Quark confinement potential examined by excitation energy of the $\Lambda_c$ and $\Lambda_b$ baryons in a quark–diquark model

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The possibility of having a diquark configuration in heavy baryons, such as $\Lambda_c$ and $\Lambda_b$, is examined by a nonrelativistic potential model with a heavy quark and a light scalar diquark. Assuming that the $\Lambda_c$ and $\Lambda_b$ baryons are composed of the heavy quark and the point-like scalar–isoscalar $ud$ diquark, we solve the two-body Schrödinger equation with the Coulomb plus linear potential and obtain the energy spectra for the heavy baryons. Contrary to our expectation, it is found that the potential determined by the quarkonium spectra fails to reproduce the excitation spectra of the $\Lambda_c$ and $\Lambda_b$ in the quark–diquark picture, while the $\Lambda_c$ and $\Lambda_b$ spectra are reproduced with half the strength of the confinement string tension than for the quarkonium. The finite size effect of the diquark is also examined and it is found that the introduction of a finite size diquark would resolve the failure of the spectrum reproduction. The $\Xi_c$ excitation energy is also calculated and is found to be smaller than $\Lambda_c$ in the quark–diquark model. This is not consistent with experimental observations.

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1. Introduction

Hunts for fundamental correlations in strongly interacting quarks and gluons inside hadrons can give hints to understanding the hadron structure and the confinement mechanism. The constituent quarks are considered to be good effective degrees of freedom for the hadron structure, in particular explaining the nucleon magnetic moment based on the spin flavor symmetry of the light quarks and low-lying energy spectra of the heavy quarkonia by quark excitations in the linear-plus-Coulomb confinement potential. The diquark as a two quark pair correlation in a hadron can be also a strong candidate for a hadron constituent [1,2]. So far, the existence of diquark correlation inside hadrons has been suggested by phenomenological findings in baryon spectroscopy, weak non-leptonic decays, parton distribution functions, and fragmentation functions, and in particular the scalar diquark with flavor, spin, and color antisymmetric configurations, the so-called good diquark, is expected to have the most attractive correlation [3]. The strong correlation in the scalar diquark is obtained by one-gluon exchange and by instanton-induced interactions, and has been found in lattice calculations [4–7]. The strong correlation can be a remnant of the Pauli–Gürsey symmetry in two-color QCD, in which quark–quark interactions for massless quarks are exactly the same in strength as quark–antiquark interactions. In three-color QCD, this symmetry is explicitly broken and the quark–quark correlations are less attractive than the quark–antiquark correlation.
To investigate the nature of the confinement potential, we focus on the excitation spectra of the heavy baryons. So far, much work has been done for baryon spectra in quark–diquark models. In Ref. [8], the mass spectra of the $p$ and $d$ excited states of non-strange baryons were investigated in a diquark–quark model based on the SU(6)$\otimes$O(3) classification [9] in investigating the fine structure by spin–orbit interactions. It was found that the detailed confinement mechanism is irrelevant for the fine structure of the intramultiplet splitting. In Ref. [10], the ground states of spin 3/2 baryons were investigated in a relativistic formulation. In Ref. [11], radial and orbital excitations of baryons were calculated in a diquark–quark model with a confinement potential reproducing meson spectra, and detailed analyses were given for light flavor baryon spectra. Recently, in Ref. [12], the ground state masses of $\Lambda_c$, $\Lambda_c^+$, and $\Lambda_b$ were calculated in a diquark QCD sum rule, in which a scalar diquark is explicitly considered as a fundamental field in the operator product expansion, and the calculation successfully reproduced the observed $\Lambda$ masses with a “constituent” diquark mass of 0.4 GeV, satisfying the standard criteria for the QCD sum rule to work well.

The system of a heavy quark and a diquark has an advantage in investigating light diquark properties. As reported in Ref. [13,14], in meson wavefunctions, the quark–antiquark components dominate and the diquark configurations are rather suppressed. This is because the diquark correlation is weaker than the quark–antiquark correlation and, once there exists an antiquark closed to a diquark, the diquark could easily fall apart and form a quark–antiquark pair. In light baryons, diquark configuration could play an important role in baryon structure, but rearrangement of the diquark would also be important due to the symmetry among the light quarks. In systems of a heavy quark and two light quarks forming a heavy baryon, thanks to the asymmetry between the light quark and the heavy quark, a diquark of two light quarks could easily emerge in the baryon.

In this brief note, taking the quark–diquark picture with a point-like diquark, we report on the excitation energy of the $\Xi_c$ and $\Lambda_b$ baryons using the linear-plus-Coulomb confinement potential suitable for the quarkonium spectra. Here we will find the puzzle that the string tension in the confinement potential for quark and diquark systems should be half as strong as that for quark and antiquark systems to reproduce the $\Lambda_c$ and $\Lambda_b$ excitation energies. This is against the universality of the confinement force. To understand the overestimation of these excitation energies, we will examine the finite size effect of the diquark, and find that introduction of the diquark size could be a solution to the above puzzle. We will also find that the excitation energy of the $\Sigma_c$ baryon should be smaller than that of $\Lambda_c$ in the quark–diquark picture, but experiments tell us the opposite. These puzzles should be solved when one takes the quark–diquark picture for heavy baryons.

