following parametric space in model III [11]:

\begin{align*}
\text{Case A} : & \quad |\lambda_{tt}| = 0.03, \quad |\lambda_{bb}| = 100, \\
\text{Case B} : & \quad |\lambda_{tt}| = 0.15, \quad |\lambda_{bb}| = 50, \\
\text{Case C} : & \quad |\lambda_{tt}| = 0.3, \quad |\lambda_{bb}| = 30,
\end{align*}

where $\delta = \pi/2$ and the values of the masses of the Higgs particles are summarized in Table 4.

It is an established fact that, in type-II THDM, the charged Higgs contribution to $B \to \tau\nu$ interferes necessarily destructively with the SM [54]. The enhancement of $Br (B \to \tau\nu)$ is possible if the absolute value of the contribution of the charged Higgs boson is two times the SM one, but then it is in conflict with $B \to D\tau\nu$. Furthermore, this version of THDM cannot explain the observed

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**Table 2.** Values of input parameters used in our numerical analysis [47].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_B$</td>
<td>5.28 GeV</td>
</tr>
<tr>
<td>$m_b$</td>
<td>4.28 GeV</td>
</tr>
<tr>
<td>$m_t$</td>
<td>1.77 GeV</td>
</tr>
<tr>
<td>$f_B$</td>
<td>0.25 GeV</td>
</tr>
<tr>
<td>$V_{tb} V_{ts}^*$</td>
<td>$45 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\alpha^{-1}$</td>
<td>137</td>
</tr>
<tr>
<td>$G_F$</td>
<td>$1.17 \times 10^{-5}$ GeV$^{-2}$</td>
</tr>
<tr>
<td>$\tau_B$</td>
<td>$1.54 \times 10^{-12}$ sec</td>
</tr>
<tr>
<td>$m_{K_{(1400)}}$</td>
<td>1.403 GeV</td>
</tr>
<tr>
<td>$\theta_K$</td>
<td>$-34^\circ$</td>
</tr>
<tr>
<td>$m_{K_{(1720)}}$</td>
<td>1.31 GeV</td>
</tr>
</tbody>
</table>

**Table 3.** The Wilson coefficients $C_i^\mu$ at the scale $\mu \sim m_b$ in the SM.

<table>
<thead>
<tr>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
<th>$C_6$</th>
<th>$C_7$</th>
<th>$C_9$</th>
<th>$C_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.107</td>
<td>-0.248</td>
<td>-0.011</td>
<td>-0.026</td>
<td>-0.007</td>
<td>-0.031</td>
<td>-0.313</td>
<td>4.344</td>
<td>-4.669</td>
</tr>
</tbody>
</table>

**Table 4.** Value of the masses of the Higgs particles.

<table>
<thead>
<tr>
<th>Masses</th>
<th>$m_{\chi^0}$</th>
<th>$m_{h^0}$</th>
<th>$m_{H^0}$</th>
<th>$m_{H^\pm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set I (GeV)</td>
<td>125</td>
<td>125</td>
<td>160</td>
<td>200</td>
</tr>
<tr>
<td>Set II (GeV)</td>
<td>125</td>
<td>125</td>
<td>160</td>
<td>160</td>
</tr>
</tbody>
</table>

discrepancy of 2.2σ in $R(D)$ and 2.7σ in $R(D^*)$ compared to their SM values. In order to resolve this situation, a detailed discussion on model III is given in Ref. [60]. The purpose of the present study is not to put precise bounds on the parameters of the various THDM versions but to check the profile of different physical observables, e.g., the lepton forward–backward asymmetry as well as the lepton polarization asymmetries in $B \rightarrow K_1 \ell^+ \ell^-$ decays.

