Revolving D-branes and spontaneous gauge-symmetry breaking

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We propose a new mechanism of spontaneous gauge-symmetry breaking in the world-volume theory of revolving D-branes around a fixed point of orbifolds. In this paper, we consider a simple model of the $T^6/\mathbb{Z}_3$ orbifold on which we put D3-branes, D7-branes, and their anti-branes. The configuration breaks supersymmetry, but the Ramond–Ramond tadpole cancellation conditions are satisfied. A set of three D3-branes at an orbifold fixed point can separate from the point, but, when they move perpendicular to the anti-D7-branes put on the fixed point, they are pulled back due to an attractive interaction between the D3- and anti-D7-branes. In order to stabilize the separation of the D3-branes at nonzero distance, we consider revolution of the D3-branes around the fixed point. Then the gauge symmetry on the D3-branes is spontaneously broken, and the rank of the gauge group is reduced. The distance can be set at will by appropriately choosing the angular momentum of the revolving D3-branes, which should be determined by the initial condition of the cosmological evolution of the D-brane configurations. The distance corresponds to the vacuum expectation values of brane moduli fields in the world-volume theory and, if it is written as $M/M_s^2$ in terms of the string scale $M_s$, the scale of gauge-symmetry breaking is given by $M$. Angular momentum conservation of revolving D3-branes assures the stability of the scale $M$ against $M_s$.

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1. Introduction

The dynamics of electroweak symmetry breaking has become even more mysterious after the discovery of the Higgs boson. It is widely believed that the standard model is merely an effective theory of the electroweak symmetry breaking and some unknown physics or dynamics should exist behind it. Many possibilities, mainly motivated by naturalness, have been examined, e.g., dynamical symmetry breaking by strong-coupling gauge interactions [1,2], radiative symmetry breaking with [3,4] or without supersymmetric extensions [5–10], the Hosotani mechanism in the gauge–Higgs unification with extra dimensions [11], and so on.

It is also important to pursue the origin of the masses of quarks and leptons, namely, how the electroweak symmetry breaking is mediated to them. In the standard model it is simply described by Yukawa couplings, and much effort has been made to derive realistic Yukawa couplings. Modelbuildings with D-branes in string theory are most attractive, because Yukawa couplings can be understood by configurations of D-branes in the 6D compact space of 10D superstring theories. Some examples
are intersecting D-brane models [12–17] or models of D-branes at singularities [18–21]. In these
models, massless states of a string with appropriate quantum numbers are identified with the Higgs
doublet fields, but it is generically difficult to obtain small vacuum expectation values of the weak
scale in the string theory setup.

In this paper, we propose a new mechanism of spontaneous gauge-symmetry breaking by assum-
ing that our 3D space consists of revolving D-branes around a fixed point in 9 + 1D space-time.
As a simple model, we study D-branes at fixed points of a $T^6/Z_3$ orbifold in type IIB superstring
theory [18]. We assume that all the moduli, except for the D-brane moduli, are stabilized. The 6D
torus is assumed to be factorizable, $T^6 = T^2 \times T^2 \times T^2$, and the action of $Z_3$ is described by the
twist vector $v = (1/3, 1/3, -2/3)$. There are three fixed points in each $T^2$ and 27 fixed points
in total. We distribute D3-branes and D7-branes and their anti-branes so as to cancel the twisted
Ramond–Ramond (R–R) tadpoles at each fixed point as well as the untwisted R–R tadpoles in the
compact space. As a result, all supersymmetries in type IIB superstring theory are broken through
the “brane supersymmetry-breaking” mechanism [22–26]. In particular, we consider a system of four
D3-branes and three anti-D7-branes discussed in Ref. [27]. Three of the four D3-branes can move
away from the fixed point in a $Z_3$-invariant way, and the separation of these D3-branes causes
spontaneous gauge-symmetry breaking of $U(2) \times U(1) \times U(1) \rightarrow U(1) \times U(1) \times U(1)$ on the world-volume
theory on the D3-branes. The rank of the gauge group is reduced due to the identification of three
D3-branes by the $Z_3$ action. The reduction of rank is actually necessary in the electroweak symmetry
breaking.

When we put anti-D7-branes on the fixed point as well as the D3-branes, there appears an attract-
tive force between the D3-branes and the anti-D7-branes, and, when D3-branes move away from
the fixed point, they are pulled back. Thus D3-branes tend to be localized at the fixed point, and
the gauge-symmetry breaking does not occur unless there exists an additional balancing force that
repels D3-branes from the fixed point. In this paper, we consider revolution of the D3-branes whose
centrifugal force balances against the attractive force.

In Sect. 2, we give a brief review of the models with D3-branes and D7-branes at the orbifold fixed
points of a $T^6/Z_3$ orbifold. We explain how the vacuum expectation values of the D-brane moduli
fields describe the displacements of the D3-branes. In Sect. 3, the world-volume theory of revolv-
ing D3-branes is introduced. The effect of revolution is an introduction of the centrifugal potential
into the rest frame of the revolving D3-branes. In Sect. 4, the spectrum in the world-volume theory
is discussed. We find that spontaneous gauge-symmetry breaking takes place and the scale of the
expectation value is stable against string-scale perturbations. In Sect. 5, we conclude and discuss
some important issues not studied in detail in the present paper.

2. D3- and D7-branes at $T^6/Z_3$ orbifold fixed points
2.1. D-brane configurations

General ideas and formulations of D-branes at orbifold fixed points are discussed in Ref. [18]. In this
section, we particularly consider the models with D3-branes, D7-branes, and their anti-branes in a
$T^6/Z_3$ orbifold. The 6D torus is assumed to be factorized into three two-tori, $T^6 = T^2 \times T^2 \times T^2$,
and the complex coordinates of the corresponding two-tori are denoted by

$$Z_i = \frac{1}{\sqrt{2}} \left( X^{2i+2} + i X^{2i+3} \right) \quad (1)$$
with \( i = 1, 2, 3 \). Here \( X^M \) with \( M = 0, 1, \ldots, 9 \) are the coordinates of 10D space-time of type IIB superstring theory. The coordinates are identified by the translations

\[
Z_i \sim Z_i + 2\pi R_i, \quad Z_i \sim Z_i + 2\pi R_i \tau, \quad (2)
\]

where \( R_i \) are the radii of the two-tori and \( \tau = \left( -1 + i\sqrt{3} \right) / 2 \). We also identify points on the torus transformed by the \( \mathbb{Z}_3 \) action

\[
Z_i \sim e^{2\pi i v_i} Z_i \quad (3)
\]

where \( v = (1/3, 1/3, -2/3) \) is the twist vector. There are three fixed points in each two-torus under the above identifications, and in total 27 fixed points in the \( T^6/\mathbb{Z}_3 \) orbifold. Since the string world-sheet fields should follow the same identifications as the coordinates \( X^M \), there is an untwisted closed string propagating in the 10D space-time, and twisted closed strings localized at each fixed point.

We introduce D3-branes whose world-volume coordinates coincide with \( X^\mu \) with \( \mu = 0, 1, 2, 3 \). In 6D compact space, the positions of these D3-branes are specified by the corresponding points that are distributed in a \( \mathbb{Z}_3 \)-symmetric way in the torus. Therefore, when D3-branes are put away from the fixed points, three D3-branes, which belong to the following regular representation of \( \mathbb{Z}_3 \), must move together. The cyclic permutation of three D3-branes in the torus is described by the matrix

\[
\gamma = \begin{pmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
1 & 0 & 0
\end{pmatrix}, \quad (4)
\]

which can be diagonalized by a unitary transformation as

\[
U \gamma U^\dagger = \begin{pmatrix}
1 & 0 & 0 \\
0 & \alpha & 0 \\
0 & 0 & \alpha^2
\end{pmatrix} \quad (5)
\]

where \( \alpha = \exp(2\pi i / 3) \). This shows that the regular representation is not an irreducible representation of \( \mathbb{Z}_3 \). A D3-brane in the irreducible representation of \( \mathbb{Z}_3 \) with one of the above three eigenvalues of \( \gamma \) is called a fractional D3-brane. It is localized at the fixed point and cannot move away from it. Only a set of three D3-branes can move. The above matrices also act as the \( \mathbb{Z}_3 \) operations on the Chan–Paton indices of the open strings whose ends are confined in the corresponding D3-brane world-volume.