### 2. Formulation

We describe the heavy baryons composed of one heavy and two light quarks by the diquark model in which the light quarks form the scalar diquark with antisymmetrized flavor, color anti-triple $\bar{3}$, and spin parity $0^+$. We regard the heavy quark (a charm or bottom quark) and the scalar diquark as elementary particles and take a quark potential model in the non-relativistic formulation. We also assume that the diquark is a point-like particle. In the center of mass frame, the Hamiltonian of the system including the rest masses of the quark and diquark is written as

$$H = m_h + m_d + \frac{p^2}{2\mu} + V(r),$$

with the heavy quark mass $m_h$, the diquark mass $m_d$, the momentum of the relative motion $p$, and the reduced mass $\mu$. We use the $c$ and $b$ quark masses as $m_c = 1.5 \text{ GeV}/c^2$ and $m_b = 4.0 \text{ GeV}/c^2$. 

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The interaction between the heavy quark and the scalar diquark is assumed to be spherical and written as a Coulomb plus linear form [15],

$$V(r) = -\frac{4\alpha}{3} \frac{\hbar c}{r} + kr + V_0,$$

(2)

with three constant parameters $\alpha$, $k$, and $V_0$. The Coulomb part proportional to $r^{-1}$ expresses the asymptotic nature of the strong interaction at short distances, and the linear part, $kr$, represents the confinement potential with the string tension $k$ of the flux tube. The constant $V_0$ adjusts the absolute value of the potential energy, turning out to be an irrelevant parameter in the present analysis because we are interested in the excitation energies of quark–diquark systems. It is known that the potential $V(r)$ given in Eq. (2) reproduces the charmonium and bottomium spectra well with appropriate spin–spin and spin–orbit interactions [16]. The scalar diquark of interest has an anti-triplet color charge and the same color charge as the antiquark. Because the color electric force should be independent of the quark flavor in the first approximation and be determined by the color charges, we may make good use of the potential determined by the quarkonium spectra for the quark–diquark system.

The energy spectrum and the wave function of the quark–diquark system can be obtained by solving the Schrödinger equation for the radial variable of the relative motion:

$$-\frac{\hbar^2}{2\mu} \frac{d^2 \chi_\ell(r)}{dr^2} + \left[ V(r) + \frac{\ell(\ell+1)\hbar^2}{2\mu r^2} \right] \chi_\ell(r) = E_\ell \chi_\ell(r),$$

(3)

where $\mu$ is the reduced mass of the heavy quark and scalar diquark, and $\chi_\ell(r) \equiv R_\ell(r)/r$ is the radial wave function for the orbital angular momentum $\ell \hbar$. The mass of the bound system for the heavy quark and the scalar diquark is given by $M = m_h + m_d + E_\ell$. Because this Schrödinger equation is a one-body equation after separating out the center of mass motion, it is simple enough to be solved numerically. We use the state label defined by $n_\ell$ with the quantum numbers $n_r$ and $\ell$ for the radial excitation and the orbital angular momentum, respectively. We are particularly interested in the excitation pattern of the quark–diquark system in the confinement potential. We focus on the energy spectrum without considering the fine structure caused by the spin–orbit interaction. The fine structure is not sensitive to the details of the confinement mechanism [8]. We may refine our model quantitatively, for instance, by fine-tuning the quark and diquark masses, or introducing scale dependence of the Coulomb part of the confinement potential. These are important next steps to be done. Here, of great interest is the global structure of the excitation spectra of the heavy baryon. Thus, as a first step, we ignore these corrections and the spin–orbit interaction.

3. Results

In this section, we present our results for the energy spectra obtained by solving the Schrödinger equation (3) with the confinement potential (2). First we show the charmonium and bottomium spectra to confirm that the confinement potential with $\alpha = 0.4$ and $k = 0.9 \text{ GeV fm}^{-1}$ reproduces the observed major excitation well. We will also see that with the same parameters the model works for the heavy mesons, $D, D_s, B, B_s$, with the light quark mass $m_q = 0.3 \text{ GeV}/c^2$ and the strange quark mass $m_s = 0.5 \text{ GeV}/c^2$. Next, finding that the confinement potential suitable for the meson spectra does not work in the quark–diquark systems to reproduce the excitation energies of $\Lambda_c$ and $\Lambda_b$, we search for the appropriate parameter set to reproduce the $\Lambda_c$ and $\Lambda_b$ spectra. We will calculate the $\Xi_c$ and $\Xi_b$ excitation energies by introducing the strange scalar diquark composed of a strange quark and a light quark. We also estimate the size of the decay width of the excited state of $\Lambda_c$ based on
**Fig. 1.** Excitation spectra of charmonium and bottomium in our model with $\alpha = 0.4$, $k = 0.9$ GeV fm$^{-1}$, $m_c = 1.5$ GeV/c$^2$, and $m_b = 4.0$ GeV/c$^2$. In the left line for each quarkonium the calculated values are shown with the state label and a number. The states are labeled by $n, \ell$ with the radial quantum number $n$ and the angular momentum quantum number $\ell$. The calculation is done without spin–spin or spin–orbit interactions. In the right line for each quarkonium we show the excitation energies of the observed quarkonia measured from the spin-weighted average of the $1s$ ground state energies. The experimental data are taken from the Particle Data Group [18].