It is important to mention here that, as an exclusive decay, there are different sources of uncertainties involved in the analysis of the above-mentioned decay. The major source of uncertainties in the numerical analysis of $B \rightarrow K_1 \ell^+ \ell^-$ ($\ell = \mu, \tau$) decays originated from the $B \rightarrow K_1$ transition form factors summarized in Table 1. However, it is also important to stress that these hadronic uncertainties have almost no influence on the various asymmetries including the forward–backward asymmetries and the lepton polarization asymmetries in $B \rightarrow K_1 \ell^+ \ell^-$ because of the cancellation among different polarization states; this makes them a good tool to probe for physics beyond the SM.

4.1. Analysis of forward–backward asymmetry

To illustrate the impact of the parametric space of the THDM on the forward–backward asymmetry $A_{FB}$, we plot $d(A_{FB})/ds$ as a function of $s$ in Fig. 1. It is argued for the zero position of $A_{FB}$ that the uncertainty in its position due to the hadronic form factors is negligible [67,68]. Therefore, the zero position of the $A_{FB}$ can serve as a stringent test for the NP effects arising from the different versions of THDM. Figures 1(a) and 1(c) describe the $A_{FB}$ for $B \rightarrow K_1 \mu^+ \mu^-$ with long-distance contributions in the Wilson coefficients for THDM types I and II, respectively. Before we discuss the attitude of
different parameters in the forward–backward asymmetry, it will be useful to have a closer look at Eqs. (12)–(14). In order to recover the SM phenomenology, one has to set the parameter $y = 0$. It can been seen that, in THDM type I, because of the different signs of $\lambda_{tt}$ and $\lambda_{bb}$, the second term in Eq. (12) makes constructive contributions whereas the third term makes a destructive contribution. In contrast to this, the effects of the Wilson coefficient $C_7$ in the type-II model are constructive and hence we expect a large deviation from the SM value in this version compared to that of type I. In $B \rightarrow K_1 \mu^+ \mu^-$ decay, Figs. 1(a) and 1(c) depict this fact. Here, we can see that, in the case of type I, the deviation of the zero position of the forward–backward asymmetry lies almost in the uncertainty band, since the only contribution of NP is coming from the Wilson coefficient $C_7$. However, in the case of type II, the zero position as well as the magnitude of the $A_{FB}$ is shifted significantly from the SM value, especially when we change the values of the charged Higgs mass in this version. Therefore, the precise measurement of the zero position of $A_{FB}$ for the decay $B \rightarrow K_1 \mu^+ \mu^-$ will be a very good observable to yield any indirect indications of NP due to the parameters of THDM and can serve as a good tool to distinguish between the different variants of it. In addition, the situation for $B \rightarrow K_1 \tau^+ \tau^-$ is shown in Figs. 1(b) and 1(d), where the shift in the value of $A_{FB}$ is small compared to the case when we have $\mu$ as the final-state leptons.

In Fig. 2, the effects of different parameters corresponding to type III of THDM are shown in the FB asymmetry in $B \rightarrow K_1 \ell^+ \ell^-$ ($\ell = \mu, \tau$). It can be seen in Figs. 2(a) and 2(b) that, in the case of $\mu$ as the final-state leptons, the zero position of the FB asymmetry is sensitive to the phase angle $\delta$ and, for $\delta = 60^\circ$, this shift is a maximum for the maximum value of $\lambda_{bb} \lambda_{tt}$. This effect can easily be understood if we look closely at Eq. (61), where it can be seen that $A_{FB}$ is proportional to the real part...
Fig. 3. The dependence of the forward–backward asymmetry on $s$ after including the charm-quark loop effects. The blue line corresponds to the case when the charm-quark loop is ignored, while the purple and golden lines correspond to the case when $\delta_i$ in Eq. (72) is taken to be 1 and $-1$, respectively.

of the combination of auxiliary functions $f_2$ and $f_8$. In Eq. (44), the term proportional $C_7$ involves the new phase $\delta$ and, for $\delta = 90^\circ$, the NP contribution coming to $C_7$ is zero and hence the deviation from the SM is small compared to the case when $\delta = 60^\circ$. In contrast to the $\mu$ case, the NP effects in $A_{FB}$ for $B \rightarrow K_1 \tau^+ \tau^-$ are too faint for the whole range of the phase $\delta$ and other parameters of THDM type III.