The twisted R–R tadpole cancellation condition of the D-branes at a fixed point is given by using the matrices of the \( \mathbb{Z}_3 \) operation acting on the various D-branes:

\[
3 \text{Tr} (\gamma_3) - 3 \text{Tr} (\gamma^\dagger_3) + \text{Tr} (\gamma_7) - \text{Tr} (\gamma^\dagger_7) = 0. \quad (6)
\]

Here, \( \gamma_3, \gamma^\dagger_3, \gamma_7, \) and \( \gamma^\dagger_7 \) are the matrices for D3-, anti-D3-, D7-, and anti-D7-branes localized at the fixed point. A simple solution to the condition of Eq. (6) with four D3-branes and three anti-D7-branes is given by

\[
\gamma_3 = \text{diag} \left[ I_2, \alpha, \alpha^2 \right], \quad \gamma^\dagger_7 = I_3, \quad (7)
\]

where \( I_N \) denotes an \( N \times N \) unit matrix [27]. Among the four D3-branes at the fixed point, a set of three can move away from the fixed point by constructing the regular representation of Eq. (5). The remaining D3 is in the irreducible representation \( I_1 \) with an eigenvalue 1 of the matrix \( \gamma \).

The above subsystem with Eq. (7) can be embedded globally in the \( T^6/\mathbb{Z}_3 \) orbifold as shown in Fig. 1. Since the codimension of D7-branes is 2 in 10D space-time, we can specify it by indicating
Fig. 1. An example of D-brane models in which all the twisted and untwisted R–R tadpoles are canceled. Each of the complex coordinates of three tori $Z_i$ is represented as a line; the blobs on each line indicate three fixed points on the torus. D7- and anti-D7-branes are not extended to the third torus. Owing to the Wilson line introduced on the anti-D7-branes, no D3-branes or anti-D3-branes are required for six fixed points inside the square with a dashed line (see text).

directions along which its world-volume is not extended. $D_{7_i}$ denotes a D7-brane whose world-volume does not include the direction of the $i$th torus described by $Z_i$. The $Z_3$-action matrices for other D-branes are given by

$$\gamma_3' = 1, \quad \gamma_3'' = \text{diag}(1, \alpha), \quad \gamma_3 = \alpha^2, \quad \gamma_7 = \alpha^2 1_3. \quad (8)$$

All of the above D-branes are fractional D-branes. We also introduce a Wilson line on the anti-D7-branes, which is described by the operation matrix

$$\gamma_W = \text{diag}(1, \alpha, \alpha^2), \quad (9)$$

corresponding to the torus shifts of Eq. (2). The twisted R–R tadpole cancellation conditions for the fixed points, which are fixed under $Z_3$ transformations up to the shift of Eq. (2), are modified by replacing $\gamma_7$ by $\gamma_W \gamma_7$ or $\gamma_W^2 \gamma_7$ in Eq. (6) [28].

The untwisted R–R tadpoles are canceled out, because the numbers of D3-branes and anti-D3-branes are the same, and the numbers of D7-branes and anti-D7-branes are the same. If moduli stabilization requires some other objects that are sources of untwisted R–R charges [29–33], these D-brane configurations are changed accordingly. In the following analysis, we simply assume that the model is constructed in a globally consistent way and focus on the subsystem of Eq. (7) near the fixed point where four D3- and three $\overline{D7}_3$-branes are localized. Since the supersymmetry is broken in the present model, the Neveu–Schwarz–Neveu–Schwarz (NS–NS) tadpoles are not canceled and the flat space-time with trivial dilaton and B-field background is no longer a solution to the equation of motion of string theory [34,35]. The effects of the NS–NS tadpoles on the background geometry are neglected for the moment and will be mentioned briefly in Sect. 5.

2.2. DBI action

The bosonic part of the world-volume low-energy effective theory of a $D_p$-brane is given by the generalized Dirac–Born–Infeld (DBI) action

$$S_p = -\tau_p \int d^{p+1}x \, e^{-\phi} \sqrt{-\det \left( G_{ab} + B_{ab} + 2\pi \alpha' F_{ab} \right)}, \quad (10)$$

where $\tau_p = 1/(2\pi)^p g_s (\alpha')^{p(p+1)/2}$ is the tension of the $D_p$-brane while string coupling $g_s = e^{\langle \Phi \rangle}$ is determined by the vacuum expectation value of the 10D dilaton field $\Phi = \langle \Phi \rangle + \phi$. The induced
The world-sheet theory of the four D-branes at the orbifold fixed point is described by the $U(2) \times U(1)_1 \times U(1)_2$ quiver gauge theory. The fields $Z_i^{(a)}$ are in the bifundamental representations and depicted by arrows between the blobs. Three lines with $i = 1, 2, 3$ correspond to complex fields on each of the three tori.

The metric on the $Dp$-brane is defined as

$$G_{ab} = \frac{\partial X^M}{\partial \xi^a} \frac{\partial X^N}{\partial \xi^b} \eta_{MN},$$

where $X^M(\xi)$ describe the embedding of the $Dp$-brane in 10D space-time. Here, we assume that the 10D space-time is flat Minkowski. Generalizations of the DBI action to $n$ $Dp$-branes have been discussed in various works.

Let us now focus on the $p = 3$ case. The low-energy effective action of $n$ D3-branes is described by 4D $\mathcal{N} = 4 U(n)$ super-Yang–Mills theory whose bosonic part is obtained by expanding the non-Abelian generalization of the DBI action in Eq. (10). The vector multiplet of 4D $\mathcal{N} = 4 U(n)$ super-Yang–Mills theory consists of $U(n)$ gauge bosons, four Weyl fermions, and six real scalar fields. The six scalar fields originate from the embedding fields $X^M(\xi)$ with $M = 4, 5, \ldots, 9$ and can be interpreted as the brane moduli fields whose vacuum expectation values describe the positions of D-branes in 6D compact space. We define the complex combinations $Z_i$ ($i = 1, 2, 3$) of these moduli fields as in Eq. (1). In the absence of other sources of supersymmetry breaking such as the anti-D7-branes, there is no potential along the moduli directions. As we will see, some of these properties survive after $T^6/\mathbb{Z}_3$ compactification, namely in the world-volume theory of D3-branes at the orbifold fixed points.

### 2.3. Gauge-symmetry breaking in the quiver gauge theory

The world-volume theory of the four D3-branes at the orbifold fixed point can be obtained by imposing the orbifolding conditions on $\mathcal{N} = 4$ SYM theory and described by the 4D $\mathcal{N} = 1 U(2) \times U(1)_1 \times U(1)_2$ quiver gauge theory as shown in Fig. 2. It has chiral multiplets in the bifundamental representations

$$
\begin{align*}
Z_i^{(1)} &\quad U(2) \quad U(1)_1 \quad U(1)_2 \\
Z_i^{(2)} &\quad 2 \quad 0 \quad -1 \\
Z_i^{(3)} &\quad 2^* \quad +1 \quad 0 \\
Z_i &\quad 1 \quad -1 \quad +1
\end{align*}
$$

The subscript $i$ with $i = 1, 2, 3$ denotes the index of three tori $Z_i$ and the superscript $(a)$ with $a = 1, 2, 3$ denotes three different types of bifundamental representations of the $U(2) \times U(1)_1 \times U(1)_2$ gauge symmetry. We use the same symbols $Z_i^{(a)}$ to describe the scalar components of the
corresponding chiral superfields. The covariant derivatives on the scalar fields are given by

\[ D_\mu Z_i^{(1)} = \left( \partial_\mu Z_i^{(1)} + ig \left( \frac{\tau^A}{2} \right) W^A_{\mu} Z_i^{(1)} - ig \frac{g}{\sqrt{2}} Z_i^{(1)} B^{(2)}_\mu \right) \]

\[ D_\mu Z_i^{(2)} = \left( \partial_\mu Z_i^{(2)} + ig \left( \frac{-\tau^A}{2} \right)^* W^A_{\mu} Z_i^{(2)} + ig \frac{g}{\sqrt{2}} Z_i^{(2)} B^{(1)}_\mu \right) \]

\[ D_\mu Z_i^{(3)} = \left( \partial_\mu - ig \frac{g}{\sqrt{2}} B^{(1)}_\mu + ig \frac{g}{\sqrt{2}} B^{(2)}_\mu \right) Z_i^{(3)} , \] (12)

where the gauge fields of U(2), U(1)\_1, and U(1)\_2 are described as $W^A_{\mu}$, $B^{(1)}_\mu$, and $B^{(2)}_\mu$ with $A = 0, 1, 2, 3$. The $\tau$ matrices are defined as $\tau^A = (1_2, \sigma^i)$ where $\sigma^i$ are the Pauli matrices. The gauge-coupling constant is defined as $g = \sqrt{2\pi g_s}$. Among the U(1) symmetries, the combination of U(1)\_1–U(1)\_2 is an anomalous U(1) symmetry whose chiral anomaly is canceled by the generalized Green–Schwarz mechanism [36].