Let us first see that the confinement potential (2) with $\alpha = 0.4$ and $k = 0.9$ GeV fm$^{-1}$ works well for the charmonium and bottomium spectra [17]. In Fig. 1, we plot the excitation energy spectra of the charmonium and bottomium measured from each ground state. The calculation is done with the potential given in Eq. (2) with $\alpha = 0.4$ and $k = 0.9$ GeV fm$^{-1}$ without spin–spin or spin–orbit interactions. The quark masses are fixed as $m_c = 1.5$ GeV/c$^2$ for the charm quark and $m_b = 4.0$ GeV/c$^2$ for the bottom quark. To compare the calculation without the spin–spin and spin–orbit interactions, the excitation energies of the observed charmonia are measured from the spin-weighted average of the $\eta_c$ and $J/\psi$ masses, $(m_{\eta_c} + 3 m_{J/\psi})/4$, which removes the effect of the spin–spin interaction perturbatively, since the matrix element of the spin–spin operator $\vec{s}_1 \cdot \vec{s}_2$ for the quark–antiquark system with total spin $S$ and angular momentum $\ell = 0$ is calculated as

$$2 \langle S | \vec{s}_1 \cdot \vec{s}_2 | S \rangle = S(S + 1) - \frac{3}{2} = \begin{cases} -\frac{3}{2} & \text{for } S = 0, \\ \frac{1}{2} & \text{for } S = 1. \end{cases}$$

In the same way, the excitation energies of the observed bottomia are measured from the spin-weighted average of $\eta_b$ and $\Upsilon$, $(m_{\eta_b} + 3 m_{\Upsilon})/4$. Figure 1 shows that the potential (2) with $\alpha = 0.4$ and $k = 0.9$ GeV fm$^{-1}$ successfully reproduces the size of the major excitations (radial and orbital excitations) both for charmonium and bottomonium.

We also calculate the excitation energies of the $D$ and $D_s$ mesons, which are composed of a charm quark and a light (up, down, or strange) quark, by using the same potential parameters, the light quark mass $m_q = 0.3$ GeV/c$^2$, and the strange quark mass $m_s = 0.5$ GeV/c$^2$. As shown in Fig. 2, we obtain 0.50 GeV and 0.77 GeV for the $D$ meson excitation energies of the $1p$ and $2s$ states from the ground state (1s), respectively. These values should be compared with the experimental data.
The experimental excitation energies are shown again from the spin-weighted averaged value of the ground states with $\ell = 0$. Because the excited states of the $D$ and $D_s$ mesons are unstable against the strong interaction with large decay widths, and the quantum numbers of some states are not well known yet, the detailed comparison is somewhat difficult. Nevertheless, it is observed that the excitation energies of the first excited states of the $D$ meson are around 0.45 to 0.57 GeV. These values are very similar to the value obtained by our calculation. Thus, we presume that the excitation of the $D$ meson can also be described by the potential with $\alpha = 0.4$ and $k = 0.9$ GeV fm$^{-1}$. In Fig. 3 we show the calculated results for the $B$ and $B_s$ mesons together with the observed spectra. We find that there is no strong discrepancy between the calculation and the observation.

### 3.2. Mass of diquark in $\Lambda_c$

With the success of the potential model for the meson spectra, we would make good use of the same potential parameters for the $\Lambda_c$ spectrum. Once we have fixed the potential parameters as $\alpha = 0.4$ and $k = 0.9$ GeV fm$^{-1}$ in the meson spectra, the diquark mass is only the parameter in the present model. The diquark mass can be fixed, for instance, by the excitation energy of $\Lambda_c$. In Fig. 4, we plot the diquark mass dependence of the excitation energy of the first excited state...
Fig. 4. The $\Lambda_c$ excitation energy of the first excited state ($1p$) from the ground state ($1s$) as a function of the diquark mass with the $c$ quark mass $m_c = 1.5 \text{ GeV}/c^2$ and the potential parameters $k = 0.9 \text{ GeV} \cdot \text{fm}^{-1}$ and $\alpha = 0.4$, which are suitable for the meson systems. The excitation energy for the observed $\Lambda_c$ is 0.330 GeV. The quark–diquark model with the confinement potential suitable for the meson systems and a reasonable diquark mass cannot reproduce the observed $\Lambda_c$ spectrum.