It is worth emphasizing that, in addition to the NP indication coming through the Wilson coefficients $C_7$, $C_9$, $C_{10}$ in $A_{FB}$, there is also a contribution from the NP arising due to the neutral Higgs boson (NHB) coming through the auxiliary function $f_8$. It is indeed suppressed compared to the contributions from $C_7$, $C_9$, $C_{10}$ and hence its effects are too mild in the FB asymmetry.

It has already been pointed out that, in $B \rightarrow K^* \ell^+ \ell^-$, the charm-loop pollution significantly modifies the results of various asymmetries in different bins of the square of momentum $s$. The perturbative charm-loop contribution is usually absorbed into the definition of $C_9^{\text{eff}}$ [55]. The long-distance contribution is difficult to estimate, and, to incorporate it, a universal correction to $C_9$ arises from the long-distance charm-loop contribution, which we parametrize as [56–59]:

$$\delta C_9^{\text{LD}} = \delta_i \frac{a + bs(c - s)}{s(c - s)}$$

with $a \in [2, 7] \text{ GeV}^4$, $b \in [0.1, 0.2]$ and $c \in [9.2, 9.5] \text{ GeV}^2$, whereas the range of the parameter $\delta_i$ is $[-1, 1]$. Because of the lack of experimental data on the decay under consideration, the purpose here is not to scan the $A_{FB}$ in different bins of $s$ but to see how much the deviation is by varying the parameters given in the range above. Being bold in giving a possible estimate of deviations in the value of $A_{FB}$ without the charm-quark loop in the SM, we have chosen $a = 3 \text{ GeV}^4$, $b = 0.15$, $c = 9.4 \text{ GeV}^2$, and taken values of $\delta_i = 1$ (purple curve) and $\delta_i = -1$ (golden curve) in Fig. 3. We can see that the maximum shift in the value of forward–backward asymmetry is around 20% from the case when charm-loop pollution is ignored. In Figs. 1 and 2, we can see that, in a certain range of THDM parameters, the deviation from the SM value is significantly larger. Therefore, in future, when we have data on these decays, it will be possible to limit the parametric space of THDM as well as of the parameters corresponding to charm-loop effects.
The dependence of $\langle A_{\text{FB}} \rangle$ on $m_{H^\pm}$ for different values of $\tan \beta$ in the left panel and on $\delta$ in the right panel for $B \to K_1 \mu^+ \mu^-$ decay in THDMs of types II and III, respectively. The values of the other parameters are given above each panel.

Besides the zero position of $A_{\text{FB}}$, its magnitude will also serve as an important tool to see the indications of NP. The average value of $A_{\text{FB}}$, after integration on $s$ in the range that is below the resonances, i.e., $4m_\ell^2 \leq s \leq 9$ GeV$^2$ for $B \to K_1 \mu^+ \mu^-$, is displayed in Fig. 4. In Fig. 4(a), the variations of $\langle A_{\text{FB}} \rangle$ with the mass of the charged Higgs boson ($m_{H^\pm}$) for different values of $\tan \beta$ are portrayed. It can be observed that, for a small value of the mass of the charged Higgs boson ($m_{H^\pm}$), $\langle A_{\text{FB}} \rangle$ is significantly enhanced by enhancing the value of $\tan \beta$ in $B \to K_1 \mu^+ \mu^-$ decay (c.f. Fig. 4(a)). However, this value becomes less sensitive to the value of $\tan \beta$ at large values of $m_{H^\pm}$. This is because the value of the parameter $y = \frac{m_\tau^2}{m_{H^\pm}^2}$ decreases and so the corresponding NP effects become small.