These scalar fields acquire the F-term potential

\[ V_F = \sum_{i, a} \left| \frac{\partial W}{\partial Z_i^{(a)}} \right|^2 , \] (13)

where $W$ is given by

\[ W = ig \epsilon_{ijk} Z_i^{(2)\beta} Z_j^{(1)\beta} Z_k^{(3)} , \] (14)

and $\beta$ is the index of the doublet representation of the U(2) group. There is also a D-term potential,

\[ V_D = \frac{1}{2} \left( (D^A_{\mu(2)})^2 + (D_1)^2 + (D_2)^2 \right) , \] (15)

where

\[ D^A_{\mu(2)} = -g \left( Z_i^{(1)\uparrow} \frac{\tau^A}{2} Z_i^{(1)} + Z_i^{(2)\uparrow} \left( \frac{-\tau^A}{2} \right)^* Z_i^{(2)} \right) , \] (16)

\[ D_1 = -g \left( Z_i^{(2)\uparrow} \frac{1}{\sqrt{2}} Z_i^{(2)} + Z_i^{(3)\uparrow} \left( \frac{-1}{\sqrt{2}} \right) Z_i^{(3)} \right) , \] (17)

\[ D_2 = -g \left( Z_i^{(3)\uparrow} \frac{1}{\sqrt{2}} Z_i^{(3)} + Z_i^{(1)\uparrow} \left( \frac{-1}{\sqrt{2}} \right) Z_i^{(1)} \right) . \] (18)

The normalization of the Lie generators is fixed by the relation $\text{Tr} \left( T^a T^b \right) = \delta^{ab}/2$. The flat directions of the potential $V_F + V_D$ are parametrized by the six parameters $\left( v_i, \theta_i \right)$ as

\[ \left\{ Z_i^{(1)} \right\} = \left( 0, v_i e^{i\theta_i} \right) , \quad \left\{ Z_i^{(2)} \right\} = \left( 0, v_i e^{i\theta_i} \right) , \quad \left\{ Z_i^{(3)} \right\} = v_i e^{i\theta_i} . \] (19)

The relative phases between $Z_i^{(a)}$ and $Z_i^{(b)}$ with different $a$ and $b$ can be absorbed by some combinations of gauge transformations of the U(1) component of U(2), U(1)\_1, and U(1)\_2, though the common phase cannot be gauged away, because all the fields are in bifundamental representations. If the set of three D-branes gets the vacuum expectation value of Eq. (19), the gauge symmetry $U(2) \times U(1)\_1 \times U(1)\_2$ is spontaneously broken down to $U(1) \times U(1)$. Expanding around the vacuum expectation value, the gauge bosons associated with the broken generators acquire the mass of $\sqrt{\sum_i g^2 v_i^2}$. 

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This has the following geometrical interpretation. These vacuum expectation values describe the configuration of three D3-branes away from the fixed point on $T^6/\mathbb{Z}_3$. They are distributed in a $\mathbb{Z}_3$-invariant way on each torus as shown in Fig. 3. Since each $Z_i^{(a)}$ is a scalar mode of an open string stretching from one D-brane to another, the mass is proportional to the distances $d$ or $s$, namely, $d/2\sqrt{2}\pi\alpha'$ or $s/2\sqrt{2}\pi\alpha'$, respectively. Writing the position of three D-branes on the $i$th torus as $(d_i e^{i\theta_i}, d_i e^{i(\theta_i+2\pi/3)}, d_i e^{i(\theta_i+4\pi/3)})$, the relative complex coordinates between them are given by, e.g., $d_i e^{i\theta_i} (e^{2\pi/3} - 1) = i\sqrt{3}d_i e^{i(\theta_i+\pi/3)}$. If we take a different pair of D-branes, the constant part of the phase $\pi/3$ is changed. The distance $s$ between the branes is related to the distance of the branes from the origin $d = \sqrt{\sum_i d_i^2}$ by the relation $s = \sqrt{3}d$, as shown in Fig. 3. The geometric interpretation of the coordinates and boson masses is made more explicit in the following calculations, but we want to stress here that the total phase $\theta_i$ represents the direction of the set of three D3-branes, as shown in Fig. 3.

Within the $\mathcal{N} = 1$ quiver gauge theory, there is no potential along the direction of increasing $v_i$ and the vacuum expectation value cannot be dynamically determined. In the next section, in order to study the dynamics of the gauge-symmetry breaking in string theory, we will include the effect of the anti-D7-branes at the fixed point as shown in Fig. 1, which breaks the supersymmetry. Furthermore, we introduce revolution of the D3-branes to balance the attractive force between the D3- and anti-D7-branes. These two effects determine the vacuum expectation value $v_i$.

3. Revolving D3-branes around anti-D7-branes

In this section, we study the dynamics of the gauge-symmetry breaking in the model of revolving D3-branes around the anti-D7-branes put at the orbifold fixed point. It is realized as a subsystem in the configuration of Fig. 1.

3.1. Attractive potential between D3- and $\overline{D7}$-branes

First we consider an effect of the anti-D7-branes. This breaks the supersymmetry of the D3-brane system. Since the anti-D7-branes are not extended along the third torus $\mathbb{Z}_3$, it can be expected that the D3-brane flat direction in Eq. (19) is lifted by the appearance of a mass-like term along $Z_3^{(a)}$. The supersymmetry-breaking potential cannot be understood by the closed-string exchange, since the situation that we have in mind is $d < l_s$: the distance $d$ between the D3-branes and the anti-D7-branes is much shorter than the string length $l_s \equiv \sqrt{\alpha'}$ (see Ref. [37]). Instead, we can estimate it by calculating one-loop corrections of open strings stretching between D3- and anti-D7-branes, whose
spectra do not preserve supersymmetry. For $|Z_3^{(a)}| \ll M_s$, the supersymmetry-breaking potential can be expanded and the mass term

$$\mu^2 \sum_a |Z_3^{(a)}|^2$$

is added to the F-term and D-term potentials $V_F + V_D$. An order estimation [27] gives the coefficient

$$\mu^2 = \frac{1}{C^2} \frac{g^2}{16\pi^2} M_s^2,$$

where $M_s \equiv 1/\sqrt{\alpha'}$ and $C$ is a constant $C \sim \mathcal{O}(1)$.

Since D3-branes are attracted to the anti-D7-branes due to the potential (20), they are bound to the anti-D7-branes at the fixed point on the third torus unless another repulsive interaction is added. Other anti-D-branes can be introduced far from the fixed point so that D3-branes are separated from the fixed point. However, it is generally difficult to stabilize the positions of D3-branes, and, furthermore, even if they are stabilized, the typical length scale is given by the string scale, since there are no other dimensionful parameters in the theory.