(1p) from the ground state (1s). The calculation is done without the spin–orbit interaction for the charm quark, which is responsible for the $LS$ splitting between $1/2^-$ and $3/2^-$. In the range of the diquark mass from 0.3 GeV/c$^2$ to 1.0 GeV/c$^2$, we obtain the excitation energy in the range 0.5 GeV to 0.44 GeV. To compare these values with the observed $\Lambda_c$ excitation energies, we take the spin-weighted average of the excitation energies of the 1p states, $1/2^-$ and $3/2^-$. The matrix element of the spin–orbit interaction operator is calculated for the state with total angular momentum $J$, orbital angular momentum $\ell$, and spin $s = 1/2$ as

$$2\langle J, \ell, 1/2 | \vec{\ell} \cdot \vec{s} | J, \ell, 1/2 \rangle = J(J + 1) - \ell(\ell + 1) - \frac{3}{4} = \left\{ \begin{array}{ll}
-2 & \text{for } J = 1/2, \ell = 1, \\
1 & \text{for } J = 3/2, \ell = 1.
\end{array} \right. \quad (5)$$

Thus, if we take the weighted sum $E_{\text{ave}} = \frac{2}{3}E_{3/2^-} + \frac{1}{3}E_{1/2^-}$, we can remove the effect of the spin–orbit interaction in perturbation theory.

The averaged value of the observed excitation energies of $\Lambda_c$ is found to be 0.330 GeV. In the following, we refer to this value as the experimental value of the 1p excitation energy of $\Lambda_c$. The experimental value 0.330 GeV is outside the range of the calculated values with the diquark mass from 0.3 GeV/c$^2$ to 1.0 GeV/c$^2$, which is an expected range for the diquark as a bound state of two constituent quarks. Thus, we conclude that the confinement potential suitable for the meson systems cannot produce the $\Lambda_c$ spectrum in the quark–diquark picture with a reasonable diquark mass. This is a very interesting finding. The confinement force in the color electric interaction may be mainly dependent on the color charge and not on the flavor of the colored object. The present result, however, shows that the confinement force is different in the quark–antiquark and quark–diquark systems in the case of a point-like diquark, although the antiquark and diquark have the same color charge. Therefore, as long as we take the quark–diquark picture, we should explain why the confinement potential is different in these systems.

3.3. Potential parameters for $\Lambda_c$ and $\Lambda_b$

We have found that the confinement potential fitted by the quarkonium spectra does not reproduce the excitation energy of the $\Lambda_c$ baryon in the quark–diquark picture. In this section, we look for
the potential parameters appropriate for the $\Lambda_c$ and $\Lambda_b$ baryons. Let us fix the diquark mass as 0.5 GeV/c^2. We have checked that we also obtain qualitatively the same results for diquark mass 0.4 GeV/c^2.

First of all, we see the $\alpha$ dependence of the $\Lambda_c$ excitation energy. In Fig. 5, we plot the excitation energy of the $1p$ state calculated with various values of the $\alpha$ parameter. The string tension $k$ is fixed to be $k = 0.9$ GeV fm^{-1}. The figure shows that the experimental excitation energy, 0.330 GeV, cannot be reproduced by $\alpha$ in the range of $\alpha = 0.0$ to 0.6 and $k = 0.9$ GeV fm^{-1}.

Next, we look for an appropriate value of the string tension $k$ for the $\Lambda_c$ spectrum. In Fig. 6, we plot the $1p$ excitation energies calculated with various values of the string tension $k$. The $\alpha$ parameter is fixed as $\alpha = 0.4$. As we see in the figure, the experimental value, 0.330 GeV, can be reproduced with $k \sim 0.5$ GeV fm^{-1}. Also calculating the $\Lambda_b$ spectrum, we find that the best value of the string tension $k$ for both $\Lambda_c$ and $\Lambda_b$ is $k = 0.47$ GeV fm^{-1}. Figure 7 shows a comparison...
Fig. 7. Comparison of calculated $\Lambda_c$ and $\Lambda_b$ excitation energies from the ground state ($1s$) with the experimental data. The potential parameters are fixed as $k = 0.47 \text{ GeV fm}^{-1}$ and $\alpha = 0.4$, which reproduce the observed data well. The masses of the $c$ quark, the $b$ quark, and the diquark are taken as 1.5 GeV/$c^2$, 4.0 GeV/$c^2$, and 0.5 GeV/$c^2$, respectively. The experimental data are taken from the Particle Data Group [18].

Fig. 8. Schematic figure of interquark flux tube in a baryon and its diquark limit. There are two possibilities for the gluon configuration in a baryon: $\Delta$-type and $Y$-type.

of the calculated results and observed spectra for $\Lambda_c$ and $\Lambda_b$, suggesting that the potential with $\alpha = 0.4$ and $k = 0.47 \text{ GeV fm}^{-1}$ reproduces the whole excitation spectra of $\Lambda_c$ and $\Lambda_b$. This value is substantially smaller than the string tension obtained for the meson systems, $k = 0.9 \text{ GeV fm}^{-1}$. This implies that the confinement strength of the quark–diquark system is almost half of that in the quark–antiquark system. This is again incompatible with universality for the confinement potential.