Likewise, we have also shown the dependence of $\langle A_{\text{FB}} \rangle$ on the CP-violating phase $\delta$ arising in model III in Fig. 4(b). Here, we have kept the mass of the charged Higgs to 300 GeV and varied the values of $\lambda_{bb}$ and $\lambda_{tt}$ for the different cases defined in Eq. (69). It can be seen that, for small values of the phase, an increase in the value of $\lambda_{tt}$ will lead to an increase in the value of the magnitude of $A_{\text{FB}}$ and, at a phase value of $90^\circ$, this value for all the three cases becomes the same. Hence, being insensitive to the uncertainties arising due to different input parameters, the deviations in the magnitude of $A_{\text{FB}}$ due to the THDM parameters are very prominent and easy to measure in the experiment, which can also help us to put constraints on the parameter space of different versions of the THDM.

4.2. Analysis of lepton polarization asymmetries

In addition to the forward–backward asymmetry, the other interesting asymmetries to obtain complementary information about NP associated with the THDM in $B \to K_1 \ell^+ \ell^-$ ($\ell = \mu, \tau$) decays are lepton polarization asymmetries, which are shown in Figs. 5–10. Since lepton polarization asymmetries depend on the different combinations of the Wilson coefficients, one can expect a large dependence of these asymmetries on different versions of the THDM, making these observables very suitable for searching for possible NP. In the case of the longitudinal lepton polarization ($P_L$), it can be seen from Eq. (66) that the contribution from the NHB encoded in the Wilson coefficients $C_{Q_1}$ and $C_{Q_2}$ is suppressed by the mass of the final-state leptons. In addition, these coefficients have a factor of $|\lambda_{tt}|^2$ in the denominator (c.f. Eqs. (3) and (4)) and, in model III, where this factor is less than 1, it lifts the suppression due to the mass of the lepton. Therefore, one can expect a large contribution from NHB in the THDM of type III. Figures 5(a) and 5(b) display the trend of $P_L$ with the square of
The dependence of the longitudinal polarization asymmetries of $B \rightarrow K_1 \ell^+ \ell^-$ on $s$ with long-distance contributions for (a) muons, (c) tauons in THDM I and for (b) muons, (d) tauons in THDM II, where the bands show the range of $\tan \beta$ from 1–30$^\circ$. In all of the graphs, the black band corresponds to the SM, and the yellow, blue, and red bands correspond to the values of $m_{H^\pm} = 300$ GeV, $m_{H^\pm} = 400$ GeV, and $m_{H^\pm} = 700$ GeV, respectively, while the values of $m_{H^0}$ and $m_A$ are set at 500 GeV.

In contrast to types I and II, in version III of THDM, one can expect the large contribution from the NHBs because of its proportionality to the inverse of $|\lambda_{tt}|^2$, which would lift the lepton mass suppression coming in the last term of the longitudinal lepton polarization asymmetry. As $|\lambda_{tt}|$ is less than one in type III, the terms proportional to the square of $|\lambda_{tt}|$ are ignorable compared to the terms linear in $\lambda_{tt} \lambda_{bb}$ and hence only the type-III THDM contributions will be prominent (c.f. Eq. (12)).

As the term $\lambda_{tt} \lambda_{bb}$ contains the phase $\delta$, $P_L$ will also be sensitive to the phase $\delta$; this can be seen in Fig. 6. In the case of muons as the final-state leptons, one can see from Figs. 6(a) and 6(b) that, at $\delta = 60^\circ$, the contribution from model III will lead to a contribution that makes the value of $P_L$ compared to the SM value; it also lies away from the uncertainty region. However, the trend is entirely different in the case of $\delta = 90^\circ$. Compared to muons, when the final-state leptons are $\tau$, the effects of type III give a positive contribution for both values of the phase. However, in this case, the effects are mild but still distinguishable from the SM.

In order to show the impact of NP coming through the THDM, we have plotted the average value of the longitudinal lepton polarization asymmetry $\langle P_L \rangle$ with the charged Higgs boson mass (Fig. 7(a))
Fig. 6. The dependence of the longitudinal polarization asymmetries of $B \rightarrow K_{1} \ell^{+} \ell^{-}$ on $s$ with long-distance contributions for muons (a) $\delta = 60^\circ$, (b) $\delta = 90^\circ$ and for taus (c) $\delta = 60^\circ$, (d) $\delta = 90^\circ$ in THDM III. The line conventions as well as the values of parameters corresponding to version III of the THDM are same as in Fig. 2.