### 3.2. A toy model of revolution

In the following, we consider D3-branes revolving around an anti-D7-brane whose centrifugal force balances the attractive force from the anti-D7-branes. As a warm-up exercise, let us consider a single complex scalar field $\phi$, which represents the moduli of the set of three D3-branes on the third torus, namely, each component of $Z_3^{(a)}$ that has the vacuum expectation value (see Eq. (19)). The Lagrangian that we consider is simply given by

$$\mathcal{L} = |\dot{\phi}|^2 - |\nabla \phi|^2 - \mu^2|\phi|^2,$$

where the mass term comes from the attraction between the D3- and anti-D7-branes. The coefficient is $\mu \sim M_s/10$. When we study time-dependent solutions, it is more convenient to consider the Hamiltonian density:

$$\mathcal{H} = |\pi_\phi|^2 + |\nabla \phi|^2 + \mu^2|\phi|^2.$$

Here, $\pi_\phi = \dot{\phi}^\dagger$. A solution to the equation of motion is given by

$$\phi = v_0 \exp(i\omega t)$$

with $\omega = \mu$. Since $\phi = (\phi_1 + i\phi_2)/\sqrt{2}$ represents the position of three D3-branes, this solution can be interpreted as a revolving D3-brane with an angular velocity $\omega$. In quantizing the system, we usually treat the zero-mode wave function equally with the other plane waves with nonzero momenta, because we consider that $\phi$ itself represents a microscopic quantum field. The amplitude of the zero mode $v_0$ fluctuates around $v_0 = 0$ and takes various values. But what if the solution is macroscopic (or classical)? In our case, $\phi$ represents the position of a D3-brane and the zero-mode wave function $\phi = v_0 \exp(i\omega t)$ has a geometrical meaning of a revolving D3-brane around the center: $(\phi_1, \phi_2) = (\sqrt{2}v_0 \cos \omega t, \sqrt{2}v_0 \sin \omega t)$. Then the solution $\phi$ must be treated classically as the coordinate of the revolving D3-brane. We thus need to expand the scalar field around the solution (24) and quantize the fluctuations, just as we do in the case of the Higgs field around a nontrivial vacuum in a double-well potential.

The next question is what determines the value of $v_0$. It cannot be fixed by the equation of motion derived from the Lagrangian (22). The situation is different from the case of the ordinary Higgs.
where it acquires a nontrivial vacuum expectation value at the minimum of the effective potential. In the present case, the would-be moduli field will acquire a nonzero value of $v_0$ as a result of the angular momentum conservation of the revolving D3-brane. In this sense, the determination of $v_0$ needs some external input (or an initial condition) provided from outside the 4D field theory. In order to see it, we first note that the Lagrangian (22) is invariant under the global $U(1)$ phase rotation $\phi \rightarrow e^{i \theta} \phi$. The corresponding Noether current

$$j_{\mu} = -i \left( \phi^* \partial_{\mu} \phi - \partial_{\mu} \phi^* \phi \right)$$

is conserved: $\nabla_{\mu} j_{\mu} = 0$. In the geometrical interpretation in terms of the D3-brane configurations, the symmetry is nothing but the rotational symmetry of the D3-brane around the center. Inserting $\phi = v_0 e^{i \omega t}$ into the current conservation, we get the conservation of the angular momentum of the revolving D3-brane around the anti-D7-branes:

$$\frac{\partial}{\partial t} \left( v_0^2 \omega \right) = 0. \quad (26)$$

The quantity $l = v_0^2 \omega$ is a constant of motion with a mass dimension 3, and is related to the angular momentum of the revolving D3-brane. The angular momentum is given by $M_{D3} \omega d^2$, where $M_{D3} \equiv \tau_3 V_{D3}$ is the mass of the D3-brane ($V_{D3}$ is the volume of the D3-brane) and $d$ is the distance from the center of the revolution. The mass $M_{D3}$ can be read from the kinetic term in the DBI action of Eq. (10), where the scalar fields $X^M$ are regarded as the coordinates of the brane embedded in the 10D space-time. The relations between $d$ and $v_0$, $d = 2 \sqrt{2} \pi \alpha' g v_0$, are obtained by identifying the mass of the ground state of the stretched open string, $d / 2 \sqrt{2} \pi \alpha'$, and gauge boson mass through the Higgs mechanism, $g v_0$. Hence, the value of $l$, which determines the value of $v_0$, needs to be fixed by the initial condition of the cosmological evolution of D3-brane configurations. At the end of the discussions in Sect. 5, we generalize the analysis to include the scale factor $a(t)$ of the Friedmann–Robertson–Walker (FRW) universe on D3-branes. The essential points discussed here are not changed.

In order to see the appearance of the centrifugal potential, we focus on spacial homogeneous solutions and write the complex scalar field as $\phi = v(t)e^{i\mu(t)}$. It can be expanded as $\phi = (v_0 + \delta v) e^{i(\omega t + \pi)} = v_0 e^{i\omega t} + \varphi$, where $\varphi \sim (\delta v + i v_0 \pi)e^{i\omega t}$. By inserting $\phi$ into the Hamiltonian density, we get

$$\mathcal{H} = |\dot{v} + ivu|^2 + \mu^2 v^2$$

$$= (\delta v)^2 + (v_0 + \delta v)^2 (\omega + \pi)^2 + \mu^2 (v_0 + \delta v)^2. \quad (27)$$

On the other hand, the current conservation requires that the combination

$$l = v^2 \dot{u} = (v_0 + \delta v)^2 (\omega + \pi) \quad (28)$$

is a constant of motion that should be fixed by an initial condition. This introduces a constraint in the system, and reduces the dynamical degrees of freedom. The Hamiltonian is written in terms of

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1 In the case of the ghost condensation [38], the derivative of a scalar field acquires a nonzero value as a result of the equation of motion. See discussions in Sect. 5 for comparison between the present model and the ghost condensation.
\[ \delta v \text{ only and becomes} \]

\[ H = (\delta \dot{v})^2 + \frac{l^2}{(v_0 + \delta v)^2} + \mu^2 (v_0 + \delta v)^2. \]  \hspace{1cm} (29) \]

Due to the conservation of the angular momentum, the phase fluctuation \( \pi \) disappears from the Hamiltonian. This is physically reasonable since the conservation of angular momentum relates the angular velocity to the radius of the revolution. Of course, this reduction of dynamical degrees of freedom is applied only to the zero mode, and nonzero momentum modes of \( \pi \) do not disappear from the Hamiltonian because only the total angular momentum is conserved. We will see this explicitly in Sect. 4. Hence the role of the angular momentum conservation is not the reduction of degrees of freedom, but the determination of the value of \( v_0 \). The minimum of the potential density \( V = \frac{l^2}{v_0^2} + \frac{\mu^2 v_0^2}{2} \) determines the value of \( v_0 \) as \( v_0^2 = l/\mu \). This is consistent with the relation \( l = v_0^2 \omega \) and the frequency of the classical solution of the revolution \( \omega = \mu \).

In the toy model discussed above, we have considered only the zero mode of the field configuration, which does not depend on the space coordinates. In obtaining the centrifugal potential for nonzero modes, we need to take into account competition between the strength of the centrifugal force and the tension on the brane. There are two limiting cases depending on which force is stronger. If the centrifugal force is stronger than the tension, each point on the brane behaves as if it is revolving independently of the other points. In this case, we can use an approximation that the conservation of the angular momentum at each point holds separately. Then each point is bound to the minimum of the potential in (29). The coefficient \( l^2 \) of the centrifugal potential can depend on position \( x \), and hence so does the minimum \( v_0(x) \). Due to the balance between the attractive force \( F_{\text{attractive}} = \mu v(x) \) and the repulsive force \( F_{\text{repulsive}} = \omega(x) v(x) \) at each point on the D3-brane, the brane revolves with the common angular frequency \( \omega(x) = \mu \) independent of the value of \( v_0(x) \). On the other hand, in the opposite case where the tension is stronger, the only conserved quantity is the total angular momentum, and the integral of (28) over the space \( L = \mu T_{D3} \int v(x)^2 d^3x \). Then the brane can rotate around itself as it revolves. The degrees of freedom on the D3-brane can contribute to the angular momentum. In the following, we consider the first situation. Then each point on the brane is strongly constrained in the minimum of the potential. Furthermore, we consider the simplest case where \( v_0(x) \) is independent of \( x \). \(^2\) General situations are left for future investigations.