The weaker confinement potential for the quark–diquark system is unlikely to be explained by a geometrical argument for the interquark flux tube configuration, as long as we take a point-like diquark. As shown in Fig. 8, there are two possibilities for the flux configuration for three-quark baryons, the so-called $\Delta$-type [19] and $Y$-type [20,21]. In the $\Delta$-type configuration, three flux tubes between pairs of quarks form the triangle structure, while in the $Y$-type configuration there is one junction from which flux tubes are strung to the quarks.

According to the one-gluon exchange calculation, the attractive interaction between quarks with the $3$ color configuration is half as strong as the interaction between a quark and an antiquark with the color singlet configuration [22,23]. In the $\Delta$-type configuration, the string tension among the quarks may be given as $k/2$. In this case, if we take the diquark limit where two light quarks come
Fig. 9. Diquark mass dependence of the ground state energy of $\Lambda_c$ (1s) measured from the $\Lambda_c$ mass with $m_d = 0.5$ GeV/$c^2$. The horizontal dotted line shows the mass difference between the observed ground states of $\Lambda_c$ and $\Xi_c$. This figure suggests that the strange diquark mass $m_{ds} = 0.76$ GeV/$c^2$ reproduces the observed $\Lambda_c$ and $\Xi_c$ mass difference.

to the same point, the string tension between the heavy quark and diquark may be $k/2 + k/2 = k$, as shown in the upper part of Fig. 8. This does not explain the reduction of the string tension in the quark–diquark system. In the $Y$-type configuration, the string tension from the junction to the quark has the same strength as the quark–antiquark system with color singlet [21]. If we take the diquark limit, the strength of the string tension between the heavy quark and diquark may be a value between $k$ and $2k$, depending on the position of the junction. For the junction at the diquark, the strength can be $k$, while the strength can be $k + k = 2k$ if the junction is located at the heavy quark. This does not again explain the weakness of the string tension for the quark–diquark system.

3.4. Strange diquark mass

Here we consider the scalar strange diquark, which is composed of a light quark (up or down) and a strange quark forming the flavor and color antisymmetric configuration (3 both for the flavor and color spaces). The strange diquark can be a constituent of $\Xi_c$ ($\Xi_b$) together with a charm (bottom) quark. Assuming the light flavor symmetry for the confinement potential (2), in which the potential is shared both for the $ud$ diquark and the strange diquark up to the constant part of Eq. (2), $V_0$, we can determine the strange diquark mass from, for instance, the mass difference between $\Lambda_c$ and $\Xi_c$. In Fig. 9, we show the diquark mass dependence of the charmed baryon mass composed of a charm quark and a scalar diquark. The charmed baryon mass is measured from the $\Lambda_c$ spectrum calculated with the diquark mass $m_d = 0.5$ GeV/$c^2$, where the $\Lambda_c$ spectrum is reproduced well. The experimental data for the difference between the $\Lambda_c$ mass and the isospin-averaged $\Xi_c$ mass is 0.18 GeV/$c^2$. This corresponds to the calculation with the strange diquark mass $m_{ds} = 0.76$ GeV/$c^2$, which is a reasonable value for the strange diquark mass.

With the strange diquark mass $m_{ds} = 0.75$ GeV/$c^2$, we calculate the ground state $\Xi_b$ mass. We find that the mass difference between the ground states of $\Lambda_b$ and $\Xi_b$ is calculated as 0.165 GeV/$c^2$. We compare this value with the experimental mass difference of 0.175 GeV/$c^2$ and find that this is in reasonably good agreement. For the ground state $\Xi_b$ mass, we take the isospin average.
Fig. 10. Excitation energy of the $\Xi_c$ 1p state in comparison with $\Lambda_c$ and the observed excitation energies. The potential parameters are fixed as $k = 0.47$ GeV fm$^{-1}$ and $\alpha = 0.4$, which reproduce the observed $\Lambda_c$ data well. The masses of the $c$ quark and the strange diquark are taken as $m_c = 1.5$ GeV/$c^2$ and $m_{ds} = 0.76$ GeV/$c^2$, respectively. The experimental data are taken from the Particle Data Group [18].

Fig. 11. Diquark mass dependence of the excitation energy of the $\Lambda_c$ 1p state from the 1s ground state. This figure shows that we obtain the smaller excitation energy with the larger diquark mass.