Fig. 7. The dependence of $\langle P_L \rangle$ on $m_{H^\pm}$ for different values of $\tan \beta$ in the left panel and on $\delta$ in the right panel for $B \rightarrow K_{1} \mu^{+} \mu^{-}$ decay in THDMs of types II and III, respectively. The values of the other parameters are given above each panel.

and with the CP-violating phase $\delta$ (Fig. 7(b)) in versions II and III of the THDM, respectively. The integration on the square of the momentum is performed in the range $s_{\text{min}} \leq s \leq 7$ GeV$^2$, i.e., well below the resonance region. Figure 7(a) shows that, for certain values of the parameters in the THDM, the shift in $\langle P_L \rangle$ is significant at small values of the mass of the charged Higgs boson. However, this shift in $\langle P_L \rangle$ diminishes at large values of $m_{H^\pm}$ due to the fact that NP entering in the Wilson coefficients comes partly through the parameter $y = m_{tt}^2 / m_{H^\pm}^2$, which becomes small at large values of $m_{H^\pm}$. Similarly, Fig. 7(b) depicts the behavior of $\langle P_L \rangle$ with the phase $\delta$ for different values of
Fig. 8. The dependence of normal polarization asymmetries of $B \to K_1 \ell^+ \ell^-$ on $s$ with long-distance contributions for (a) muons, (c) tauons in THDM I and for (b) muons, (d) tauons in THDM II, where the bands show the range of $\tan \beta$ from 1–30$^\circ$. In all of the graphs, the black band corresponds to the SM, and the yellow, blue, and red bands correspond to the values of $m_{H^\pm} = 300$ GeV, $m_{H^\pm} = 400$ GeV, and $m_{H^\pm} = 700$ GeV, respectively, while the values of $m_{H^0}$ and $m_A$ are set at 500 GeV.

$\lambda_{tt}$ and $\lambda_{bb}$. It can be observed that, for large values of $\delta$ along with $\lambda_{tt} = 0.3$ and $\lambda_{bb} = 30$, the average value of $P_L$ that is likely to be measured at some of the ongoing and future experiments is significantly modified.

Figure 8 shows the trend of normal lepton polarization asymmetry ($P_N$) in $B \to K_1 \ell^+ \ell^-$ decay in versions I and II of the THDM. It can be seen that the most prominent change in the value of $P_N$ for $B \to K_1 \mu^+ \mu^-$ decay comes at small values of the mass of the charged Higgs boson, which is shown by the yellow band. However, at large values of the mass of the Higgs boson, the variations due to $\tan \beta$ are small and the value also approaches the SM results. This is because the value of the variable $y$ decreases due to an increase in the value of $m_{H^\pm}$ and so the NP content becomes small in the Wilson coefficients.

Figures 9(a),(b) and 9(c),(d) show the dependence of normal lepton polarization asymmetries with the square of the momentum transfer when we have $\mu$ and $\tau$ as the final-state leptons, respectively, in THDM version III for different values of the CP-violating phase $\delta$. The black solid band corresponds to the SM results by including the uncertainties involved in different input parameters like the form factors, etc. One can notice that the value of $P_N$ is quite sensitive to Cases A and C (c.f. Eq. (71)), which are depicted by dashed and dot-dashed lines in Fig. 9 for $\delta = 60^\circ$ and $90^\circ$.