### 3.3. Revolving D3-brane in DBI action

When the embedding \( X^\mu \) represents revolution of a D3-brane, we can see that the DBI action (10) is reduced to the toy model discussed above. In the static gauge, a revolving D3-brane on the third torus is described by the following set of coordinates:

\[ X^\mu = \xi^\mu, \quad Z_3 = \left( \tilde{d} e^{i\varphi} + \varphi \right)/\sqrt{\tau_3} \]  \hspace{1cm} (30) \]

where \( \tilde{d} \equiv \sqrt{\tau_3} d \). We set \( Z_1 = Z_2 = 0 \) for simplicity since we are interested in revolution of the D3-brane on the third torus. Then the induced metric on the world-volume of the D3-brane is given

\(^2\) If the attractive potential deviates from the harmonic potential, the angular frequency \( \omega(x) \) depends on \( v_0(x) \). A single brane favors revolution with the same angular velocity in order to avoid ripping. Hence \( v_0(x) \) may have a tendency to be aligned.
by
\[ G_{ab} = \eta_{ab} + \frac{1}{\tau_3} \tilde{D}_a \varphi^\dagger \tilde{D}_b \varphi \]  
(31)
where
\[ \tilde{D}_0 \varphi = \frac{\partial \varphi}{\partial \xi^0} + i d \omega e^{i \omega_0} \xi^0, \quad \tilde{D}_i \varphi = \frac{\partial \varphi}{\partial \xi^i}. \]  
(32)

Setting the B-field, dilaton, and gauge fields to zero, the DBI Lagrangian becomes
\[ L_{DBI} = -\tau_3 \sqrt{-\det G_{ab}} = -\tau_3 + \left| \tilde{D}_0 \varphi \right|^2 - \left| \nabla \varphi \right|^2 + O(\varphi^4). \]  
(33)

Including the supersymmetry-breaking mass term (20), this is the same as the toy model of the massive free scalar field of Eq. (27) by identifying the parameter as \( \tilde{d} = \sqrt{2} v_0 \). Then we obtain the relation \( d = 2 \sqrt{2} \pi a' g v_0 \). A generalization to \( n \) D3-branes is straightforward.

3.4. Effects of D7-branes and revolution of D3-branes in \( T^6/\mathbb{Z}_3 \)

The effects of the anti-D7-branes and the revolution of D3-branes in the world-volume field theory of the D3-branes can be summarized by adding a potential term \( l^2/v_0^2 + \mu^2 v_0^2 \) to the world-volume field theory of D3-branes. Here \( v_0 \) is proportional to the distance between the anti-D7- and D3-branes.

In the rotating reference frame where D3-branes are at rest, the branes feel the centrifugal potential \( l^2/|\phi|^2 \). Here, \( \phi \) is the moduli field representing the complex coordinate of the D3-branes. Originally it comes from the kinetic term but we can regard the term as one of the potential terms in the rotating reference frame. The discussion can be straightforwardly generalized to the case of revolving D3-branes on the \( T^6/\mathbb{Z}_3 \) orbifold. Since the three D3-branes are interacting with each other, they exchange angular momenta so that only the sum of each angular momentum of the D3-branes on the third torus is conserved. The conserved current is then the sum of each angular momentum of the D3-branes:
\[ j_\mu = -i \sum_a \left( Z_3^{(a)*} \partial_\mu Z_3^{(a)} - \partial_\mu Z_3^{(a)*} Z_3^{(a)} \right). \]  
(34)

Following the same argument in the toy model, the effects of the anti-D7-branes and the revolution of three D3-branes are to generate the attractive and repulsive potentials, respectively:
\[ V_M(Z) = \frac{(3l^2)}{\sum_a \left| Z_3^{(a)} \right|^2} + \mu^2 \sum_a \left| Z_3^{(a)} \right|^2. \]  
(35)

The coefficient 3 in front of \( l \) is introduced for later convenience. In order to apply this to nonzero modes, as we discussed at the end of Sect. 3.2, the centrifugal force must be stronger than the tension. In the next section, we discuss the low-energy spectrum of the D3–anti-D7-brane system with the above potential. Since the potential \( V_M(Z) \) depends only on the combination \( \sum_a \left| Z_3^{(a)} \right|^2 \), the mass term appears only in the breathing mode, which changes the magnitude of the same combination. In the rotating reference frame on the third torus, without loss of generality, we can choose the classical solution of D3-branes \( Z_i^{(a)} \) as
\[ \langle Z_3^{(1)} \rangle = \begin{pmatrix} 0 \\ v_0 \end{pmatrix}, \quad \langle Z_3^{(2)} \rangle = \begin{pmatrix} 0 \\ v_0 \end{pmatrix}, \quad \langle Z_3^{(3)} \rangle = v_0, \quad \langle Z_1^{(a)} \rangle = \langle Z_2^{(a)} \rangle = 0. \]  
(36)
Here, we have assumed that there is no separation of D3-branes in the first and second tori. This satisfies the D-flat condition. Note that we are now in the rotating reference frame of the D3-branes and the vacuum expectation value is set stationary without the phase. The breathing mode $\sigma$ is defined as that to change the magnitude of $\sum_a |Z_3^{(a)}|^2$:

$$v_0 \rightarrow v_0 + \frac{\sigma}{\sqrt{6}}.$$  \hspace{1cm} (37)

The normalization of the $\sigma$ field is chosen so that the kinetic term becomes canonical. Inserting this into the potential $V_M$, we can read the mass $m_\sigma$ of the breathing mode as $m_\sigma = 2\mu$. Hence it becomes as heavy as the string scale. The other modes are massless.

Finally in this section, let us play with the numerics of various quantities. A D3-brane is a gigantic macroscopic object and its mass is obtained as $M_{D3} = \tau_3 V_{D3}$. If we set $M_s = 10^{18}$ GeV and $V = (10^{-33} \text{ eV})^{-3}$ (i.e., the present particle horizon), the mass becomes $M_{D3} = 10^{207}$ eV. Of course, this is much larger than the corresponding “mass” obtained from the critical density $\rho_{\text{crit}}$ of our current universe $\rho_{\text{crit}} V_{D3} \sim 10^{89}$ eV. This is nothing but the cosmological-constant problem, since the energy scale of the tension of the D3-brane is typically as large as the string scale and much larger than the energy scale of the dark energy in the present universe, i.e., meV. The total angular momentum of the D3-brane is thus given by

$$L = M_{D3} \omega d^2 \sim (gv_0)^2 M_s V_{D3}.$$  \hspace{1cm} (38)

In the second equality, we have dropped the numerical factors and used $\omega = \mu = M_s$ and $d^2 = (gv_0)^2 / M_s^4$. When we put, e.g., $gv_0 = 100$ GeV, the angular momentum is roughly $L \sim 10^{148}$. Hence, we can treat the revolution of the D3-brane classically. Though the angular velocity $\omega$ is as huge as the string scale, the velocity of the revolution is nonrelativistic if the vacuum expectation value is much smaller than the string scale. For example, if $gv_0 \sim 100$ GeV, the radius of the revolution is $d \sim gv_0 / M_s^2$. Then the velocity of the revolving D3-brane is given by $\omega d \sim gv_0 / M_s \sim 10^{-16}$, and the motion is quite nonrelativistic. The acceleration is given by $\omega^2 d \sim gv_0$, and it is again much smaller than the string scale. This justifies the validity of the DBI action and its expansions.

4. Gauge-symmetry breaking by revolving D3-branes

In this section, we investigate the low-energy spectrum of the world-volume theory with revolving D3-branes on the third torus, and calculate the masses of various fields. We also discuss the stability of the vacuum expectation value.

If the centrifugal potential is stronger than the tension, we can treat each point on the brane as bound at the minimum of the potential (35).\(^3\) In such a situation, the Lagrangian that we are going to study becomes

$$\mathcal{L} = \left| D^\mu Z_i^{(a)} \right|^2 - (V_F + V_D + V_M).$$  \hspace{1cm} (39)

$V_F$ and $V_D$ are the F- and D-term potentials. The potential $V_M$ is the supersymmetry-breaking potential induced by the anti-D7-branes and the revolution of D3-branes discussed in the previous section.