3.5. Excitation energy of $\Xi_c$

With the strange diquark mass $m_{ds} = 0.76$ GeV/$c^2$, we calculate the excitation energy of the $\Xi_c$ 1p state from its 1s ground state. As shown in Fig. 10, the excitation energy is obtained as 0.313 GeV. Comparing the experimental data shown in the figure, we find that the calculated value underestimates the observed masses of the 1p states. The size of the deviation from the observation is as small as the accuracy of the present model, and we do not intend to discuss this discrepancy further quantitatively. Nevertheless, what puzzles us is that the calculated excitation energy of $\Xi_c$ is smaller than that of $\Lambda_c$, while the experimental observation suggests that they are reversed in order. As shown in Fig. 11, in this potential model the 1p $\Lambda_c$ state with the larger diquark mass has a smaller excitation energy, which fits our intuition that the heavier object is harder to be excited. Unanticipatedly, in nature the excitation energy of $\Xi_c$ is larger than that of $\Lambda_c$, although $\Xi_c$ has a heavier component than $\Lambda_c$. This is opposite to our model and intuition. Thus, the present result is qualitatively against the observation. This disagreement could be solved by introducing the mixing with the $\Xi'_c$ state in which two light quarks are symmetric in the flavor space. Nevertheless, the $\Xi'_c$ state is a higher excited state than $\Xi_c$. Usually, once the mixing of two states is introduced, level repulsion would take place and the lower
0.2
0.4
0.6
0.8
1
0
0.5
1
1.5
2
2.5
2md
E1p
\(a_0\)
\(a_0\)
\(a\)
\(b\)
V(r)
\(r\) [fm]
Fig. 12. Potential (6) with the centrifugal potential for \(\ell = 1\) as a function of \(r\). The energy \(E_{1p}\) is the mass of \(\Lambda_c\) in the \(1p\) state measured from \(m_c + m_d\). The classical turning points are located at \(a_0 = 0.38\) fm and \(a = 1.42\) fm. The end of the tunnel is calculated as \(b = 2.27\) fm.

state could be pushed down. This is the opposite direction to explain the smaller excitation energy of \(\Xi_c\) than \(\Lambda_c\).

3.6. Decay width of \(\Lambda_c\) excited state

We play one more game with \(\Lambda_c\) in the quark–diquark model with a point-like diquark. We estimate the decay width of the \(1p\) excited state of \(\Lambda_c\). The decay widths of the observed \(\Lambda_c\) states are as narrow as 2.6 MeV for \(\Lambda(2595)\) with \(1/2^-\) and less than 0.97 MeV for \(\Lambda(2625)\) with \(3/2^-\), which are extremely small in the strong decay. The main decay mode for both \(\Lambda_c\) is \(\pi\pi\Lambda_c\), and \(\pi\Lambda_c\) is forbidden by isospin symmetry.

In the flux tube model, pair creation of a quark and an antiquark induces the decay of a hadron into two lighter hadrons. In the quark–diquark model for \(\Lambda_c\), by the creation of a quark and an antiquark inside of the flux tube, the \(\Lambda_c\) baryon decays into a \(D\) meson and a nucleon, where the \(c\) quark forms the \(D\) meson with the created antiquark and the diquark forms the nucleon with the created quark. But, the \(DN\) system has a larger rest mass than the \(\Lambda_c\) excited state, so that it cannot decay into \(DN\).

With the diquark–antidiquark pair production, the \(\Lambda_c\) excited state decays into \(\Lambda_c\) made of the charm quark and the created diquark and a tetraquark formed by the created antidiquark and the diquark originally in the excited \(\Lambda_c\); the tetraquark may be a sigma meson and decay into two pions.

Here let us estimate the decay width of \(\Lambda_c\) in the \(1p\) state into \(\Lambda_c\pi\pi\) by considering pair creation of diquark and antidiquark in the flux tube. We model this situation by taking the potential to be

\[
V(r) = \begin{cases} 
-\frac{4}{3} \frac{a}{r} h c + k r + V_0 & \text{for } r < b, \\
0 & \text{for } r > b, 
\end{cases}
\]

where \(b\) is defined by the position where the potential energy reaches the energy for the diquark–antidiquark pair creation, \(V(b) = 2m_d\). For larger distances than \(b\), thanks to the pair creation of diquark and antidiquark, there are no interactions and the potential becomes zero. The constant \(V_0\) is determined so as to reproduce the absolute value of the \(\Lambda_c\) ground state mass by the potential (2) together with the center of mass energy \(m_c + m_d\). In the present case, \(V_0 = -0.039\) GeV. In Fig. 12, we plot the potential (6) together with the centrifugal potential for \(\ell = 1\). The energy \(E_{1p}\) is the mass of \(\Lambda_c\) with \(\ell = 1\) measured from \(m_c + m_d\). The positions \(a_0\) and \(a\) are the classical turning points. The excited \(\Lambda_c\) state with \(E_{1p}\) decays by penetrating through the potential barrier from \(r = a\)
to \( r = b \). We calculate the decay width based on Gamow theory for nuclear \( \alpha \) decay. There, the tunneling probability is calculated by the Wentzel–Kramers–Brillouin (WKB) approximation:

\[
P = \exp \left[-\frac{2}{\hbar c} \int_a^b \sqrt{2\mu c^2 (V(r) - E_1 p)} \, dr \right] = 0.048.
\]  

(7)