To be clearer about the influence of the THDM parameters, we have plotted the average value of normal lepton polarization asymmetry, $\langle P_N \rangle$, against the mass of the charged Higgs boson ($m_{H^\pm}$) and CP-violating phase $\delta$ in Figs. 10(a) and 10(b), respectively, for $B \to K_1 \mu^+ \mu^-$ decay. In Fig. 10(a),
Fig. 9. The dependence of normal polarization asymmetries of $B \rightarrow K_{1} \ell^{+} \ell^{-}$ on $s$ with long-distance contributions for muons (a) $\delta = 60^{\circ}$, (b) $\delta = 90^{\circ}$ and for taus (c) $\delta = 60^{\circ}$, (d) $\delta = 90^{\circ}$ in THDM of type III. In all of the graphs, the solid band corresponds to the SM uncertainties. The dashed, dotted, and dot-dashed lines correspond to Cases A, B, and C (c.f. Eq. (71)), respectively.

Fig. 10. The dependence of $\langle P_{N} \rangle$ on $m_{H^{\pm}}$ for different values of $\tan \beta$ in the left panel and on $\delta$ in the right panel for $B \rightarrow K_{1} \mu^{+} \mu^{-}$ decay in THDMs of types II and III, respectively. The lines with crosses, boxes, and triangles correspond to the values of $\tan \beta$ equal to 1, 2, and 3, respectively. The values of the other parameters are given above each panel.

one can notice that, at small values of $m_{H^{\pm}}$, the average value of normal lepton polarization asymmetry is very sensitive to the value of $\tan \beta$ in type-II THDM. We can see that, on increasing the value of $\tan \beta$, the value of $\langle P_{N} \rangle$ increases from $-0.105$ to $-0.125$ when the value of $m_{H^{\pm}}$ is fixed at 300 GeV. However, at large values of $m_{H^{\pm}}$, the value is no longer sensitive to the parameters of type-II THDM. Likewise, the average value of $P_{N}$ is also sensitive to the parameters of type-III THDM, which is depicted in Fig. 10(b). In this figure, we have integrated on $s$ in the range $s_{\text{min}} \leq s \leq 3$ GeV$^{2}$ because
the most visible effects comes in this bin of \( s \). In Fig. 10(b), it can be noticed that \( \langle P_N \rangle \) is quite sensitive to the parameters \( \lambda_{tt}, \lambda_{bb}, \) and \( \delta \). We can see that the value of \( \langle P_N \rangle \) becomes more negative when \( \lambda_{tt} \) is decreased from 0.3 to 0.03 and the corresponding \( \lambda_{bb} \) increases from 30 to 100. It is very likely that the measurement of \( P_N \) and its average value will help us to distinguish the NP effects coming through different versions of the THDM.

In Eq. (68), we can see that the transverse lepton polarization asymmetry \( (P_T) \) is not only \( m_\ell \) suppressed but is also proportional to the imaginary part of the different combinations of the Wilson coefficients. Therefore, its value is expected to be too small to measure experimentally; thus, we have not shown it graphically in the present study.

5. Conclusion

The experimental results on angular observables in the rare decay \( B \to K^{*} \ell^+ \ell^- \) have shown some deviations from the SM predictions [61–65] and these observables are investigated in detail in the literature [57–59,66]. It has been pointed out that, in certain observables like \( P'_5 \), where the deviations from SM predictions are 2–3\( \sigma \), it is possible to accommodate certain NP, and it would be interesting if such an analysis were done in different versions of the two-Higgs-doublet model. However, the purpose here is to give an overview of the NP coming through the allowed parametric space of the THDM on the forward–backward asymmetry and different lepton polarization asymmetries in \( B \to K_1 \ell^+ \ell^- \) decays. We observed that the forward–backward asymmetry and the different lepton polarization asymmetries show a clear signal of the THDM model of all three types. However, the CP-violation asymmetry is only nonzero in type III of the THDM because of the presence of a new phase, \( \delta \); this will be discussed in a separate study (I. Ahmed et al., manuscript in preparation). Therefore, the precise measurement of this asymmetry along with the one calculated here will help us to get the constraints on the phase \( \delta \) as well as other parameters of the THDM.

To sum up, the further data to be available from LHCb and the future super B-factories will provide a powerful testing ground for the SM and also put some constraints on the two-Higgs-doublet model parameter space.

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