\(^3\)Note that, in the opposite situation, where the centrifugal potential is comparable to or weaker than the tension of the brane, a different treatment of the effect of revolution is necessary by expanding the action of the system around the time-dependent background with only the total angular momentum being conserved. In this case, the term linear in the time-derivative cannot be removed and the effect of Lorentz violation on the D3-branes appears. Detailed investigations will be made in a separate paper.

\(^4\)The convention of the signature of the metric is different from that used in the DBI action.
This term determines the position of the D3-branes revolving around the fixed point. The solution of the stationary condition of the potential is given by Eq. (36) with $v_0^2 = l/\mu$, and then the gauge symmetry is spontaneously broken from $U(2) \times U(1)_1 \times U(1)_2$ to $U(1) \times U(1)$.

4.1. **Gauge boson masses**

First let us calculate the masses of the gauge bosons. Since the gauge-symmetry breaking is from $U(2) \times U(1)_1 \times U(1)_2$ to $U(1) \times U(1)$, four of the six gauge bosons, $W^A_\mu$ of $U(2)$, $B^{(1)}_\mu$ of $U(1)_1$, and $B^{(2)}_\mu$ of $U(1)_2$, become massive by the Higgs mechanism. The other two gauge fields, associated with two unbroken generators, remain massless. With the normalizations of the charges in the covariant derivatives of Eq. (12), the following two combinations

$$A^{(1)}_\mu = \frac{1}{\sqrt{2}} W^3_\mu - \frac{1}{2} (B^{(1)}_\mu + B^{(2)}_\mu), \quad A^{(2)}_\mu = \sqrt{\frac{2}{3}} W^0_\mu + \frac{1}{\sqrt{6}} W^3_\mu + \frac{1}{2\sqrt{3}} (B^{(1)}_\mu + B^{(2)}_\mu)$$

are massless. The other four combinations become massive. They are classified into two different types. The first type is associated with the broken generators of $U(2)$ and is given by

$$W^\pm_\mu = \frac{1}{\sqrt{2}} (W^1_\mu \pm i W^2_\mu).$$

It acquires the mass of $g v_0$ and is charged under the above unbroken $U(1)$ symmetries. The other type of combination is orthogonal to the massless gauge fields (40) and is written as

$$Z^{(1)}_\mu = \frac{1}{\sqrt{2}} (B^{(1)}_\mu - B^{(2)}_\mu), \quad Z^{(2)}_\mu = \frac{1}{\sqrt{3}} (W^0_\mu - W^3_\mu) - \frac{1}{\sqrt{6}} (B^{(1)}_\mu + B^{(2)}_\mu).$$

It acquires masses of $\sqrt{3} g v_0$ and is neutral under the remaining two $U(1)$ symmetries. The gauge boson $Z^{(1)}_\mu$ corresponds to an anomalous $U(1)$ gauge symmetry of $U(1)_1 \times U(2)_2$. These two mass values can be geometrically understood as the distances of the stretched open strings between the D3-branes. Namely, $g v$ corresponds to the distance $d$ of the D3-branes from the center and $\sqrt{3} g v_0$ corresponds to the distance $s$ between the D3-branes in Fig. 3.

4.2. **Scalar boson masses**

Next let us calculate the mass spectrum of the scalar fields. As we discussed at the end of the previous section, since the potential $V_M$ is a function of a single combination of fields, only the breathing mode of Eq. (37) acquires mass through the potential $V_M$ and the other modes are not affected by the supersymmetry-breaking potential $V_M$. We now consider the fluctuations of the scalar fields along the third torus $Z^{(a)}_3$. Define the fluctuations as $Z^{(a)}_3 = \{Z^{(a)}_3\} + \phi^a$, where the vacuum expectation value is defined in Eq. (36) and $\phi^a$ are decomposed into three types of scalar fields $\sigma$, $\rho$, and $\pi$:

$$\varphi^1 = \left(\frac{\rho^1}{\sqrt{2}}\right), \quad \varphi^2 = \left(\frac{\sigma^2 + i\pi^2}{\sqrt{2}}\right), \quad \varphi^3 = \frac{1}{\sqrt{2}} \left(\sigma^3 + i\pi^3\right).$$

The mass terms of these scalars can be obtained by expanding the potential $V_F + V_D + V_M$ around the classical solution of Eq. (36). The F-term potential $V_F$ does not generate any mass term, because the classical solution is nonvanishing only along the third direction $Z^{(a)}_3$. The potential $V_M$ generates

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5 It is neutral under the combination of $(A^{(1)}_\mu - \sqrt{3} A^{(2)}_\mu)/2$. 

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the mass term for the breathing mode in $\sigma$ only, and the other modes of $\sigma$, $\pi$, and $\rho$ are not affected by $V_M$.

The mass matrix of the $\sigma$ modes is obtained as follows. By redefining the fluctuations of $\sigma^a$ by the unitary transformation

\[
\begin{pmatrix}
\tilde{\sigma}^1 \\
\tilde{\sigma}^2 \\
\tilde{\sigma}^3
\end{pmatrix}
= U
\begin{pmatrix}
\sigma^1 \\
\sigma^2 \\
\sigma^3
\end{pmatrix},
\quad U = \begin{pmatrix}
\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{3}}
\end{pmatrix},
\]

only the first component $\tilde{\sigma}^1$, which corresponds to the breathing mode, acquires mass proportional to $\mu$. Since this mode originally corresponds to the flat direction of the D-term potential, $V_D$ does not generate potential for the breathing mode. On the other hand, the other two combinations get their masses from the D-term potential only, and are not affected by $V_M$. Their masses are proportional to $g v_0$ and independent of $\mu \sim M_X/10$. Indeed, the mass matrix of “Higgs bosons”, $\sigma^a$, is given by

\[
\frac{1}{3}(2\mu)^2 \begin{pmatrix}
1 + 2\epsilon & 1 - \epsilon & 1 - \epsilon \\
1 - \epsilon & 1 + 2\epsilon & 1 - \epsilon \\
1 - \epsilon & 1 - \epsilon & 1 + 2\epsilon
\end{pmatrix},
\]

where $\epsilon \equiv (g v_0/2\mu)^2/3$. The mass of the breathing mode $\tilde{\sigma}^1$ is 2$\mu$ and becomes very heavy. The other two modes $\tilde{\sigma}^2$ and $\tilde{\sigma}^3$ have equal masses of $\sqrt{3}g v_0$.

The masses of the $\rho$ modes are given as follows. Among four real scalar fields in $\rho^a$, two combinations are would-be Nambu–Goldstone bosons of the gauge-symmetry breaking of SU(2) into U(1). Since the broken generators acting on $Z_3^{(1)}$ and $Z_3^{(2)}$ are $\sigma^a/2$ and $(-\sigma^a)^*/2$ with $a = 1, 2$, respectively, the combinations of

\[
\frac{1}{\sqrt{2}} (\rho^1_R + \rho^2_R), \quad \frac{1}{\sqrt{2}} (\rho^1_I - \rho^2_I)
\]

are eaten by the longitudinal modes of $W^\pm_\mu$ gauge bosons. Here, we have written the real and imaginary parts of $\rho^a$ as $\rho^a = (\rho^a_R + i\rho^a_I)/\sqrt{2}$. The other two combinations of

\[
\frac{1}{\sqrt{2}} (\rho^1_R - \rho^2_R), \quad \frac{1}{\sqrt{2}} (\rho^1_I + \rho^2_I)
\]

are “charged Higgs bosons” with mass $g v_0$.

For the $\pi^a$ modes, two of them are the would-be Nambu–Goldstone bosons of the gauge-symmetry breaking of four U(1) symmetries (corresponding to two diagonal generators of U(2), U(1)$_1$, and U(1)$_2$) into U(1) $\times$ U(1), and become the longitudinal modes of $Z_3^{(1)}$ and $Z_3^{(2)}$. The last one is massless. It is the Nambu–Goldstone boson associated with the breaking of the global U(1) symmetry, by which the phases of the scalar fields $Z_3^{(a)}$ ($a = 1, 2, 3$) are rotated simultaneously as $Z_3^{(a)} \rightarrow e^{i\theta} Z_3^{(a)}$. This mode corresponds to changing the positions of the revolving D3-branes without changing their relative positions and the distance from the origin. This global symmetry is the anomalous U(1)$_R$ symmetry of the full theory with the superpotential of Eq. (14), and, accordingly, the corresponding scalar field becomes massive.