For the confined particle in the classical orbit, the numbers of trials of the penetration per unit time may be calculated as \( v_0/(2R) \), with the length of the classical orbit \( R = a - a_0 \) and the velocity of the particle \( v_0 \). We calculate the velocity by \( v_0 = \sqrt{(E_{\text{kin}})/(2\mu)} \) with the expectation value of the kinetic energy \( \langle E_{\text{kin}} \rangle \) evaluated with the obtained wavefunction, \( \langle E_{\text{kin}} \rangle = 0.125 \text{ GeV} \). Finally, the decay width is estimated as

\[
\Gamma = \frac{\hbar}{\tau} = \frac{\hbar c}{R} \sqrt{\frac{\langle E_{\text{kin}} \rangle}{2\mu c^2}} P = 3.8 \text{ [MeV]}.
\]  

(8)

The size of the obtained value is consistent with the observed decay widths of the \( 1p \) states. It is noted that, since the initial state with \( \ell = 1 \) has negative parity, the relative angular moment between \( \Lambda_c \) and the \( \pi\pi \) system in the final state should be an odd number, such as \( \ell = 1 \), for \( \pi\pi \) coming from a sigma meson. Nevertheless, in this model we do not consider spin and parity for the decay reaction. Thus, we could have a further suppression factor for parity-conserved configurations.

A similar calculation is also done using the potential with \( k = 0.9 \text{ GeV fm}^{-1} \). In the calculation we set the mass of the \( 1p \) state to be the observed mass, which is not an eigenenergy of the potential with \( k = 0.9 \text{ GeV fm}^{-1} \). We find the decay width \( \Gamma = 31 \text{ MeV} \), which is one order of magnitude larger than the observed width. This fact also supports the smaller string tension for the heavy quark and diquark system.

### 3.7. Estimation of the diquark size effect

As reported in Refs. [7,24], the size of the diquark can be as large as 1 fm. Here we estimate the effect of the diquark size on the excitation spectrum of \( \Lambda_c \). Let us set the positions of the up, down, and charm quarks to be \( x_1, x_2, \) and \( x_3 \), respectively. Assuming that the interaction strength of the inter-quark force is half of the force between quark and antiquark, we may write down the color electric potential between the charm quark and the light quarks as

\[
V_{fs} = -\frac{4}{3} \hbar c \alpha \frac{\rho}{2} \left( \frac{1}{|x_3 - x_1|} + \frac{1}{|x_3 - x_2|} \right) + \frac{k}{2} \left( |x_3 - x_1| + |x_3 - x_2| \right),
\]  

(9)

where \( \alpha \) and \( k \) are the coupling strengths appearing in the quark–antiquark potential. Introducing a Jacobi coordinate defined by \( \rho = x_1 - x_2 \) and \( r = x_3 - \frac{1}{2} (x_1 + x_2) \), we write the potential as

\[
V_{fs} = -\frac{4}{3} \hbar c \alpha \frac{\rho}{2} \left( \frac{1}{|r - \frac{1}{2} \rho|} + \frac{1}{|r + \frac{1}{2} \rho|} \right) + \frac{k}{2} \left( |r - \frac{1}{2} \rho| + |r + \frac{1}{2} \rho| \right).
\]  

(10)

We recover the potential (2) in the limit of \( x_1 = x_2 \), that is \( \rho = 0 \). Assuming that two light quarks form a diquark, we do not regard \( |\rho| \) as a dynamical variable, but rather as a parameter for the diquark size. The modulus of \( r, r = |r| \), represents the distance between the diquark and the charm quark.

Now we try two ways of estimating the diquark size effect. First we use perturbation theory by regarding the diquark size to be small. Let us expand the linear potential in terms of the diquark size.
\( \rho \) and take the leading correction:

\[
\Delta V^{(1)}_{fs} = kr \left( 1 - \cos^2 \theta \right) \frac{\rho^2}{r^2},
\]

(11)

where \( \theta \) is the angle between \( \rho \) and \( r \). We calculate the finite size effect for the \( \Lambda_c \) excitation energy between the 1s and 1p states as a first-order perturbation:

\[
\Delta E = \langle 1P|\Delta V^{(1)}_{fs}|1P \rangle - \langle 1S|\Delta V^{(1)}_{fs}|1S \rangle.
\]

(12)

To calculate the matrix elements, we use the wave functions obtained by solving the Schrödinger equation for the unperturbed Hamiltonian with the original parameters for the quarkonia, \( \alpha = 0.4 \) and \( k = 0.9 \text{ GeV fm}^{-1} \), and we assume that \( \rho \) orients to the z direction and take \( \ell_z = 0 \) for the 1p state. Then we find the finite size correction calculated by perturbation theory to be

\[
\Delta E = -\left( 0.13 \text{ GeV}^2 \text{ fm}^{-2} \right) \rho^2.
\]

(13)

This shows that the finite size effect reduces the excitation energy, and the magnitude of the correction is of the order of 0.1 GeV for the diquark size of 1 fm, although 1 fm should not be a small enough value for perturbative calculation but we could learn the order of magnitude of the size effect. In this calculation the angle between \( \rho \) and \( r \) is a dynamical variable and we do not fix the orientation of the diquark against the charm quark direction.

In the second estimation, we fix the angle between \( \rho \) and \( r \) to be orthogonal, but we solve the Schrödinger equation fully. With this assumption the potential is written as

\[
V^{(2)}_{fs}(r) = -\frac{4}{3} \hbar c \alpha \frac{1}{\sqrt{r^2 + \frac{1}{4} \rho^2}} + k \sqrt{r^2 + \frac{1}{4} \rho^2},
\]

(14)

where \( \rho \) is the parameter for the diquark size and this potential is a function of the distance between the charm quark and diquark, \( r \). It can be understood from this potential that, for \( r < \rho \), the charm quark is located between two light quarks and, consequently, color forces between the charm quark and the light quarks have opposite directions and the forces are to be neutralized. Thus, around the center the color electric force between the charm quark and the diquark is less attractive than the case of the point-like diquark. For \( r > \rho \), \( \rho \) would be negligible and we recover the original potential for the point-like diquark. Solving the Schrödinger equation with \( \alpha = 0.4 \) and \( k = 0.9 \text{ GeV fm}^{-1} \), we find the \( \Lambda_c \) excitation energy to be 0.341 GeV for \( \rho = 0.5 \text{ fm} \) and 0.273 GeV for \( \rho = 1.0 \text{ fm} \), which should be compared with 0.464 for \( \rho = 0.0 \text{ fm} \). Again we find that the finite size correction reduces the excitation energy and the diquark size 0.5 fm reproduces a reasonable value for the excitation energy, which is consistent with the finding for the finite size diquark in Refs. [7,24]. The reason for the suppression of the excitation energy for the finite size diquark is the following: As discussed above, for \( r < \rho \) the attraction between the charm quark and the diquark gets weaker, and in such a case the distribution of the \( \Lambda_c \) wave function spreads out more than the case of the point-like diquark. This makes the centrifugal force less effective. Thus, the orbital excitation energy is reduced for the finite size diquark. For final conclusions, we should investigate the \( \Lambda_c \) spectrum more precisely, for instance by considering orbital and radial excited states more and the \( \Lambda_b \) spectrum. For a quantitatively more precise argument, we should perform a three-body calculation by regarding three quarks as dynamical objects; this is, however, beyond the scope of this brief report. Here we just mention that the finite size effect can be a solution of the overestimation problem for the excitation energies, and that the diquark should have color polarization.
4. Summary and conclusion

We have examined the excitation energy of the $\Lambda_c$ and $\Lambda_b$ baryons in the quark–diquark model. In this model, $\Lambda_c$ and $\Lambda_b$ are composed of a heavy quark and a scalar $ud$ diquark with the $\bar{3}$ configuration for the flavor and color spaces, and these constituents are confined by the Coulomb and linear potential.

We have found that the confinement potential suitable for the meson system overestimates the $\Lambda_c$ and $\Lambda_b$ excitation energies, if we assume a point-like diquark. We also find that, in order to reproduce the $\Lambda_c$ and $\Lambda_b$ spectra, the string tension should be half as strong as in the meson case. This implies that if one takes the quark–diquark model for the $\Lambda_c$ and $\Lambda_b$ baryons with a point-like diquark, one should take such a smaller string tension for the quark–diquark system. This raises the interesting question of why the quark–diquark system should have such a smaller string tension, because the color electric confinement potential is considered to be universal for the same color configuration.

We have estimated the decay width of the $\Lambda_c$ excited state based on Gamow theory of the nuclear alpha decay using the WKB approximation. We have found that the result obtained in this model is consistent with experiment, and this supports the smaller string tension for the quark–diquark model.

The overestimation of the $\Lambda_c$ excitation energy with the confinement potential suitable for the meson spectra could be resolved by introducing a finite size for the diquark. In this case, for the short distance between the heavy quark and the diquark, the color forces between the heavy quark and the light quarks in the diquark have opposite directions and cancel. Thus the interaction between the heavy quark and the diquark for short distances is less attractive. This makes the relative distance of the quark and diquark larger in the $\Lambda_c$ wave function, and the centrifugal force becomes less effective. We have estimated the finite size effect on the $\Lambda_c$ excitation energy in two ways. One is to use perturbation theory, and the other is to fix the diquark orientation. Both calculations can be done within a simple two-body model. With these calculations, we have found that the finite size effect reduces the excitation energy, and the magnitude of the reduction would be 0.1 GeV to 0.2 GeV for a diquark size of 1 fm. This is a reasonable value to solve the overestimation. Thus, this finding can be one of the confirmations of the finite size diquark in heavy baryons and implies that the diquark should have color polarization. For a more precise argument for the diquark size, we certainly need full three-body calculations.

We have also calculated the excitation energy of $\Xi_c$ as a system of a charm quark and a strange diquark. We have found that the excitation energy of $\Xi_c$ is moderately smaller that that of $\Lambda_c$. This is consistent with our intuition that a heavier particle is harder to be excited, but it is inconsistent with the experimental observation in which the $\Xi_c$ excitation energy is slightly larger than that of $\Lambda_c$.

We hope that these findings give us a good chance to reconsider the nature of the confinement force in baryons for excitation spectra of heavy baryons and the feature of diquarks in heavy baryons.

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