4.3. Stability of the symmetry-breaking scale
The following arguments indicate the stability of the mass spectrum of the scalar fields against radiative corrections. Spontaneous symmetry breaking occurs due to the potential $V_M$ in Eq. (39). If gauge
interactions were absent, we could consider the upper and lower components of the doublet fields $Z_3^{(a)}$ with $a = 1, 2$ as independent complex scalar fields. The action is then invariant under the $U(5)$ rotation of 5 complex scalar fields, and the $U(5)$ invariant potential $V_M$ has a minimum where $U(5)$ is spontaneously broken down to $U(4)$. Then the symmetry breaking of the global $U(5)$ symmetry produces one massive “Higgs” boson and nine massless Nambu–Goldstone bosons. In the presence of gauge interactions, four of these nine Nambu–Goldstone bosons are absorbed into the gauge bosons through the Higgs mechanism, and the remaining five scalars become massive as pseudo-Nambu–Goldstone bosons (pNGBs), because the corresponding $U(5)$ symmetries are explicitly broken by the gauge interactions. The radiative correction to the mass of the pNGB is protected by the approximate symmetry. The mass spectrum of the pNGB is generally written as the Dashen formula \[ m^2 = \frac{1}{f^2} \langle [Q, [Q, H]] \rangle, \] where $f$ is the decay constant and $Q$ is the broken generator corresponding to the fluctuation of the pNGB around the minimum, and is stable under radiative corrections so long as the coupling is weak \[40–43\]. In our scenario, the vacuum expectation value $v_0$ is determined in terms of the initial angular momentum of the D3-branes that is conserved. Hence, it is fixed by the input of the initial condition, not by the field theory itself. This guarantees the stability of the value of the distance between the revolving D3-branes and the fixed point where the anti-D7-branes are localized. In this geometrical way, the symmetry-breaking scale is invariant.\[6\] Furthermore, since the mass of the $\tilde{\sigma}_1$ field is much heavier than the other scalar fields, quantum fluctuations of the vacuum expectation value are highly suppressed in the low-energy effective theory. This is another indication of the stability of the scale of spontaneous gauge-symmetry breaking.

### 4.4. Other fields and fermions

So far we have considered only the scalar fields $Z_3^{(a)}$ on the third torus. Let us now see how the masses for other fields $Z_i^{(a)}$ with $i = 1, 2$ are generated. They were originally moduli fields corresponding to the motion of D3-branes along the first and second tori. When $Z_3^{(a)}$ have the vacuum expectation values of Eq. (36), they acquire their masses through the interactions described by the F-term potential (13). For their fermionic superpartners, they acquire the same masses through the Yukawa couplings, since the gauge-symmetry breaking by the vacuum expectation value (36) does not break supersymmetries. All the fields, which are associated with the first and second tori, obtain masses of the order of $g v_0$. Thus all the D-brane moduli fields corresponding to the first and second tori are stabilized. They may also obtain their masses through one-loop effects of open strings between D3- and anti-D7-branes, which are not supersymmetric.

Yukawa couplings between the scalar fields $Z_i^{(a)}$ and the fermion fields in the chiral multiplets come from the superpotential of Eq. (14). Because of the $\epsilon$-tensor in the superpotential, the Yukawa couplings must contain at least one field associated with each of the three tori. Hence, since the scalar fields $Z_3^{(a)}$ with the nonzero vacuum expectation values couple to the fermions associated with the first and second tori, the fermions in the first and second tori become massive in pairs. In particular, the Yukawa coupling of the “Higgs boson”, $\tilde{\sigma}_1$, with the fermions is proportional to their masses. But this is not the case for the light scalar fields, $\tilde{\sigma}_2$ and $\tilde{\sigma}_3$. Note that the fermion fields associated

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\[6\] In Sect. 5, we discuss the possible decay of the revolution radius (i.e., the vacuum expectation value) through emissions of massless closed-string states.
with the third torus do not obtain their masses through Yukawa couplings. A brief discussion towards more realistic model constructions of the standard model is given in Sect. 5.

5. Conclusions and discussions

In this paper, we have proposed a new mechanism of spontaneous gauge-symmetry breaking in string theory with revolving D-branes around an orbifold fixed point.

In orbifold compactification of the string theory, two types of D-branes are known: ordinary D-branes and fractional D-branes. The ordinary D-branes can move away from the fixed points by forming an invariant set under the orbifold projections, while the fractional D-branes are localized at the fixed points. When the ordinary D-branes move away from the fixed point, the gauge symmetry is spontaneously broken with reduction of the rank of the gauge group. If the separation scale of the D-branes is shorter than the string length and their relative motion is nonrelativistic, the dynamics can be well described by the low-energy world-volume theories of D-branes [44]. The separation length corresponds to the vacuum expectation value of the D-brane moduli fields. Unless supersymmetry is broken, the moduli fields have a flat potential along the vacuum expectation values. If we consider configurations of D-branes with broken supersymmetries, the flat directions will be lifted up by the one-loop corrections, whose mass scales are generically given by the string scale with a small correction of the one-loop suppression factors. Such effects are responsible for the attractive forces between D-branes and anti-D-branes.

In the present paper, we consider a particular model of D3- and anti-D7-branes at a fixed point of a $T^6/Z_3$ orbifold. In order to stabilize the separation of D3-branes from the fixed point, we introduced revolution of D3-branes around anti-D7-branes localized at the fixed point. The revolution can be described as the time-dependent phase of the classical solution of D-brane moduli fields. In the rest frame of the revolving D3-branes, the centrifugal potential appears. This force can balance the attractive force between the D3- and anti-D7-branes. The distance of the revolving D3-branes from the fixed point is determined by the value of the angular momentum, which should be given by the initial condition of the cosmological evolution of the D-brane configuration. An important point is that the scale of the distance, namely the vacuum expectation value of the moduli fields, is determined in terms of the angular momentum and can be taken much lower than the string scale. The situation is quite similar to the revolution radius of the earth around the sun, where it was determined by the initial condition of solar-system evolution. We can imagine that the early universe is filled with a gas of D-branes and anti-D-branes. They collide and some of them are annihilated, but, due to the conservation of angular momentum, some branes revolve around others.

We calculated the mass spectra of various moduli fields. The breathing mode, in which all the nonzero expectation values of the moduli fields change simultaneously, becomes superheavy as the string scale $M_s$ because the mass comes from the balance of the attractive potential between D3 and $\overline{D7}$ and the repulsive centrifugal potential. On the other hand, the other scalar modes acquire their masses through gauge-symmetry breaking, and hence their masses are of the order of the vacuum expectation value $v_0 \ll M_s$. They are stable under radiative corrections due to their pseudo-Nambu–Goldstone-boson nature. It is interesting to note that the stability of the scale $v_0$ is related to the stability of the geometrical distance of the revolving D3-branes from the fixed point. The distance is classically stable due to the conservation of angular momentum.

Now several important remarks are in order. The first remark is about the possible loss of the angular momentum of the revolving D3-branes by emitting massless closed-string states (gravitons
or a 4-form R–R field). If the effects are large, the revolution frequency $\omega$ decreases and the D3-branes fall rapidly into the fixed point. As we discussed at the end of Sect. 3, the velocity $\beta = \omega d \sim \alpha v_0/M_s$ and the acceleration $\alpha = \omega^2 d \sim \omega v_0$ are very small and the emission may be expected to be tiny. However, this is not the case since both the tension and the R–R charge are very large. Here let us estimate the emission rate of the gravitational radiation into the bulk from the revolving D3-brane by using the formula in Ref. [45]. The energy emission rate per unit time and unit volume of the D3-brane is estimated by $M_s^5 (\omega d)^4$. A huge factor comes from the tension of the D3-brane. The rate is very large in comparison with the kinetic energy of the revolving D3-branes per unit volume $t_\gamma (\omega d)^2/2 \sim M_s^4 (\omega d)^2$, even though $\omega d \sim (\omega/M_s)(v/M_s)$ is very small. If this is really the case, the stability of the vacuum expectation value will be lost. However, the formula of the emission rate may not be applicable to the present case since the typical frequency of the radiation, which is of the same order of the angular velocity of the revolution $\omega \approx M_s/10$, is close to the cutoff scale of the low-energy effective theory, $M_s$. Also, it is not certain whether radiation itself is possible since the length scale of the compact space is comparable to the typical wavelength of the emitted radiation. Furthermore, the NS–NS tadpoles are not canceled in this nonsupersymmetric configuration of D-branes and we may need to take into account the effects of the backreaction to the space-time metric, dilaton, and B-fields, which may drastically change the emission rate of the radiation. So more detailed analysis will be necessary to give correct estimates of the emission rate.

The second remark concerns the possibility of a modification of gravity in the IR region. One may expect that the revolution of D3-branes affects the gravity in the IR region since our model is similar to the ghost condensation [38]. In both the ghost condensation and our setting, the time-dependent classical solutions of the scalar fields play an important role. In the ghost condensation, $X^2 = |\phi|^2\phi|\phi|$ has a nonvanishing vacuum expectation value at the minimum $X_0^2$ of the function $P(X^2)$. Then the field can be expanded around the time-dependent solution $\phi = X_0 t$, which breaks the Poincaré symmetry. Writing $\phi = X_0 t + \varphi$, we have $X^2 = (X_0 + \varphi)^2 - (\nabla \varphi)^2$. Then the kinetic term $P(X^2)$ becomes $P(X^2) \sim P(X_0^2) + \left[2 P''(X_0^2) X_0^2 \right] \varphi^2$. The $(\nabla \varphi)^2$ term is absent, and the IR dispersion relation is drastically modified as $\omega^2 \propto p^4$. In our case of revolving D-branes, the time-dependent solution appears not because it gives a minimum of the kinetic term but because it is due to the centrifugal potential in the rest frame of the D-branes. Because of this, if the centrifugal potential is stronger than the tension of the brane and each point of the brane is bound strongly at the minimum of the potential, the spectrum of the fluctuation seems to keep the Lorentz invariance and is different from the case of the ghost condensation. But the Lorentz invariance is generally broken due to the revolution, especially if the tension is comparable to the centrifugal potential. We will come back to this issue in the near future.

The third remark regards the construction of more realistic models of particle physics. There have been many efforts to construct realistic models in the system of D-branes at orbifold fixed points or other singular points in general (see, e.g., Refs. [18–21]). Though the aim of this paper is not to propose a realistic model, some interesting structures of realistic models are included in the model that we have discussed above. For example, following Ref. [18], we can identify the subgroup SU(2) in U(2) as the standard-model SU(2)_L group and a nonanomalous combination of U(1) symmetries, $Q_Y = -(Q/2 + Q_1 + Q_2)$, as U(1)_Y in the standard model. Here $Q$, $Q_1$, and $Q_2$ are charges of U(1) in U(2), U(1)_1, and U(1)_2, respectively. In the identification, the value of the Weinberg angle becomes $\sin^2 \theta_W \approx 0.27$. Since the charges are normalized to take values of $\pm 1$, we can identify the scalar component of $Z_3^{(1)}$ as a Higgs doublet field ($Q_Y = +1/2$), the fermion components of $Z_{i=1,2}^{(2)}$ as the left-handed lepton doublets of the first and second generations ($Q_Y = -1/2$), and
the fermion components of $Z_{i=2,1}^{(3)}$ as the right-handed neutrinos of the first and second generations ($Q_Y = 0$). In this simple setting, the Yukawa couplings originate from the superpotential of Eq. (14) and give large masses to neutrinos. It would be interesting to try to construct more realistic models with quarks and leptons in the framework of spontaneous gauge-symmetry breaking by the revolution of D-branes.

The fourth remark is about a realization of an inflationary universe with revolving D-branes. In the present paper, we treated the bulk space-time as flat Minkowski, but of course it receives huge backreaction effects, since the configuration is not supersymmetric. In the following, instead of solving the bulk 10D Einstein equation, we study the time evolution of the induced metric on the D3-branes [46]. Since D3-branes are isotropic and homogeneous, the time evolution of the metric will be described by the FRW universe\(^7\) with the scale factor $a(t)$. In the FRW universe, the solution to the equation of motion is changed from Eq. (24) to

$$\phi \sim a^{-3/2} v_0 e^{i\omega t}. \quad (49)$$

This solution is valid when the oscillation frequency $\omega = \mu$ is larger than the Hubble parameter $H = \dot{a}/a$. In the opposite case $H > \mu$, the field is frozen so that the prefactor is almost constant. Such a scale-factor dependence is understandable since the oscillation in a harmonic potential behaves as nonrelativistic particles and the energy density $\mu^2 v^2$ must decay as $a^{-3}$. Now suppose that D3-branes, while they are rotating, start rolling down along the potential from a larger value of $v$ to the stabilized point $v_0$. The current $j_0$ is proportional to $a^{-3} v^2 \omega$. Due to an additional factor $\sqrt{|g|} = a^3$ in the current conservation $\nabla^\mu j_\mu = a^{-3} \partial^\mu (a^3 j_\mu) = 0$, the same quantity $l = v^2 \omega$ is conserved. The Hamiltonian density $\rho$ for $\mu > H$ becomes

$$\rho = a^{-3} (v^2 \omega^2 + \mu^2 v^2) = a^{-3} (l^2 / v^2 + \mu^2 v^2). \quad (50)$$

The minimum of the potential is given by the same value $v_0 = l/\mu$. The pressure density $p$ is given by changing the sign of the second term as

$$p = a^{-3} \left( l^2 / v^2 - \mu^2 v^2 \right). \quad (51)$$

This is because the first term (centrifugal potential) originates from the kinetic energy while the second one comes from the attractive interaction between D3- and anti-D7-branes. Denoting the ratio of $v$ to $v_0 = \sqrt{l/\mu_0}$ by $v/v_0 \equiv e^{\theta/2}$, $\rho$ and $p$ can be written as $\rho = 2a^{-3} l \mu \cosh \theta$ and $p = -2a^{-3} l \mu \sinh \theta$. Then the equation of state becomes

$$w = \frac{p}{\rho} = -\tanh \theta. \quad (52)$$

It is interesting that the equation of state of the revolving motion interpolates between $w = -1$ and $w = 0$. At the final stage of the evolution, D3-branes are stabilized at $v = v_0$ and the equation of state is $w = 0$. The energy density is then given by $\rho = 2l \mu a^{-3}$. On the other hand, in the initial stage of cosmic evolution, the second term $\mu^2 v^2$ in $\rho$ and $p$ dominates the first term and the equation of state is $w = -1$. In this case, the field is frozen and the prefactor $a^{-3}$ is replaced by a constant. Then the energy density becomes $\rho = \mu^2 v^2$, which gives the Hubble constant of the inflationary universe caused by the revolving D3-branes. In this scenario, the radial direction of the revolving

\(^7\) Our assumption here is that the metrics on different D3-branes are strongly correlated with each other so that they can be described by a single common metric.
D-branes, $\tilde{\sigma}_1$, plays the role of the inflaton field, similar to the brane-inflation model [37]. Similar angular motions of D-branes in the DBI inflation scenario has been discussed in Refs. [47–49].

The final remark is about possible modifications of the background geometry of the compact space due to the introduction of three-form fluxes and additional (anti-)D-branes for the moduli stabilization. The form of the angular momentum conservation might be modified as well as the attractive potential. However, as long as the rotational symmetry is preserved and the angular momentum is conserved, the essential mechanism will not be changed much. The nontrivial modification of the background geometry could make it easier to obtain the small scale of the gauge-symmetry breaking in comparison to the string scale, as is the case in the warped geometries [50].

Each of the issues mentioned above is very interesting, but needs more careful and detailed investigations. We want to come back to these issues in the future.

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